Backus/November 30, 1998

Interest-Rate Options

0. Overview

- Fixed Income Options
- Option Fundamentals
- Caps and Floors
- Options on Bonds
- Options on Futures
- Swaptions

1. Fixed Income Options

- Options imbedded in bonds:
  - Callable bonds
  - Putable bonds
  - Convertible bonds
- Options on futures
  - Bond futures
  - Eurocurrency futures
- OTC options
  - Caps, floors, and collars
  - Swaptions

2. Option Basics

- Big picture
  - Options are everywhere
    - Stock options for CEOs and others
    - Corporate equity: call option on a firm
    - Mortgages: the option to refinance
  - Options are like insurance
    - Premiums cover the down side, keep the up side
    - Customers like this combination
    - Insurer bears risk or shares it
      (diversification or reinsurance)
  - Managing cost of insurance
    - Out-of-the-money options are cheaper
      (insurance with a big deductable)
    - Collars: sell some of the up side
    - Aggregate: basket option cheaper than basket of options
  - Managing option books
    - Customer demands may result in exposed position
    - Particular exposure to volatility:
      puts and calls both rise with volatility
    - Hedging through replication is another route

2. Option Basics (continued)

- Option terminology
  - Basic terms
    - Options are the right to buy (a *call*) or sell (a *put*) at a fixed price (*strike price*)
    - The thing being bought or sold is the *underlying*
    - This right typically has a fixed *expiration* date
    - A short position is said to have *written* an option
  - Kinds of options
    - *European* options can be exercised only at expiration
    - *American* options can be exercised any time
    - *Bermuda* options can be exercised at specific dates (eg, bonds callable only on coupon dates)

2. Option Basics (continued)

- Features of options
  - Leverage (cheap source of exposure)
  - Nonlinear payoffs
    * Payoffs vary with underlying
      (in- and out-of-the money)
    * Translates into variable duration
      (convexity rears its ugly head)
    * Creates risk management hazards
  - Volatility has positive effect on both puts and calls
    * Another risk management hazard!
  - They're state-contingent claims
    (no way around it, but nothing new either)

3. Approaches to Valuation

- Why use a pricing model? No choice — the instruments demand that we value uncertain cash flows (state-contingent claims).
- What pricing model? Good question.
- Interest rate trees
  - Been there... (and it hasn't changed)
  - We'll return to them shortly
- The Black-Scholes benchmark (Black's formula)
  - Underlying: an arbitrary bond with (say) maturity m
  - Parameters: n-period European call with strike price k
  - Formula:

$$\text{Call Price} = pN(x) - d_n k N(x - n^{1/2}\sigma)$$

with

$$\begin{aligned} p &= \text{current price of underlying} \\ f &= \text{forward price of underlying} \\ d_n &= n\text{-period discount factor} \\ N &= \text{normal cdf} \\ x &= \frac{\log(f/k) + n\sigma^2/2}{n^{1/2}\sigma} \end{aligned}$$

3. Approaches to Valuation (continued)

- Remarks on Black-Scholes for fixed income
  - Formula based on log-normal price of underlying
    $\Rightarrow$ normal (continuously compounded) spot rates
    $\Rightarrow$ possibility of negative spot rates

  - Volatility σ varies systematically with maturities of option and underlying ("term structure of volatility")
    Sample swaption volatility matrix (%):

Option	Swap Maturity			
Maturity	1 yr	2 yr	5 yr	10 yr
1 m	15.50	16.00	16.75	15.25
3 m	17.50	18.50	18.25	18.25
1 yr	21.50	21.25	19.25	16.50
5 yr	20.00	19.00	17.50	15.50

    Source: Tradition, Inc, global swap broker, Jan 2, 1996.
    Remark: "hump" is typical

  - Despite problems, a common benchmark (dealers often quote volatility instead of price)

3. Approaches to Valuation (continued)

- Properties of Black-Scholes option prices
  (most of these generalize to other settings)
  - The Delta:

    $$\text{Delta} \;=\; \frac{\Delta \text{Call Price}}{\Delta p} \;=\; N(x),$$

    which varies between zero and one (nonlinear).
  - Volatility is the only unobservable
    (we use call prices to "imply" it)
  - If volatility rises, so does the call price (puts, too)

4. Caps, Floors, and Collars

- Terminology:
  - A *cap* pays the difference between a reference rate and the cap rate, if positive. (Series of call options on an interest rate)
  - A *floor* pays the difference between the floor rate and a reference rate, if positive. (Series of put options on an interest rate)
  - A *collar* is a long position in a cap plus a short position in a floor.
- Contract terms:
  - Cap and/or floor rate
  - Reference rate (typically LIBOR)
  - Frequency of payment
  - Notional principal (amount on which interest is paid)
- Approaches:
  - Apply Black's formula
  - Interest rate tree (we did this earlier)
  - An uncountable number of other models

4. Caps, Floors, and Collars (continued)

- Example 1: two-year semiannual 7% cap on 6-m LIBOR

  Payments shifted back one period:

  $$\frac{(r-7\%)^+/2}{1+r/2} \times \text{Notional Principal}$$

  (three such semi-annual payments, excluding the first)

  Short rate tree (same as before):

			8.913
		7.869	
	6.587		6.913
6.036		5.869	
	4.587		4.913
		3.869	
			2.913

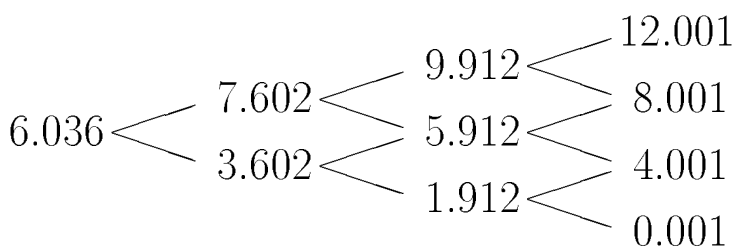
  Price path for cap (for 100 notional):

			0.916
		0.858	
	0.416		0.000
0.202		0.000	
	0.000		0.000
		0.000	
			0.000

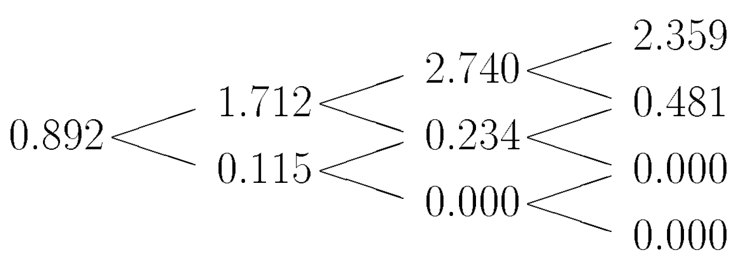
4. Caps, Floors, and Collars (continued)

- Example 1 (continued): the effects of volatility

  Short rate tree ($\sigma = 2\%$, same implied spot rates):



  Price path for cap (for 100 notional):



  Remarks:

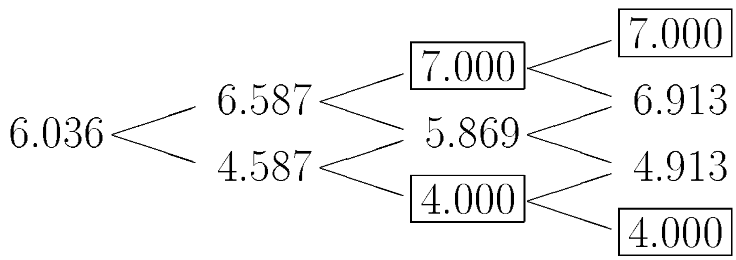
  – Volatility increase cap prices (they're bets on extreme events, and higher σ makes them more likely)

  – Similar in this respect to Black-Scholes

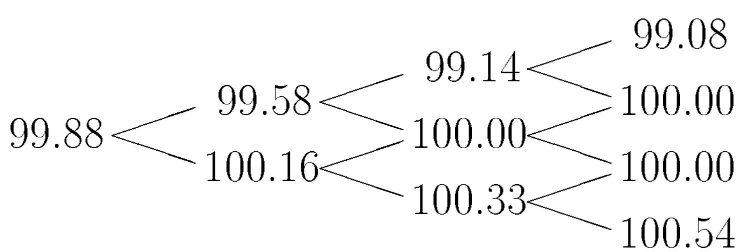
4. Caps, Floors, and Collars (continued)

- Example 2: 2-Year FRN with collar (7% cap, 4% floor)

  Effective interest rates (boxes indicate cap/floor binds):



  Price path for note:



  Remarks:

  – Differences from 100 indicate impact of collar

  – Giving up low rates partially offsets cost of cap ($0.12 = 100 - 99.88 < 0.20$)

  – Issuers would generally adjust cap and floor to get a price of 100

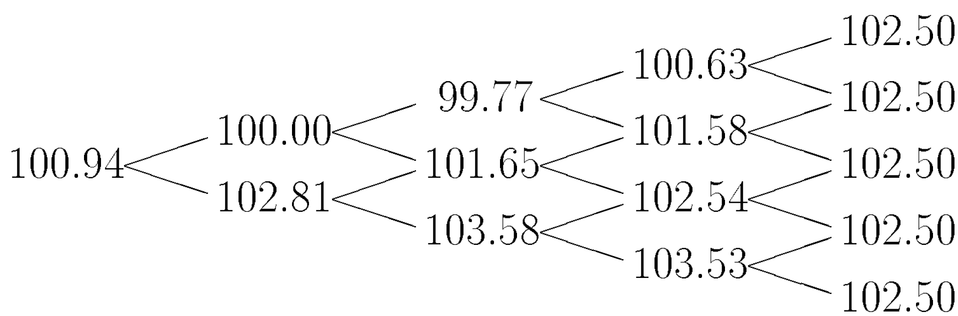
5. Options on Bonds

- Earliest and most common interest-rate option?
- Examples of callable corporate bonds
  - Apple Computer Corporation's 6-1/2s, issued February 10, 1994, due February 15, 2004. Callable at "make whole."
  - Ford Motor Company's 6.11 percent bonds, issued September 22, 1993, due January 1, 2001. Callable at "make whole under special circumstances."
  - Intel Overseas Corporation's 8-1/8s, issued April 1, 1987 (really), due March 15, 1997, callable at par. Par in this situation means par plus accrued interest: the firm pays the relevant interest as well as the face value. The bonds were called March 15, 1994, at 100.
  - Texas Instruments' 9s of July 99, issued July 18, 1989, due July 15, 1999. Callable on or after July 15, 1996, at par.

5. Options on Bonds (continued)

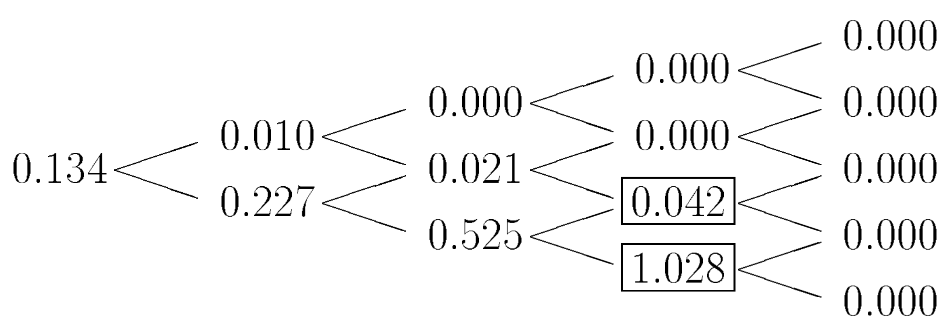
- Example: call option on 2-year 5% bond
  (the usual rate tree)

  Price path of bond:



- 18-month European option (callable at 102.5 — "par")

  Price path is:



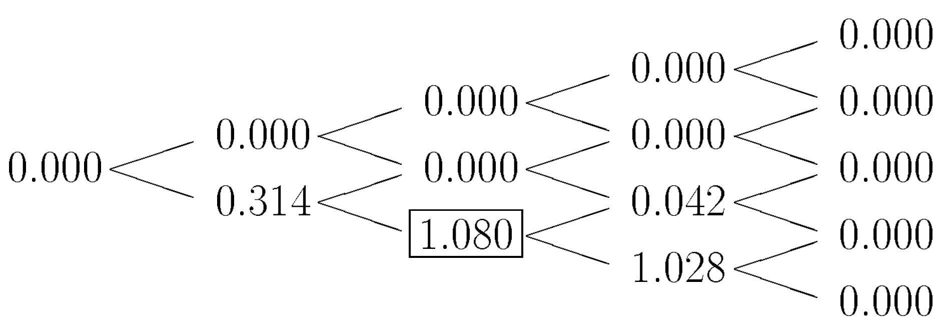
  Nodes in boxes indicate cash flows from exercise, other nodes indicate value in earlier periods.

5. Options on Bonds (continued)

- American option has greater value
  (can exercise either at expiration, or earlier if better)

- Approach:
  - Start at expiration, work backwards
  - At each node, choose better of "exercise" or "hold"

- Cash flows from immediate exercise:



- Node with box:
  - If hold:

$$\text{Value} = \frac{0.5}{1 + 0.03869/2} \times (0.042 + 1.028) = 0.525$$

  - If exercise:

$$\text{Value} = 1.080$$

  This is better: we take it.

5. Options on Bonds (continued)

- American option (continued)
- Node (0,1) (one down move from start):
  - If hold:

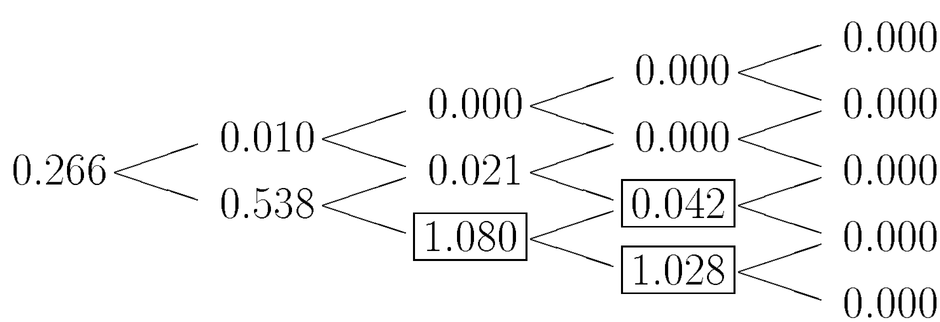
$$\text{Value} = \frac{0.5}{1 + 0.04587/2} \times (0.000 + 1.080) = 0.538$$

  - If exercise:

$$\text{Value} = 0.314$$

  Hold is better in this case, so we write 0.538 here.

- Complete price path:



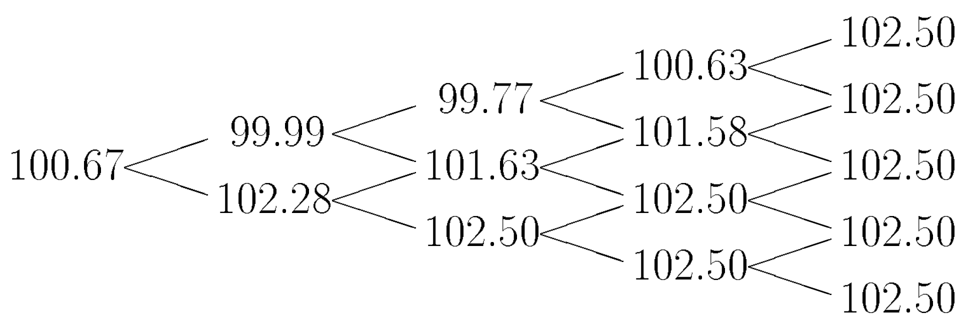
Boxes indicate nodes where option is exercised.

- Summary
  - Worth more than European call
  - Valued recursively (as usual)

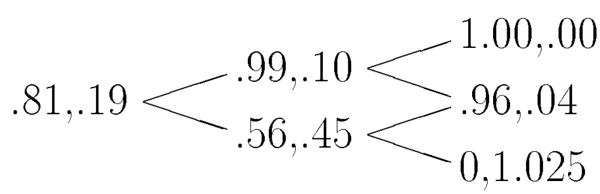
6. Callable Bonds

- Example: Callable bond based on previous example:
  2-year 5% bond with 18-month American call

  Price path:



- Interest-sensitivity 1: replication with (x_a, x_b) units, resp, of underlying bond and one-period zero



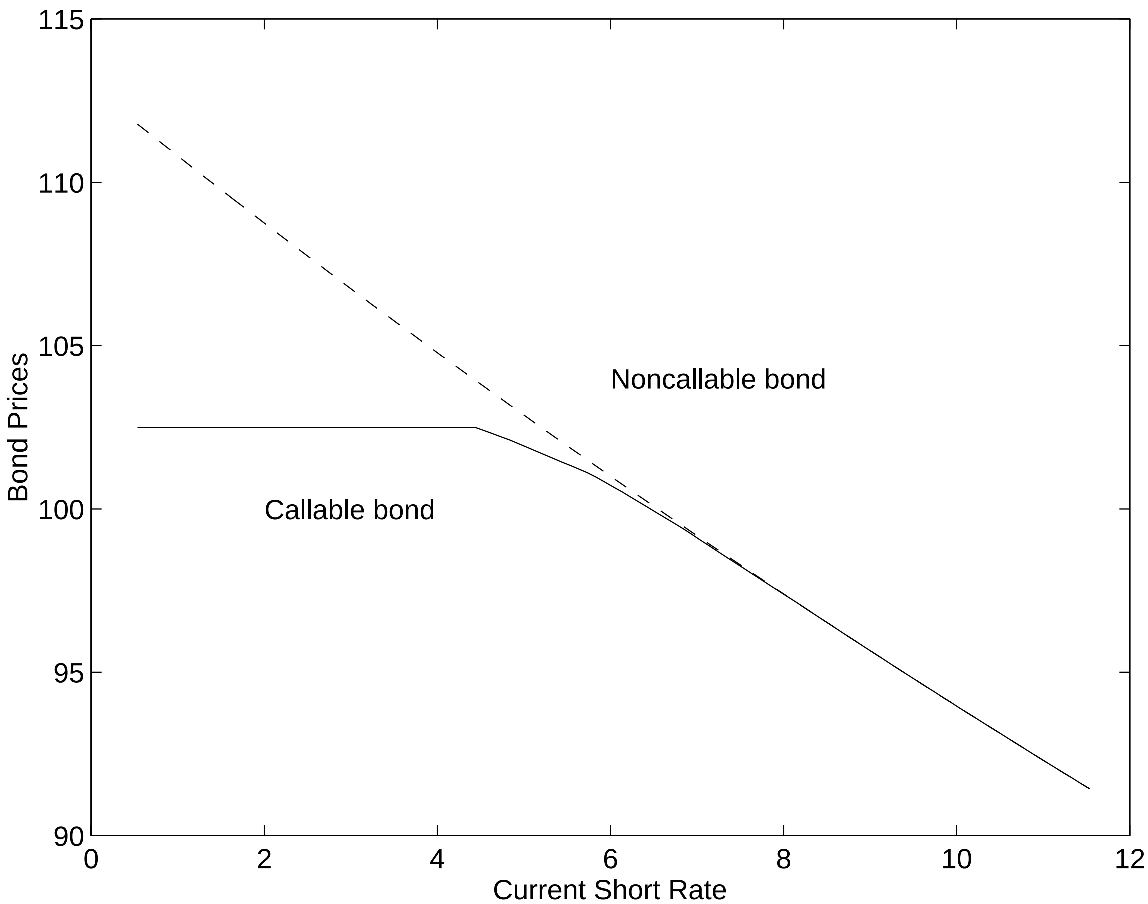
  Eg, the callable bond is equivalent, in the initial node, to 0.81 units of the underlying noncallable bond and 0.19 units of a one-period zero.

  Remarks:

  – The callable has shorter duration than the noncallable
  – How much shorter varies throughout the tree

6. Callable Bonds (continued)

- Interest-sensitivity 2: price-yield relation
  - How does price vary if we shift the whole short rate tree up and down?
  - Below we graph price against initial short rate
  - Slope used to compute "effective duration"



6. Callable Bonds (continued)

- Dumb ideas
  - Yield to first call date for bonds in the money
  - Yield to worst: find call date with highest yield
  - Remarks:
    * These approaches ignore the intrinsic difficulties of valuing uncertain cash flows
    * They're dumb for exactly that reason
    * Our approach: call decision varies through the tree
- Option-Adjusted Spread (OAS)
  - Consider the valuation of a callable bond
  - Suppose market price is p
  - Compute spread s added to the short rate tree required to reproduce the market price
  - Positive spread means the market values the bond more highly than the model

7. Options on Futures

- Options available on major futures contracts
  - Government bond contracts
  - Eurocurrency contracts
  - Brady bond futures
- Same strengths as the underlying futures
  - Highly liquid markets
  - Low transaction costs

8. Swaptions

- Swaptions: options on swaps
  - Option to enter a swap
  - Option to extend a swap
  - Option to terminate a swap
  - European, American, and Bermuda
- Properties
  - Similar to bond options (swap = bond - FRN, or reverse)
  - Exposure to long-dated volatility
  - Currently the OTC option standard

Summary

- Options are ubiquitous.
- Their nonlinear payoffs pose challenges to valuation and risk management.
- Nonlinearity translates in this context into nonlinear price-yield relations — convexity, in other words.
- Black-Scholes is less well suited for fixed income than other securities, but remains a common benchmark nonetheless.
- American options are valued recursively: at each node, we decide whether to exercise or hold.
- Shortcuts don't work: yield-to-call is meaningless.