Backus/February 3, 1998

Propectus

1. Fixed income is a fascinating part of finance . . .

2. . . . but it's quantitative

3. . . . and takes time and effort to master

4. Assignments are critical learning experiences

5. Do them in groups (it's easier)

6. We'll emphasize international markets

7. Home page has "text" and "overheads" (like these)

8. Useful references:
   - Garbade, *Fixed Income Analytics*
   - Fabozzi, *Bond Markets, Analysis, and Strategy*

9. Read the syllabus!

Theme 1: Debt Markets are Global

- Bond Markets
  (amounts outstanding, billions of US dollars, 1995)

Category	Outstanding
Total	24,110.0
Private	8,776.7
Public	14,197.5
US	10,726.0
Japan	4,958.6
Germany	1,906.4

  Source: IMF.

- International capital flows of all kinds are booming

Theme 2: Debt Markets are Derivatives Markets (and vice versa)

Exchange-Traded Derivatives
(annual turnover, millions of contracts traded, 1995)

Category	Turnover (mm)
Interest rate futures	561.0
Interest rate options	225.5
Currency futures	98.3
Currency options	23.2
Stock market index futures	114.8
Stock market index options	187.3
North America	455.0
Europe	353.3
Asia-Pacific	126.5
Other	275.4
Total	1,210.1

Source: BIS.

Theme 2: Derivatives (continued)

All Derivatives
(notional outstandings, billions of US dollars, 1995)

Category	Over-the-Counter	Exchanges
Interest rate	26,645	15,669
Currency	13,095	120
Equity and stock indexes	579	442
Commodities	318	142

Tell Figlewski: futures and options = fixed income!

Remark: OTC derivatives tied to global interbank market

Theme 3: Debt Markets are Emerging

- Net Capital Flows to Emerging Markets (billions of US dollars, 1995)

Total	193.7
Direct Investment	71.7
Portfolio (Debt and Equity)	37.0
Loans	85.1

- Gross Private Issues of Debt and Equity (billions of US dollars, 1995)

Debt	
Total amount	501.7
Share of emerging markets (%)	11.6
Equity	
Total amount	44.2
Share of emerging markets (%)	25.3

- Summary of emerging markets:
  - Significant and growing share
  - Increasing use of public markets

Fixed Income Analytics at Work

Example 1: Bell Atlantic

- Stylized balance sheet (typical of nonfinancial corps) (year-end 1996, billions of dollars)

Assets	
PP&E	16
Liabilities and Shareholders' Equity	
Debt	6
Shareholders' Equity	8
Stuff	2

- Debt notes:
  - Primarily fixed rate, with maturities through 2033
  - Accounting: coupons charged against income
  - 1.5b callable, and some putable
  - Derivatives: 0.2b interest rate swaps (receive fixed)

- Question: Is long debt less risky than short?

  Answer 1: Yes, interest expense is predictable

  Answer 2: No, market value varies more

Fixed Income Analytics at Work (continued)

Example 2: Intel

- Balance sheet summary
  (year-end 1996, billions of dollars)

Assets	
Cash and Securities	7.9
PP&E	8.5
Stuff	1.2
Liabilities and Shareholders' Equity	
Debt	0.7
Shareholders' Equity	16.9

- Securities and debt notes:
  - Everything swapped into dollar-LIBOR (floating rate)
  - Accounting at market value: interest ***and changes in market value*** included in revenue and expense

- Question: Are floating rate (short) securities less risky?

  Answer 1: Yes, market value is stable

  Answer 2: No, interest income/expense unpredictable

Fixed Income Analytics at Work (continued)

Example 3: Banc One

- Stylized balance sheet
  (year-end 1996, billions of dollars)

Assets	
Loans	73
Cash and Securities	29
Liabilities and Shareholders' Equity	
Deposits	72
Debt	21
Shareholders' Equity	8

- Summary and comments:
  - Highly levered (like all commercial banks)
  - Assets shorter than liabilities
    $\Rightarrow$ vulnerable to fall in rates
  - Swaps used to moderate interest sensitivity
  - Accounting: mixture of market value ("available for sale") and historical cost ("held to maturity")

Fixed Income Analytics at Work (continued)

Example 4: Emerging Markets

- General characteristics of emerging markets
  - Debt easier to issue than equity
  - Often comes with more stringent disclosure requirements than domestic issues
  - Typically denominated in major currency (dollars, say)
  - Borrowers are sovereigns and firms with strong credit, hard-currency revenues
  - Foreign-currency denomination adds currency risk to to the usual credit risk (economies and currencies often implode together)
- Examples:
  - Par Bonds, Mexico (Bradies)
  - Globals, Mexico (eurobonds)
  - Grupo Carso SA, Mexico, floating rate eurobonds
  - Brazilian "C" Bonds (Bradies)
  - Argentinian FRB's
  - ICICI, India, eurobonds (144A)
  - Ministry of Finance, Russia (144A)

Outline

Part I: Bonds and Close Relatives

1. Fixed Income Securities
   assets whose value depends on interest rates

2. Bond Arithmetic
   calculating spot rates, yields, etc

3. Macrofoundations of Interest Rates
   monetary policy and other factors

4. Quantifying Interest Rate Risk
   duration and beyond, activist investment strategies

5. Interest Rate Swaps
   also floaters and inverse floaters

6. Risk Management, Accounting, and Control
   market and book value, disasters and their sources

Outline (continued)

Part II: Interest Rate Derivatives

7. State-Contingent Claims
   analytical framework for derivative valuation

8. Forwards and Futures
   bond and interest rate futures

9. Options
   analytics of options, callable bonds, caps and floors

10. Corporate Bonds
    introduction to credit risk

11. Emerging Market Debt
    Brady bonds, eurobonds, trends

12. Mortgages
    Mortgages, mortgage-backed securities, structured notes

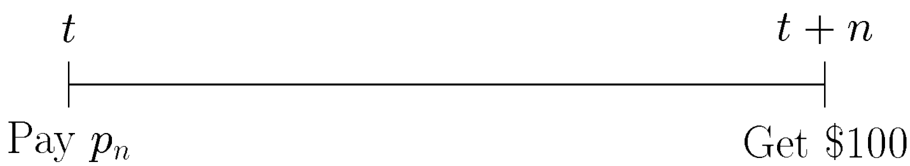
Bond Arithmetic

0. Overview

- Zeros and coupon bonds
- Spot rates and yields
- Day count conventions
- Replication and arbitrage
- Forward rates
- Yields and returns

1. Zeros or STRIPS

- A *zero* is a claim to \$100 in n periods (price = p_n)



- A *spot rate* is a yield on a zero:

$$p_n = \frac{100}{(1 + y_n/2)^n}$$

- US treasury conventions:
  - price quoted for principal of 100
  - time measured in half-years
  - semi-annual compounding

1. Zeros (continued)

- A *discount factor* is a price of a claim to one dollar:

$$d_n = \frac{p_n}{100} = \frac{1}{(1 + y_n/2)^n}$$

- Examples (US treasury STRIPS, May 1995)

Maturity (Yrs)	Price ($)	Discount Factor	Spot Rate (%)
0.5	97.09	0.9709	5.99
1.0	94.22	0.9422	6.05
1.5	91.39	0.9139	
2.0	88.60	0.8860	6.15

2. Yield Conventions

- A yield convention is an arbitrary set of rules for computing yields (like spot rates) from discount factors

- US Treasuries use semiannual compounding:

$$d_n = \frac{1}{(1 + y_n/2)^n}$$

  with n measured in half-years

- Other conventions with n measured in years:

$$d_n = \begin{cases} (1+y_n)^{-n} & \text{annual compounding} \\ (1+y_n/k)^{-kn} & \text{“k” compounding} \\ e^{-ny_n} & \text{continuous compounding } (k \to \infty) \\ (1+ny_n)^{-1} & \text{“simple interest”} \\ (1-ny_n) & \text{“discount basis”} \end{cases}$$

- All of these formulas define rules for computing the yield y_n from the discount factor d_n, but of course they're all different and the choice among them is arbitrary. That's one reason discount factors are easier to think about.

3. Coupon Bonds

- Coupon bonds are claims to fixed future payments (c_n, say)

- They're collections of zeros and can be valued that way:

$$\begin{aligned}\text{Price} &= d_1 c_1 + d_2 c_2 + \cdots + d_n c_n \\ &= \frac{c_1}{(1+y_1/2)} + \frac{c_2}{(1+y_2/2)^2} + \cdots + \frac{c_n}{(1+y_n/2)^n}\end{aligned}$$

- Example: Two-year "8-1/2s"
  Four coupons remaining of 4.25 each

$$\begin{aligned}\text{Price} &= 0.9709 \times 4.25 \; + \; 0.9422 \times 4.25 \\ &\quad + 0.9139 \times 4.25 \; + \; 0.8860 \times 104.25 \\ &= 104.38.\end{aligned}$$

- Two fundamental principles of asset pricing:
  - Replication: two ways to generate same cash flows
  - Arbitrage: equivalent cash flows should have same price

4. Spot Rates from Coupon Bonds

- We computed the price of a coupon bond from prices of zeros

- Now reverse the process with these coupon bonds:

Bond	Maturity (Yrs)	Coupon	Price
A	0.5	8.00	100.97
B	1.0	6.00	99.96

- Compute discount factors "recursively":

$$100.97 = d_1 \times 104 \quad \Rightarrow \quad d_1 = 0.9709$$

$$\begin{aligned} 99.96 &= d_1 \times 3 + d_2 \times 103 \\ &= 0.9709 \times 3 + d_2 \times 103 \quad \Rightarrow \quad d_2 = 0.9422 \end{aligned}$$

- Spot rates follow from discount factors:

$$d_n = \frac{1}{(1 + y_n/2)^n}$$

4. Spot Rates from Coupon Bonds (continued)

- Do zeros and coupon bonds imply the same discount factors and spot rates?

- Example: Suppose Bond B sells for 99.50, implying $d_2 = 0.9377$ (B seems cheap)
  - Replication of B's cash flows with zeros:
  $$\begin{aligned} 3 &= x_1 \times 100 \quad \Rightarrow \quad x_1 = 0.03 \\ 103 &= x_2 \times 100 \quad \Rightarrow \quad x_2 = 1.03 \end{aligned}$$
  - Cost of replication is
  $$\text{Cost} = 97.09 \times 0.03 + 94.22 \times 1.03 = 99.96$$
  - Arbitrage strategy: buy B and sell its replication
    Riskfree profit is 99.96 – 99.50 = 0.46

- **Proposition.** If (and only if) there are no arbitrage opportunities, then zeros and coupon bonds imply the same discount factors and spot rates.

- Presumption: markets are approximately "arbitrage-free"

- Practical considerations: bid/ask spreads, hard to short

5. Yields on Coupon Bonds

- Spot rates apply to specific maturities

- The *yield-to-maturity* on a coupon bond satisfies

$$\text{Price} = \frac{c_1}{(1+y/2)} + \frac{c_2}{(1+y/2)^2} + \cdots + \frac{c_n}{(1+y/2)^n}$$

- Example: Two-year 8-1/2s

$$104.38 = \frac{4.25}{(1+y/2)} + \frac{4.25}{(1+y/2)^2} + \frac{4.25}{(1+y/2)^3} + \frac{104.25}{(1+y/2)^4}$$

The yield is $y = 6.15\%$.

- Comments:
  - Yield depends on the coupon
  - Computation: guess y until price is right

6. Par Yields

- We've found prices and yields for given coupons

- Find the coupon that delivers a price of 100 (par)

$$\text{Price} \ = \ 100 \ = \ (d_1 + \cdots + d_n)\text{Coupon} + d_n 100$$

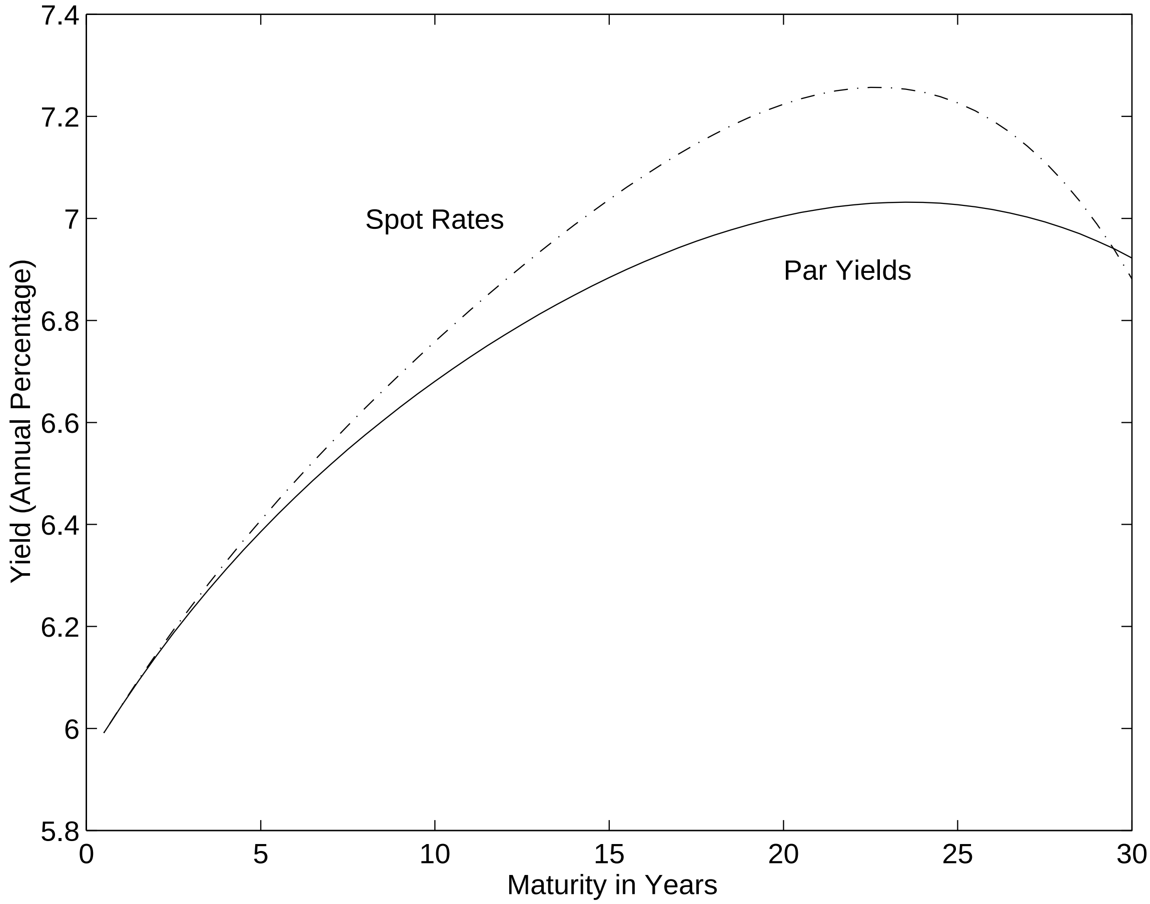
The annualized coupon rate is

$$\begin{aligned}\text{Par Yield} &= 2 \times \text{Coupon} \\ &= 2 \times \frac{1 - d_n}{d_1 + \cdots + d_n} \times 100\end{aligned}$$

- This obscure calculation underlies the initial pricing of bonds and swaps

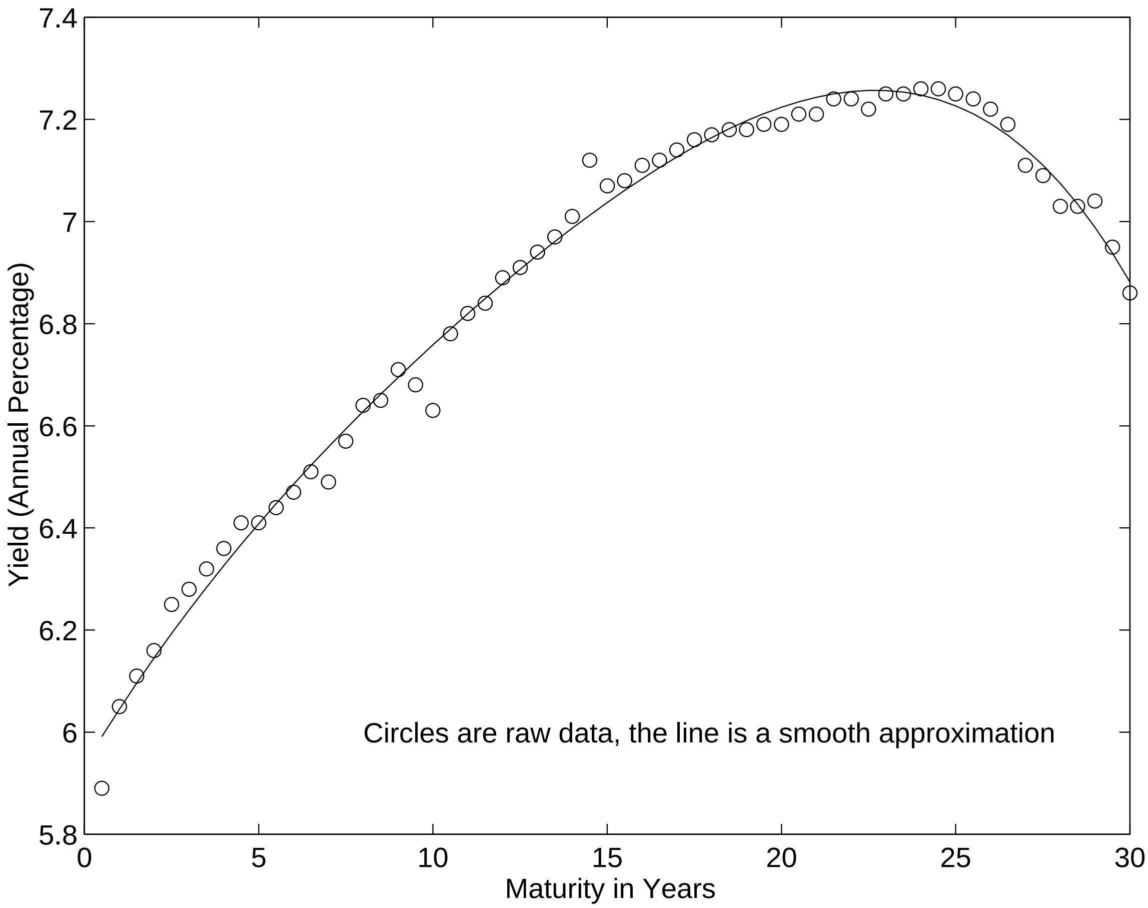
7. Yield Curves

- A *yield curve* is a graph of yield y_n against maturity n
- May 1995, from US Treasury STRIPS:



8. Estimating Bond Yields

- Standard practice is to estimate spot rates by fitting a smooth function of n to spot rates or discount factors



- "Noise": bid/ask spread, stale quotes, liquidity, coupons, special features
- Reminder that the frictionless world of the proposition is an approximation

9. Day Counts for US Treasuries

- Overview
  - Bonds typically have fractional first periods
  - You pay the quoted price plus a pro-rated share of the first coupon (accrued interest)
  - Day count conventions govern how prices are quoted and yields are computed

- Details
  - Invoice price calculation (what you pay):

$$\begin{aligned}\text{Invoice Price} &= \text{Quoted Price} + \text{Accrued Interest} \\ \text{Accrued Interest} &= \frac{u}{u+v}\text{Coupon} \\ u &= \text{Days Since Last Coupon} \\ v &= \text{Days Until Next Coupon}\end{aligned}$$

  - Yield calculation ("street convention"):

$$\begin{aligned}\text{Invoice Price} &= \frac{\text{Coupon}}{(1+y/2)^{w}} + \frac{\text{Coupon}}{(1+y/2)^{w+1}} \\ &+ \cdots + \frac{\text{Coupon} + 100}{(1+y/2)^{w+n-1}} \\ w &= v/(u+v) \\ n &= \text{Number of Coupons Remaining}\end{aligned}$$

9. Day Counts for US Treasuries (continued)

- US Treasuries use "actual/actual" day counts for u and v (ie, we actually count up the days)

- Example: 8-1/2s of April 97 (as of May 95)

Issued	April 16, 1990
Settlement	May 18, 1995
Matures	April 15, 1997
Coupon Freq	Semiannual
Coupon Dates	15th of Apr and Oct
Coupon Rate	8.50
Coupon	4.25
Quoted Price	104.19

Time line:

$$u =$$

$$v =$$

$$n =$$

9. Day Counts for US Treasuries (continued)

Price calculations:

$$\begin{aligned} \text{Accrued Interest} &= \\ \text{Invoice Price} &= \end{aligned}$$

Yield calculations:

$$\begin{aligned} w &= \\ d &= 1/(1+y/2) \qquad \text{(to save typing)} \\ 104.95 &= d^w(1+d+d^2+d^3)\,4.25 + d^{w+3}100 \\ \Rightarrow \quad & d = 0.97021 \\ & y = 6.14\% \end{aligned}$$

(The last step is easier with a computer)

10. Other Day Count Conventions

- US Corporate bonds (30/360 day count convention) (roughly: count days as if every month had 30 days)

  Example: Citicorp's 7 1/8s

Settlement	June 16, 1995
Matures	March 15, 2004
Coupon Freq	Semiannual
Quoted Price	101.255

  Calculations:

  $$\begin{aligned} n &= \\ u &= \\ v &= \\ w &= \\ \text{Accrued Interest} &= \\ \text{Invoice Price} &= \end{aligned}$$

  $$\text{Invoice Price} = d^w \left(\frac{1 - d^n}{1 - d} \right) \text{ Coupon } + d^{w+n-1} 100$$

  $$\Rightarrow \qquad y = 6.929\%$$

  Remark: term in brackets is a formula

10. Other Day Count Conventions (continued)

- Eurobonds (30E/360 day count convention)
  (ie, count days as if every month has 30 days)

  Example: IBRD 9s, dollar-denominated

Settlement	June 20, 1995
Matures	August 12, 1997
Coupon Freq	Annual
Quoted Price	106.188

  Calculations:

$$n =$$

$$u =$$

$$v =$$

$$w =$$

$$\text{Accrued Interest} =$$

$$\text{Invoice Price} =$$

$$\text{Invoice Price} = d^w \left(\frac{1 - d^n}{1 - d} \right) \text{ Coupon } + d^{w+n-1} 100$$

$$\Rightarrow \quad y = 5.831\%$$

  Remark: annual compounding, $d = 1/(1 + y)$

10. Other Day Count Conventions (continued)

- Eurocurrency deposits
  (generally actual/360 day count convention)

  Example: 6-month dollar deposit in interbank market

Settlement	June 22, 1995
Matures	December 22, 1995
Rate (LIBOR)	5.375

  Cash flows: pay (say) 100, get 100 plus interest

  Interest computed by

$$\begin{aligned} \text{Interest} &= \text{LIBOR} \times \frac{\text{Actual Days to Payment}}{360} \\ &= 5.375 \times \left(\frac{183}{360}\right) = 3.018. \end{aligned}$$

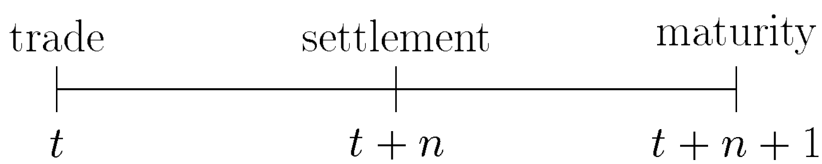
  Remarks

  – Can be denominated in any currency

  – "Interest" is analogous to $y/2$ in treasury formulas

11. Forward Rates

- A one-period ***forward rate*** f_n at date t is the rate paid on a one-period investment arranged at t (“trade date”) and made at $t+n$ (“‘settlement date”)



- Representative cash flows (scale is arbitrary)

t	$t+n$	$t+n+1$
0	$-F$	100

with $F = 100/(1+f_n/2)$ (verify that rate is f_n)

- Replication with zeros

t	$t+n$	$t+n+1$
$-p_{n+1}$		100
xp_n	$-100x$	

Choose x to replicate cash flows of forward contract

$$0 = -p_{n+1} + xp_n$$

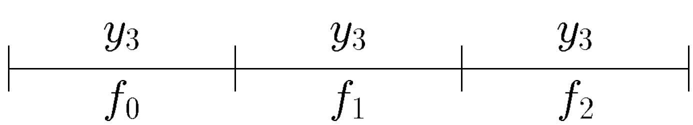
$$\Rightarrow \quad 1 + f_n/2 = \frac{p_n}{p_{n+1}}$$

11. Forward Rates (continued)

- Sample forward rate calculations:

Maturity	Price ($)	Spot Rate (%)	Forward Rate (%)
0.5	97.09	5.99	5.99
1.0	94.22	6.05	6.10
1.5	91.39	6.10	6.20
2.0	88.60	6.15	6.29

- Forward rates are the marginal cost of one more period:

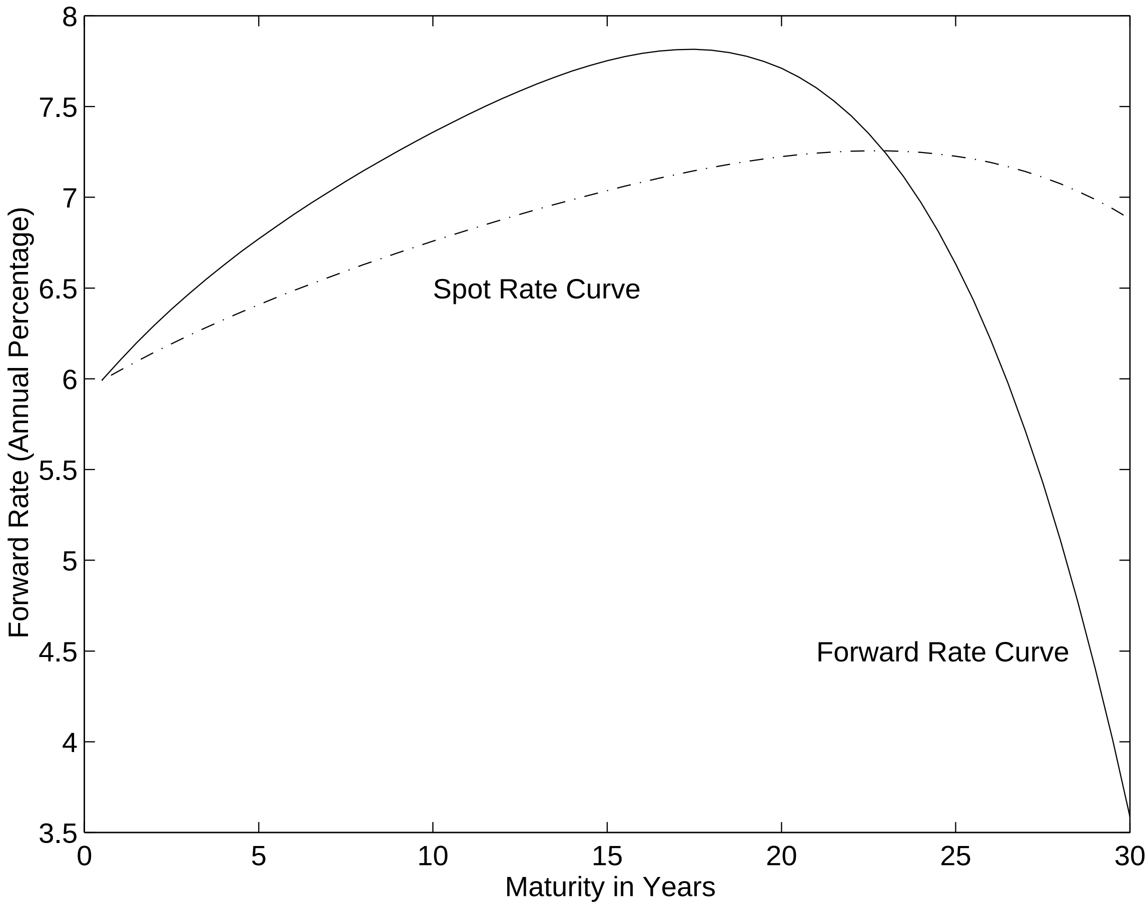


- Spot rates are (approximately) averages:

$$
\begin{aligned}
y_n &\cong n^{-1}(f_0 + f_1 + \cdots + f_{n-1}) \\
&\cong n^{-1} \sum_{j=1}^{n} f_{j-1}
\end{aligned}
$$

11. Forward Rates (continued)

- Forward and spot rates in May 1995:



12. Yields and Returns on Zeros

- Example: Six-month investments in two zeros

Zero	Maturity	Price	Spot Rate (%)	Forward Rate (%)
A	0.5	97.56	5.00	5.00
B	1.0	94.26	6.00	7.00

- Scenarios for spot rates in six months

Spot Rates (%)	Scenario 1	Scenario 2	Scenario 3
y_1	8.00	5.00	2.00
y_2	9.00	6.00	3.00
	"up 3"	"no change"	"down 3"

- One-period returns on zeros

$$1 + h/2 \;=\; \frac{\text{Sale Price}}{\text{Purchase Price}}$$

(h for holding period, which is six months here)

- Scenario 2 returns

$$\text{(A)} \quad 1 + h/2 \;=\; \frac{100}{97.56} \;=\; 1.025 \quad \Rightarrow \quad h = 0.0500$$

$$\text{(B)} \quad 1 + h/2 \;=\; \frac{97.56}{94.26} \;=\; 1.035 \quad \Rightarrow \quad h = 0.0700$$

12. Yields and Returns on Zeros (continued)

- Six-month returns (h):

	Return on A (%)	Return on B (%)
Scenario 1	5.00	4.02
Scenario 2	5.00	7.00
Scenario 3	5.00	10.08

- Remarks
  - Return on A is the same in all scenarios
    Standard result when holding period equals maturity:

$$(1 + h/2)^n \;=\; \frac{100}{p_n} \;=\; (1 + y/2)^n$$

  - Return on B depends on interest rate movements

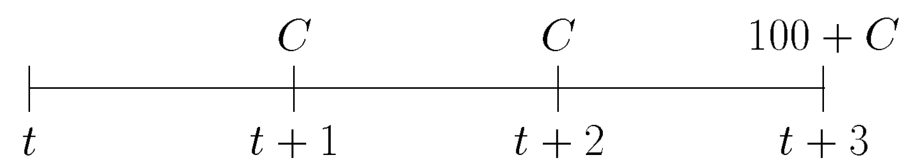
13. Yields and Returns on Coupon Bonds

- One-period returns

$$1 + h/2 \;=\; \frac{\text{Sale Price + Coupon}}{\text{Purchase Price}}$$

(buy and sell just after coupon payment)

- Return when held to maturity
Needed: return r on reinvested coupons
  - Three-period example



$$(1{+}h/2)^3 \;=\; \frac{(1+r)^2C + (1+r)C + C + 100}{\text{Purchase Price}}$$

  - Return depends on reinvestment rate r (arbitrary)
  - No simple connection between return and yield

- Bottom line: yields are not returns

Summary

- Bond prices and discount factors represent the time-value of money
- Spot rates do, too
- Conventions govern the calculation of spot rates from discount factors
- Yields on coupon bonds are a common way of representing prices, but are not otherwise very useful
- Cash flows of coupon bonds can be "replicated" with zeros, and vice versa
- Replication and arbitrage relations apply to frictionless markets, but hold only approximately in practice
- Yields and returns aren't the same thing