Differential information and timing ability

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This paper discusses performance evaluation when the analyst has access to the composition of the portfolio. In the U.S., master trustees and custodians have this information. With this assumption the paper derives techniques for evaluating superior performance. The techniques unlike conventional measures can detect superior performance even when the manager is attempting to market time.

1. Introduction

Recently, a number of papers have appeared which call into question many of the standard methods of performance measurement. Questions have been raised about the use of standard methods to measure the timing ability of managers, to identify selection ability, and to differentiate between the two.¹ By building on a framework developed by Dybvig and Ross (1985), we are able to develop a set of performance measures which both correctly identify the accurate use of information and can be computed in terms of observable variables.

The principal reason that these measures overcome the Dybvig and Ross problem is that we assume the existence of additional information available to the outside observer. Dybvig and Ross assume that the outsider observer is furnished only information on the time series of returns. Most of the literature on portfolio evaluation such as Sharpe (1966), Friend, Blume and Crockett (1970), Jensen (1969), Admati and Ross (1985), and Admati et al. (1986) develop measures based on only observed returns.² We assume the observer knows the portfolio proportions at frequent intervals over time (e.g., daily) as well as a time series of returns for each asset. The information on

²The exceptions to this are Cornell (1979) and some of the measures employed by Grinblatt and Titman (1985) where the assumption is made that portfolio weights are available.
portfolio composition clearly exists in master trust accounts and custodian accounts. In fact, it is the master trustee or custodian who normally provides the information to the pension fund trustees or mutual fund for the purpose of evaluating fund managers. Pension funds are important consumers of portfolio evaluation services and they are clearly in a position to obtain the type of information needed to use our proposed measures. In addition, any mutual fund or trust account is in a position to compute these measures should they be requested by customers.

This paper is divided into three sections. In section 2, we deal directly with one problem initially studied by Dybvig and Ross (1985): can we discern superior timing ability by management in the absence of selection ability? In this section, we show that security market line (SML) analysis can be employed to measure timing, if the proper information is available, and the problems raised by Dybvig and Ross can be overcome. In addition, this section serves as a review of the Dybvig and Ross methodology which will be used throughout the rest of this paper. In section 3, we propose one solution to measuring timing and selectivity when a decision maker receives both market-wide signals and signals on the performance of individual securities. This section defines market timing and selection using traditional ideas of what these measures are. In section 4, we develop measures of timing and selectivity under a more stylized description of the decision-making firm. If the signal on any stock can be viewed as either a signal about the stock market in general or a signal which is unique to the security, then the payoff from each type of signal can be clearly differentiated by an investor observing the transactions of management.

2. The detection of timing ability

In a very important article, Dybvig and Ross (1985) have shown that an informed manager making correct timing decisions between a risky portfolio and a riskless asset can appear to have inferior performance in terms of SML analysis. That is, a manager who acts correctly on the basis of true information can have a negative abnormal return on his portfolio.

The analysis is particularly important because it implies that all prior analysis of portfolio performance using SML analysis is suspect. Since almost all commercial and academic studies use SML analysis, this implies that most prior studies of portfolio performance, and in particular, of the performance of mutual funds must be discarded. Even studies that did not use SML analysis but use betas to categorize comparable funds such as Friend, Blume and Crocket (1970) must be discarded. This follows since, as

Hwang (1988) in a Ph.D. dissertation has obtained portfolio positions from a master trust account for several pension funds and has used the measures advocated in this paper to evaluate portfolio performance.
shown later, managers' timing leads to a misestimation of the average beta by the observer.

The Dybvig and Ross analysis is both correct and insightful when betas are computed using traditional simple regression on portfolio returns. The purpose of this section is to develop a measure that can separate managers who have superior timing ability from those who do not. We shall first briefly review the major results of Dybvig and Ross and then derive our measure of performance due to timing.

2.1. The Dybvig and Ross model

Dybvig and Ross (D–R) assume that the manager can allocate funds between a risky portfolio and a riskless asset and that returns on the risky portfolio are generated by the following process:

\[ x = r + \pi + s + e, \]

where \( r \) is the riskless rate of interest; \( \pi \) is the risk premium; \( s \) is a signal observed by the manager with \( s \), a particular value for the signal; and \( e \) is an unobservable noise term with \( e \), a particular value of \( e \); \( s \) and \( e \) are assumed to be uncorrelated with each other and each have a mean of zero. The variance of \( e \) is \( \sigma_e^2 \).

Dybvig and Ross make the assumption that \( r \) is zero since this assumption simplifies the analysis without affecting the results. We shall make the same assumption throughout the remainder of the paper. Assuming the manager tries to maximize the expected utility from an exponential utility function of the form \(-\exp(Ax)\), D–R show that the fraction of funds which the manager places in the risky portfolio \( \gamma(s) \) is given by

\[ \gamma(s_i) = (\pi + s_i)/\sigma_e^2 A. \]

Dybvig and Ross show that the abnormal returns to any portfolio using security market line (SML) analysis is

\[ \delta = E[\gamma(s)x] - \frac{\text{Cov}(x, \gamma(s)x)}{\text{Var}(x)} E(\pi + s + e) \]

or

\[ \delta = E[\gamma(s)x] - \beta_y E(\pi + s + e). \]

They proceed to show that this can be negative even if the manager
possesses and acts correctly on real information as long as $\pi^2 > \sigma_i^2 + \sigma_f^2$. Similarly, a manager can have perverse information (act on $-s$ rather than $s$) and still show positive performance. In fact, the only case in which abnormal return analysis can be guaranteed to lead to correct answers is where the manager has no information ($s = 0$). The driving force behind the D–R proof is that the investor does not have the same information set as the manager and performs the analysis without being able to observe $s$, or $\gamma(s)$. The problems pointed out by D–R carry over logically to ex post performance evaluation. Assume that returns follow an i.i.d. distribution and that investors can only observe the returns on a managed portfolio over time. Then all the D–R analysis discussed above holds exactly with realizations substituted for expectations.

However, as we shall show, the problem raised by D–R can be eliminated by assuming the investor gains access to additional data beyond that assumed by D–R. This is data which, while not costless, can be obtained by at least some investors. A manager makes decision on $\gamma(s)$ at some interval (a day, week, or month) and changes his portfolio accordingly. The frequency of the interval is unimportant though obviously it must be considered in doing empirical work. If the investor has access to the portfolio composition of the manager’s portfolio, then both $\gamma(s)$ and $\beta$, can be observed by the investor at any point in time. With this information we can reformulate the D–R analysis to find an appropriate metric for timing ability. This is the subject to which we now turn.

2.2. A market timing measure

In this section, we will propose a metric to measure market timing ability and will show that the metric produces a correct indication of whether the manager does or does not have timing ability. We will retain unless otherwise stated the assumptions and terminology of D–R. The proposed metric is

$$d = E[R_p] - E[R_{\beta=\beta}],$$

(1)

D–R results hold in the limit for a large sample. Since ex-post evaluation normally involves a sample, the D–R results should be modified because of possible dependence between random variables in the sample.

The ability to observe investment proportions reduces the differential information aspects of the problem because the investor can now in effect observe management’s interpretation of the information it receives. The impact of this is made clear in the following analysis.

We are not the first one to propose this measure. This is equivalent to the Cornell measure if one assumes stationarity of returns, although in any application the results would be very different. Finally, Grinblatt and Titman (1985) reformulate Cornell’s measure in a form that is identical to our eq. (1).
where \( R_p \) is the return on the managed portfolio; and \( R_{\beta=\bar{\beta}} \) is the return that would have been obtained if the manager operated at the average level of beta at all points in time.

It is convenient to assume that the market portfolio is an all equity portfolio.\(^7\) This assumption does not affect the results of the proof but makes it easier to follow. D–R have an ex ante perspective or an infinitely long ex post perspective. In the ex post perspective \( E(R_p) \) is the average return on the portfolio. \( E(R_{\beta=\bar{\beta}}) \) is the average beta on the portfolio times the average return on the market.

The beta on the stock portfolio is one since the market is assumed to be the stock portfolio.\(^8\) The beta in any period on the manager’s portfolio which is a combination of the stock portfolio and T-bills is simply the proportion in the stock portfolio or in D–R terms \( \gamma(s_i) \). Thus, the average beta is the average \( \bar{\gamma}(s_i) \). From the definition of \( \gamma(s_i) \), \( \bar{\beta} = E[\gamma(s_i)] = \pi/\sigma^2_A \). If \( x \) is the random return of the market, then the expected return on the manager’s portfolio is

\[
E[R_p] = E[\gamma(s)x].
\]

Likewise,

\[
E[R_{\beta=\bar{\beta}}] = \bar{\beta} E(x) = E(\gamma(s)) E(x).
\]

With these two substitutions our measure becomes\(^9\)

\[
d = E[\gamma(s)x] - E(\gamma(s)) E(x) = \text{Cov}(\gamma(s), x).
\]

As shown in D–R, if the manager acts on the signal \( s_i \), then \( \gamma(s_i) \) is

\[
(\pi + s_i)/\sigma^2_A.
\]

Substituting this expression into (4),

\[
d = E\{[(\pi + s)/\sigma^2_A - \pi/\sigma^2_A][\pi + s + e - \pi]\}
\]

\[
= (1/\sigma^2_A)(\sigma^2_e) > 0.
\]

\(^7\)In D–R’s terminology, \( \alpha \) is the proportion of the market portfolio that is invested in risky assets. Thus we are assuming \( \alpha = 1 \).

\(^8\)The analysis throughout this paper is consistent with a one-factor model. For convenience in this section, we assume the beta on the pure stock portfolio which is mixed with the riskless asset is one. The extension of the model to a stock portfolio different from one is trivial. In later sections we recognize different betas on individual assets.

\(^9\)Grinblatt and Titman (1985, Proposition 3) state that eq. (4) is equivalent to eq. (1). In utilizing these metrics for measures of portfolio performance, the covariance could be estimated from the sample period.
Since, from the form of the utility function, $A$ must be positive for any risk averse investor, and since $\sigma_x^2$ and $\sigma_y^2$ must be positive, $d$ is positive for any manager who receives and correctly acts on the signal $(s_i)$.\textsuperscript{10} If the manager uses the signal perversely, then the proportion placed in the stock portfolio is $(\pi - s_i)/\sigma_x^2 A$. In this case, eq. (7) becomes

$$d = (1/\sigma_x^2 A)(-\sigma_y^2).$$  

For the reasons given above $d$ must be negative in this case. If, on the other hand, the signal contains no information (is random), then Cov$(y(s), x) = 0$. Thus, the proposed measure correctly identifies timing information. In order to obtain our measure one needs to know the composition of the portfolio at each point in time. While this information is not generally available to all observers, it is available to at least some subset of market participants. For example, all pension trustees can obtain it from master trust accounts.

One further issue should be discussed. Is our measure affected by a noisy signal? The situation we have in mind is where there are multiple signals and the manager only picks up one of them or alternatively the manager misjudges the signal (interprets it with error). Consider the following situation. At a point in time the manager believes the expected return on the stock portfolio is

$$\bar{x} = \pi + s_i,$$

and the true mean is

$$\bar{x} = \pi + s_i + k.$$

The process generating return is

$$x = \pi + s_i + k + \epsilon.$$ 

$k$ can either be considered as a signal the manager does not see or $s_i$ can be viewed as a misinterpretation of the true signal $s_i + k$. Further assume that $k$ is independent of $s_i$ and $\epsilon$. The manager's perception of the return process is

\textsuperscript{10}We have used the D-R assumption of exponential utility throughout this paper. The analysis is easiest to perform in this case, for portfolio proportions are linear functions of the signal $(s)$. However, the conclusions hold under more complex definitions of the impact of a signal on returns or the forms of the utility function exhibited by the management. For example, $d$ will have the correct sign as long as a utility function exhibiting non-increasing absolute risk aversion.
\[ x = \pi + s_i + \epsilon', \] where \( \epsilon' \) will in fact be equal to \( k + \epsilon \). Employing the methodology of D–R yields

\[ \gamma(s_i) = \frac{(\pi + s_i)}{[(\sigma_i^2 + \sigma_k^2)A]} \].

Continuing with an \textit{ex ante} perspective, we have

\[ d = \mathbb{E}\left[ \left( \frac{\pi + s}{(\sigma_i^2 + \sigma_k^2)A} - \frac{\pi}{(\sigma_i^2 + \sigma_k^2)A} \right) (\pi + s + k + \epsilon - \pi) \right] 
= \frac{1}{(\sigma_i^2 + \sigma_k^2)A} \sigma_i^2 > 0. \]

Proofs analogous to those presented above hold when the manager acts perversely or has no information. Thus, our measure to detect manager's timing ability is unaffected by the manager reacting to the signal with error.

3. Timing and selectivity

In the previous section, we examined the measurement of timing when there was a single risky asset and a riskless asset. In this section, we explore timing and selectivity. To do this we introduce two risky assets. The generalization to \( N \) risky assets is straightforward. We assume two types of signals, a market-wide signal and individual security signals. In this section we will derive timing and selectivity measure within their traditional meanings. In a later section, we will develop measures that isolate the value of the signals directly. Continuing with the simplification of zero return on the riskless asset, the returns on the two assets are given by \( x_1 \) and \( x_2 \), where

\[ x_1 = \pi_1 + \beta_1 s_0 + s_1 + \epsilon_1, \quad x_2 = \pi_2 + \beta_2 s_0 + s_2 + \epsilon_2, \]

where \( \pi_1 \) is the expected risk premium of asset 1; \( \pi_2 \) is the expected risk premium of asset 2; \( s_0 \) is the market-wide signal with \( s_0 \), a particular value of \( s_0 \); \( s_i \) is the individual signal unique to security \( i \) with \( s_i \), a particular value of \( s_i \); \( \epsilon_i \) is the residual return (\( \epsilon_i \) has an expected return of zero and a variance of \( \sigma_i^2 \)); and \( \beta_i \) is the sensitivity of the return on security \( i \) to a market signal.

It is assumed that the signals have a mean of zero and are uncorrelated. Thus, the market-wide signal is independent of the unique security signal. Furthermore, we will assume that the residuals are uncorrelated with the
signal and with each other. Once again we will follow D–R and assume that the investors have the exponential utility function described in section 2.

Letting $\gamma_1$ and $\gamma_2$ be the proportions in the risky securities and continuing the assumption of a zero return on the riskless asset, the return on the total portfolio is

$$\gamma_1(\pi_1 + \beta_1 s_{0t} + s_{1t} + \varepsilon_1) + \gamma_2(\pi_2 + \beta_2 s_{0t} + s_{2t} + \varepsilon_2).$$

Making this substitution, using moment generating functions, and taking logs, the utility of wealth at the end of the period $U(W_i)$ is

$$U(W_i) = A\gamma_1(\pi_1 + \beta_1 s_{0t} + s_{1t}) + A\gamma_2(\pi_2 + \beta_2 s_{0t} + s_{2t}) - A^2 \text{Var}(\gamma_1 \varepsilon_1 + \gamma_2 \varepsilon_2).$$

Taking the derivative with respect to $\gamma_1$ and $\gamma_2$, respectively, setting the resulting expression equal to zero, and solving for $\gamma_1$ and $\gamma_2$ yields

$$\gamma_1 = \frac{\pi_1 + \beta_1 s_{0t} + s_{1t}}{A\sigma_1^2}, \quad \gamma_2 = \frac{\pi_2 + \beta_2 s_{0t} + s_{2t}}{A\sigma_2^2}.$$

Before analyzing market timing and selectivity a couple of comments are in order. First, note that the proportion of wealth invested in any security ($\gamma_i$) depends on the market signal ($s_{0t}$) as well as the security signal ($s_{it}$). A positive $s_{0t}$ will in general change the ratio of $\gamma_1$ and $\gamma_2$. Thus, market-wide signals can change the relative proportion invested in the two securities. Likewise, security signals can affect the aggregate amount invested in all risky assets. For example, a positive $s_{1t}$, causes the amount invested in security one to increase without affecting security two. However, it also causes the aggregate amount invested in all risky securities to increase. Thus, security specific information leads to changes in the aggregate amount invested in risky assets. This was recognized by D–R and Mayers and Rice (1979). Both authors assumed away these effects. In this section, we will

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11Since we allow for a market-wide signal, the assumption of independence in $\varepsilon_i$ is analogous to the assumption of zero correlation in residuals from a single index model.
follow their procedures and define timing in terms of the allocation between risky and riskless assets. In the next section, we will make a complete break with traditional meanings of timing and selectivity, and present an evaluation scheme that separates general effects from the effect of security-specific signals.

The aggregate return is a function of the proportion invested in securities one and two. Letting $R$ be the return as a function of the proportion in each security, we can formulate three return measures. The first, shown in eq. (14), is the actual return earned:

$$R(\gamma_1, \gamma_2) = E[\gamma_1 x_1 + \gamma_2 x_2].$$  

(14)

The second, shown in eq. (15), is the return that would have been earned if the total proportion of funds invested in stocks was held constant (at the average level) but the proportion invested in each individual stock was allowed to vary. Since $\gamma_1 + \gamma_2$ varies over time, we maintain constant proportions in stocks by multiplying the actual proportions by the average proportions $\bar{\gamma}_1 + \bar{\gamma}_2$:

$$R \left[ \frac{\gamma_1}{\bar{\gamma}_1 + \bar{\gamma}_2}, \frac{\gamma_2}{\bar{\gamma}_1 + \bar{\gamma}_2} \right] (\bar{\gamma}_1 + \bar{\gamma}_2)$$

$$= (\bar{\gamma}_1 + \bar{\gamma}_2) E \left[ \frac{\gamma_1}{\bar{\gamma}_1 + \bar{\gamma}_2} x_1 + \frac{\gamma_2}{\bar{\gamma}_1 + \bar{\gamma}_2} x_2 \right].$$  

(15)

The third, shown in eq. (16), is the return that would have been earned if the amount invested in each stock was held at the average level:

$$R(\bar{\gamma}_1, \bar{\gamma}_2) = E(\gamma_1) E(x_1) + E(\gamma_2) E(x_2).$$  

(16)

As we will show, timing is measured by eq. (14) minus eq. (15) and selectivity is measured by eq. (15) minus eq. (16). Let us examine each in turn. First consider what happens if a manager has no timing ability. We will use the conventional meaning of no timing, namely, that the aggregate amount invested in risky securities is independent of the return on the securities. In other words, greater return on securities is not associated with greater investment in risky assets.\textsuperscript{12} This definition of no timing ability in equation form is:

\textsuperscript{12}Recall that earlier we discussed how individual security signals could affect the aggregate amount invested in securities. Thus the conventional definition has ambiguity. We will deal with this in the next section.
The left-hand side of this equation is analogous to eq. (4) which was our timing measure when we considered timing alone. We know that

$$\text{Cov}(A, B) = E(AB) - E(A)E(B).$$

Thus, the definition of no timing ability becomes

$$E\left(\frac{y_1 + y_2}{y_1 + y_2} x_1 + \frac{y_2}{y_1 + y_2} x_2\right) = 0,$$

or, after rearranging,

$$E[y_1 x_1 + y_2 x_2] = E(y_1 + y_2) E\left(\frac{y_1}{y_1 + y_2} x_1 + \frac{y_2}{y_1 + y_2} x_2\right).$$

Thus, with no timing ability in the conventional meaning of timing, our proposed measure of timing ability, eq. (14) minus eq. (15), is identically zero. The selectivity measure is positive, negative, or zero depending on whether the signals are correctly utilized in decision making. With no timing, eqs. (14) and (15) are identical. Thus, the difference between eq. (15) and eq. (16) is identical to the difference between (14) and (16). If the signals are used correctly, this difference is

$$\frac{\sigma_s^2 + \beta_1^2 \sigma_s^2 + \sigma_s^2 + \beta_2^2 \sigma_s^2}{A\sigma_{e_1}^2 + A\sigma_{e_2}^2}.$$

Therefore, the selectivity measure has the correct sign if the information is used correctly.\(^{13}\)

Our proposed measure also leads to correct conclusions if there is zero selectivity but timing exists. One case of zero selectivity is that the proportion of investment in all risky assets which is held in any particular asset is a constant.\(^ {14}\) Employing this traditional meaning for selectivity

\(^{13}\)By direct analogy, misinformation has the wrong sign.

\(^{14}\)The more general case with zero selectivity involves the proportion of risky assets invested in any one asset being independent of the return on that asset and the timing decision (the proportion in the aggregate of risky assets). The mathematics are more complex but the results are identical.
\[ \gamma_1 = K \gamma_2 \] where \( K \) is some constant. With this substitution eqs. (15) and (16) are identical and equal to

\[ \frac{(K \pi_1 + \pi_2)^2}{A \sigma^2_{e_1}}. \]

Thus, the proposed selectivity measure is zero when there is no selectivity. Since eqs. (15) and (16) are identical, we can examine the difference between eqs. (14) and (16) to analyze the timing measuring. As before, this difference if the signal is correctly used is

\[ \frac{\sigma^2_{e_1} + \beta^2_1 \sigma^2_{s_0} + \sigma^2_{e_2} + \beta^2_2 \sigma^2_{s_0}}{A \sigma^2_{e_1}}. \]

If the signal is ignored, then this measure is zero, and finally if the signal is used perversely, this measure is negative. So, the timing measure has the correct sign. Note that employing eqs. (14), (15), and (16) the timing and selectivity measures proposed in this section can be computed for any manager as long as the composition of the investment portfolio can be observed. The generalization to \( N \) securities is straightforward.

4. An alternative approach to timing and selectivity

In the last section of this paper, we developed measures of timing and selectivity under a particular and widely used definition of selectivity and timing. In particular, we assumed that timing involved the allocation between risky and riskless assets. Selectivity was what was left to performance after timing. In fact, as pointed out earlier, part of what this analysis attributes to timing is really the response to signals on individual securities rather than signals about securities in general. In this section of the paper, we shall develop a new and less conventional set of measures. Now timing will be defined as a response to the economy-wide signal \( s_{or} \) and selectivity the response to signals on individual securities. Our framework and measures are completely different from those used in previous research [e.g. Dybvig and Ross (1985), Grinblatt and Titman (1985), Mayers and Rice (1979)] where individual security signals can affect timing measures.

To aid in developing the new measures we will define a highly stylized model of a firm whose performance we wish to analyze. We will initially assume that the firm is only concerned with two risky assets but shortly will generalize to the case of \( N \) risky assets. We will continue with the return models developed in the last section of this paper. We will change the way signals are produced and hence our definition of timing. Assume the analytical functions of our stylized firm can be divided into two separate
groups. One group (we could call them ‘economists’) produces forecasts of the economy and the market (signals $s_0$ in our world). The second group (we could call them ‘security analysts’) received the signal $s_{ot}$ and produces forecasts about individual stocks $s_{1t}$ or $s_{2t}$ given knowledge of $s_{0t}$.

We then define timing as decisions reached on the basis of $s_{0t}$ and selectivity as decisions reached on the basis of $s_{1t}$ and $s_{2t}$. All reaction and only reaction to $s_{0t}$ is called timing. Recall that from our last section a signal $s_{ot}$ also caused us to change the proportions invested in stocks one and two. This second-order effect was designed as part of selectivity. Here we identify any change in return resulting from $s_{0t}$ as timing. Similarity, any change resulting from a signal $s_{1t}$ or $s_{2t}$ is called selectivity.

The advantage of this procedure is that it results in a completely unambiguous division of returns into two components. If the reader objects to us referring to these as timing and selectivity, he can think of them as payoff from processing economy-wide data and payoff from processing data unique to each firm.

Let us examine the payoff from the combination of timing and selectivity. This is simply

$$E[\gamma_1 x_1 + \gamma_2 x_2] - [E(\gamma_1) E(x_1) + E(\gamma_2) E(x_2)],$$

where $\gamma_1$, $\gamma_2$, $x_1$, and $x_2$ are defined earlier.

Substituting in the values for $\gamma_1$, $\gamma_2$, $x_1$, and $x_2$ developed earlier, the overall manager’s payoff becomes

$$d_0 = \frac{\beta_1^2 \sigma_{s0}^2}{A\sigma_{x1}^2} + \frac{\sigma_{s1}^2}{A\sigma_{x1}^2} + \frac{\beta_2^2 \sigma_{s0}^2}{A\sigma_{x2}^2} + \frac{\sigma_{s2}^2}{A\sigma_{x2}^2}. \quad (17)$$

The measure we propose for timing should relate the change in proportions of any stock solely to the market signal and its impact on that stock. Timing can be thought of as:

$$E[\gamma_1 (x_1 - s_1) + \gamma_2 (x_2 - s_2)] - [E(\gamma_1) E(x_1 - s_1) + E(\gamma_2) E(x_2 - s_2)]. \quad (18)$$

The individual stock signal has been removed from earnings so that only timing remains. While, as one will shortly show, the above measure is correct, the form of eq. (18) is not observable. To implement this measure we

\[15\] In the popular parlance of security analysis, this is known as top-down security analysis. An alternative model is bottom-up analysis. In this case security analysts make forecasts on individual stocks, and market forecasts are developed from the aggregate of these forecasts. In this latter case, after developing $s_0$, it could be removed from each individual forecast to produce the individual $s_{ot}$. 

have to recast (18) in terms of a set of observable variables. The above expression can be written as

$$\text{Cov}\{\gamma_1, (x_1-s_1)\} + \text{Cov}\{\gamma_2, (x_2-s_2)\}.$$  

This is mathematically equivalent to\(^{16}\)

$$\text{Cov}(\gamma_1, (\beta_1/\beta_2)x_2) + \text{Cov}(\gamma_2, (\beta_1/\beta_2)x_1).$$

This form of the measure is not only observable, but it has intuitive appeal. It points out that the only reason the fraction of wealth invested in any stock should change with the return on a second stock is because they are both affected by a common timing signal. Adjustment by the ratios of the betas is necessary to correctly incorporate the differential sensitivity (of the two stocks) to the market signal. Substituting the values for \(y_1\) and \(y_2\), and \(x_1\) and \(x_2\) in this expression yields

$$d_T = \frac{\beta_1^2 \sigma_{s_0}^2}{A\sigma_{e_1}^2} + \frac{\beta_2^2 \sigma_{s_0}^2}{A\sigma_{e_2}^2}.$$  \(19\)

for the manager who perceives a signal \(s_0\), and correctly acts on it. If there is no perceived signal, this equals zero. If the manager acts perversely on any timing signal \(s_0\), this becomes

$$d_T = -\frac{\beta_1^2 \sigma_{s_0}^2}{A\sigma_{e_1}^2} - \frac{\beta_2^2 \sigma_{s_0}^2}{A\sigma_{e_2}^2}.$$  

So, our timing measure correctly identifies the manager's ability to act on \(s_0\). Note that, unlike conventional measures, the sign of the timing measure only depends on the response to market-wide signals. With conventional measures the response to unique security signals can be picked up in the timing measure.

The measure of selectivity is simply the overall payoff to management, eq. (17), minus the reward to timing, eq. (19), or

$$d_s = \sigma_{s_1}^2/A\sigma_{e_1}^2 + \sigma_{s_2}^2/A\sigma_{e_2}^2.$$  \(20\)

Once again this measure has the properties we desire. It is independent of \(s_0\). If the manager fails to act on individual signals, it equals zero. If the manager acts correctly for both stocks, it is positive. If the manager acts

\(^{16}\)This follows from the definition of \(x_1\) and \(x_2\), together with the assumption that \(s_0, s_1, s_2\) are all uncorrelated.
perversely for both stocks, it is negative. Finally, if the actions are correct for one and perverse for the other, it can be positive or negative depending on the impact of the actions on the portfolio return. Thus, these measures separate market signals and security signals. Furthermore, they are easily calculated by an outside party who has access to portfolio composition but not to the information signals on which managers act.

While these measures have been developed for the two-security case, they can easily be generalized to an \( N \)-security case:

\[
d_T = \frac{1}{N - 1} \sum_{i=1}^{N} \sum_{j=1 \atop j \neq i}^{N} \text{Cov}(\gamma_i, (\beta_i/\beta_j) x_j). \tag{21}
\]

The overall return due to timing and selectivity is simply

\[
d_o = E \left( \sum_{i=1}^{N} \gamma_i x_i \right) - \sum_{i=1}^{N} E(\gamma_i) E(x_i). \tag{22}
\]

The selectivity measure is simply the overall return minus the timing measure. Substituting in for \( \gamma_i \) and \( x_i \), timing is equal to

\[
d_T = \sum_{i=1}^{N} \beta_i^2 \sigma_{\delta i}^2 / A \sigma_{\delta i}^2, \tag{23}
\]

and selectivity is equal to

\[
d_S = \sum_{i=1}^{N} \sigma_i^2 / A \sigma_i^2, \tag{24}
\]

Eqs. (23) and (24) are interesting because they show that the selectivity and timing measures derived have the economic characteristics we desire. Eqs. (21) and (22) are important because institutions which have data on the composition of portfolios over time can measure the selection and timing ability of any manager.

5. Conclusion

Recently, standard measures of performance evaluation have come under increased attack. Dybvig and Ross have shown that, if the manager possesses private information, then outsiders (parties not privy to that information) can reach an incorrect evaluation of management ability. We have shown that, under the assumptions of Dybvig and Ross, if an outsider has access to
portfolio composition, portfolio evaluation can be done in a consistent and correct manner. We have argued that at least one important group of outsiders, pension-fund managers, are in a position to gain access to the transaction data necessary. We have been able to develop measures for identifying both selectivity and timing ability when a manager possesses both or neither. Two different sets of measures were developed because the meanings of timing and selectivity are ambiguous. In section 3, we follow common convention in identifying timing with a switch between stocks and T-bills. In section 4, we recognize that signals about individual stocks can affect this allocation and develop measures that separate out the impact of information about all stocks from information about individual stocks. The choice between the measures proposed in section 3 and those proposed in section 4 is largely a matter of taste. Both lead to the same conclusion about overall performance. They differ in allocating performance between selectivity and timing. All of the measures we propose can be computed by an outsider who has access to transaction data. This data is available to an important group of outsiders. It allows this group of outsiders to reach the same conclusions about management performance which they would reach, had they had access to all of the inside information which management employed.

References
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