IMPROVED FORECASTING THROUGH THE DESIGN OF HOMOGENEOUS GROUPS*

EDWIN J. ELTON AND MARTIN J. GRUBER†

The purpose of this paper is both to discuss the need to disaggregate economic data into meaningful groups in order to better understand and forecast the future course of economic phenomena, and to illustrate with a specific example that such disaggregation can lead to improved results.

The reasons for placing observations into homogeneous groups has already been documented by the authors but will be reviewed briefly in the first section of this paper.[1] The next section will be concerned with the general procedure for grouping observations. The remainder of the paper will discuss in some detail the improvement in forecasting ability that comes from a specific application of grouping procedures to the problem of forecasting earnings per share for a large group of manufacturing concerns. Forecasts prepared on the basis of statistically grouped data will be compared with forecasts made on data grouped on traditional industrial criteria as well as with forecasts prepared by mechanical extrapolation techniques.

THE HETEROGENEITY OF HOMOGENEOUS GROUPS

In this section we will discuss the reasons for grouping observations and will show that the grouping of observations is not unique but rather is determined by the general objective in grouping and the specific problem under study.

The reasons for grouping observations fall into two categories: (*a*) to isolate homogeneous units that should act alike, and (*b*) to isolate units which should have the same structural relationship between two or more variables.

Although, as the discussion below will make clear, these reasons are not mutually exclusive, nevertheless this dichotomy serves a useful purpose in understanding why we group.

THE NEED FOR HOMOGENEOUS GROUPS

One often tries to form groups of homogeneous units that have had or will continue to have the same value for one or more variables. For example, the SIC industrial codes can be viewed as forming groups of firms which are homogeneous with respect to end product.

The SIC codes have also been used to group firms for purposes of security and portfolio analysis. Security analysts typically evaluate a stock by comparing its performance against other stocks in the same industry. For example, a company may be judged to be a good buy when its price-earnings ratio is low compared with the average price-earnings ratio for its industry. The assumption being made is that the price-earnings ratio for an industry is a good predictor of the price-earnings ratio of each stock in the indus-

*The research for this paper was supported by grants from the Institute for Quantitative Research in Finance and by TIAA-CREF. We would like to thank Steve Replen for computational assistance.

†Associate professors of finance at the Graduate School of Business, New York University.

[1] Edwin J. Elton and Martin J. Gruber, "Homogeneous Groups and the Testing of Economic Hypotheses," *Journal of Financial and Quantitative Analysis* 4 (January 1970): 581–602.

try. Similarly, the typical portfolio manager who attempts to obtain risk diversification by buying stocks from different industries is acting as if he believed that the SIC industrial classifications were homogeneous with respect to market risk.

The second reason for forming homogeneous groups is to find a sample of observations (firms) within which the same structural relationship between two or more variables exists. There are two related conditions under which the failure to properly select a sample can result in the misspecification of the relationships between two or more variables.

The first is where an omitted variable is correlated with both the dependent variable and one or more independent variables.[2] The failure to hold the effects of an omitted variable constant will result in biased regression coefficients, biased correlation coefficients, and both regression and correlation coefficients which are extremely sample sensitive.[3]

The problem of omitted variables has been recognized by several authors, and the solution accepted has often been to accept SIC industrial classification as a suitable metric for homogeneity (regardless of the variable with respect to which homogeneity was being sought). For example, Modigliani and Miller restricted their study of the effect of financial risk on the cost of capital to one industry in an attempt to hold business risk constant.[4] Similarly, many of the early studies of dividend policy on cost of capital or stock price felt that they could eliminate omitted variable bias by restricting their study to a single industry.[5]

The second case where homogeneous groups are needed is where the magnitude or sign of a relationship between two variables can be affected by the value a third variable takes on. For example, suppose we were examining a sample of firms which finance their investments solely from internally generated funds. Then, one would expect a positive relationship between stock price and payout for firms which earned a low return on their marginal investment and a negative relationship for firms which earned a high return. Pooling the data could result in a multiple regression which found no relationship between payout and price, although in fact two very different relationships were present.

Once again, the way to avoid the problem of nonhomogeneous relationships has typically been to accept SIC industrial classification as a suitable metric for homogeneity and so to confine regression analysis to data within one industry. This has been the practice in almost all stock-pricing and cost-of-capital models.

These three reasons for grouping overlap to a considerable extent. For example, one may try to find a group of firms which are homogeneous with respect to risk so that security analysts can ignore risk in their analysis and so that portfolio managers can treat the group as one firm in determining appropriate diversification. Second, if risk is difficult to measure, then one may try to find a grouping that allows the relationship between the two other variables to be studied (e.g., stock price and growth)

[2] Often the omitted variable cannot be included in the regression equation because it cannot be specified exactly.

[3] For a full explanation of the source of bias, see Elton and Gruber (n. 1 above).

[4] F. Modigliani and M. Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment," *American Economic Review* 48 (June 1958): 261–97.

[5] See, e.g., Myron Gordon, *The Investment, Financing, and Valuation of the Corporation* (Homewood, Ill.: Richard D. Irwin, Inc., 1962).

without having to worry about the effect of risk. On the other hand, if a measure of risk can be found, one might consider entering it into the regression equation as a second independent variable (rather than confining the study to a homogeneous risk group) unless the relationship between stock price and growth changes for different levels of risk. If it does change, we should group for the third reason outlined above.

HOMOGENEITY AND TRADITIONAL INDUSTRY GROUPINGS

Both the practice of security and portfolio analysis and the empirical testing of theory in finance has been based on either implicit or occasionally explicit groupings of firms. The groupings employed by almost all authors have been the same—SIC industrial classification. In other words, the assumption has been made that classification by end product is a suitable technique for grouping in almost all studies in finance.[6] But we have a host of evidence that grouping by industries is not particularly suitable for most of the purposes for which it is employed. Let us review some of the evidence.

As mentioned earlier, industry groupings are often used as a basis for comparative analysis by the security analyst. For example, the analyst will typically compare price-earnings (P/E) ratios for a stock with the typical P/E ratio for the industry. Breen has tested the profitability of buying high-growth low-P/E-ratio stocks.[7] His study defined low P/E ratios both in absolute terms and relative to the industry average P/E ratio.[8] Buying stocks with low P/E ratios relative to the industry means yielded a lower rate of return than buying stocks with a low P/E ratio relative to the overall means. This indicates that using traditional industries as a homogeneous group did not aid in formulating a strategy to produce high rates of return. This does not indicate that a proper grouping did not exist, but it does indicate that the SIC industrial classifications were not appropriate.

Similar conclusions can be drawn with respect to portfolio analysis. Cohen and Pogue attempted to see if introducing industry effects into the Sharpe model could improve portfolio performance.[9] Sharpe assumed that the interrelationships between the price movements of stocks could be expressed in terms of their movement with an index of general stock-price movement. Cohen and Pogue added a second influence—the movement of stocks with their respective industrial average.[10] Cohen and Pogue's model did not outperform Sharpe's simple market model, indicating that no

[6] See, e.g., ibid.; Ronald Wippern, "Financial Structure and the Value of the Firm," *Journal of Finance* 21 (December 1966): 615–33; F. D. Arditti, "Risk and the Required Return on Equity," *Journal of Finance* 22 (March 1967): 19–36; and Martin J. Gruber, *The Determinants of Common Stock Prices* (College Park: Pennsylvania State University, 1971).

[7] William Breen, "Low Price-Earnings Ratios and Industry Relatives," *Financial Analyst Journal* 24, no. 4 (July/August 1968): 125–27.

[8] Since differences in growth rates were adjusted for in the Breen study, we can consider differences in P/E ratios as due to risk plus market imperfections. Adjustment by the industry-average P/E can be viewed as an attempt to correct for risk differentials between industries.

[9] See K. Cohen and J. Pogue, "An Empirical Evaluation of Alternative Portfolio Selection Models," *Journal of Business* 40, no. 1 (January 1967): 163–93; and William Sharpe, "A Simplified Model for Portfolio Analysis," *Management Science* 9, no. 2 (January 1963): 277–93.

[10] Cohen and Pogue actually tested two models. In the first model, the industrial averages were assumed to be uncorrelated except for their common movement with the overall market. In the second, the industrial averages were assumed to have covariance above that due to movements with the general market.

new information was gained by viewing stocks as members of traditional industries.[11]

We can also question the use of industrial classification as a method of holding certain variables constant. In finance, the most common variable researchers desire to hold constant is business risk. Wippern has used an analysis of variance to test differences in the stability of net operating income (his measure of business risk) within and between industries.[12] He found that there was virtually no difference and concluded that his testing "provides clear evidence that industry groups do not provide an adequate basis on which to insure homogeneity of basic business uncertainty." If, in fact, an industry was a homogeneous risk class, then the relationships, as hypothesized by Modigliani and Miller, between cost of capital and financial risk should be reasonably stable across samples of firms from the same industry. Keenan has demonstrated the extreme sample sensitivity of the Modigliani and Miller models with small random changes in a one-industry sample, indicating clearly that the industries he tested are not homogeneous risk classes.[13] A different homogeneous group must be found in order to hold business risk constant.[14]

The normal solution to the problem of holding the effect of omitted variables constant has been to introduce additional variables (as proxies of the previously omitted variable) into the regression equation. Problems can still arise because some variable correlated with both the dependent and one or more independent variables has been omitted or because the nature of the relationship between some of the variables changes as the magnitude of an included variable changes. Such problems would be evidenced by extreme sensitivity of the regression parameters to changes in the sample.

Keenan has used analysis of covariance in examining the models presented by Barges, Benishay, Gordon, and Modigliani and Miller.[15] In this study, Keenan has shown that the parameters of all the models which he tested are so extremely sample sensitive that the removal of one or two firms from the sample can change the sign of the regression parameter even when all firms in the sample are drawn from the same universe. A similar result is contained in Durand when he demonstrates that a stock-pricing model can yield statistically significant differences in parameters when it is estimated across geographical subsectors of an industry.[16]

The purpose of this section has been both to show the necessity of finding homogeneous groups of observations and to demonstrate that one particular

[11] Benjamin King ("Market and Industry Factors in Stock Price Behavior," *Journal of Business* 39 [January 1966]: 139–90) provides evidence that traditional industrial groupings explain part of the historical variability in the firms' rate of return. The results of his study suggest that some traditional industrial grouping might be appropriate for portfolio selection.

[12] See Wippern (n. 6 above).

[13] M. Keenan, "Toward a Positive Theory of Equity Valuation" (Ph.D. diss., Carnegie Institute of Technology, 1967).

[14] I. Friend and M. Puckett, "Dividends and Stock Prices," *American Economic Review* 54 (September 1964): 656–82.

[15] See Keenan (n. 15 above); Alexander Barges, *The Effect of Capital Structure on the Cost of Capital* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963); H. Benishay, "Determinants of Variability in Earnings Price Ratios of Corporate Equities" (Ph.D. diss., University of Chicago, 1960); Gordon (n. 5 above); and Modigliani and Miller (n. 4 above).

[16] David Durand, *Bank Stock Prices and the Bank Capital Problem*, National Bureau of Economic Research, Occasional Paper no. 54 (New York: National Bureau of Economic Research, 1957).

grouping (specifically that based on SIC industrial classification) is not an appropriate grouping for all purposes. The grouping of observations that is appropriate for any study depends on the objective of the study and the nature of the process under investigation. One must first decide on why one wants homogeneous groups and then, with that objective in mind, select a variable or group of variables with respect to which homogeneity is desired. For example, in seeking a group of firms which are homogeneous with respect to business risk, one might well be content to find firms which have had the same amount of variation in past operating earnings.[17] This, rather than industrial classification, should be the basis of grouping.

In the next section of this paper, we will discuss the way homogeneous groups of observations can be formed once a set of variables with which homogeneity is desired has been selected. We will then present a case study that illustrates the improvement in forecasting ability which can be obtained through the use of grouping techniques.

AN ALTERNATIVE METHOD OF FORMING HOMOGENEOUS GROUPS

An alternative to grouping on the basis of traditional industries is to group on the basis of a variable or set of variables which are deemed relevant for the problem under study. If N variables are chosen as relevant for grouping, each firm can be viewed as a point in N-dimensional space. The distance between firms can be measured by simple Euclidean distance and firms grouped according to their distance from other points.[18] This grouping can be accomplished by using cluster analysis, which, in its present state of development, consists of a group of heuristics for partitioning points in N-dimensional space into groups. The clustering algorithm used in this study combines points which are close together, in order to minimize the sum of the squared distance between each point and its group centroid.[19]

In order for this process to yield reasonable groupings, the method of measuring interpoint distances in the N-dimensional space must be meaningful.

If we locate all firms in N-dimensional space and clustered on the basis of the Euclidean distance between firms, then the groups that would result would depend on the scale of the original variables and the extent of their orthogonality. For example, the squared distance between two points is:

$$D_{jk}^2 = \sum_{i=1}^{N}(P_{ji} - P_{ki})^2 , \qquad (1)$$

where (1) D_{jk} = the Euclidean distance firms j and k, and (2) P_{ji} = the value of variable i for firm j. If two variables P_1 and P_2 are perfectly correlated, then it is obvious from (1) that their influence is counted first as $P_{j1} - P_{k1}$ and then as $P_{j2} - P_{k2}$, or double counted. The greater the correlation of P_1 with other variables, the greater the effect of the common influence.

Further, from (1) it is obvious that the scale of the data influences the distance measure. If each P_{j1} is in units of thousands rather than millions, then the

[17] Assuming such variation is a proxy for the basic uncertainty associated with earnings.

[18] We will take up the problem of using Euclidean distance in the unadjusted N-dimensional space shortly.

[19] Other objective functions assumed are maximization of the squared distance between group centroids and the minimization of average squared distance between all points within a group. These are equivalent to the one in the text. See Elton and Gruber (n. 1 above) for proofs of the equivalence as well as for a discussion of clustering techniques which assume different objective functions.

influence of P_{j1} is 1,000-fold higher.

Unfortunately, in most economic problems, the variables are multicollinear and the scaling of the variables is arbitrary. It would be very undesirable if multicollinearity and the scaling of variables affected final groupings.

Fortunately, a technique exists for decomposing a set of variables (axes) into a new set of variables that are both orthogonal and insensitive to the unit of original measurement. The first step is to perform a principal components analysis of the correlation matrix of the raw data. This produces a new set of variables (axes) which are a linear transformation of the original variables and which are uncorrelated with each other. The new variables (axes) are ordered by their ability to account for the original joint variation in the data. That is, the first principal component explains the greatest amount of variation of the original data. These new variables now define an N-dimensional space in terms of orthogonal directions.[20]

The value of any new variable for any firm can be found by simply multiplying the relevant factor loadings by the normalized value of the original variables for the firm.[21] By repeating this procedure for each new variable, we can locate any firm in an orthogonal N-dimensional space.

However, the distances defined in the principal components space will still be determined by the amount of correlation in the original data, despite the fact that the space is defined in terms of orthogonal dimensions. The variance of the values of any new variable will be a function of the ability of that component to account for the original joint variation in the data. The more highly correlated the original variables, the better a particular component can account for original variance, and so the larger the interpoint distances will be in that dimension. This again would result in the double counting of correlated variables and is likely to lead to the overpowering of important firm differences. To overcome this problem, one need only divide each new variable by its standard deviation (eigen value) across all firms.[22] This will produce a set of firm measurements and differences which are both insensitive to the correlation and scale of the original variables and so can be used to group firms.[23]

FORECASTING EARNINGS— AN APPLICATION

This section illustrates the application of grouping techniques to a particular problem and shows that these techniques can improve forecasting ability. The par-

[20] One may use the information produced by the principal components analysis to decrease the dimensionality of the space in which firms are examined. The first P components where $P < N$ may explain so much of the original variation (e.g., 99 percent or more) that one is willing to assume that these P dimensions capture the relevant differences between firms.

[21] There is one set of factor loadings for each new variable. The values of each original variable are normalized across all firms to a mean of zero and a standard deviation of one.

[22] This procedure can be simplified by dividing the eigen vectors by the eigen values before computing factor scores. For more rigorous proof of the analysis in this section, see Donald Farrar, "Multivariate Measures of Profile Similarity for the Objective Stratification of Economic Data" (working paper, Alfred P. Sloan School of Management, M.I.T., 1968).

[23] If all adjusted principal components are used to group firms, one can pick up and misinterpret large amounts of random noise from the last few components (which usually explain very little of the variance in the original data). To overcome this problem, one will usually use a number of principal components which is smaller than the number of variables included in the analysis. There is no optimum way to decide on the number of components to use and the ultimate justification for our choice must rest with the usefulness of our results.

ticular problem we chose to study was the forecasting of earnings per share for industrial corporations.

This problem was selected for several reasons:

1. Almost every valuation and cost of capital model reported in the literature employs an earnings or earnings-growth variable.[24] Furthermore, the results of these models have proven to be extremely sensitive to the way the earnings or growth variable was defined.

2. We had already explored a series of more naïve techniques for forecasting earnings per share, and so a bench mark existed against which to judge the results of this study.[25]

3. We felt that the determinants (forecasts) of earnings per share were not homogeneous across all companies and that improvement in forecasts would result from the substitution of statistical grouping techniques for groupings based on final product.

The first steps which had to be taken in the study were the selection of criteria with respect to which we wished our groups to be homogeneous and the selection of a set of variables which could be used to forecast earnings per share within each group.

These are not really independent problems. We felt that earnings per share could be forecast by relating the change in earnings per share to corporate variables. The definition of the variables used to predict earnings per share are included in Appendix B. In general, this list includes measures of the type and size of sources of funds (e.g., 1, 2, 3, 4), measures of uses of funds (e.g., 11, 12), measures of profitability (e.g., 14, 20), measures of historical growth rates (e.g., 15, 16, 17), and measures of liquidity (e.g., 6, 7, 18).[26] While these variables should, in general, be good predictors of earnings per share, the way in which different firms responded to change in any variable might differ. For example, a decrease in profitability for a firm with a cyclical earnings pattern may mean a very different thing from a decrease in profitability for a firm which has demonstrated a steady growth in earnings.

We wanted to place firms into groups which had had the same earnings-growth pattern over time. This was done because we felt that, if firms have demonstrated the same pattern of growth over time (management reacted the same way to changes in economic condition), then differences in such things as profitability or liquidity would be likely to trigger the same reaction on management's part and so have the same effect on future earnings.

Having decided on the variables to use in forming homogeneous groups for the purpose of forecasting, the next step was to design our basic sample.

The sample could have been formed by randomly selecting a predetermined number of firms.[27] However, it was desirable to have a large sample of firms from each of several traditional industries so that (*a*) the dispersion of firms

[24] Benishay (n. 15 above); Gordon (n. 5 above); Gruber (n. 6 above); Modigliani and Miller (n. 4 above); Wippern (n. 6 above).

[25] See Edwin J. Elton and Martin J. Gruber, "Earnings Forecasters and Expectational Data," *Management Science* (in press).

[26] The particular definition of an influence (such as profitability) that we use is, of course, somewhat arbitrary. The actual list of definitions was obtained from standard definitions reported in the literature and from the suggestions made by officers of thirty large financial institutions. These suggestions were made after hearing the research proposal on which this study was based.

[27] Our universe is biased in favor or large firms, since we restrict it to firms included on the compustat tape.

from traditional industries across our homogeneous groups could be studied, and (*b*) we could compare forecasts based on assuming that each traditional industry is a homogeneous group with forecasts based on our pseudoindustries (statistically homogeneous groups). To accomplish this, stratified sampling was employed.[28] We selected nine large four-digit-industry classifications at random from among those included on the compustat tape and included all firms with a suitable history from each of these traditional industries.[29] Sixty-one additional firms were then selected at random from the compustat tape. These firms were included so that we could examine whether they would cluster with traditional industries or segments of traditional industries or whether they would remain as outliers. Our final sample consisted of 180 firms representing forty-four industries. Our sample classified by both traditional and pseudoindustry is presented in Appendix A.

The next step in the study was to find homogeneous groups of firms. Annual growth rates in earnings per share were computed using earnings-per-share data for the years 1948–63.[30] This gave us fifteen growth rates for each of our 180 companies. Principal components analysis was then performed on the correlation matrix of the raw data. The results of the principal components analysis (both eigen vectors and eigen values) are reproduced in Appendix C. As can be seen from the eigen values in Appendix C, the first eleven principal components accounted for 88 percent of the variation of the raw data, while the next four only accounted for 16 percent. The decision was made to cluster the data in terms of eleven orthogonal axes (principal components).[31] As discussed earlier, if principal component scores for each company were computed simply by multiplying the eigen vector (principal component loadings) for each component by the standardized growth rates for each company, the resultant scores would still be a function of the amount of correlation in the raw data. To correct this, each of the first eleven principal components was standardized by dividing its eigen vector by the square root of its eigen value (which is the same as the standard deviation of the principal component scores on that component). The resultant standardized principal component loadings are displayed in Appendix D. The principal component scores were calculated for each firm in terms of these standardized principal component loadings. We now had a score for each firm on each of eleven standardized principal components.[32] We viewed each

[28] All firms for which the compustat service did not record earnings in one or more years from 1953 to 1966 or which reported negative earnings from 1953 to 1962 were eliminated from the sample. This was done so that the final results could be compared with the outcome from mechanical techniques reported in Elton and Gruber (n. 25 above). This also biases our sample in favor of large, stable firms.

[29] This meant that the probability of a firm being selected in the first part of our sample was a function of the number of firms in the industry to which it belonged.

[30] Data for the years 1964, 1965, 1966 are not used in the analysis, for the clustering patterns obtained are to be used to test forecast accuracy for these years.

[31] The decision as to how many of the principal components to preserve must, to some extent, be arbitrary. Preserving all components standardized to unit variance would pick up and magnify the large amount of random fluctuations contained in the last few principal components. Using too few principal components would ignore important dimensions of the original data.

[32] We can express this analytically as follows. Let G_i stand for the vector of standardized growth rates for firm i over time; F_j stand for the vector of principal component loadings for component j (the eigen vector for component j); and λ_j, the eigen value for component j. Then the standardized principal components loadings in Appendix D are de-

of the 180 firms as a point in eleven dimensional orthogonal space and employed clustering techniques to group them in terms of Euclidean distance. Specifically, we grouped firms by sequentially combining a firm (or a previously formed group) with the firm or group to which it was closest in terms of Euclidean distance. This sequential process was continued until further aggregation involved a "large" change in within-group distances. At this point, there were ten groups. The composition of the groups we obtained are presented in Appendix A.

THE FORECAST PROCEDURE

Having obtained pseudoindustries, we are now ready to examine the usefulness of these groupings for forecasting purposes and to answer the specific question, "Do these grouping techniques yield better forecasts than aggregation along traditional industry lines?"

The first step in the experimental design was to select the method for forecasting earnings per share within each group. The specific procedure adopted involved running a cross-sectional, stepwise regression within each group and using the regression equations to estimate future earnings. In running the regression, the dependent variable was defined as the change in earnings between the end of 1960 and 1961; while the independent variables were those presented in Appendix B, calculated as of the end of 1960. Independent variables were added to the regression until no excluded variables were statistically significant at the 1 percent level.[33]

The regression equations which meet the above criteria differed both across pseudoindustries and traditional industries. Most equations contained some measure of sources of funds (1, 2, 3, or 4), profitability (14 or 20), and/or past growth rates (15, 16, 17). Though the measure of each of these influences which entered the forecasting equation differed from one industry to the next, each entered with a plus sign. This is what one would expect since the higher the profitability, the higher the historic growth rate; or the more funds raised by a firm, the higher one would expect future growth to be. All of the regressions had statistically significant coefficients of determination. However, the performance of the models should be determined by their forecasting ability (which we discuss below) rather than by their R^2.

After establishing the parameters of the regression equation, we forecast the value of earnings for 1964. This forecast was prepared by calculating the value of the independent variables as of 1963, substituting in the regression equations discussed above and adding the estimated change in earnings to the actual earnings for 1963.[34]

rived from the raw principal component loadings in Appendix C by calculating F_j/λ_j. Furthermore, the principal component score of firm i on component j (or the location of firm i in the jth dimension) is equal to G_iF_j/λ_j.

[33] Alternative procedures might be used for establishing the best forecasting equations. However, since our emphasis is on establishing the usefulness of pseudoindustries and since a large number of forecasting equations had to be established for pseudoindustries (ten) and for traditional industries (nine), we felt such a procedure was justified.

[34] The question may arise as to why we estimated earnings for 1964 rather than for 1962. Most of the independent variables used in our regressions were constructed in terms of three-year averages of data (in order to damp out random fluctuations). If we had recalculated the independent variables as of 1961 in order to forecast for 1962, two out of three of the observations used in constructing the independent variables would be the same as those used in running the regression. To avoid this problem, the period ending in 1963 was used in defining

The next step was to select a procedure for comparing forecast techniques. It was desirable both to select an external criterion against which our forecasts could be compared and to set up a procedure for analyzing the statistical significance of results.

In an earlier study we compared nine techniques for forecasting earnings, using mechanical extrapolation techniques against each other and against analysts' estimates.[35] The results of this study showed that one technique dominated the other mechanical techniques at a statistically significant level. Furthermore, the performance of this technique, an exponentially weighted moving average with an arithmetic trend, could not be differentiated from the performance of the security analysts at the three large financial institutions studied (see Appendix E).[36] Forecasts using this technique should be a useful bench mark against which to judge the performance of our within-group regression.

In order to measure differences in the performance of our three forecasting techniques (the exponentially weighted moving average with an arithmetic trend, regressions within traditional industries, and regressions within pseudoindustries), we examined the frequency functions of the differences in the squared error between various pairs of forecasts. As an example, consider the determination of the frequency function used to compare the exponential with the regression within traditional industries. One observation determining this frequency function would be the squared error in the exponentially weighted forecast for company A minus the squared error for the regression forecast for company A. When we repeated this for all possible companies, we have one frequency function from which we can judge the comparative performance of the two forecasting techniques. If the frequency function had all positive or all negative values then this would indicate that one technique always had a lower squared error than a second technique and that dominance existed. Given the size of our sample, this would be unlikely to occur. What could and did happen was that some frequency functions had mostly

---

the independent variables. If this gap in time introduces a bias, it should increase the inaccuracy of our regression results rather than work in favor of our results.

[35] The sample for this earlier study was identical with the sample used in this study. The nine forecasts were prepared in the following manner: (1) the previous year's earnings plus the previous year's change in earnings; (2) a four-year moving average; (3) a moving average of optimum length; (4) a linear regression on time; (5) a log linear regression on time; (6) an exponentially weighted moving average with an arithmetic growth trend; (7) an exponentially weighted moving average with a geometric growth trend; (8) the same as (6) except an arithmetic growth in the trend was added; (9) the same as (7) except a geometric growth in the trend was added. See Elton and Gruber (n. 25 above) for a fuller description of the results and a detailed description of the procedure used to determine optimum weights for the exponential weighting.

[36] The results of the comparison of the mechanical extrapolation technique with the forecasts of security analysts are contained in Appendix E. The test used to analyze possible statistical differences in the forecasts is discussed in the next several paragraphs of the text. None of the differences between analysts' forecasts and the mechanical forecasts were statistically significant. Several comments are in order concerning the analysts' estimates. The analysts' estimates come from three financial institutions. These institutions were not selected at random; rather, we would expect them to be among those institutions that had produced the best earnings projections. The investment-advisory service was selected after analysts in a number of financial institutions indicated that this was the service in whose projections they placed the greatest faith. Furthermore, the other two institutions represented the only ones among those contracted which were willing to expose their forecasts to rigorous testing. Since this had potential repercussions within their own firms, this indicated some confidence in their projections.

positive or negative values and had a mean significantly different from zero. When the mean is significantly different from zero, we can state that it is highly unlikely that the techniques being compared forecasted equally well, and we will say that one technique is dominated by a second.[37]

The first comparisons we made were over all firms for which we had either type of within-industry forecast. Since neither the classifications by SIC industrial code nor by pseudoindustries included all firms in our sample (nor did the two classifications include the same firms), we could not directly compare pseudoindustry forecasts with forecasts using SIC industrial classifications. Instead, we compare forecasts based on regressions within traditional industry groupings against mechanical forecasts for the same firms and forecasts based on regressions within our pseudoindustries against mechanical forecasts for these firms.[38] The results are shown in table 1. The forecast prepared using regression analysis within pseudoindustries outperforms the mechanical forecast at the 5 percent level of significance. On the other hand, the forecast prepared using regression analysis within traditional industries is outperformed by the mechanical technique at the 0.1 percent level. The results indicate the dominance of pseudoindustry forecasts over both mechanical forecasts and forecasts prepared on the basis of traditional industries.

There is some possibility that these results arose because of differences in the pattern of earnings between firms which were selected as members of pseudoindustries and firms which were members of traditional industries. For example, mechanical techniques might just work better for those firms that are members of traditional industries than they do for those firms that are not members of traditional industries but are members of pseudoindustries. To avoid this possibility, tests were repeated on only those firms which had been grouped as members of both traditional industries and

[37] From the central limit theorem, we can state that the distribution of the mean of our frequency functions is normally distributed with mean equal to the mean of the frequency function and standard deviation equal to the standard deviation of the frequency function divided by the square root of the number of observations. That the frequency functions under question were derived from differencing two variables should not bother the reader. The central limit theorem states that, as the number of observations increases, the distribution of the mean is normally distributed, no matter what the original frequency function.

[38] Only SIC industries and pseudoindustries with five or more firms were included in preparing forecasts. Extremely small industries would not allow enough degrees of freedom for the regression analysis.

TABLE 1

FORECAST COMPARISON—ALL FIRMS

Forecasting Techniques Compared	Mean Difference in Squared Error (a-b)*	Standard Error of Mean	Significance Level of the Mean Difference (%)
a) Forecasts based on traditional industry groupings; *b*) Mechanical forecasts	0.8366	.2360	0.1
a) Forecasts based on pseudoindustries; *b*) Mechanical forecasts	−0.0907	.0445	5

* If the number is negative, technique *a* dominates technique *b*.

pseudoindustries.[39] The results (reported in table 2) once again support the dominance of forecasts based on pseudoindustries. Forecasts based on pseudoindustries were statistically better than forecasts based on traditional industries at the 0.1 percent level and forecasts based on the mechanical model at the 10 percent level. Once again, the mechanical technique dominates forecasts based on traditional industries at the 0.1 percent level.

When the data were decomposed and forecasts prepared by each of the three methods examined for each traditional industry, the forecasts based on pseudoindustries outperformed the forecasts based on traditional industries for eight out of the nine traditional industries with the one reversal not being statistically significant.[40]

The results in this study have been based on an intensive analysis of forecasts for one year (1964). Although forecasts based on pseudoindustries outperform those based on traditional industries, mechanical methods, and subjective analysts' estimates at a statistically significant level for this year, it is possible that the results could differ for other years. However, there is no a priori reason to believe that this will happen. Furthermore, previous tests have shown that the ranking of alternative mechanical techniques against each other and against different analysts' estimates are reasonably stable over time.

CONCLUSION

In this paper we have (1) discussed the need to disaggregate economic data

[39] The sample of firms that are included in both groups consists of ninety-eight firms, as seen from Appendix A.

[40] It is interesting to note the extent to which traditional industries are rearranged among pseudoindustries. We have noted the SIC number of the traditional industry, the number of firms in our sample from that industry, and the number of pseudoindustries into which the traditional industries split (see table below).

Traditional Industry No.	No. Firms in Each Industry	No. Pseudoindustries among Which the Traditional Industry Is Split
2800	21	7
2830	15	2
2912	16	5
3000	5	3
3310	15	7
3400	4	4
3550	7	4
3560	8	3
3714	7	4

TABLE 2

FORECAST COMPARISON—COMMON FIRMS

Forecasting Techniques Compared	Mean Difference in Squared Error	Standard Error of Mean	Significance Level of the Mean Difference (%)
a) Forecasts based on traditional industry groupings b) Mechanical forecasts	0.8862	.2546	0.1
a) Forecasts based on pseudoindustries b) Mechanical forecasts	−0.1185	.0692	10
a) Forecasts based on pseudoindustries b) Forecasts based on traditional industry groupings	−1.0047	.2549	0.1

into meaningful groups in order to both better understand and predict economic phenomena; (2) presented a technique which can be used to partition observations into groups which are homogeneous with respect to a predetermined set of criteria; and (3) demonstrated that this technique can lead to better estimates of earnings per share than a grouping based on SIC industrial classifications. In addition, the forecasts based on our statistical groupings were shown to outperform mechanical extrapolation techniques, which we had previously shown to perform about as well as security analysts.[41]

[41] See Elton and Gruber (n. 25 above).

APPENDIX A

Standard Industrial Classification Code	Pseudoindustry	Name of Company
Chemicals:		
2800 006900	P6	Air Reduction Co.
2800 013000	P2	Allied Chemical Corp.
2800 021900	P6	American Cyanamid Co.
2800 053841	...	Atlas Chemical Industries, Inc.
2800 131500	P5	Celanese Corp.
2800 144244	...	Chemetron Corp.
2800 171700	P7	Commercial Solvents Corp.
2800 215701	P2	Diamond Shamrock Corp.
2800 225000	P4	Dow Chemical
2800 229300	P3	E. I. Dupont de Nemours & Co.
2800 236400	P3	Eastman Kodak Co.
2800 281740	P4	FMC Corp.
2800 351700	P4	Hercules, Inc.
2800 359900	P4	Hooker Chemical Co.
2800 381200	P3	Interchemical
2800 413600	P8	Koppers Co.
2800 443000	P4	MacAndrews & Forbes
2800 482800	P3	Minnesota Mining & Manufacturing Co.
2800 491010	P4	Monsanto Co.
2800 512900	P5	National Lead Co.
2800 588500	P1	Pittsburgh Plate Glass Co.
2800 627900	P3	Rohm & Haas Co.
2800 734100	P6	Union Carbide Corp.
Drugs:		
2830 026000	P3	American Home Products Corp.
2830 091000	P4	Bristol-Myers Co.
2830 313600	P3	Gillette Co.
2830 397700	P4	Johnson & Johnson
2830 406300	P4	Kendall Co.
2830 471000	P4	Merck & Co.
2830 479000	P3	Miles Laboratories, Inc.
2830 543200	P3	Norwich Pharmacal Co.
2830 565800	P4	Parke, Davis & Co.
2830 579000	P4	Pfizer Uchasco & Coct Inc.
2830 591800	P3	Plough, Inc.
2830 619550	P3	Richardson-Merrell, Inc.
2830 648000	P3	G. D. Searle Co.
2830 665500	P3	Smith Kline & French Laboratories, Inc.
2830 693600	P4	Sterling Drug, Inc.
Machinery specialty:		
3550 028100	...	American Machine & Foundry Co.
3550 079300	P2	Black & Decker Manufacturing Co.
3550 089200	P1	Briggs & Stratton
3550 195800	...	Crompton Knowles Corp.
3550 252200	P1	Ex-Cell-O Corp.

Standard Industrial Classification Code	Pseudoindustry	Name of Company
Machinery specialty—*Continued*		
3550 399200	....	Joy Manufacturing Co.
3550 477470	P6	Midland Ross Corp.
3550 555300	P4	Otis Elevator Co.
3550 555700	P1	Outboard Marine Corp.
3550 746200	P4	United Shoe Machinery Corp.
Machinery industrial:		
3560 021000	P6	American Chain & Cable Co., Inc.
3560 081700	P2	Blaw-Knox Co.
3560 147700	P2	Chicago Pneumatic Tool Co.
3560 294800	P2	Gardner-Denver Co.
3560 378400	P2	Ingersoll-Rand Co.
3560 471900	P4	Mesta Machine Co.
Automobile suppliers:		
3714 085100	P10	Borg-Warner Corp.
3714 159700	P6	Clevite Corp.
3714 203800	P6	Dana Corp.
3714 237236	P10	Eaton Yale & Towne, Inc.
3714 259600	P4	Federal Mogul Corp.
3714 406000	P10	Kelsey Hayes Co.
3714 718900	P9	Timken Roller Bearing Co.
Oils:		
2912 053377	P8	Atlantic Richfield Co.
2912 152800	P2	Cities Service Co.
2912 187700	P4	Continental Oil Co.
2912 373250	P5	Imperial Oil Ltd.
2912 407900	....	Kerr McGee Corp.
2912 452180	P2	Marathon Oil
2912 583700	P4	Phillips Petroleum Co.
2912 606500	P6	Quaker State Oil Refining
2912 656500	P2	Shell Oil Co.
2912 659300	P2	Signal Oil Gas Co.
2912 662800	P2	Sinclair Oil Corp.
2912 686300	P2	Standard Oil Co. of Indiana
2912 686700	P7	Standard Oil Co. of Ohio
2912 699600	P2	Sun Oil Co.
2912 701220	P2	Sunray DX Oil Co.
2912 736700	P2	Union Oil Co. of California
Tire and rubber:		
3000 047430	P4	Armstrong Rubber
3000 266300	P2	Firestone Tire & Rubber Co.
3000 317900	P4	B. F. Goodrich Co.
3000 318100	P5	Goodyear Tire & Rubber Co.
3000 738651	P2	Uniroyal, Inc.
Steels:		
3310 011600	P9	Allegheny Ludlum Steel Corp.
3310 046800	P7	Armco Steel Corp.
3310 076700	P7	Bethlehem Steel Corp.
3310 127400	P1	Carpenter Steel Co.
3310 187900	P4	Continental Steel Corp.
3310 189500	P10	Copperweld Steel
3310 214000	....	Detroit Steel Corp.
3310 322500	P9	Granite City Steel Co.
3310 378800	P7	Inland Steel Co.
3310 381710	P10	Interlake Steel Co.
3310 409600	....	Keystone Steel Wire Co. of Delaware
3310 441200	P6	Lukens Steel Co.
3310 464750	P9	McLouth Steel Corp.
3310 616800	P7	Republic Steel Corp.
3310 655400	....	Sharon Steel Corp.
3310 692700	P2	Steel Company of Canada
3310 752200	P7	U.S. Steel Corp.
3310 801700	P9	Youngstown Sheet & Tube Co.

Standard Industrial Classification Code	Pseudoindustry	Name of Company
Machinery fabricating:		
3400 073200........	P10	Belden Corp.
3400 252000........	....	Eversharp, Inc.
3400 298300........	P2	General Cable Corp.
3400 516300........	P6	National Standard Co.
3400 659900........	P4	Signode Corp.
Other firms:		
1042 358930........	P1	Homestake Mining
1311 016400........	P2	Amerada Petroleum Corp.
1311 663600........	P2	Skelly Oil Co.
1311 702300........	P2	Superior Oil Co.
2000 405900........	P3	Kellogg Co.
2052 349910........	P4	Helme Products
2063 016100........	P4	Amalgamated Sugar Co.
2121 298700........	P2	General Cigar Co., Inc.
2200 164400........	....	Collins Aikman Corp.
2510 226200........	P4	Drexel Enterprises
2600 644600........	P3	Scott Paper Co.
2700 462200........	P5	McCall Corporation
2844 057610........	P3	Avon Products
2899 188091........	P4	Conwood Corp.
2899 232728........	....	Eagle Picher Industries
2899 263000........	P4	Ferro Corp.
2899 503230........	P3	Nalco Chemical Co.
2912 048900........	P2	Ashland Oil Refinery
2950 279100........	P6	Flintkote Co.
3241 466600........	P3	Medusa Portland Cement Co.
3321 133800........	P7	Central Foundry Co.
3430 195000........	....	Crane Company
3511 059000........	P5	Babcock & Wilcox Co.
3511 169700........	P4	Combustion Engineering, Inc.
3522 014600........	P6	Allis-Chalmers Manufacturing Co.
3522 383300........	P6	International Harvester Co.
3522 788800........	P3	Wickes Corp.
3531 131000........	P5	Caterpillar Tractor Co.
3531 156800........	P10	Clark Equipment Co.
3531 617760........	P6	Rex Chainbelt
3533 225900........	P2	Dresser Industries, Inc.
3533 336700........	P5	Halliburton Co.
3540 151600........	....	Cincinnati Milling Machine Co.
3540 488200........	....	Monarch Machine Tool Co.
3540 504400........	P1	National Acme Co.
3540 663900........	P4	Skil Corp.
3540 701100........	....	Sundstrand Corp.
3540 768800........	....	Warner & Swasey
3569 694600........	P6	Stewart-Warner Corp.
3569 704900........	....	Symington Wayne Corp.
3570 004000........	P4	Addressograph-Multigraph
3570 382700........	P3	International Business Machines Corp.
3570 507600........	P4	National Cash Register Co.
3570 587000........	P3	Pitney-Bowes, Inc.
3570 798830........	....	Xerox Corp.
3600 299800........	P4	General Electric Co.
3600 359025........	P3	Honeywell, Inc.
3600 386100........	P4	International Telephone & Telegraph
3600 608100........	P6	Radio Corp. of America
3600 784700........	....	Westinghouse Electric Corp.
3610 244900........	P8	Emerson Electric Co.
3610 316200........	P6	Globe Union Inc.
3610 463600........	P1	McGraw-Edison Co.
3610 545800........	P2	Ohio Brass Co.
3610 614500........	P2	Reliance Electrical Engineering
3622 202300........	P2	Cutler-Hammer, Inc.
3622 624500........	P4	Robertshaw Controls

APPENDIX A—*Continued*

Standard Industrial Classification Code	Pseudoindustry	Name of Company
3622 683500........	P2	Square D Company
3630 461000........	P4	Maytag Co.
3630 563100........	....	Packard Bell Electronics
3630 662900........	P2	Singer Co.
3670 253900........	....	Fairchild Camera & Instrument Corp.
3670 305610........	P2	General Signal Co.
3679 369705........	....	IRC Inc.
3679 447500........	P6	P. R. Mallory Co.
3679 682600........	P1	Sprague Electric Co.
3711 303010........	P1	General Motors Corp.
3713 199900........	P6	Cummins Engine
3713 291100........	....	Fruehauf Corp.
3713 787000........	P9	White Motors Co.
3725 586900........	....	Piper Aircraft Corp.
3811 039080........	P2	Ametex Inc.
3811 131900........	P8	Cenco Instruments
3811 627000........	P6	Rockwell Manufacturing Co.
3999 338600........	P5	Hammond Corp.
3999 386000........	....	International Silver Co.
3999 628700........	....	Ronson Corp.
3999 724300........	P8	M. E. Torrington Co.

APPENDIX B

DEFINITION OF VARIABLES*

1. $\dfrac{\text{Debt }(t) - \text{Debt }(t-1)}{\text{Total assets }(t-1)}$

2. $\dfrac{\text{Equity }(t) - \text{Equity }(t-1) - \text{Retained earnings }(t)}{\text{Total assets }(t-1)}$

3. $\dfrac{\text{Retained earnings }(t)}{\text{Total assets }(t-1)}$

4. $\dfrac{\text{Preferred stock }(t) - \text{Preferred stock }(t-1)}{\text{Total assets }(t-1)}$

5. $\dfrac{\text{Depreciation }(t)}{\text{Total assets }(t-1)}$

6. $\dfrac{\text{Cash }(t)}{\text{Current liabilities }(t)}$

7. $\dfrac{\dfrac{\text{Receivables }(t) + \text{Receivables }(t-1)}{2}}{\text{Sales }(t)}$

8. $\dfrac{\text{Long-term debt }(t)}{\text{Market-value stock }(t)}$

*All variables except 15, 16, 17, 19, and 21 are defined as three-year averages of the definition given above. The values of t in establishing the regression equation for 1961 were 1958, 1959, and 1960. The values for t when preparing the forecast for 1964 were 1961, 1962, and 1963.

9. $\dfrac{\text{Long-term debt }(t) - \text{Long-term debt }(t-1)}{\text{Market-value stock }(t) - \text{Market-value stock }(t-1)}$

10. $\dfrac{\text{Preferred }(t)}{\text{Market-value common }(t)}$

11. $\dfrac{\text{Total dividends }(t)}{\text{Earnings available }(t) + \text{Depreciation }(t)}$

12. $\dfrac{\text{Capital expenditure }(t)}{\text{Change in total assets }(t) - (t-1)}$

13. Total assets (t)

14. $\dfrac{\text{Operating income }(t)}{\text{Long-term debt }(t) + \text{Preferred }(t) + \text{Book common }(t)}$

15. Five-year average growth in earnings per share†

16. Five-year average growth in sales per share†

17. Five-year average of the product of retention rate and the rate of return on common equity†

18. $\dfrac{\text{Cash }(t)}{\text{Total assets }(t)}$

19. STD deviation of growth in earnings over five years†

20. $\dfrac{\text{Operating income }(t)}{\text{Net sales }(t)}$

21. Growth in market price, five years†

22. $\dfrac{\text{Current liabilities }(t) - \text{Current liabilities }(t-1)}{\text{Total assets }(t-1)}$

23. $\dfrac{\text{Total assets }(t) - \text{Total assets }(t-1)}{\text{Total assets }(t-1)}$

†These variables were calculated using per-share data. All data were adjusted for stock dividends and stock splits.

APPENDIX C

Eigen Vectors

Growth Rate for Year	Principal Component Number														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1949	.4633	−.0436	−.2144	−.1485	−.0709	−.0302	−.0366	−.0155	−.0445	−.4846	−.3921	−.4698	−.1873	−.1751	−.1732
1950	.3634	−.0304	.3992	−.0433	.4006	.2838	.0897	.0790	−.1266	.1368	−.1630	−.1394	.3372	−.1759	.4789
1951	−.0414	−.2524	.2505	−.1678	−.3140	−.3051	.4988	.1307	−.4215	−.2753	.0805	.1132	.3238	−.0302	−.0891
1952	.0118	−.4894	−.2154	−.1430	−.1694	.0265	.3115	.2461	.0874	.3906	.1055	−.2752	−.3997	.0327	.3142
1953	−.0848	−.1153	−.5094	.0884	.2906	−.0448	.3093	−.3192	.1595	−.3859	.0685	.2603	.1033	−.1038	.4012
1954	.1464	−.2096	−.4371	.3293	.0897	.3173	.0591	.3851	−.0622	.1199	−.0470	.0258	.4395	.1575	−.3714
1955	−.0070	.2023	−.3570	.0475	.1765	−.3711	−.1162	−.0552	−.6790	.3540	−.1135	−.0264	−.0612	−.1803	.0826
1956	−.0848	−.4968	−.0176	−.1938	.1042	−.2214	−.4511	−.1564	.1440	.0674	.2849	−.1627	.3224	−.4085	−.1362
1957	.0998	−.0462	−.0727	.0060	−.4359	.5088	.0380	−.6521	−.2533	.1584	.0824	−.0709	.0966	−.0309	−.0167
1958	.2926	.1507	−.0107	.4506	−.2768	−.0617	−.2166	.2138	−.0987	−.2281	.5815	−.1232	−.0254	−.0671	.3065
1959	−.1399	.1542	−.0719	−.4476	.3707	.3721	.0588	.1343	−.2737	−.2078	.5024	−.1422	−.1651	.0352	−.1748
1960	.0710	−.5154	.1550	.1362	.1960	.0223	−.3401	−.1209	−.3301	−.2111	−.0758	.1824	−.2446	.5113	.0741
1961	−.4236	−.1602	.1245	.3296	−.0387	.3233	−.0398	.1995	−.1584	−.1757	−.2097	.0999	−.2582	−.5804	−.0092
1962	−.3452	.0921	−.2133	−.3363	−.3196	.1267	−.3659	.2516	−.0526	−.1582	−.2313	−.0768	.3005	.1955	.4235
1963	−.4436	.0239	.1074	.3574	.1629	−.1142	.1602	−.1772	−.0026	−.0649	−.0078	−.6973	.1453	.2414	.0019
Eigen values	2.006	1.774	1.604	1.409	1.115	1.043	.985	.956	.867	.779	.664	.571	.452	.402	.375

APPENDIX D

Normalized Eigen Vectors

Growth Rate for Year	Principal Component Number										
	1	2	3	4	5	6	7	8	9	10	11
1949	.3271	−.0327	−.1693	−.1251	−.0672	.−0296	−.0369	−.0159	−.0478	−.5492	−.4813
1950	.2566	−.0228	.3152	−.0365	.3794	.2780	.0904	.0808	−.1360	.1550	−.2001
1951	−.0292	−.1895	.1978	−.1414	−.2974	−.2988	.5027	.1337	−.4526	−.3120	.0988
1952	.0083	−.3675	−.1701	−.1205	−.1604	.0260	.3139	.2517	.0939	.4427	.1295
1953	−.0599	−.0866	−.4023	.0745	.2752	−.0439	.3117	−.3264	.1713	−.4373	.0841
1954	.1034	−.1574	−.3452	.2774	.0550	.3108	.0596	.3938	−.0668	.1359	−.0577
1955	−.0049	.1519	−.2819	.0400	.1672	−.3635	−.1171	−.0564	−.7292	.4012	−.1393
1956	−.0599	−.3730	−.0139	−.1633	.0987	−.2168	−.4546	−.1599	.1546	.0764	3497
1957	.0705	−.0347	−.0574	.0051	−.4129	.4983	.0383	−.6668	−.2720	.1795	.1012
1958	.2066	.1132	−.0084	.3796	−.2622	−.0604	−.2183	.2186	−.1060	−.2585	.7138
1959	−.0988	.1158	−.0568	.3771	.3511	.3644	.0593	.1373	−.2939	−.2355	.6167
1960	.0501	−.3870	.1224	.1147	.1856	.0218	−.3428	−.1236	−.3545	−.2392	−.0931
1961	−.2991	−.1203	.0983	.2777	−.0367	.3167	−.0401	.2040	−.1662	−.1991	−.2575
1962	−.2437	.0692	−.1684	.2833	−.3027	.1241	−.3688	.2573	−.0565	−.1793	−.2839
1963	−.3132	.0179	.0848	.3011	.1543	.1119	.1615	−.1812	−.0028	−.0735	−.0096

APPENDIX E

RELATIVE PERFORMANCE OF SECURITY ANALYSTS AND ADDITIVE EXPONENTIAL MODEL (BASED ON SQUARED ERROR)*

	Investment Advisory Service	Brokerage House	Pension Fund
Sample size	213	177	84
Security analysts	.0231	.0342	.0310
Additive exponential	.0221	.0352	.0441
t-value	+0.12	−0.34	−1.23

* A positive value indicates that the mechanical technique outperformed the analysts' estimates.