

Interest-Rate Options

0. Overview

- Fixed Income Options
- Option Fundamentals
- Caps and Floors
- Options on Bonds
- Swaptions

1. Fixed Income Options

- Options imbedded in bonds:
 - Callable bonds
 - Puttable bonds
 - Convertible bonds

- Options on futures
 - Bond futures
 - Eurocurrency futures

- OTC options
 - Caps, floors, and collars
 - Swaptions

2. Option Basics

- Big picture
 - Options are everywhere
 - * Stock options for CEOs and others
 - * Corporate equity: call option on a firm
 - * Mortgages: the option to refinance
 - Options are like insurance
 - * Premiums cover the down side, keep the up side
 - * Customers like this combination
 - * Insurer bears or shares risk (reinsurance, diversification)
 - Managing cost of insurance
 - * Out-of-the-money options are cheaper (insurance with a big deductible)
 - * Collars: sell some of the up side
 - * Aggregate: basket option cheaper than basket of options
 - Managing option books
 - * Customer demands may result in exposed position
 - * Particular exposure to volatility:
 - puts and calls both rise with volatility
 - * Hedging through replication is another route

2. Option Basics (continued)

- Option terminology
 - Options are the right to buy (a *call*) or sell (a *put*) at a fixed price (*strike price*)
 - The thing being bought or sold is the *underlying*
 - This right typically has a fixed *expiration* date
 - A short position is said to have *written* an option
 - *European* options can be exercised only at expiration
 - *American* options can be exercised any time
 - *Bermuda* options can be exercised at specific dates (eg, bonds callable only on coupon dates)

2. Option Basics (continued)

- Features of options
 - Leverage (cheap source of exposure)
 - Nonlinear payoffs
 - * Translates into variable duration
(convexity rears its ugly head)
 - * Creates risk management hazards
 - Volatility has positive effect on puts and calls
 - * Another risk management hazard!
 - A state-contingent claim
(no way 'round it, but nothing new either)

3. Modeling Approaches

- Why pricing models? No choice.
- What pricing model? Good question.
- Interest rate trees
 - Been there . . . (and it hasn't changed)
- The Black-Scholes benchmark
 - Parameters: n -period European call on $(n+m)$ -period zero at strike price k .
 - Formula:

$$\text{Call Price} = p_{n+m}N(x_1) - d_n k N(x_2)$$

with

p_n = price of n -period zero

d_n = n -period discount factor

N = normal cdf

$x_1 = [\log(f/k) + n\sigma^2/2] / n^{1/2}\sigma$

$x_2 = x_1 - n^{1/2}\sigma$

$f = 100 \times p_{n+m}/p_n$ (forward price)

3. Modeling Approaches (continued)

- Black-Scholes (cont'd): Properties

- The Delta:

$$\text{Delta} = \frac{d\text{Call Price}}{dp_{n+m}} = N(x_1),$$

which varies between zero and one (nonlinear).

- Volatility is the only unobservable
(so we use call prices to infer it)

- Price increases in volatility

- Remarks on Black-Scholes for fixed income

- Obviously interest rates aren't fixed

- But formula based on log-normal zero prices
⇒ normal (continuously compounded) spot rates
⇒ possibility of negative spot rates

- Volatility varies systematically with (m, n)
(the “term structure of volatility”)

- Despite problems, a common benchmark

4. Caps, Floors, and Collars

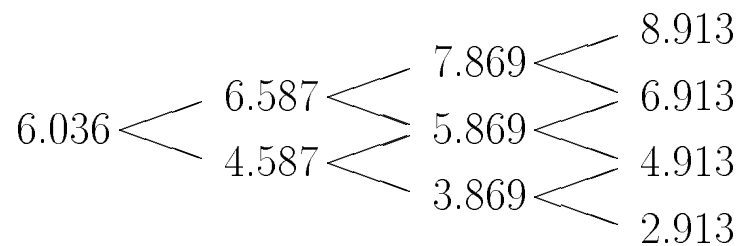
- An interest rate cap pays the difference between the cap rate and a reference rate, if positive.
- Contract terms:
 - Cap rate
 - Reference rate (typically LIBOR)
 - Frequency of payment
 - Notional principal (amount on which interest is paid)
- Example: two-year, semi-annual, 7% cap on 6-m LIBOR, Notional principal 250mm.

Effective Cash Flow at $t+n$:

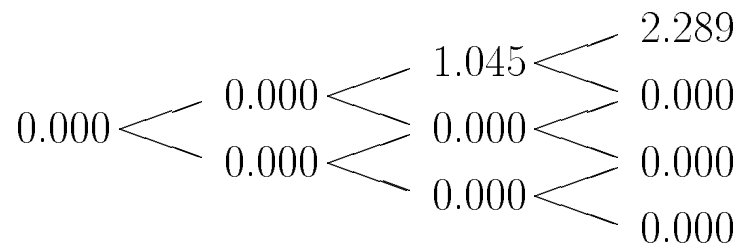
$$(r_{t+n} - 0.07)^+ \times \frac{1}{1 + r_{t+n}/2} \times \frac{\text{Notional Principal}}{2}$$

4. Caps, Floors, and Collars (continued)

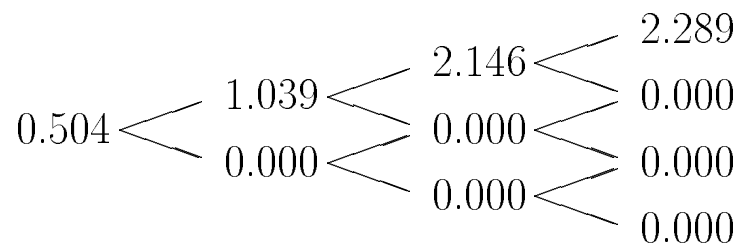
- Rate tree (our usual)



- Cash flow tree



- Price path



4. Caps, Floors, and Collars (continued)

- A floor pays the difference between the floor rate and a reference rate, if positive
eg, a 3% floor pays 0.8% if LIBOR is 2.2%
- A collar is a combination of a cap and a floor
- Collars on FRNs limit interest payments,
reduce cost by returning part of the “up” side
- Options on eurocurrency futures can be used to hedge caps (calls) and floors (puts)

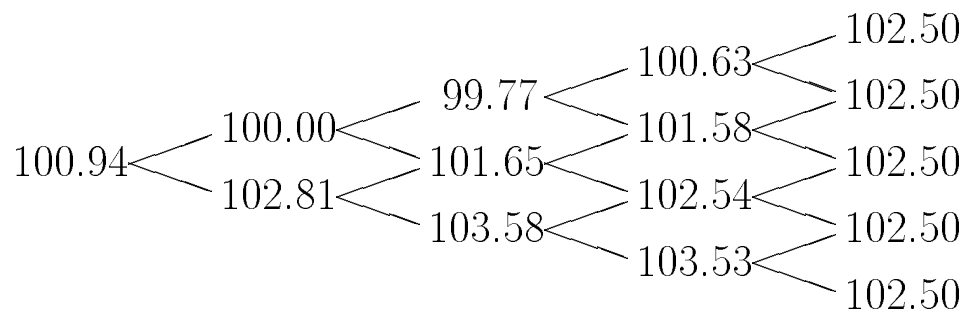
5. Options on Bonds

- Earliest and most common interest-rate option?
- Examples of corporate callable bonds
 - Apple Computer Corporation’s 6-1/2s, issued February 10, 1994, due February 15, 2004. Callable at “make whole.”
 - Ford Motor Company’s 6.11 percent bonds, issued September 22, 1993, due January 1, 2001. Callable at “make whole under special circumstances.”
 - Intel Overseas Corporation’s 8-1/8s, issued April 1, 1987 (really), due March 15, 1997, callable at par. Par in this situation means par plus accrued interest: the firm pays the relevant interest as well as the face value. The bonds were called March 15, 1994, at 100.
 - Texas Instruments’ 9s of July 99, issued July 18, 1989, due July 15, 1999. Callable on or after July 15, 1996, at par.

5. Options on Bonds (continued)

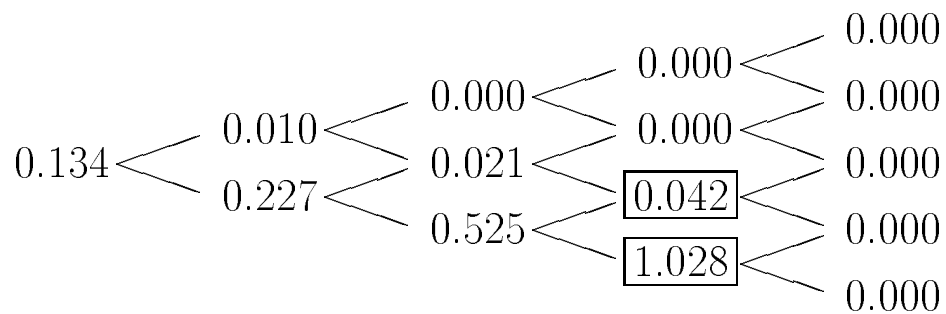
- Example: call option on 2-year 5% bond
(the usual rate tree)

Price path of bond:



- 18-month European option (callable at 102.5)

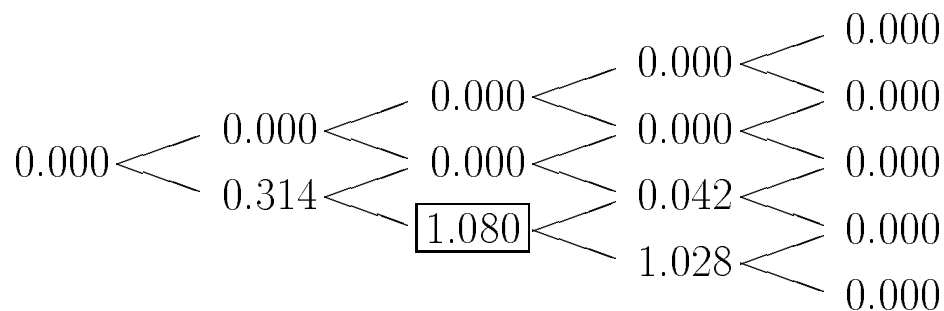
Price path is:



Nodes in boxes indicate cash flows from exercise, other nodes the value in earlier periods.

5. Options on Bonds (continued)

- American option has greater value
(can exercise at expiration, or earlier if better)
- Approach: start at expiration, work backwards
At each node, choose better of “exercise” or “hold”
- Cash flows from immediate exercise:



- Node with box:

– If hold:

$$\text{Value} = \frac{0.5}{1 + 0.03869/2} \times (0.042 + 1.028) = 0.525$$

– If exercise:

$$\text{Value} = 1.080$$

This is better: we take it.

5. Options on Bonds (continued)

- American option (continued)
- Node (0,1) (one down move from start):

– If hold:

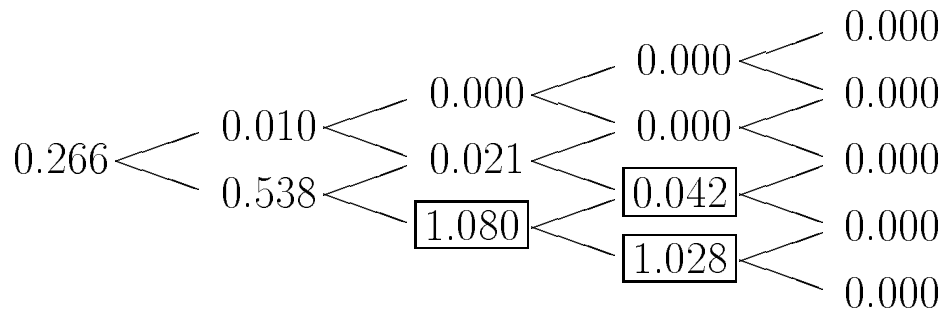
$$\text{Value} = \frac{0.5}{1 + 0.04587/2} \times (0.000 + 1.080) = 0.538$$

– If exercise:

$$\text{Value} = 0.314$$

Hold is better in this case, so we write 0.538 here.

- Complete price path:



Boxes indicate nodes where option is exercised.

- Summary
 - Worth more than European call
 - Valued recursively

6. Swaptions

- Options on swaps
- Just like bond options
(swap = long bond, short FRN that trades at par)
- European, American, Bermuda
- Currently the OTC standard
- Good source of implied volatilities (%):

| | Swap | | | |
|--------|-------|-------|-------|-------|
| Option | 1 yr | 2 yr | 5 yr | 10 yr |
| 1 m | 15.50 | 16.00 | 16.75 | 15.25 |
| 3 m | 17.50 | 18.50 | 18.25 | 18.25 |
| 1 yr | 21.50 | 21.25 | 19.25 | 16.50 |
| 5 yr | 20.00 | 19.00 | 17.50 | 15.50 |

Source: Tradition, Inc, global swap broker, Jan 2, 1996.

Summary

- Options are ubiquitous.
- Their nonlinear payoffs pose challenges to valuation and risk management.
- Nonlinearity translates in this context into nonlinear price-yield relations — convexity, in other words.
- Black-Scholes is less well suited for fixed income than other securities, but remains a common benchmark nonetheless.
- American options are valued recursively: at each node, we decide whether to hold or exercise.
- Shortcuts don't work: yield-to-call is meaningless.