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# The balance of power in closely held corporations☆

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**Abstract**

We analyze a closely held corporation characterized by the absence of a resale market for its shares. We show that the founder of the firm can optimally choose an ownership structure with several large shareholders to force them to form coalitions to obtain control. By grouping member cash flows, a coalition internalizes to a larger extent the consequences of its actions and hence takes more efficient actions than would any of its individual members. The model has implications for the optimal bundling of cash flow and voting rights, and for the optimal number and size of shareholders. © 2000 Elsevier Science S.A. All rights reserved.

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## 1. Introduction

The vast majority of firms in developing and transitional economies do not have their shares traded in an exchange. Even in the U.S., out of almost 4 million corporations that filed taxes in 1993 (Statistics of Income), a mere 7,842 corporations were listed in the NYSE, Nasdaq and Amex combined (1994 Nasdaq Fact Book).

Closely held corporations typically have an ownership structure comprised of several significant shareholders.[1] However, the corporate finance literature has focused on firms with either a dispersed ownership structure (Berle and Means, 1932; Grossman and Hart, 1980) or a single controlling shareholder (Jensen and Meckling, 1976; Shleifer and Vishny, 1986; Burkart et al., 1997). In the former case, shareholders are too small and disorganized to impose their will. As a result, control resides in the hands of the manager. In the latter, the dominant shareholder dictates corporate policy either by managing the firm directly or by closely monitoring the managing team. Remaining shareholders lack either the power or the incentives to oppose the controlling shareholder's decisions.

In this paper, we focus on firms in which shareholders are large enough not to surrender control to the manager even though no individual shareholder is large enough to control the firm alone. In these firms, corporate policy is the result of interaction among shareholders.

In the model we develop, an initial owner chooses an ownership structure with multiple large shareholders to prevent a single shareholder from taking unilateral actions that might hurt other shareholders. For instance, in the presence of multiple large shareholders, a decision to divert funds from the firm requires the consent of a coalition of shareholders. Since a coalition of shareholders diverts fewer funds than would any of its individual members, the initial owner can commit to low levels of fund diversion by seeking the participation of other large shareholders. In other words, by *diluting* her power, the initial owner commits not to undertake unilateral actions.

In our model, a firm's initial owner, in need of external capital, sells votes and cash flows to wealth-constrained outside investors. Once the ownership structure is determined, coalitions compete to seize control of the firm. Shareholders value control because it allows them to enjoy private benefits. We assume that these benefits come at the expense of noncontrolling shareholders and, moreover, that such benefits are inefficient to extract. The outcome of the control contest is influenced by the ownership structure, i.e., by the number of votes and the size of the cash flows of each shareholder. Therefore, when deciding the

[1] Gomes and Novaes (1999) find that 86.9% of the closely held corporations listed in the National Survey of Small Business Finances (NSSBF) with annual sales above 10 million dollars have at least one large shareholder and 57.2% have more than one large shareholder.

ownership structure, the initial owner takes into account both the need to raise funds and the influence of the ownership structure on the outcome of the control contest.

Our results about ownership are driven by two opposing effects, which we call the *alignment* and *coalition formation* effects. The alignment effect is the positive relation that exists between the cash flow stake of the controlling coalition and total firm value. The greater the cash flow possessed by the controlling coalition, the more this coalition internalizes the costs of its actions. Hence, the fewer costly private benefits it extracts. This effect is similar to that described in Jensen and Meckling (1976).

The second and novel effect arises from shareholder equilibrium behavior at the time of coalition formation. Once votes and cash flows are distributed, many different coalitions have sufficient voting power to control the firm. However, out of these coalitions, the one with the smallest cash flow stake wins the control contest because it has the largest group of shareholders (in term of cash flows) from whom to expropriate. This implies that, conditional on having sufficient voting power to control the firm, the coalition formation effect minimizes the cash flow stake of the winning coalition.

Consider the initial owner's choice between retaining absolute control and diluting control of the firm. Note that, through the price of the securities sold, the initial owner bears any inefficiency caused by diversion. If the initial owner has enough wealth to finance the firm and retain a large portion of the cash flows, say 80%, then it is optimal for her to retain absolute control. By doing so, the winning coalition, which in this case will be the initial owner herself, will have a considerable fraction of the cash flows.

However, if the initial owner's wealth allows her to retain a smaller fraction, say only 55%, then retaining control is not optimal. Consider an alternative ownership structure in which the initial owner sells $33\frac{1}{3}\%$ of the cash flows and votes to each of two investors. With this ownership structure, two shareholders will form a coalition to control the firm. With $66\frac{2}{3}\%$ of the cash flows, the controlling body will take more efficient actions than one with only 55%. Clearly, the initial owner would like the controlling coalition to include all three shareholders (the alignment effect). However, once votes and cash flows are distributed, shareholders have incentives to form a controlling coalition that expropriates the largest set of shareholders (the coalition formation effect), and consequently, one shareholder is left out.

Not all ownership structures in which control is diluted generate the same value for the firm. Indeed, by studying how control can be diluted optimally, our analysis yields a number of results about ownership structure.

First, we define a control structure as the collection of coalitions with sufficient power to control the firm. We find that for *any* possible control structure, there is a one-share–one-vote ownership structure that maximizes efficiency. The intuition behind this finding is that deviations from a one-share–one-vote structure create shareholders with a high ratio of votes to cash flows. Thus, the coalition formation effect tends to include these shareholders in the controlling coalition. As a result, this coalition ends up with a small cash flow stake, which is a bad outcome in terms of efficiency.

Following the initial contributions from Grossman and Hart (1988) and Harris and Raviv (1988), an extensive literature on the optimality of one-share–one-vote has developed. Our finding differs from this literature in that it is not derived in the context of takeovers and applies to closely held corporations.

Second, we establish the relation between efficiency and the number of shareholders. In particular, we show that the larger the *equilibrium* number of shareholders, the smaller the value of the firm. Third, we show that the best ownership structure is one with either a single large shareholder or shareholders of roughly the same size. The intuition for these two results is similar: as cash flows are distributed among more shareholders, or as they are distributed more unevenly, it will be easier to find a coalition with sufficient voting power and a relatively small cash flow stake. A finding similar to our third result is obtained by Zwiebel (1995) in a model in which the ownership structure is determined by investors who allocate their wealth across firms to receive a larger share of the private benefits.

This paper is related to a growing literature on the expropriation of minority shareholders. Shleifer and Vishny (1997) argue that, in most countries, the relevant agency problem that corporate governance should address is the expropriation of minority shareholders by controlling shareholders rather than the expropriation of all shareholders by the manager. A number of studies suggest that large shareholders function as a mechanism to mitigate such expropriation. La Porta et al. (1999) and Pagano and Roel (1998) argue that other large shareholders reduce diversion by monitoring the controlling shareholder. Gomes and Novaes (1999) focus on how ex-post bargaining problems among large shareholders protect minority shareholders by preventing large shareholders from undertaking actions that would reduce minority shareholders' payoffs. Gomes and Novaes' analysis and ours are complementary since we focus on the formation process of the controlling coalition without considering ex-post bargaining problems, while they concentrate on bargaining problems and do not consider strategic issues relating to coalition formation. However, the two analyses are similar in that they endogenize the process by which large shareholders mitigate diversion rather than starting with an exogenous monitoring technology. Our paper contributes to this literature by introducing control dilution as a mechanism to reduce diversion.

Interestingly, our result that control dilution is an effective mechanism to reduce diversion counters the suggestions in La Porta et al. (1999) and Bebchuk (1999) that argue that, in countries with poor investor protection, control should be concentrated to prevent someone seizing it without fully paying for it. However, this scenario cannot occur in our model since, once the initial owner sells the securities, there is no market for them. In addition, the initial owner is not worried about losing control since she receives full payment for the benefits of control that shareholders expect to receive.

The rest of the paper is organized as follows. Section 2 lays out the model. In Section 3, we analyze the coalition formation process. In Section 4, we take the control structure (i.e., the voting distribution) as given and show the optimality of a one-share–one-vote ownership structure. In Section 5, we completely solve for the optimal ownership structure (i.e., the number of shareholders, the control structure, and the cash flow distribution). Section 6 discusses contracts that specify the composition of the board. In the model, we do not consider these contracts even though they can be shown to improve efficiency. In this section we argue why our model is still valid and provide rough evidence that coalitions form to expropriate funds from the shareholders that are left out. Section 7 concludes. Proofs are relegated to the appendix.

## 2. The model

The timing of events is shown in Fig. 1. At date 0, an entrepreneur has the opportunity of setting up a firm at a cost $K < 1$. We assume that the entrepreneur's wealth is not sufficient to cover the entire setup cost, so she must obtain outside finance. We denote the pool of potential investors by $N$. Each investor has a finite level of wealth $W_i$, $i \in N$. We order these investors such that $W_1 \geqslant W_2 \geqslant \cdots \geqslant W_{\#N}$. We assign the number 0 to the entrepreneur and denote her initial wealth by $W_0$. At this date, the entrepreneur chooses a subset of investors to become shareholders of the firm and sells a number of votes and cash flow rights to each. We allow the entrepreneur to sell a different number of votes and cash flows to any shareholder; that is, we do not impose a one-share–one-vote ownership structure. The entrepreneur might decide to sell out or stay as a shareholder. The ownership structure is then described by a set of shareholders $I$, (with $I - \{0\} \subseteq N$), a number of votes $v_i \geqslant 0$, and a fraction of the firm's cash flow $c_i > 0$, for each shareholder $i \in I$ with $\sum_{\ell \in I} c_\ell = \sum_{\ell \in I} v_\ell = 1$. We restrict $c_i > 0$ to simplify the proofs. Allowing the entrepreneur to set $c_i = 0$

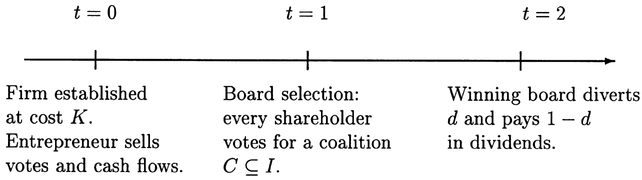


Fig. 1. Timing of events.

will not affect the results. We define $\boldsymbol{v} = \{v_i\}_{i \in I}$ and $\boldsymbol{c} = \{c_i\}_{i \in I}$ as the profile of votes and cash flow rights.

We assume that shareholders are not allowed to trade their shares. This assumption captures the closely-held nature of the corporations we are studying. These corporations do not have their shares listed in an exchange. Furthermore, they usually have contracts in place that significantly restrict the transferability of shares. Moreover, absent this restriction, only ownership structures with one significant shareholder would be stable. Thus, by restricting the transferability of shares the entrepreneur can choose from a larger set of ownership structures. However, analyzing the tradeoff between this benefit and the potential costs of the lack of transferability is beyond the scope of this paper.

At date 1, shareholders meet to elect the board of the firm. Each shareholder votes for *one* of the potential boards. We do not restrict shareholders to vote for coalitions to which they belong. In fact, we assume that a shareholder can vote for any coalition $C \subseteq I$. The winning coalition is the one that receives the most votes, with the number of votes that a given coalition receives computed by summing the votes of the shareholders that voted for it. In the case of a tied vote, the coalition supported by the shareholder with the lowest index wins. For instance, if the entrepreneur stays as a shareholder and there is a 50–50 tie, the coalition she supports is the winner. This rule might reflect the power of the entrepreneur as the founder of the firm. We apply this tie-breaking assumption merely to simplify the voting game. In practice, however, when there is a tie or a 'deadlock' a shareholder can ask a court for 'involuntary dissolution' of the corporation or a judge can impose a buyout at a judicially determined price (Hamilton, 1996).

At date 2, the board diverts $d$ of the revenue and pays out the remaining portion as dividends. We assume that funds are lost when revenue is diverted. The cost of diversion is embodied in the diversion technology $B(\cdot)$. For a level of diversion $d$, the diverted amount received by the board is only $B(d)$, which satisfies

$$B(0) = 0,\ B'(0) = 1,\ B'(1) = 0, \text{ and } B''(\cdot) < 0. \tag{1}$$

This diversion technology corresponds, in essence, to Jensen and Meckling's (1976) idea of extraction of nonpecuniary benefits. The same set of assumptions for the diversion technology can be found in Burkart et al. (1998).

We assume that the diverted amount is distributed among board members in proportion to their cash flow rights. Shareholders that do not belong to the board receive only what is left of the revenue. Designating the winning coalition as $C$ and the level of diversion chosen by this coalition as $d$, the date 2 payoff to shareholder $i$ is given by

$$\pi_i = \begin{cases} c_i(1-d) + \frac{c_i}{\sum_{\ell \in C} c_\ell} B(d) & \text{if } i \in C, \\ c_i(1-d) & \text{otherwise.} \end{cases} \tag{2}$$

We adopt this ad-hoc sharing rule to focus on the conflict between board members and minority shareholders, rather than on the conflict among board members. With this rule, members of *any* possible board unanimously agree on the level of diversion (see comments after Lemma 1 below). In addition, other reasonable sharing rules such as dividing the private benefits equally among board members or dividing them in proportion to the number of votes do not guarantee existence of equilibrium in the voting game.

Finally, we assume that the interest rate is zero and that all agents are risk neutral. The objective of the entrepreneur is to maximize the sum of the amount collected at date 0 from investors and, if she stays as a shareholder, her expected date 2 payoff. The amount investors are willing to pay and the entrepreneur's date 2 payoff depend on the identity of the winning coalition and its subsequent actions. Therefore, in designing the ownership structure, the entrepreneur must consider how this structure will affect the control contest and the level of diversion that the winning coalition will carry out. We examine these questions in the next section.

## 3. Characterization of the equilibrium

In this section, we analyze the subgame that starts at date 1. We take as given the ownership distribution, that is, the set of shareholders $I$ and the cash flow and vote profiles, $\mathbf{c}$ and $\mathbf{v}$, respectively. We are interested in two aspects of the equilibrium: the identity of the winning coalition and the amount of diversion that it carries out.

The answer to this last question is given in the following lemma.

**Lemma 1.** *Suppose that coalition $C$ is the board and that $\mathbf{c}$ is the cash flow profile. The level of diversion chosen by $C$, $d(C, \mathbf{c})$, satisfies*

$$B'(d(C, \mathbf{c})) = \sum_{\ell \in C} c_{\ell}. \tag{3}$$

*In addition, $d(C, \mathbf{c})$ is decreasing in the cash flow stake of the winning coalition.*

The optimal diversion level for shareholder $i$ is found by maximizing his payoff given in Eq. (2). Eq. (3) is the resulting first-order condition (the second-order conditions for a maximum are satisfied). Since this first-order condition is the same for every board member, they unanimously agree to divert $d(C, \mathbf{c})$. Furthermore, since $B''(\cdot) < 0$, Eq. (3) implies that diversion decreases with the cash flow stake of the winning coalition.

Lemma 1 is related to the alignment effect we discussed in the introduction. Specifically, the winning coalition more fully internalizes the consequences of its actions and hence engages in less diversion the greater its cash flows. Therefore, since diversion is inefficient, firm value increases with the cash flow stake of the board.

Using pure-strategy subgame-perfect equilibrium as our solution concept for the voting game leaves us with a large number of equilibria. In fact, when no shareholder has more than 50% of the votes, *any* coalition can be the winning board. To see this fact, take any coalition and consider the case in which all shareholders vote for that coalition. This is an equilibrium since no shareholder can change the result of the election by unilaterally deviating. To narrow this set of equilibria, we use a cooperative refinement. In the voting game at date 1, we require that no coalition of shareholders can jointly deviate and thus strictly increase the payoff to each coalition member. That is, we require the equilibrium to be a *strong equilibrium* in the sense of Aumann (1959).

Intuitively, the coalition of board members must have sufficient voting power to be elected. In particular, since this coalition expropriates the coalition of non-board members, it must be able to beat this latter coalition at the shareholders' meeting. This motivates the concept of a strong coalition, which proves useful in characterizing the set of equilibria.

*Definition* 1. A *strong coalition* relative to the voting distribution $\boldsymbol{v}$ is a coalition in which either

1. The sum of the votes of its members is strictly larger than 50% or
2. The sum of the votes is exactly 50% and the shareholder with the minimum index is a member.

A *weak coalition* relative to $\boldsymbol{v}$ is one that is not strong relative to $\boldsymbol{v}$.

In other words, a strong coalition can impose its will regardless of how the rest of the shareholders vote. In particular, when all the members of a strong coalition vote for a given board, that board is elected.

We define $S(\boldsymbol{v})$ as the collection of all strong coalitions relative to a voting distribution $\boldsymbol{v}$. That is,

$$S(\boldsymbol{v}) = \{X \subseteq I\colon X \text{ is strong relative to } \boldsymbol{v}\}.$$

The following proposition characterizes the set of equilibria.

*Proposition 1. For the subgame that starts at date* 1, *an equilibrium always exists for any set of shareholders, I, and any cash flow and vote distributions,* $\boldsymbol{c}$ *and* $\boldsymbol{v}$, *respectively. Moreover, in all equilibria, the winning coalition,* $C^*$, *is a strong coalition with the minimum cash flow of all strong coalitions. That is,*

$$C^* \in \operatorname*{argmin}_{C \in S(\boldsymbol{v})} \sum_{\ell \in C} c_\ell. \tag{4}$$

Many coalitions could satisfy Eq. (4). In what follows, we assume that any of these coalitions is equally likely to win.

The first part of Proposition 1 guarantees existence of an equilibrium for every possible ownership structure. This guarantee is necessary to have a well-defined game. The second part of Proposition 1 is intuitively appealing. The coalition of board members expropriates wealth from the coalition of non-board members. Hence, board members must have sufficient voting power to beat the coalition of non-board members, i.e., the winning board must be strong. Furthermore, the coalition with the smallest cash flow stake of all the strong coalitions wins. This coalition wins because, of all the strong coalitions, it is the one whose members benefit the most from fund diversion since they have a "bigger" set (in terms of cash flows) of non-board members from whom to expropriate. This result is what we call the coalition formation effect. By selecting the strong coalition with the smallest cash flow stake, the coalition formation effect provides a counterweight to the entrepreneur's objective of maximizing the cash flow of the winning coalition. This tension drives most of the results in our analysis.

Two intermediate results are needed to prove Proposition 1. We state them as lemmas and then sketch the proof of Proposition 1.

*Lemma 2. For a given cash flow distribution,* $\mathbf{c}$, *the following hold:*

(i) *A shareholder prefers being on the board to not being on the board.*
(ii) *For any two boards in which a shareholder can potentially participate, he prefers the one with the smaller cash flow stake and is indifferent if the cash flow of the coalitions is the same.*

Part (i) of Lemma 2 is straightforward: a shareholder prefers expropriating to being expropriated. The explanation for part (ii) is the following. Since a controlling coalition with a smaller cash flow stake diverts more funds than one with a larger cash flow stake (Lemma 1), a shareholder gets a larger fraction of the firm's revenue when he participates in the coalition with the smaller cash flow stake.

The next lemma identifies properties of strong coalitions. These properties follow directly from the definition.

*Lemma 3. For a given voting distribution,* $\mathbf{v}$, *the following hold:*

(i) *A coalition is strong if and only if its complement (the coalition formed by the members that do not belong to it) is weak.*
(ii) *Any two strong coalitions have at least one shareholder in common.*

With these two lemmas, we now explain the result of Proposition 1. First, a weak coalition cannot be supported in equilibrium as the winning board. If such a coalition won, non-board shareholders could profitably deviate by voting for the coalition formed by themselves. By Lemma 3(i), the deviating coalition is strong, and hence it would be elected as the winning board. In addition, by Lemma 2(i), each of the members of the deviating coalition would be better off because, as a result of the deviation, they would go from being expropriated to expropriating others.

Second, a strong coalition that does not minimize the sum of cash flows among the strong coalitions cannot be supported as the winning board. If such a coalition were the winning board, it would be profitable for the members of any strong coalition with a smaller cash flow stake to vote for the coalition formed by themselves. Since the deviating coalition is strong, it would win. Moreover, all the members of the deviating coalition would be better off. First, those who did not belong to the winning board at the proposed equilibrium would become board members. Hence, by Lemma 2(i), they would be better off. Second, those who did belong to the winning board at the proposed equilibrium would become board members of a board with a smaller cash flow stake. Thus, by Lemma 2(ii), they too would be better off.

Finally, there is always at least one equilibrium in which the coalition that minimizes the sum of cash flows among the strong coalitions, $C^*$, wins. Indeed, if everyone votes for $C^*$, no coalition can profitably deviates. A weak coalition does not deviate because it cannot change the results of the election. To see that a strong coalition does not deviate, suppose towards a contradiction that a strong coalition profitably deviates. Since deviating shareholders are better off, the deviation causes a new board to be elected. Note that the new board need not be the same as the deviating coalition. Since any two strong coalitions have at least one shareholders in common (Lemma 3(ii)), then one of the deviating shareholders must be a member of $C^*$. Because this shareholder is better off deviating, he must be a member of the new board (Lemma 2(i)). Furthermore, according to Lemma 2(ii), this new board must have less cash flow than $C^*$. Hence, the new board is weak (by definition, no strong coalition can have less cash flow than $C^*$) and diverts more than $C^*$ (by Proposition 1). Since the new board is weak, but the deviating coalition is strong, there must be a deviating shareholder not in the new board. Since the new board diverts more than $C^*$, this shareholder cannot be better off. Therefore, no coalition deviates from the proposed equilibrium.

Proposition 1 can be further simplified by noting that a strong coalition with at least one strong proper subset never wins. The reason is that such a coalition has more cash flow than its strong proper subset and thus it never has the minimum cash flow stake among the strong coalitions. Therefore, to find a winning coalition, we can restrict attention to strong coalitions with no strong proper subsets. We call these coalitions relevant strong coalitions.

*Definition 2.* A *relevant strong coalition* relative to a voting distribution $\boldsymbol{v}$ is a strong coalition whose proper subsets are all weak. In addition, a *relevant shareholder* is one that belongs to at least one relevant strong coalition.

According to this discussion, Eq. (4) can be rewritten as

$$C^* \in \underset{C \in R(\boldsymbol{v})}{\operatorname{argmin}} \sum_{\ell \in C} c_\ell, \tag{5}$$

where $R(\boldsymbol{v})$ is the collection of all relevant strong coalitions relative to $\boldsymbol{v}$, or

$$R(\boldsymbol{v}) = \{X \subseteq I\text{: } X \text{ is a relevant strong coalition relative to } \boldsymbol{v}\}.$$

Relevant strong coalitions are those that compete for control. These coalitions have the minimum number of shareholders necessary to amass sufficient votes to seize control. Members of a relevant strong coalition can obtain control and share the private benefits among themselves. Therefore, they have no incentive to add shareholders to their coalition.

An element of the range of $R(\cdot)$ is a collection of only those coalitions able to seize control of the firm. Note that many voting distributions generate the same collection of relevant strong coalitions. Interchanging any two such voting distributions (but keeping the distribution of cash flows fixed) does not affect the identity of the winning coalition, the amount of fund diversion, or the final payoff to each shareholder. That is, all the information about the voting distribution is summarized by the collection of relevant strong coalitions it generates. In the following discussion, we refer to each of these collections as a control structure.

*Definition 3.* A *control structure* is a collection of all the relevant strong coalitions relative to some voting distribution.

To clarify this term, we provide an example. Consider the voting distribution (0.2, 0.35, 0.45). The collection of strong coalitions relative to this voting distribution is $S((0.2, 0.35, 0.45)) = \{\{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$. However, coalition $\{1, 2, 3\}$ is not relevant strong, because some of its proper subsets are strong coalitions. Hence, the collection of relevant strong coalitions for this voting distribution is $R((0.2, 0.35, 0.45)) = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$. Similarly, starting from the voting distribution $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, we find that $R((\frac{1}{3}, \frac{1}{3}, \frac{1}{3})) = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$. Therefore, we say that the voting distributions $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and (0.2, 0.35, 0.45) generate the same control structure $\{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$.

This section has described the identity of the winning coalition and the amount of funds it diverts as a function of the ownership structure. The following sections are devoted to solving the entrepreneur's problem of choosing the set of shareholders, the control structure, and the cash flow distribution at date 0. Note that, by the previous discussion, we can think of the entrepreneur's problem at date 0 as choosing a control structure rather than a voting distribution.

The first best solution in this model is obtained when the entrepreneur has sufficient wealth to pay for the entire setup cost. In this case, she keeps 100% of the cash flows and votes, does not divert funds, and receives a payoff of 1 at date 2. Alternatively, if there is an investor with wealth greater than 1, the entrepreneur can sell the entire firm to him for an amount of 1. In all other cases, the entrepreneur has to consider ownership structures with more than one shareholder. We focus on such cases in the following sections.

We solve the entrepreneur's problem in a number of steps. Although the problem is interesting only in cases in which wealth constraints prevent the entrepreneur from obtaining the first best solution, the solution method we follow does not explicitly consider wealth constraints until the last step. In the next section, we take any control structure (with any number of relevant shareholders) as given and find the associated cash flow distribution that maximizes firm value. In Section 5, we take the number of relevant shareholders as given and find both the control structure and its associated cash flow distribution that maximize firm value. We then use the results we obtain in these two sections to solve the complete problem with wealth constraints. These preliminary results both lead to the solution and provide insights into the optimal ownership structure.

## 4. One-share–one-vote

In this section, we apply Proposition 1 to analyze the optimal bundling of cash flows and votes. The previous discussion suggests that the entrepreneur's problem can be stated as one of choosing the number of shareholders, the cash flow distribution, and the control structure. In this section, however, we do not yet consider this complete problem. Rather, we find the cash flow profile that maximizes the value of the firm given a particular control structure.

Because diverting funds is inefficient, maximizing the value of the firm is equivalent to minimizing fund diversion. Lemma 1 indicates that this is accomplished by maximizing the cash flow stake of the board. Since by Proposition 1, the winning coalition is the one with the minimum cash flow of all relevant strong coalitions, the optimal cash flow distribution, $\boldsymbol{c}^*$, for a given control structure, $\mathscr{R}$, is the solution to the following problem:

$$\max_{\boldsymbol{c}} \min_{C \in \mathscr{R}} \sum_{\ell \in C} c_{\ell}. \tag{6}$$

Given a control structure, the goal is to distribute cash flows to maximize the cash flow stake of the relevant strong coalition with the minimum cash flow.

To illustrate the main result of this section, consider the following example. Take the control structure $\mathscr{R} = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$. First, consider the cash flow distribution (0.2, 0.2, 0.6). The cash flows of the relevant strong coalitions $\{1, 2\}$, $\{1, 3\}$, and $\{2, 3\}$ are 0.4, 0.8, and 0.8, respectively. Therefore, by Proposition 1, coalition $\{1, 2\}$, which has 0.4 of the cash flow, would win. Next, consider the cash flow distribution $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. All the relevant strong coalitions have cash flows of $\frac{2}{3}$. Therefore, each has an equal probability of winning. Clearly, this cash flow distribution is better than the previous one, which yielded a winning coalition with a cash flow of only 0.4. Moreover, this cash flow distribution maximizes firm value since it solves Eq. (6) for this control structure.

In this section, we are interested in the relation between the optimal cash flow distribution and the voting distributions that generate the control structure, $\mathscr{R}$. In this example, the voting distribution $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ generates the control structure $R((\frac{1}{3}, \frac{1}{3}, \frac{1}{3})) = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$. This control structure is the one which with we started. Hence, there exists a one-share–one-vote ownership structure, $\boldsymbol{v}^* = \boldsymbol{c}^* = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, that maximizes efficiency for the given control structure $\mathscr{R}$. Note that this is not the only ownership structure that maximizes efficiency in this control structure. Nor is it the case that any one-share–one-vote structure is optimal. This example simply suggests that there is one one-share–one-vote ownership structure that maximizes efficiency.[2] Proposition 2 generalizes this example to *any* control structure.

*Proposition 2. Fix any control structure, $\mathscr{R}$, and let $\boldsymbol{c}^*$ be the cash flow distribution that maximizes firm value for this control structure. The voting distribution, $\boldsymbol{v}^* = \boldsymbol{c}^*$, generates $\mathscr{R}$, i.e., $R(\boldsymbol{v}^*) = \mathscr{R}$.*

This proposition states that, for any control structure, there is a one-share–one-vote ownership structure, $\boldsymbol{v}^* = \boldsymbol{c}^*$, such that $\boldsymbol{c}^*$ is the optimal cash flow distribution for the given control structure, and $\boldsymbol{v}^*$ generates the given control structure.

We show in the appendix that the given control structure, $\mathscr{R}$, and the control structure, $R(\boldsymbol{v}^*)$, are the same. Here, however, we focus on why the coalitions in

[2] The example, though simple, might appear coincidental because, with three shareholders, there are is a limited number of control structures. We provide two additional examples. First, starting with the control structure $\mathscr{R}' = \{\{2, 3, 4, 5\}, \{1, 5\}, \{1, 4\}, \{1, 3\}, \{1, 2\}\}$, we obtain (3/7, 1/7, 1/7, 1/7, 1/7) as the optimal cash flow distribution. It can easily be checked that the voting distribution (3/7, 1/7, 1/7, 1/7, 1/7) in turn generates the control structure $\mathscr{R}'$. Another example is the control structure $\mathscr{R}'' = \{\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\},\{1, 5\}, \{2, 3, 5\}, \{2, 4, 5\}, \{3, 4, 5\}\}$ with its corresponding optimal cash flow distribution (2/7, 1/7, 1/7, 1/7, 2/7), which again generates $\mathscr{R}''$.

$\mathcal{R}$ are relevant strong relative to $\boldsymbol{v}^*$. This result follows from the tension between the alignment effect, which requires that the winning coalition have the largest possible cash flow stake, and the coalition formation effect, which selects the relevant strong coalition with the smallest cash flow stake.

First, notice that *any* one-share–one-vote ownership structure guarantees a winning coalition with more than 50% of the cash flows. The reason is that the winning coalition must be one of the relevant strong coalitions that, by definition, have more than 50% of the votes. Thus, with a one-share–one-vote ownership structure, the winning coalition must have more than 50% of the cash flows as well. Therefore, the value for the problem in Eq. (6) must be more than 50% because the entrepreneur can always choose a cash flow distribution equal to one of the voting distributions that generate $\mathcal{R}$. Moreover, since the winning coalition minimizes the cash flow of all relevant strong coalitions, then all relevant strong coalitions have more cash flows than the winning coalition, that is, more than 50% of the cash flow. Therefore, for the voting distribution $\boldsymbol{v}^* = \boldsymbol{c}^*$, all coalitions in $\mathcal{R}$ have more than 50% of the votes, i.e., all coalitions in $\mathcal{R}$ are strong relative to $\boldsymbol{v}^*$.

Second, we show that the coalitions in $\mathcal{R}$ are relevant strong relative to $\boldsymbol{v}^*$. We prove that, for the optimal cash flow distribution, when any shareholder is removed from any of the coalitions in $\mathcal{R}$ the sum of cash flows drops below 50%. This result implies that, for $\boldsymbol{v}^* = \boldsymbol{c}^*$, all the coalitions in $\mathcal{R}$ have weak proper subsets and hence are relevant strong. To establish this result, we need the following lemma.

*Lemma 4. Let shareholder i be any member of coalition $C \in \mathcal{R}$. There exists a coalition $D \in \mathcal{R}$ such that $C \cap D = \{i\}$.*

This lemma indicates that, for every coalition in $\mathcal{R}$, there is another coalition in $\mathcal{R}$ such that these two are "almost" disjoint. This result follows from the fact that the relevant strong coalitions have the minimum number of shareholders that ensures sufficient voting power.

Take any coalition $C \in \mathcal{R}$, and any shareholder $i \in C$. Also, take the coalition $D \in \mathcal{R}$ that satisfies the above lemma. Coalition $C - \{i\}$ and $D$ are disjoint. Hence the sum of their cash flows must be less than 100%. Since $D$ is relevant strong, it has more than 50% of the cash flows at the solution. Consequently, $C - \{i\}$ must have less than 50% of the cash flows. Since $\boldsymbol{v}^* = \boldsymbol{c}^*$, this implies that $C - \{i\}$ is weak relative to $\boldsymbol{v}^*$. Since this is true for any $i \in C$, it follows that every proper subset of $C$ is weak.

The intuition for this part is that the desire to get as large a fraction of the private benefits creates relevant strong coalitions that are almost disjoint (Lemma 4). This sets an upper bound to the cash flow that any relevant strong coalition can have at the solution. If one relevant strong coalition is allocated a large fraction of the cash flow, there will be an almost disjoint relevant strong coalition with a small fraction. But this allocation cannot be the solution, because the coalition with a small cash flow stake would be the one elected. This upper bound implies that when one shareholder in any of the relevant strong coalitions is removed, the cash flow stake drops below 50%.

As mentioned, Proposition 2 differs from the previous literature on the optimal bundling of cash flow and votes in that our result is not derived in the context of takeovers. Nevertheless, the intuition is similar. Equilibrium behavior at the time of coalition formation brings stakes together in a similar way that share buying by the parties competing for control did in the previous literature.

## 5. Ownership structure: the number and size of shareholders

In the last section, we discussed the properties of the best cash flow distribution for a *given* control structure. In this section, we investigate the entrepreneur's complete problem of choosing, at date 0, the set of shareholders, the control structure, and the cash flow distribution.

In principle, to solve the entrepreneur's complete problem, we need to consider every possible number of shareholders and, for a given number, all possible control structures. For each of these cases, we need to solve a problem similar to that in Eq. (6) but adding the investor's wealth constraints. To simplify this procedure, we state Proposition 3. Note that in this proposition, we do not consider wealth constraints yet. Rather we fix the number of relevant shareholders and find the control structure and cash flow distribution that maximize the value of the firm.

*Proposition 3. Suppose the number of relevant shareholders is $I$. In the ownership structure that maximizes the value of the firm, there are no nonrelevant shareholders, the $I$ relevant shareholders receive $(\mathbf{c}, \mathbf{v})$, and the cash flow stake the winning coalition is $\lambda$, where*

- *If $I = 1$, $\mathbf{c} = \mathbf{v} = 1$ and $\lambda = 1$.*
- *If $I \geqslant 3$ is odd, $\mathbf{c} = \mathbf{v} = (\frac{1}{I}, \frac{1}{I}, \ldots, \frac{1}{I})$ and $\lambda = \frac{1}{2} + \frac{1}{2I}$.*
- *If $I \geqslant 3$ is even, $\mathbf{c} = \mathbf{v} = (\frac{1}{I+1}, \frac{1}{I+1}, \ldots, \frac{2}{I+1}, \ldots, \frac{1}{I+1})$ and $\lambda = \frac{1}{2} + \frac{1}{2(I+1)}$, where the shareholder that has $2\frac{1}{I+1}$ of the votes and cash flows can be anyone.*

Note that, according to the definition of a strong coalition, $I = 2$ is not a possibility. Also, to lighten notation, rather than providing the optimal control structure, we provide one of the voting distributions generating such a control structure.

Small shareholders decrease efficiency because they tend to be included in the winning coalition but contribute little cash flow to it. To maximize the size of the smallest shareholder, cash flow is spread evenly. The difference between the odd and even cases is due to the tie-breaking rule. However, the result and the intuition for both cases are similar.

Some comments relating to Proposition 3 are in order. First, as the number of relevant shareholders increases, efficiency is reduced. As cash flows and votes are distributed among more people, it is easier for a coalition with just enough votes and cash flows to form and be elected. Indeed, this may be one explanation for the fact that closely held corporations tend to have just a few shareholders.

Second, a firm's ownership structure contain either one large shareholder or many shareholders of roughly equal size. Similar results can be found in both Zwiebel (1995) and Burkart et al. (1997).

Third, by Proposition 2, we know that, for a given control structure, there is a one-share–one-vote ownership structure that maximizes efficiency. However, it is not the case that *any* one-share–one-vote ownership structure is optimal. Nevertheless, the efficiency loss of using any one-share–one-vote structure instead of the optimal structure goes to zero as the number of relevant shareholders increases. Recall that any one-share–one-vote ownership distribution produces a winning coalition with more than 50% of the cash flow. But by Proposition 3, the cash flow of the winning coalition goes to 50% as the number of relevant shareholders increases. This result parallels those found for the optimality of a one-share–one-vote structure when ownership is dispersed. However, we are interested in cases with a small number of shareholders. In these cases, the efficiency loss of using any one-share–one-vote ownership structure could be significant.

Finally, with Proposition 3, the entrepreneur's complete problem is now easily solved. Proposition 3 takes the number of relevant shareholders as given and assumes that shareholders have enough wealth to buy the fractions of cash flow and votes assigned to them. However, in the complete problem the entrepreneur must consider wealth constraints. Nevertheless the value of $\lambda$ from Proposition 3 is the maximum the entrepreneur can get when wealth constraints are introduced. This is because the introduction of constraints cannot increase the value of a maximization problem. We use this result below.

To simplify notation, we solve the entrepreneur's problem for a particular diversion technology $B(d) = d - \frac{1}{2}d^2$. Qualitatively, the results are similar for any $B(\cdot)$ satisfying the assumptions in (1). Furthermore, we assume that the entrepreneur does not retain any cash flow or votes. We focus on the case in which the investment required $K \leqslant \frac{17}{18}$. The case in which $K > \frac{17}{18}$ is less interesting, since the value created by the firm is not high enough to support more than one significant shareholder.

In this example, the entrepreneur's choice of ownership structure is given by:

1. If $W_1 \geqslant \frac{13}{18}$, the best ownership structure is one in which shareholder 1 has complete control (i.e., more than 50% of the votes) and as large a cash flow

stake as he can afford. The rest of the cash flow and votes are sold to other investors; however, neither the number of investors nor their size matters.
2. If $W_1, W_2$, and $W_3$ belong to the interval $[\frac{17}{54}, \frac{13}{18})$, the optimal ownership structure is one in which each shareholder receives $\frac{1}{3}$ of both votes and cash flow.

This mapping from $(W_1, \ldots, W_N)$ to the optimal ownership structure could be extended; however, the two cases above are sufficient to illustrate the main points.

If the wealthiest investor has wealth $W_1 \geqslant 1$, then the first best solution is obtained by selling the entire firm to this investor. However, if first best solution cannot be achieved, the entrepreneur should not necessarily sell the firm to more than one relevant shareholder. Actually, by Proposition 3, we know that the best the entrepreneur can do with three (or more) relevant shareholders is to have a winning coalition with two-thirds of the cash flows. Hence, as long as there is one investor with enough wealth to buy more than two-thirds of the cash flows, the ownership structure that gives this investor absolute control is optimal.

To calculate the necessary wealth level to buy absolute control and two-thirds of the cash flows, notice that the diversion technology implies an equilibrium diversion level of $d^* = 1 - c$, where $c$ is the cash flow of the winning coalition. The dominant shareholder elects himself to the board and receives the entire private benefit. Hence, the wealth needed is given by

$$c(1 - d^*) + d^* - \frac{d^{*^2}}{2} = \frac{2}{3}\left(1 - \left(1 - \frac{2}{3}\right)\right) + \left(1 - \frac{2}{3}\right) - \frac{(1 - \frac{2}{3})^2}{2} = \frac{13}{18}.$$

When none of the potential shareholders has sufficient wealth to buy at least two-thirds of the cash flow, then an ownership structure with one relevant shareholder is not optimal. By Proposition 3, we know that the more relevant shareholders there are, the larger the efficiency loss. Hence, the natural step for the entrepreneur is to consider ownership structures with three relevant shareholders. Also, by Proposition 3, we know that the best ownership structure with three relevant shareholders is one in which both votes and cash flows are distributed equally among shareholders, that is $\boldsymbol{v} = \boldsymbol{c} = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. With this ownership structure, any coalition of two shareholders can become the winning board. Hence, a shareholder is in the board with probability $\frac{2}{3}$, and when in the board, receives half the private benefits. All potential winning boards have cash flows of $\frac{2}{3}$, implying that $d^* = 1 - \frac{2}{3} = \frac{1}{3}$. At date 0, each of the future shareholders pays for the cash flow and private benefits he expects to receive:

$$\frac{1}{3}(1 - d^*) + \frac{2}{3}\,\frac{1}{2}\left(d^* - \frac{d^{*^2}}{2}\right) = \frac{17}{54}.$$

We make a few comments regarding this example. First, when $W_1$ is in the interval $[\frac{13}{18}, 1)$, the optimal ownership structure has one relevant and fairly large shareholder and (potentially many) nonrelevant minority shareholders. Note that this ownership structure is not one of those prescribed in Proposition 3, because wealth constraints force the entrepreneur to deviate from an ownership with 100% of both votes and cash flows. That is, when wealth constraints are introduced, the resulting ownership structures are not necessarily those prescribed in Proposition 3. Nevertheless, this proposition specifies upper bounds that help reaching a solution.

Second, the entrepreneur chooses an ownership structure with three relevant shareholders, even when having one relevant shareholder and many nonrelevant shareholders is feasible.[3] That is, our argument is *not* that the entrepreneur should add more relevant shareholders when wealth constraints prevent him from having only one relevant shareholder. Instead, the entrepreneur switches to three shareholders for efficiency reasons. By distributing power among three shareholders so that no individual shareholder has absolute power, the entrepreneur forces shareholders to form a coalition to obtain control.

Finally, an ownership structure with equal-sized investors is not a consequence of equal wealth. Rather, it is the best way to avoid having small shareholders who contribute little cash flow to the winning coalition.

## 6. Discussion

In our model, the winning coalition is elected by a specific voting game and, once elected, can take any action. We have not considered mechanisms that alter the voting game or restrict the actions that the board can take. For instance, the corporate charter (or a shareholder agreement) may provide minority shareholders with veto rights on certain important policy issues, it may state that the board must be elected using a cumulative voting rule guaranteeing a place on the board for minority owners, or it may explicitly include the composition of the board.

Such initiatives improve efficiency in our model. Indeed, the first best solution can be obtained by contractually requiring all shareholders to have a place on the board. In such circumstances, our analysis remains valid for at least two reasons.

First, if a contract gives every shareholder a seat on the board or veto power over all decisions, a coalition of shareholders would have incentives to invalidate such a contract. One way to invalidate this contract is to argue in court that it alters the statutory scheme (the system specified by law according to which corporations should be governed). Although U.S. courts now uphold a variety

[3] It can be shown that this is the case when $W_1$ is in the interval $[2 - K - \sqrt{2 - 2K}, \frac{13}{18})$ (this interval is not empty because $K < \frac{17}{18}$), and $W_1$, $W_2$, and $W_3$ belong to $[\frac{17}{54}, \frac{13}{18})$.

of contracts that alter the governance system in closely held corporations, this has not always been the case. In fact, as Hamilton (1996) puts it: "Cases such as these, whether or not correctly decided, illustrate the pervasive impact of the statutory scheme on judicial thought and the *danger of assuming that simply because all parties in interest agree to a variation in the statutory scheme, the variation is valid*" (p. 195, emphasis added). In our model, even a small probability that the agreement among shareholders will be invalidated and that the board will be determined by an election is enough to render our analysis valid.

Second, even if it is feasible to give all shareholders a seat on the board, this alternative can be costly for reasons we do not model in this paper. In general, there is a trade-off between managerial flexibility and the degree of protection afforded to minority shareholders. If the corporate charter (or shareholder agreement) does not protect minority shareholders, they are vulnerable to inefficient exploitation. On the other hand, too strong a protection, may significantly restrict the controlling coalition's ability to govern the firm, and may also raise the probability of costly deadlocks among owners. Indeed, this trade-off is discussed in the legal literature on closely held corporations. For instance, Easterbrook and Fischel (1991) notice that: "Drafters of the organizing documents of a closely held corporation cannot avoid a trade-off. On the one hand, they must provide some protection to minority investors to ensure that they receive an adequate return on the minority shareholder's investment if the venture succeeds. On the other hand, they cannot give the minority too many rights, for the minority might exercise their rights in opportunistic fashion to divert returns." (p. 238)

Finally, court cases and discussions in corporate law books indicate that the situation in which only a subset of shareholders obtain control and expropriate those who are left out is common.

An example of a court case discussing coalition formation and expropriation is the Levy versus Markal Sales Corporation, 1994 case. Markal Sales Corporation was founded in 1960 by Levy. Some years later, the corporation's ownership structure consisted of three shareholders: Levy with 40% of the stock, Gust with another 40%, and Bakal with the remaining 20% (there were no deviations from one-share–one-vote). In 1980, Gust and Bakal voted to fire Levy as an employer of the corporation and effectively excluded him from management. However, Levy kept his shares. Soon after, Markal was offered the opportunity to be a sales representative for Apple Computers. However, instead of offering the opportunity to Markal Sales Corporation, Gust and Bakal formed their own corporation, G/B Marketing, to represent Apple. Clearly, by doing so, Gust and Bakal diverted resources from Markal by taking a project that belonged to the corporation. After establishing G/B Marketing, Gust and Bakal diverted additional resources from Markal: several G/B salespeople were on Markal payroll, expenses for business trips related to G/B were charged to Markal, and G/B rented office space from Markal at a price significantly below market value.

On a more general level, O'Neal (1987), in his discussion of shareholder disputes in closely held corporations, observes "The most frequently occurring conflict of interest is between active shareholders, i.e. shareholder-officers or employees, and shareholders who are not active in the business" (p. 122). Furthermore, he explicitly acknowledges the role of a power contest and the difficulties in enforcing contracts among shareholders:

> Holders of a majority of the voting shares in a corporation, through their ability to elect and control a majority of the directors and to determine the outcome of shareholders' votes on other matters, have tremendous power to benefit themselves at the expense of minority shareholders.... Traditionally, American courts have been reluctant to interfere in the internal affairs of corporations, even when minority shareholders claim they are being squeezed out or otherwise oppressed... Furthermore, many courts apparently feel that there is a legitimate sphere in which controlling shareholders can act in their own interest even if the minority suffers (p. 125)

## 7. Conclusions

In this paper we analyze a model of a closely held corporation with non-transferable shares and potentially more than one significant shareholder. In our model the ownership structure determines which group of owners seizes control over the firm. The model shows that it may be in the initial owner's interest to dilute her own power by distributing votes among several large shareholders. This dilution of power commits the initial owner to form a coalition to obtain control, and thus creates a controlling body that has more cash flow, and that diverts less. In other words, we propose dilution of power as a mechanism to commit to low levels of diversion. In addition, when we consider the optimal way to dilute control, we obtain several implications for the ownership structure of the firm.

The model suggest interesting topics about closely held corporations for future research. As discussed in the last section, among these topics is the design of the optimal corporate governance mechanism in the presence of many active shareholders.

The analysis of this paper can also be extended to study the decision by firms to go public. It can be shown that, in this setting, letting the shareholders freely trade shares will render an ownership structure with many significant owners unstable. Therefore, firms that go public forgo a mechanism to mitigate diversion. This is a cost that firms must weigh against the potential benefits of going public.

## Appendix

*Proof of Lemma 1.* In the text.

*Proof of Lemma 2.* The payoff to any shareholder $i$ when coalition $C$ wins is given by

$$\pi_i(C) = \begin{cases} c_i(1-d(C)) + \frac{c_i}{\sum_{\ell \in C} c_\ell} B(d(C)) & \text{if } i \in C, \text{ and} \\ c_i(1-d(C)) & \text{otherwise,} \end{cases} \quad \text{(A.1)}$$

where $d(C)$ is given in Lemma 1. We are suppressing the dependence of $\pi_i$ and $d$ on $\boldsymbol{c}$ to lighten notation.

Part (i): Take any shareholder $i$ and any coalitions $C$ and $D$ such that $i \in C$ and $i \notin D$,

$$\pi_i(C) \geqslant c_i(1-d(D)) + \frac{c_i}{\sum_{\ell \in C} c_\ell} B(d(D)) > c_i(1-d(D)) = \pi_i(D), \quad \text{(A.2)}$$

where the first inequality follows because $d(C)$ maximizes the payoff to every shareholder $i \in C$ when the board is $C$. The second inequality is strict because $c_i > 0$ means that $\sum_{\ell \in D} c_\ell < 1$ and, therefore, $B(d(D)) > 0$.

Part (ii): Take any shareholder $i$ and any coalitions $C$ and $D$ that satisfy $\sum_{\ell \in D} c_\ell > \sum_{\ell \in C} c_\ell$ and $i \in C \cap D$,

$$\begin{aligned} \pi_i(C) &\geqslant c_i(1-d(D)) + \frac{c_i}{\sum_{\ell \in C} c_\ell} B(d(D)) \\ &> c_i(1-d(D)) + \frac{c_i}{\sum_{\ell \in D} c_\ell} B(d(D)) = \pi_i(D), \end{aligned} \quad \text{(A.3)}$$

where the first inequality follows because $d(C)$ maximizes the payoff to every shareholder $i \in C$ when the board is $C$, and the second inequality follows from $\sum_{\ell \in D} c_\ell > \sum_{\ell \in C} c_\ell$ and $c_i > 0$.

If $\sum_{\ell \in D} c_\ell = \sum_{\ell \in C} c_\ell$ and $i \in C \cap D$, then $d(D) = d(C)$ (by Lemma 1) and therefore $\pi_i(D) = \pi_i(C)$.

*Proof of Lemma 3.* Part (i): Take any $\boldsymbol{v}$ and any $C \in S(\boldsymbol{v})$. Then either (a) $\sum_{\ell \in C} v_\ell > \frac{1}{2}$, or (b) $\sum_{\ell \in C} v_\ell = \frac{1}{2}$ with tie-breaker belonging to $C$. In case (a), $C^c \notin S(\boldsymbol{v})$ because $\sum_{\ell \in C^c} v_\ell < \frac{1}{2}$. In case (b), $C^c \notin S(\boldsymbol{v})$ because $\sum_{\ell \in C^c} v_\ell = \frac{1}{2}$ and the tie-breaker is not a member of $C^c$ (he belongs to $C$). The proof of the other direction is similar.

Part (ii): Take any $\boldsymbol{v}$, any $C \in S(\boldsymbol{v})$, and any $D \in S(\boldsymbol{v})$. Suppose (towards a contradiction) that $C \cap D = \emptyset$. This implies that $D \subseteq C^c$. By Part (i), $C^c \notin S(\boldsymbol{v})$ and, therefore, $D \notin S(\boldsymbol{v})$ (subsets of weak coalitions must be weak). Contradiction.

*Proof of Proposition 1.* Take any $I$, $\boldsymbol{c}$, and $\boldsymbol{v}$. We divide the proof into three steps. Step 1 proves existence and describes one possible winning coalition. Steps 2 and 3 rule out all other coalitions.

*Step* 1: Existence.

The proof is by construction. Let $C^*$ be as described in Eq. (4). Suppose *all* shareholders vote for coalition $C^*$ at date 1. We show that this is an equilibrium.

Suppose that coalition $D \notin S(\boldsymbol{v})$ deviates. Since coalition $D^c$ does not deviate, it keeps voting for $C^*$. Since $D^c \in S(\boldsymbol{v})$ (by Lemma 3(i)), the result of the election is unchanged. Therefore, this is not a possible deviation since the members of the deviating coalition are indifferent.

Now suppose that $D \in S(\boldsymbol{v})$ deviates and that, as a result of the deviation, coalition $E$ is elected as the board. Let $i \in C^* \cap D$ (by Lemma 3(ii), $i$ exists).

First, if $\pi_i(E) \leqslant \pi_i(C^*)$, coalition $D$ does not deviate because $i$ is not better off. Second, suppose $\pi_i(E) > \pi_i(C^*)$. Since $i \in C^*$, this inequality implies that $i \in E$ (by Lemma 2(i)). And, since $i \in E \cap C^*$, the same inequality implies that $\sum_{\ell \in E} c_\ell < \sum_{\ell \in C^*} c_\ell$ (by Lemma 2(ii)). Note that $E \notin S(\boldsymbol{v})$ (because no strong coalition has strictly less cash flow than $C^*$). Take $k \in D$ and $k \notin E$ ($k$ exists, because otherwise a strong coalition ($D$) would be a subset of a weak coalition ($E$), which is impossible). We consider two cases: $k \in C^*$ and $k \notin C^*$. In the first case, since $k \notin E$, $\pi_k(E) < \pi_k(C^*)$ (by Lemma 2(i)). In the second case, $\pi_k(E) = c_k(1 - d(E)) < c_k(1 - d(C^*)) = \pi_k(C^*)$. Here the inequality follows because $\sum_{\ell \in E} c_\ell < \sum_{\ell \in C^*} c_\ell$ means that $d(E) > d(C^*)$ (by Lemma 1). Therefore, since $k \in D$ is worse off as the result of the deviation, coalition $D$ does not deviate.

*Step* 2: Weak coalitions are never supported as boards in equilibrium.

Suppose that $C \notin S(\boldsymbol{v})$ is the winning coalition. Consider the following deviation: all the members of $C^c$ vote for $C^c$ at date 1.[4] Since $C^c \in S(\boldsymbol{v})$ (by Lemma 3(i)), $C^c$ is elected as a result of the deviation. But for any $i \in C^c$, $\pi_i(C^c) > \pi_i(C)$ (by Lemma 2(i)). Thus, coalition $C^c$ deviates.

*Step* 3: Strong boards with more cash flow than the minimum among all strong coalitions are never supported in equilibrium.

Suppose that $C \in S(\boldsymbol{v})$ is the winning coalition and that there is $D \in S(\boldsymbol{v})$ such that $\sum_{\ell \in D} c_\ell < \sum_{\ell \in C} c_\ell$. Consider the following deviation: all the members of $D$ vote for $D$ at date 1. Since $D \in S(\boldsymbol{v})$, coalition $D$ is elected as the result of the deviation. Now, take any $i \in D$. If $i \notin C$, then $\pi_i(D) > \pi_i(C)$ (by Lemma 2(i)). And, if $i \in C$, then $\pi_i(D) > \pi_i(C)$ (by Lemma 2(ii)). Thus, every member of $D$ is better off and, as a result, coalition $D$ deviates.

[4] It is possible that, at the proposed equilibrium, some of the members of $C^c$ are already voting for coalition $C^c$ so, in fact, they do not deviate. However, it is impossible that all the members of $C^c$ are voting for $C^c$ at the proposed equilibrium, because if this were the case, the winning coalition would be $C^c$ and not $C$. To simplify exposition, we say that the "members of $C^c$ deviate", but it should be understood that only those who are not already voting for $C^c$ deviate.

*Proof of Lemma 4.* Take any control structure $\mathscr{R}$, any $C\in\mathscr{R}$ and any $i\in C$. Also take $\boldsymbol{v}$ such that $R(\boldsymbol{v})=\mathscr{R}$. Since $C\in R(\boldsymbol{v})$, then $C-\{i\}\notin S(\boldsymbol{v})$ (recall that relevant strong coalitions are strong coalitions with weak proper subsets). Thus, $(C-\{i\})^c=C^c\cup\{i\}\in S(\boldsymbol{v})$. Therefore, there must be $D\subseteq C^c\cup\{i\}$ such that $D\in R(\boldsymbol{v})$.[5] By construction, $D-\{i\}\subseteq C^c$ and, therefore, $C\cap(D-\{i\})=\emptyset$. Now, since $C\in S(\boldsymbol{v})$ and $D\in S(\boldsymbol{v})$ then $C\cap D\neq\emptyset$ (by Lemma 3(ii)). Finally, $C\cap(D-\{i\})=\emptyset$ and $C\cap D\neq\emptyset$ imply that $C\cap D=\{i\}$.

*Proof of Proposition 2.* Take a control structure $\mathscr{R}$. Let $\boldsymbol{c}^*(\mathscr{R})$ be the solution to the maximization problem in Eq. (6), which we rewrite here

$$\max_{c}\min_{C\in\mathscr{R}}\sum_{\ell\in C}c_\ell, \tag{6}$$

We denote the value of this maximization problem by $\lambda(\mathscr{R})$. For most of this appendix, we will suppress the dependence of $\boldsymbol{c}^*$ and $\lambda$ on $\mathscr{R}$. The objective of the proof is to show that for $\boldsymbol{v}^*=\boldsymbol{c}^*$, $R(\boldsymbol{v}^*)=\mathscr{R}$.

*Step* 1: For any coalition $C\in\mathscr{R}$, $\sum_{\ell\in C}c^*_\ell>\frac{1}{2}$.

Note that, at the solution to (6), all relevant strong coalitions have cash flows greater than or equal to $\lambda$. Thus, to prove this step, it is enough to show that $\lambda>\frac{1}{2}$. Take any voting distribution $\bar{\boldsymbol{v}}$ such that $R(\bar{\boldsymbol{v}})=\mathscr{R}$. We must have one of two cases: (a) all $C\in R(\bar{\boldsymbol{v}})$ are such that $\sum_{\ell\in C}\bar{v}_\ell>\frac{1}{2}$ or (b) there is at least one $C\in R(\bar{\boldsymbol{v}})$ such that $\sum_{\ell\in C}\bar{v}_\ell=\frac{1}{2}$ with the tie-breaker belonging to $C$.

In case (a), consider the cash flow distribution $\bar{\boldsymbol{c}}=\bar{\boldsymbol{v}}$. In this case, for any $C\in R(\bar{\boldsymbol{v}})$, $\sum_{\ell\in C}\bar{c}_\ell=\sum_{\ell\in C}\bar{v}_\ell>\frac{1}{2}$. Therefore,

$$\lambda\geqslant\min_{C\in\mathscr{R}}\sum_{\ell\in C}\bar{c}_\ell>\frac{1}{2}. \tag{A.4}$$

The first inequality follows because $\bar{\boldsymbol{c}}$ is not necessarily the solution to (6).

In case (b), it is easy to check that $\bar{\boldsymbol{v}}$ can be perturbed to obtain $\boldsymbol{v}^p$ such that $R(\boldsymbol{v}^p)=\mathscr{R}$ and that all $C\in R(\boldsymbol{v}^p)$ are such that $\sum_{\ell\in C}v^p_\ell>\frac{1}{2}$.[6] Hence, we can apply the argument of the previous paragraph to $\boldsymbol{v}^p$.

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[5] We have used the following fact: for any $\boldsymbol{v}$, $F\in S(\boldsymbol{v})\Rightarrow\exists G\subseteq F$, with $G\in R(\boldsymbol{v})$. We prove this result by induction on the number of elements of $F$. Suppose $F\in S(\boldsymbol{v})$ and $\#F=1$, then $F\in R(\boldsymbol{v})$ because the only proper subset of $F$ is $\emptyset$, which is weak. Suppose that the statement holds for $\#F=n$ and consider the case in which $F\in S(\boldsymbol{v})$ and $\#F=n+1$. If $\forall j\in F, F-\{j\}\notin S(\boldsymbol{v})$, then clearly all proper subsets of $F$ are weak. Therefore, $F\in R(\boldsymbol{v})$. If, instead, $\exists\bar{j}$ st $F-\{\bar{j}\}\in S(\boldsymbol{v})$ then, since $\#(F-\{\bar{j}\})=n$, there is $G\subseteq F-\{\bar{j}\}$ s.t. $G\in R(\boldsymbol{v})$. But $G\subseteq F-\{\bar{j}\}\subset F$.

[6] Suppose that $tb$ is the index of the tie-breaker. Consider the perturbed voting vector $v^p_{tb}=\bar{v}_{tb}+\varepsilon$ and $v^p_\ell=\bar{v}_\ell+\varepsilon/(I-1)$ for $\ell\in I-\{tb\}$. It can be shown that, for a sufficiently small $\varepsilon$, the desired result holds.

*Step* 2: For any coalition $C \in \mathscr{R}$ and any proper subset $D \subset C$, $\sum_{\ell \in D} c^*_\ell < \frac{1}{2}$.

Suppose not, i.e., that there is $E \in \mathscr{R}$ and $F \subset E$ such that $\sum_{\ell \in F} c^*_\ell \geqslant \frac{1}{2}$. Take $i \in E$ such that $F \subseteq E - \{i\}$ ($i$ exists because $F$ is a proper subset of $E$). Thus,

$$\sum_{\ell \in E - \{i\}} c^*_\ell \geqslant \sum_{\ell \in F} c^*_\ell \geqslant \frac{1}{2}. \tag{A.5}$$

By Lemma 4, there is $G \in \mathscr{R}$ such that $E \cap G = \{i\}$. But,

$$\begin{aligned} \sum_{\ell \in I} c^*_\ell &\geqslant \sum_{\ell \in E - \{i\}} c^*_\ell + \sum_{\ell \in G - \{i\}} c^*_\ell + c^*_i \\ &= \sum_{\ell \in E - \{i\}} c^*_\ell + \sum_{\ell \in G} c^*_\ell \\ &> 1, \end{aligned} \tag{A.6}$$

which is a contradiction. The first inequality uses the fact that, since $E \cap G = \{i\}$, then $E - \{i\}$, $G - \{i\}$, and $\{i\}$ are pairwise disjoint sets. The last inequality uses Eq. (A.5), and the fact that, since $G \in \mathscr{R}$, by step 1, $\sum_{\ell \in G} c^*_\ell > \frac{1}{2}$.

*Step* 3: The voting distribution $\boldsymbol{v}^* = \boldsymbol{c}^*$ is such that $R(\boldsymbol{v}^*) = \mathscr{R}$.

First, we show $\mathscr{R} \subseteq R(\boldsymbol{v}^*)$. Take any coalition $C \in \mathscr{R}$. By step 1, $\sum_{\ell \in C} v^*_\ell = \sum_{\ell \in C} c^*_\ell > \frac{1}{2}$. This means that $C \in S(\boldsymbol{v}^*)$. By step 2, for any $D \subset C$, $\sum_{\ell \in D} v^*_\ell = \sum_{\ell \in D} c^*_\ell < \frac{1}{2}$, or equivalently, $D \notin S(\boldsymbol{v}^*)$. Therefore, $C \in R(\boldsymbol{v}^*)$.

Second, we show that $R(\boldsymbol{v}^*) \subseteq \mathscr{R}$. Take $\bar{\boldsymbol{v}}$ such that $R(\bar{\boldsymbol{v}}) = \mathscr{R}$. We show that $C \notin R(\bar{\boldsymbol{v}}) \Rightarrow C \notin R(\boldsymbol{v}^*)$. Take $C \notin R(\bar{\boldsymbol{v}})$. By definition of a relevant strong coalition, either (a) $C \notin S(\bar{\boldsymbol{v}})$, or (b) $C \in S(\bar{\boldsymbol{v}})$ and there is $D \subset C$, such that $D \in S(\bar{\boldsymbol{v}})$.

Case (a) implies $C^c \in S(\bar{\boldsymbol{v}})$. Therefore, there must be $E \subseteq C^c$ such that $E \in R(\bar{\boldsymbol{v}})$ (see footnote 5). But, since we have shown that $R(\bar{\boldsymbol{v}}) \subseteq R(\boldsymbol{v}^*)$, then $E \in R(\boldsymbol{v}^*)$. Therefore, $E \in S(\boldsymbol{v}^*)$ and consequently, $E^c \notin S(\boldsymbol{v}^*)$. Finally, note that $C \subseteq E^c$. This implies that $C \notin S(\boldsymbol{v}^*)$ (a subset of a weak coalition is weak) and hence, $C \notin R(\boldsymbol{v}^*)$.

In case (b), since $D \in S(\bar{\boldsymbol{v}})$, there is $F \subseteq D$ such that $F \in R(\bar{\boldsymbol{v}})$ (again, see footnote 5). But since $R(\bar{\boldsymbol{v}}) \subseteq R(\boldsymbol{v}^*)$, then $F \in R(\boldsymbol{v}^*)$. Now $F \in R(\boldsymbol{v}^*)$ only if $F \in S(\boldsymbol{v}^*)$. But $F \subseteq D \subset C$ and $F \in S(\boldsymbol{v}^*)$ imply that $C \notin R(\boldsymbol{v}^*)$.

*Proof of Proposition 3.* This proof is organized in a series of steps.

*Step* 1: For $I = 1$, $\boldsymbol{c} = \boldsymbol{v} = 1$ with corresponding $\lambda = 1$.

This ownership structure is the first best ownership structure.

*Step* 2: For any control structure $\mathscr{R}$ with $I$ relevant shareholders, $\lambda(\mathscr{R}) \leqslant \frac{1}{2} + \frac{1}{2I}$.

Take any $\mathscr{R}$ with $I$ relevant shareholders. Let $i_{\min}$ be the relevant shareholder with the minimum cash flow at the solution $\boldsymbol{c}^*(\mathscr{R})$. Since $i_{\min}$ is a relevant shareholder, then there is $C \in \mathcal{R}$ such that $i_{\min} \in C$. By Lemma 4, there is $D \in \mathcal{R}$ such that $C \cap D = \{i_{\min}\}$. Now,

$$\begin{aligned} 1 = \sum_{\ell \in I} c_\ell^* &\geqslant \sum_{\ell \in C - \{i_{\min}\}} c_\ell^* + \sum_{\ell \in D - \{i_{\min}\}} c_\ell^* + c_{i_{\min}}^* \\ &= \sum_{\ell \in C} c_\ell^* + \sum_{\ell \in D} c_\ell^* - c_{i_{\min}}^* \\ &\geqslant 2\lambda - c_{i_{\min}}^*. \end{aligned} \tag{A.7}$$

The first inequality uses the fact that, since $C \cap D = \{i_{\min}\}$, then $C - \{i_{\min}\}$, $D - \{i_{\min}\}$, and $\{i_{\min}\}$ are pairwise disjoint sets. The last inequality follows because $C \in \mathcal{R}$ and $D \in \mathcal{R}$ and, at the solution, all relevant strong coalitions have cash flows greater than or equal to $\lambda$. Therefore,

$$\forall \mathcal{R}, \quad \lambda(\mathcal{R}) \leqslant \frac{1}{2} + \frac{c_{i_{\min}}^*(\mathcal{R})}{2}. \tag{A.8}$$

The result follows from the above equation and the fact that $c_{i_{\min}}^* \leqslant 1/I$ (recall $c_{i_{\min}}^*$ is the minimum of $I$ positive terms that add up to 1).

*Step* 3: For $I \geqslant 3$ and $I$ odd, $\boldsymbol{c} = \boldsymbol{v} = (1/I, \ldots, 1/I)$ with corresponding $\lambda = \frac{1}{2} + 1/2I$.

It is easy to check that, for the ownership structure above, $\lambda = \frac{1}{2} + 1/2I$. Therefore, by step 2, this structure is optimal.

*Step* 4: For any control structure $\mathcal{R}$ with $I$ even relevant shareholders, $\lambda(\mathcal{R}) \leqslant \frac{1}{2} + 1/2(I+1)$.

Take any $\mathcal{R}$ with $I$ even relevant shareholders and let $p(\cdot)$ be the permutation that orders $\boldsymbol{c}^*(\mathcal{R})$ as follows

$$c_{p(1)}^* \leqslant c_{p(2)}^* \cdots \leqslant c_{p(I)}^*. \tag{A.9}$$

We first show that

$$\lambda(\mathcal{R}) \leqslant \frac{1}{2} + \frac{c_{p(I)}^* - c_{p(1)}^*}{2}. \tag{A.10}$$

Consider the following coalitions

$$\begin{aligned} &C = \{p(2), \ldots, p(2n), \ldots, p(I-2)\}, \text{ and} \\ &D = \{p(3), \ldots, p(2n+1), \ldots, p(I-1)\}. \end{aligned} \tag{A.11}$$

Note that, since $\sum_{\ell \in C \cup \{p(I)\}} c_\ell^* \geqslant \sum_{\ell \in D \cup \{p(1)\}} c_\ell^*$ (because $c_{p(2)}^* \geqslant c_{p(1)}^*, \ldots,$ and $c_{p(I)}^* \geqslant c_{p(I-1)}^*$) and $\sum_{\ell \in C \cup \{p(I)\}} c_\ell^* + \sum_{\ell \in D \cup \{p(1)\}} c_\ell^* = 1$, then

$$\sum_{\ell \in C \cup \{p(I)\}} c_\ell^* \geqslant \frac{1}{2}. \tag{A.12}$$

Moreover, we show that this inequality is strict. In fact, for any coalition $A$, we have that $\sum_{\ell \in A} c^*_\ell \neq \frac{1}{2}$. To prove this fact, take any voting distribution that generates the given $\mathscr{R}$. Now, either (a) $A$ is strong, or (b) $A^c$ is strong. In case (a), by step 1 in the proof of Proposition 2, $\sum_{\ell \in A} c^*_\ell > \frac{1}{2}$. Similarly, in case (b), $\sum_{\ell \in A^c} c^*_\ell > \frac{1}{2}$ and, therefore, $\sum_{\ell \in A} c^*_\ell < \frac{1}{2}$. Therefore, since $\sum_{\ell \in C \cup \{p(I)\}} c^*_\ell \neq \frac{1}{2}$, it follows from equation Eq. (A.12) that

$$\sum_{\ell \in C \cup \{p(I)\}} c^*_\ell > \frac{1}{2}. \tag{A.13}$$

Note that, since $c^*_{p(3)} \geqslant c^*_{p(2)}, \ldots,$ and $c^*_{p(I-1)} \geqslant c^*_{p(I-2)}$, then $\sum_{\ell \in D} c^*_\ell \geqslant \sum_{\ell \in C} c^*_\ell$. Therefore,

$$\sum_{\ell \in D \cup \{p(I)\}} c^*_\ell \geqslant \sum_{\ell \in C \cup \{p(I)\}} c^*_\ell > \frac{1}{2}. \tag{A.14}$$

Now take $\boldsymbol{v}^* = \boldsymbol{c}^*$. By Proposition 2, $R(\boldsymbol{v}^*) = \mathscr{R}$ and, therefore, Eq. (A.14) implies that $D \cup \{p(I)\} \in S(\boldsymbol{v}^*)$ and $C \cup \{p(I)\} \in S(\boldsymbol{v}^*)$. Now,

$$\begin{aligned} 1 &= \sum_{\ell \in C} c^*_\ell + \sum_{\ell \in D} c^*_\ell + c^*_{p(I)} + c^*_{p(1)} \\ &= \sum_{\ell \in C \cup \{p(I)\}} c^*_\ell + \sum_{\ell \in D \cup \{p(I)\}} c^*_\ell - (c^*_{p(I)} - c^*_{p(1)}) \\ &\geqslant 2\lambda - (c^*_{p(I)} - c^*_{p(1)}), \end{aligned} \tag{A.15}$$

where the third line follows because $C \cup \{p(I)\} \in S(\boldsymbol{v}^*)$ and $D \cup \{p(I)\} \in S(\boldsymbol{v}^*)$ and, therefore, these coalitions must have cash flows greater than of equal to $\lambda$. Eq. (A.10) follows from the last line.

Now consider two cases: (1) $c^*_{p(I)} \leqslant c^*_{p(1)} + \frac{1}{I+1}$ and (2) $c^*_{p(I)} > c^*_{p(1)} + \frac{1}{I+1}$ In case (1),

$$\lambda \leqslant \frac{1}{2} + \frac{c^*_{p(I)} - c^*_{p(1)}}{2} \leqslant \frac{1}{2} + \frac{1}{2(I+1)}, \tag{A.16}$$

where the first inequality follows from Eq. (A.10).

In case (2), note that

$$1 = \sum_{\ell \in I - \{p(I)\}} c^*_\ell + c^*_{p(I)} > (I-1)c^*_{p(1)} + c^*_{p(1)} + \frac{1}{I+1}. \tag{A.17}$$

Hence, $c^*_{p(1)} < 1/(I+1)$, and therefore,

$$\lambda \leqslant \frac{1}{2} + \frac{c^*_{p(1)}}{2} < \frac{1}{2} + \frac{1}{2(I+1)}, \tag{A.18}$$

where the first inequality follows from Eq. (A.8).

*Step* 5: For $I \geqslant 3$ and even, $\boldsymbol{c} = \boldsymbol{v} = (1/(I+1), \dots, 2/(I+1), \dots, 1/(I+1))$ (every shareholder has cash flow and votes $1/(I+1)$ except one — who can be anyone — who has $2/(I+1)$), with corresponding $\lambda = \frac{1}{2} + 1/2(I+1)$.

It is easy to show that, for the above ownership structure, $\lambda = \frac{1}{2} + 1/2(I+1)$. Therefore, by step 4, this ownership structure is optimal.

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