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Interest-Rate Forwards and Futures

0. Overview

- Leading Futures Contracts
- Forward Contracts
- Futures Contracts
- Bond Futures
- Eurocurrency Futures

1. Leading Futures Contracts

- Contracts ranked by dollar volume:

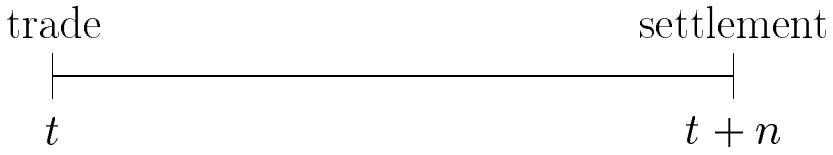
Contract	Open Interest	Monthly Volume	
	Contracts (mm)	Contracts (mm)	Dollars (bb)
Eurodollar	1.325	5.044	5,044
Euroyen	0.439	1.247	1,122
10-yr JGB	0.132	0.989	890
3-m sterling	0.212	0.941	724
Euromark	0.370	1.014	628
30-yr US T-bond	0.305	5.834	583
PIBOR	0.146	0.536	492
10-yr Notionnel	0.231	2.584	237
S&P 500	0.157	1.035	232
German Bund	0.139	1.134	176
Nikkei 225	0.149	0.950	172
10-yr US T-note	0.177	0.935	93

Source: Galitz, *Financial Engineering*, numbers for 1992.

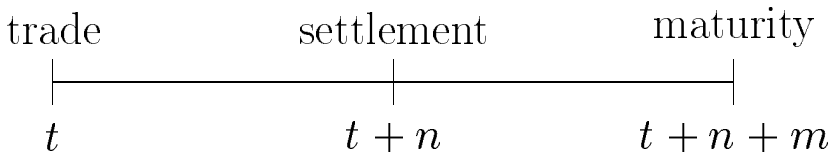
- Remarks
  - Fixed income contracts dominate
  - Eurocurrencies first, then government bonds
  - Bond contracts have greater volume to open interest than euros: short-term trading v buy-and-hold

2. Forward Contracts

- A *forward contract* is an agreement to exchange assets at a future date for a price arranged now.
- Terminology: trade date is when trade is made, settlement date is when assets are exchanged:



- With interest rate contracts, a third date is the maturity of the asset being exchanged (typically for cash):



  Convention (usually): m is maturity at settlement.

- Interest rate forwards and futures differ primarily in the magnitudes of n and m:
  - Forward rate agreements: n and m are typically single-digit months.
  - Bond futures: short settlement and long maturity (eg, n of 3 months and m of 10 years).
  - Eurocurrency futures: long settlement and short maturity (n out to 10 years and beyond, m of 3 months).

2. Forward Contracts (continued)

- Forward contract on a zero: agree at t to buy m-period zero at $t+n$ for price F. (This should look familiar!)

- Cash flows:

t	$t+n$	$t+n+1$
0	$-F$	100

- Replication with zeros (p_n = price of n-period zero):

t	$t+n$	$t+n+m$
$-p_{n+m}$		100
xp_n	$-100x$	

Choose x to replicate cash flows of forward contract:

$$0 \;=\; -p_{n+m} + xp_n$$

- Solution:

$$x \;=\; \frac{p_{n+m}}{p_n} \quad \Rightarrow \quad F \;=\; \frac{p_{n+m}}{p_n} \times 100$$

2. Forward Contracts (continued)

- Forward contract on a bond: agree at t to buy an m-period bond at $t + n$ for price F.

- Coupons complicate the analysis, but the idea is the same as the zero: replicate with long position in the bond and short positions sufficient to offset the purchase price and (in this case) the coupons between now and settlement.

- Key ingredient: replication includes a long position in the bond, so you're indirectly gaining exposure to the bond.

2. Forward Contracts (continued)

- Floating rate agreements (FRAs)
- Contract terms
  - In an "$n \times m$" (both quoted in months), m is what we've called $m + n$
  - Fixed "contract" rate (C)
  - Floating "reference" rate (r) (fixed at settlement)
  - Notional principal
  - Cash flow at settlement:

$$\text{Payment} = \frac{(C - r)(m - n)/12}{1 + r(m - n)/12} \times \text{Principal}$$

    (plus the usual eurocurrency day count adjustments)
- Example: 6×12, 1mm notional
  - Contract rate = 6%
  - Reference rate = 6-month LIBOR
  - If 6-month LIBOR is 5% at settlement,

$$\text{Payment} = \frac{(.06 - .05)/2}{1 + .05/2} \times \$1{,}000{,}000 = \$4,878$$

- Contract rate is essentially the forward rate between $t + 6$ and $t + 12$

2. Forward Contracts (continued)

- Using forwards to modify interest sensitivity
  - Duration not defined: duration is the proportional change in the price and is not defined for contracts (like swaps, forwards, and futures) that have no net value.
  - Quantify interest sensitivity as we did with swaps:
    * Use the DV01
    * Compute duration for long and short positions separately
  - Bottom line:
    * Forward contracts on long-maturity bonds are useful tools for modifying duration: you add (say) a long position in a long bond, and short a position of equal value in a short bond.
    * Forward contracts on short bonds — FRAs, for example — are useful protection against near-term changes in short rates.

3. Essential Features of Futures Contracts

- Similar to forward contracts
- Easy to short: big advantage over bonds
- Fixed contract terms
- Liquid: often easier/cheaper to trade than bonds
- Low credit risk: guaranteed by the exchange
- Trades public: good source of market information
- Cash flows:
  - No payment due on trade date
  - . . . but money is set aside in margin account
  - Margin account varies from
    - Daily changes in contract price
    - Interest on the account
    - Margin calls
- Daily "mark to market"
  - Reduces credit exposure of exchange
  - Complicates cash flows and valuation (slightly)

4. Treasury Futures

- US treasury bond/note contracts (CBOT)
  (foreign government bond contracts are similar)

- Standard features
  - Contract size: $100,000 face value
  - Contracts expire quarterly (Mar/Jun/Sep/Dec)
  - Delivery: short position
    * Can deliver any time in the contract month
    * Delivers $100,000 face value of bonds, gets cash.
  - Eligible (*contract grade*) bonds:
    * 30-year bond contract: US treasury bonds with maturity at least 15 years from delivery date
    * 10-year note contract: US treasury notes with maturity 6.5 to 10 years from delivery date
    * 5-year note contract: US treasury notes with original maturity no more than 5.25 years and maturity on first day of delivery month of at least 4.25 years
  - Wild-card option: price at close (2pm) good till 8pm
  - Timing option: futures price fixed on last trading day

4. Treasury Futures (continued)

- Valuing contracts
  - Summary: if F is quoted futures price,

    $$\text{Value} = F \times \text{Conversion Factor} + \text{Accrued Interest}$$

  - Conversion factor mitigates tendency to deliver lowest-coupon bond (the cheapest one eligible)
  - Compute conversion factor as follows:
    - Compute maturity at first delivery date, round down to nearest 3-month interval
    - Compute invoice price by discounting coupon and principal at 8% (refer to set 1, yield-to-maturity)
    - Subtract accrued interest
    - Divide by 100 to get conversion factor
  - Examples for Dec 97 contracts
    - 10.75 due Feb 2016: $n = 36$, Invoice = 126.00, Accrued = 0 , CF = 1.2600.
    - 10.375% due December 2020: $n = 46$, Invoice = 124.80, Accrued = 0 , CF = 1.2480.
    - 10.375% due April 2021: $n = 47$, $w = 0.5$, Invoice = 127.46, Accrued = 2.59 , CF = 1.2487.

4. Treasury Futures (continued)

- Interest sensitivity: standard approach
  - Easy way to add/subtract exposure to long bond
  - In earlier terminology: n is small and m is large
  - DV01 based on the cheapest to deliver bond:

$$\text{DV01 of Futures} = \frac{\text{DV01 of Bond}}{\text{Conversion Factor}}$$

  - Remark: don't ask!
- Complications
  - Sensitivity to specific part of the yield curve
  - Difference between cash and futures price may vary over time, esp when volatility changes

5. Eurocurrency Futures

- Contracts on 3-month LIBOR in major currencies (CME, LIFFE, SIMEX)
- Standard features
  - Contract size: interest on $1,000,000
  - Contracts expire quarterly (Mar/Jun/Sep/Dec) out ten years or more (third Wednesday)
  - Quoted index:

$$\text{Index} \;=\; 100 \times (1 - \text{Yield})$$

  - Effective price of a contract:

$$\text{Price} \;=\; 1mm \times (1 - \text{Yield}/4)$$

  - Cash settlement at

$$\text{Settlement Price} \;=\; 1mm \times (1 - \text{3-m LIBOR}/4)$$

    Note: no strange delivery options!
  - Strips: combinations of contracts with different maturities
- Uses
  - Helpful for hedging FRNs, swaps, etc
  - Source of market information on forward rates
  - In earlier terminology: n can be large, but m is small

Summary

- Forward contracts for bonds are equivalent to a combination of a long position in a long bond and a short position of equal value in a short bond.

- Futures contracts differ from forwards in the daily mark to market.

- Fixed income futures include government bond and eurocurrency contracts.

- Bond futures are truly ugly contracts.

- Like other derivatives, forwards and futures offer leverage: you can arrange great exposure with less money down than buying the underlying instrument.