Disaster Risk and Business Cycles

by François Gourio

Discussion by David Backus
NBER EFG Meeting | SF Fed | February 5, 2010
Plan of attack

Gourio summary

The nature of business cycle risk
  ▶ Disasters implied by options
  ▶ Cyclical behavior of asset returns

Challenges posed by the model (and met by the paper)
  ▶ Computation
  ▶ Tallarini’s result
  ▶ The Barro-King problem

Gourio revisited
Gourio summary

Business cycle model + “disasters” + recursive preferences

Disasters

- Large adverse shock to productivity — and capital
- Probability varies over time
- Magnified by recursive preferences

A disaster generates

- Sharp declines in investment, consumption, output — and maybe employment

A rise in the disaster probability generates

- Lower riskfree rate, higher risk premiums
- Lower investment, employment, output; higher consumption?
The nature of risk business cycle risk
Disasters implied by options
Disasters implied by options

![Diagram showing implied volatility for consumption and option models over different time horizons.](image-url)

- Consumption model (12 months)
- Option model (12 months)
- Option model (3 months)
- Consumption model (3 months)
Cyclical behavior of term spread
Cyclical behavior of equity returns

- S&P 500
- S&P 500 minus Short Rate
- NYSE Composite
- Nasdaq Composite

Cross-correlation with GDP:

Leads GDP
Lags GDP

Lag Relative to GDP

Backus (NYU)
Gourio, “Disaster Risk”
Challenges posed by the model
Computation

Problem: standard LQ methods independent of risk

Ditto (log)-linearization around “deterministic steady state”

Lots of alternatives

- Tallarini uses risk-sensitive control (IES = 1)
- Close relative: log-linear approximation of value function
- Fernandez-Villaverde et al., Justiano-Primiceri, Rudebusch-Swanson: perturbation
- Campanale, Castro, and Clementi: splines
- Bloom, Croce, Krueger-Kubler: discrete approximation of state space

Gourio: discrete approximation
Tallarini’s result

Tallarini found

- Change in risk or risk aversion had little impact on quantities

Extensions

- Given a log-linear approximation of the value function, any change in risk or risk aversion generates identical decision rules
- Approximately true even in some severely nonlinear environments
- Gourio: exactly true if risk changes productivity and capital proportionately and risk is constant

Risk and risk aversion do affect asset returns, esp risk premiums
Tallarini intuition: the recursive business cycle model

Bellman equation

\[ J(k_t, z_t) = \max_{c_t, n_t} V \left\{ c_t (1 - n_t)^\lambda, \mu_t [J(k_{t+1}, z_{t+1})] \right\} \]

subject to:

\[ k_{t+1} = g(i_t, k_t) = g[f(k_t, z_t n_t) - c_t, k_t] \]

plus productivity process & initial conditions

Ingredients

- \( V \): time aggregator; eg, \( V(x, y) = [(1 - \beta)x^\rho + \beta y^\rho]^{1/\rho} \)
- \( \mu \): risk preference; eg, \( \mu(x) = [Ex^\alpha]^{1/\alpha} \)
- \( f \): production function
- \( g \): law of motion (allows adjustment costs)

If ingredients hd1 \( \Rightarrow \) \( J \) is hd1
Tallarini intuition: mechanics of certainty equivalents

Example: let $\log x \sim N(\kappa_1, \kappa_2)$

Expectations and certainty equivalents for lognormals

\[
E(x) = \exp(\kappa_1 + \kappa_2/2)
\]
\[
E(x^\alpha) = \exp(\alpha \kappa_1 + \alpha^2 \kappa_2/2)
\]
\[
\mu(x) = \left[ E(x^\alpha) \right]^{1/\alpha} = \exp(\kappa_1) \exp(\alpha \kappa_2/2)
\]

Effect of risk same as discount factor $\beta$
Tallarini intuition: scaling

Bellman equation (reminder)

$$J(k_t, z_t) = \max_{c_t, n_t} V \left\{ c_t (1 - n_t)^\lambda, \mu_t [J(k_{t+1}, z_{t+1})] \right\}$$

subject to: $$k_{t+1} = g(i_t, k_t) = g[f(k_t, z_t n_t) - c_t, k_t]$$

Scaled version $$[\tilde{k}_t = k_t / z_t, \tilde{c}_t = c_t / z_t]$$

$$J(\tilde{k}_t, 1) = \max_{\tilde{c}_t, n_t} V \left\{ \tilde{c}_t (1 - n_t)^\lambda, \mu_t [(z_{t+1}/z_t)J(\tilde{k}_{t+1}, 1)] \right\}$$

subject to: $$\tilde{k}_{t+1} = g[f(\tilde{k}_t, n_t) - \tilde{c}_t, \tilde{k}_t](z_t/z_{t+1})$$

Note: proportional shock to $k$ and $z$ leaves $\tilde{k}$ unchanged
The Barro-King problem

Shocks to anything but current productivity generate opposite movements in some of: \((c, i, y, n)\)

Example: shocks to disaster probability

Resolutions

- Keep shocks small relative to productivity
- Adjustment costs (?)
- These things aren’t as highly correlated as you think
Barro-King: comovements in US data

Challenges

Backus (NYU)
Gourio revisited

Contributions

- Risk is an interesting business cycle shock
- Improves behavior of prices and quantities

Open questions

- Would stochastic volatility do the job?
- Relation to Justiano-Primiceri?
- Relation to Bloom: market price or technology?
- Where does risk come from?
- How should monetary policy respond to risk?
Related work (some of it)

Asset pricing with disasters
- Barro et al., Benzoni-Collin-Dufresne-Goldstein, Dreschler-Yaron, Gabaix et al., Longstaff-Piazzesi, Rietz, Wachter, Backus-Chernov-Martin

Cyclical behavior of asset returns
- Ang-Piaassezi-Wei, Atkeson-Kehoe, Barsky, Campbell-Cochrane, Fama-French, Gilchrist-Zakrajsek, King-Watson, Mueller, Backus-Routledge-Zin

Business cycle models with recursive preferences
- Campanale-Castro-Clementi, Croce, Fernandez-Villaverde et al., Kaltenbrunner-Lochstoer, Rudebusch-Swanson, Tallarini, Uhlig, Backus-Routledge-Zin

Other related work
- Bloom, Justiano-Primiceri
Preferences

Equations

\[ U_t = V[u_t, \mu_t(U_{t+1})] \]
\[ u_t = c_t(1 - n_t)^\lambda \]
\[ V(u_t, \mu_t) = [(1 - \beta)u_t^\rho + \beta\mu_t^\rho]^{1/\rho} \]
\[ \mu_t(U_{t+1}) = (E_t U_{t+1}^\alpha)^{1/\alpha} \]

Interpretation

\[ IES = 1/(1 - \rho) \]
\[ CRRA = 1 - \alpha \]
\[ \alpha = \rho \Rightarrow \text{additive preferences} \]
Kreps-Porteous pricing kernel

Marginal rate of substitution

$$m_{t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{\rho-1} \left( \frac{U_{t+1}}{\mu_t(U_{t+1})} \right)^{\alpha-\rho}$$

If $\alpha = \rho$

- Second term disappears
- No roles for volatility or predictable consumption growth
Linear approximation: two flavors

Problem: find decision rule $u_t = h(x_t)$ satisfying

$$E_t F(x_t, u_t, w_{t+1}) = 1, \quad w_t \sim N(0, \kappa^2)$$

Judd + many others

- Taylor series expansion of $F$
- $n$th moment shows up in $n$th-order term
- Linear approximations independent of risk

Modern finance

- Taylor series expansion of $f = \log F$ in

$$E_t \exp[f(x_t, u_t, w_{t+1})] = 1$$

- All moments show up even in linear approximation
Linear approximation: example

Linear “perturbation” method

- Linear approximation of $F$

\[
F(x_t, u_t, w_{t+1}) = F + F_x(x_t - x) + F_u(u_t - u) + F_w w_{t+1}
\]

\[
E_t F = 1 \Rightarrow u_t - u = (1 - F)/F_u - \left(F_x/F_u\right)(x_t - x)
\]

- Decision rule doesn’t depend on variance of $w$ (or higher moments)

“Affine” finance method

- Linear approximation of $f = \log F$

\[
f(x_t, u_t, w_{t+1}) = f + f_x(x_t - x) + f_u(u_t - u) + f_w w_{t+1}
\]

\[
E_t \exp(f) = 1 \Rightarrow u_t - u = -\left(f + f_w^2 \kappa_2/2\right)/f_u - \left(f_x/f_u\right)(x_t - x)
\]

- Note impact of variance $\kappa_2$ (higher moments would show up, too)
## Disasters implied by options

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<th>Cons Growth Process Based on</th>
<th>Macro Data</th>
<th>Option Prices</th>
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<td>Skewness</td>
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