

## Final Review

(December 8, 1998)

### State-Contingent Claims

1. We say uncertain cash flows are state-contingent: they depend on the state or situation.
2. In fixed income analytics, states are generally nodes in an interest rate tree. Our trees are binomial: each node has two branches.
3. We value assets starting at the end of the tree and applying, node by node,

$$\text{Price} = q_u c_u + q_d c_d + \text{Current Cash Flow}$$

$$q_u = q_d = 0.5 \times d_1 = 0.5/(1 + r/2),$$

where  $q_s$  and  $c_s$  are the (one-period) state prices and “cash flows” (or values) and  $d = 1/(1 + r/2)$  is the one-period discount factor. The second equation is the 50-50 rule, an arbitrary but common way of dividing each discount factor into components for each state.

Given these principles, we can value almost anything.

4. Parameters are generally chosen to reproduce current spot rates (Duffie’s formula!) and interest rate volatility.
5. In binomial models, even complex assets can be replicated with two other assets.
6. Models are not reality. Even the best model is only an approximation to a complex world.

### Forwards and Futures

1. A forward contract is an agreement to exchange assets at a future date at a price set now.
2. We can replicate a forward contract on a bond with a long position on a long bond and short position(s) of equal value on short bond(s) (long a long and short a short). Interest sensitivity comes almost entirely from the long position.
3. Futures differ from forwards primarily in the daily “mark-to-market.”
4. The most popular fixed income futures are those on

- long bonds, which offer exposure to long rates (to modify duration, for example);
- eurocurrency rates, which offer long-run exposure to future short rates.

Both kinds are available in all major currencies.

5. Forward and futures contracts are useful tools for interest rate risk management. They offer exposure to specific segments of the forward rate curve.

## Options

1. Options are “nonlinear.”
2. Black-Scholes requires modification in fixed income settings, with the result that other methods (like our trees) are often used instead.
3. Common option-like fixed income instruments include interest rate caps and floors, instruments like callable bonds with “embedded options,” and options on futures (long bonds, eurocurrency rates).
4. Callable bonds are equivalent to a long position in a noncallable bond and a short position (remember: the issuer gets to call) in a call option on the same bond.
5. American options can be exercised prior to expiration. They must be valued recursively, which means we need to decide at each node whether it’s better to call or hold until next period.
6. Nonlinearity shows up several ways: the cash flow is nonlinear in the underlying, the price-yield relation is nonlinear (convexity), and the replicating portfolio changes as we move through the tree.

## Credit Risk on Corporate Debt

1. Corporate debt trades at higher yields than treasuries, the effect of nonzero default rates or credit risk.
2. The difference between corporate and treasury yields is sometimes referred to as the credit spread.
3. Ratings are professional assessments of credit risk.
4. Risk on corporate bonds includes sensitivity both to treasury yields and to the spread between treasuries and corporates, which is highly variable.
5. Credit derivatives and structured products are increasingly popular tools for managing credit risk.

## Emerging Market Debt

1. International capital flows to emerging economies come in many forms, one of which is public debt denominated in major currencies.
2. Brady bonds were issued to replace nonperforming bank loans. They started the most recent boom in this market, which has since seen issues by sovereign and private firms in international bond markets (eurobonds).
3. Credit (“sovereign”) risk plays a central role. The stripped yield and sovereign spread on Brady bonds are measures of the premium markets charge for credit risk.

## Mortgages

1. The US mortgage market is enormous.
2. Prepayments give mortgages negative convexity: at high yields they have long duration, at low yields they have short duration. In this respect, they resemble callable bonds.
3. Passthroughs have helped to develop a secondary market in mortgages, a leading example of securitization.
4. Collateralized mortgage obligations (CMOs) illustrate the possibilities of carving whole mortgages into pieces desired by different kinds of investors: sequential pay tranches, planned amortization classes, interest- and principal-only strips, etc.