Backus/November 16, 1998

Interest-Rate Forwards and Futures

0. Overview

- Leading Futures Contracts
- Forward Contracts
- Futures Contracts
- Bond Futures
- Eurocurrency Futures

1. Leading Futures Contracts

- Contracts ranked by dollar volume:

Contract	Open Interest	Monthly Volume	
	Contracts (mm)	Contracts (mm)	Dollars (bb)
Eurodollar	1.325	5.044	5,044
Euroyen	0.439	1.247	1,122
10-yr JGB	0.132	0.989	890
3-m sterling	0.212	0.941	724
Euromark	0.370	1.014	628
30-yr US T-bond	0.305	5.834	583
PIBOR	0.146	0.536	492
10-yr Notionnel	0.231	2.584	237
S&P 500	0.157	1.035	232
German Bund	0.139	1.134	176
Nikkei 225	0.149	0.950	172
10-yr US T-note	0.177	0.935	93

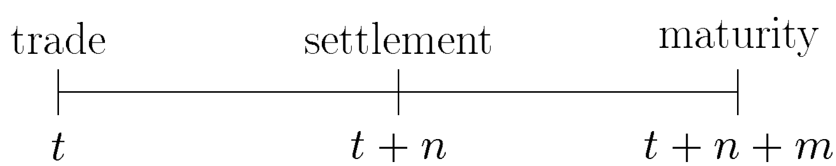
Source: Galitz, *Financial Engineering*, numbers for 1992.

- Remarks
  - Fixed income contracts dominate
  - Eurocurrencies first, then government bonds
  - Bond contracts have greater volume to open interest than euros: short-term trading v buy-and-hold

2. Forward Contracts

- A *forward contract* is an agreement to exchange assets at a future date for a price arranged now.
- Terminology: *trade date* is when trade is made, *settlement date* is when assets are exchanged:

- With interest rate contracts, a third date is the maturity of the asset being exchanged (typically for cash):



  Convention (usually): m is maturity at settlement.

- Interest rate forwards (futures, too) differ in the magnitudes of n and m:
  - Forward rate agreements: n and m are typically single-digit months.
  - Bond futures: short settlement and long maturity (eg, $n = 3$ months and m of 10 years).
  - Eurocurrency futures: long settlement and short maturity (n out to 10 years and beyond, $m = 3$ months).

2. Forward Contracts (continued)

- Forward contract on a zero:
  - Terms: agree at t to buy m-period zero at $t+n$ for price F.
  - This should look familiar!
  - No cash flows on trade date (like a swap)

- Timing of cash flows:

t	$t+n$	$t+n+m$
0	$-F$	100

- Replication with zeros (p_n = price of n-period zero):

t	$t+n$	$t+n+m$
$-p_{n+m}$		100
xp_n	$-100x$	

  Choose x to replicate cash flows of forward contract:

$$0 \;=\; -p_{n+m} + xp_n$$

- Solution:

$$x \;=\; \frac{p_{n+m}}{p_n} \quad \Rightarrow \quad F \;=\; 100 \times \frac{p_{n+m}}{p_n}$$

2. Forward Contracts (continued)

- Forward contract on a coupon bond
  - Terms: agree at t to buy an m-period bond at $t+n$ for price F
  - Cash flows:

t	$t+n$
0	$-F_t$
0	P_{t+n}

    Ie, price of bond at $t+n$ is P_{t+n} (unknown at t), but contract locks in price of F_t (set at t).

  - Coupons complicate the analysis, but the idea is the same: replicate forward with long position in the bond and short positions sufficient to offset the purchase price and (in this case) the coupons between now and settlement.
  - Key ingredient: replication includes a long position in the bond, so you're indirectly gaining exposure to the bond.

2. Forward Contracts (continued)

- Using forwards to modify interest sensitivity
  - Duration not defined: duration is the proportional change in price and is not defined for contracts (like swaps, forwards, and futures) that have no net value.
  - Quantify interest sensitivity as we did with swaps:
    * Use the DV01
    * Compute duration for long and short positions separately
    * Short cut: ignore the short position (this approach is common for bond futures)
  - Bottom line:
    * Forward contracts on long-maturity bonds are useful tools for modifying duration: you add (say) a long position in a long bond, and short a position of equal value in a short bond.
    * Forward contracts on short bonds — FRAs, for example — are useful protection against near-term changes in short rates.

3. Forward Rate Agreements (FRAs)

- Contract terms
  - In an "$n \times m$" (both quoted in months), m is what we've called $m + n$
  - Fixed "contract" rate (C)
  - Floating "reference" rate (r) (fixed at settlement) (typically ($m - n$)-month LIBOR)
  - Notional principal
  - Cash flow at settlement:

    $$\text{Payment} \;=\; \frac{(C - r)(m - n)/12}{1 + r(m - n)/12} \times \text{Principal}$$

    (plus the usual eurocurrency day count adjustments)
  - Equivalent to paying this in m months:

    $$\text{Payment} \;=\; [(C - r)(m - n)/12] \times \text{Principal}$$
  - Remark: swap with one payment!

3. FRAs (continued)

- Example: 6×12, 1mm notional
  - Contract rate = 6%
  - Reference rate = 6-month LIBOR
  - If 6-month LIBOR is 5% at settlement,

$$\text{Payment} \;=\; \frac{(.06 - .05)/2}{1 + .05/2} \times \$1{,}000{,}000 \;=\; \$4,878$$

- Contract rate is the forward rate
  - Approach like swaps
    - Consider payments in 12 months
    - Add principal to both sides
  - Value now of fixed payment in 12 months:

$$d_2 \times (1 + C/2) \times 100$$

  - Value now of floating payment in 12 months:

$$d_1 \times \frac{(1 + r/2)100}{1 + r/2} \;=\; d_1 \times 100$$

  - Equate values to find C:

$$d_1\ 100 \;=\; d_2\ (1 + C/2)\ 100 \quad \Rightarrow \quad 1 + C/2 \;=\; d_1/d_2$$

  (C is the first forward rate)

4. Futures Overview

- Features of futures
  - Standardized contracts
  - Liquid, low transactions costs
  - Easy to short
  - Low credit risk
  - Trades public: good source of market information
  - Differ from forwards in daily "mark-to-market"
- Cash flows on futures
  - No payment due on trade date
  - . . . but money is set aside in margin account
  - Margin account varies due to
    * Daily changes in contract price
    * Margin calls
    * Interest on the account
- Daily "mark to market"
  - Reduces credit exposure of exchange
  - Complicates cash flows and valuation (slightly)

5. Bond Futures

- US treasury bond/note contracts (CBOT)
  (foreign government bond contracts are similar)
- Standard features
  - Contract size: $100,000 face value
  - Contracts expire quarterly (Mar/Jun/Sep/Dec)
  - Delivery controlled by short position, which
    * Can deliver any time in the contract month
    * Delivers $100,000 face value of bonds, gets cash.
  - Eligible ("contract grade") bonds:
    * 30-year bond contract: US treasury bonds with maturity at least 15 years from first delivery date
    * 10-year note contract: US treasury notes with maturity 6.5 to 10 years from first delivery date
    * 5-year note contract: US treasury notes with original maturity no more than 5.25 years and maturity on first day of delivery month of at least 4.25 years
  - Wild-card option: price at close (2pm) good till 8pm
  - Timing option: futures price fixed on last trading day

5. Bond Futures (continued)

Settlement for delivered bonds

- Invoice price (what long position pays):

$$\text{Invoice Price} = F \times \text{Conv Factor} + \text{Acc Interest}$$

  (F is quoted futures price)

- Conversion factor:
  - Why? Make more bonds deliverable, avoid squeezes
  - Problem: people would always deliver the low-coupon bonds, since they're cheaper
  - Solution: scale up price by conversion factor, the ratio of the quoted price of a bond to the price of an 8% bond, using yields of 8%
  - Computed this way:
    * Compute maturity at first delivery date, rounded down to nearest 3-month interval $\Rightarrow$ n (number of coupons remaining) and w (length of initial period, 0.5 or 1)
    * Compute

$$\text{Conversion Factor} = \frac{\text{Quoted Price at y=8\%}}{\text{Quoted Price of 8\% Bond}}$$

      (Denominator is 100)

5. Bond Futures (continued)

Calculating conversion factors

- Formula:

$$\text{Quoted Price} = d^w \left(\frac{1-d^n}{1-d}\right) \text{Coupon} + d^{w+n-1} 100 - (1-w)\text{Coupon}$$

  (Just like our earlier work on bond yields, except that we have rounded maturity down to the nearest 3 months.)

- Examples for the Dec 98 long bond contract:
  - 11 1/4 of 2/15/15: $n = 32$ (Feb 99 less than 3 months), $w = 1$, Quoted Price = 129.04, CF = 1.2904.
  - 10 5/8 of 8/15/15: $n = 33$, $w = 1$, CF = 1.2382.
  - 9 7/8 of 11/15/15: $n = 34$, $w = 0.5$, CF = 1.1711.
  - 9 1/8 of 5/15/18: $n = 39$, $w = 0.5$, CF = 1.1093.
  - 5 1/2 of 8/15/28: $n = 59$, $w = 1$, CF = 1.1093.

- Remarks
  - Crude adjustment
  - Effects vary with yields: if we used yields higher (lower) than 8%, we would tend to choose bonds with higher (lower) duration

5. Bond Futures (continued)

The Basis

- Basis is difference between cash price of bond and price through the futures contract:

$$\text{Basis} = \text{Bond Price} - F \times \text{Conversion Factor}$$

- Question: do we use invoice or quoted price?
  Answer: doesn't matter (accrued interest in both)

- Analysis for 11/2/98 quotes for the Dec 98 long bond contract ($F = 128.656$, 33 contract grade bonds in all):

Bond	Quoted Price	Conv Fac	Basis
11 1/4 of 2/15/15	166.297	1.290	0.280
10 5/8 of 8/15/15	159.563	1.238	0.260
9 7/8 of 11/15/15	151.000	1.171	0.331
9 1/8 of 5/15/18	144.688	1.109	1.969
5 1/2 of 8/15/28	104.891	0.718	12.46

  Remark: try to reproduce these numbers, to make sure you understand where they come from.

- The basis is zero at delivery: the short position delivers the cheapest bond, which should sell for the same price in the cash and futures markets.

5. Bond Futures (continued)

Interest sensitivity

- Rule-of-thumb:

$$\text{DV01 of Futures} = \frac{\text{DV01 of Bond}}{\text{Conversion Factor}}$$

- Assumption: basis doesn't change
  - Ignores short position in forward replication (small, but not zero)
  - Also ignores daily mark to market, which induces a sensitivity of the basis to changes in yields and volatility
  - Basis risk!

- Convexity: since the cheapest to deliver varies with yields, bond futures tend to have negative convexity
  (In case you ask: as rates fall, cheapest bond shifts to lower duration.)

6. Eurocurrency Futures

- Contracts on 3-month LIBOR in major currencies (CME, LIFFE, SIMEX)
- Standard features
  - Contract size: interest on $1,000,000
  - Contracts expire quarterly (Mar/Jun/Sep/Dec) out ten years or more (third Wednesday)
  - Quoted index:

$$\text{Index} = 100 \times (1 - \text{Yield})$$

  - Effective price of a contract:

$$\text{Price} = 1mm \times (1 - \text{Yield}/4)$$

  - Cash settlement at

$$\text{Settlement Price} = 1mm \times (1 - \text{3-m LIBOR}/4)$$

    Note: no strange delivery options!
  - Strips, bundles, and packs: combinations of contracts with different maturities
- Uses
  - Helpful for hedging FRNs, FRAs, swaps, etc
  - Source of market information on forward rates
  - In earlier terminology: n can be large, but m is small

Summary

- Forward contracts for bonds are equivalent to a combination of a long position in a long bond and a short position of equal value in a short bond.

- Futures contracts differ from forwards in the daily mark to market.

- Fixed income futures include government bond and eurocurrency contracts.

- Bond futures are truly ugly contracts, but useful tools for managing interest rate risk: liquid, easy to short, low transaction costs.

- Like other derivatives, forwards and futures offer leverage: you can arrange great exposure with less money down than buying the underlying instrument.