

## Tax and Liquidity Effects in Pricing Government Bonds

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### ABSTRACT

Daily data from interdealer government bond brokers are examined for tax and liquidity effects. We use two approaches to create cash flow matching portfolios of similar securities and look for pricing discrepancies associated with liquidity or tax effects. We also look for the presence of tax and liquidity effects by including a liquidity term when fitting a cubic spline to the after-tax yield curve. We find evidence of tax timing options and liquidity effects. However, the effects are much smaller than previously reported and the effects of liquidity are primarily due to high volume bonds with long maturities.

CASH FLOWS OF NON-CALLABLE Treasury securities are fixed and certain, simplifying the pricing of these assets to a present value calculation using the current term structure of interest rates. It is well known, however, that pricing errors exist when government securities are priced by discounting the cash flows by any set of estimated spot rates even for non-flower bonds without option features. A number of theories have been offered to explain these pricing discrepancies. Explanations include economic influences such as liquidity effects, tax regime effects, tax clienteles, tax timing options, and the use of bonds in the overnight repurchase market. Another potential source of pricing errors is data problems that arise from nonsynchronous trading and the fact that the prices found in common data sets may be estimates from a model or the best guess of a trader. It is difficult to distinguish between these various explanations because securities rarely exist that are affected by only one of the effects. For example, illiquid securities are likely to be associated with pricing errors due to nonsynchronous trading and may also have coupons that would lead to considerable tax effects. In addition, it is difficult to sort out the effect of model prices or dealer estimates on pricing errors.

The purpose of this study is to try to separate out the various factors that lead to errors in the pricing of government securities. We examine a new

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data set from the interdealer market for Treasury securities which provides us with three advantages over previous work. First, we have access to trading volume for each Treasury security. Trading volume is a more robust measure of asset liquidity than other proxies used in previous studies such as age and type of security. Second, the data are recorded on a daily basis, which provides us with a large number of observations within the same economic environment. Previous authors have used more limited data, and this has led them to study only one potential source of pricing error. Our much larger data set allows us to distinguish between the effects of various economic influences, such as liquidity, tax effects, and repo specials. Third, access to daily data also enables us to focus on more recent price data. As discussed later, the accuracy of bond price data has improved substantially in recent years. Studies using monthly data include observations over time periods in which the price data are less accurate in order to obtain a large number of observations. Having many cross sections of accurate data allows us to reduce the impact of data problems on measurements of the effects of taxes and liquidity. Thus, having access to daily data from the interdealer broker market gives us a unique opportunity to examine the effects of liquidity and taxes on a broad range of maturities.

Our evidence suggests that liquidity is a significant determinant in the relative pricing of Treasury bonds, but its role is much less than previously reported and primarily associated with highly liquid bonds with long maturities. In addition, we confirm the work of Green and Ødegaard (1997) in that we find tax clienteles do not substantially impact bond prices. However, we stop short of declaring that taxes are irrelevant in the Treasury market. Our arbitrage tests provide evidence that tax timing options do have value, and we also discuss the shortcomings of procedures to estimate the tax rate of the marginal investor. Nonetheless, we find the effects of both liquidity and taxes to be quite small, which suggests that a broader sample can be used to estimate empirical term structure models. Practitioners fitting the yield curve commonly restrict their data sets to bonds they believe have small liquidity and tax effects. Our evidence suggests many more bonds can be included, which should reduce estimation error.

The effect of liquidity on the expected return of stocks is studied by Amihud and Mendelson (1986) and Silber (1991). In the corporate bond market, Fisher (1959) shows that liquidity is one of the determinants of the yield spread between corporate bonds and Treasury securities. In the Treasury market, Amihud and Mendelson (1991), Warga (1992), Garbade (1996), Garbade and Silber (1979), and Kamara (1994) study aspects of liquidity and expected returns. The effects of tax clienteles and the tax rate of the marginal investor in the government bond market are examined by Green and Ødegaard (1997), Litzenberger and Rolfo (1984a), and Schaefer (1982). In addition, Ronn and Shin (1997), Jordan and Jordan (1991), Constantinides and Ingersoll (1984), and Litzenberger and Rolfo (1984b), study the importance of tax timing options. The effect of repo specialness is studied by Duffie (1996) and Jordan and Jordan (1997).

The paper is divided into five sections. In the first section we discuss the details of the data. The second section discusses the data used in previous studies and compares our data set to prior data sets. Since we have access to a robust measure of liquidity, the third section examines the reasonableness of the proxies used by others for measuring liquidity. The fourth section examines which factors are important in explaining pricing discrepancies by using arbitrage tests and errors from empirical term structure models. The fifth section reports our conclusions.

## I. The Data

The primary data set contains trade prices of Treasury bills, notes, and bonds in the government interdealer market. According to the Federal Reserve Bulletin, roughly 60 percent of all Treasury security transactions occur between dealers. Treasury dealers trade with one another through intermediaries called interdealer brokers. Dealers use intermediaries rather than trading directly with each other in order to maintain anonymity. Dealers leave firm quotes with brokers along with the largest size at which they are willing to trade. The minimum trade size is one million dollars, and normal units are in millions of dollars. Six of the seven brokers,<sup>1</sup> representing about 70 percent of the market, use a computer system managed by GovPX Inc. The GovPX network is tied to each trading desk and displays the highest bid and lowest offer across the four brokers on a terminal screen. When a dealer hits the bid or takes the offer, the broker posting the quote takes a small commission for handling the transaction. In addition to current price quotes, the GovPX terminal reports the last trade timed to the nearest second, as well as the cumulative daily volume for each bond. If the bond has not traded that day, GovPX reports the last day the bond traded.

The data set we examine consists of daily snapshot files provided by GovPX. The daily files contain information on the first trade, the high and low trade, and the last trade (prior to 6:00 p.m. EST) stamped to the nearest second, as well as whether the last trade occurred at the bid or offer price. The files also provide daily volume information for each listed security. We have daily data from June 17, 1991, through September 29, 1995. In order to make the data more manageable, for some of the exercises we consider a smaller sample consisting of three subsamples of 90 trading days. The subsamples are taken from different months in different years so that any calendar effects will influence each subsample differently. We report the results for the combined sample unless the results differ across the subsamples. In addition to the snapshot files, GovPX provided us with three consecutive days of bid-ask spread information in the interdealer market at approximately 10 a.m. each day.

<sup>1</sup> The brokers monitored by GovPX are Garban Ltd., EJV Brokerage Inc., Fundamental Brokers Inc., Liberty Brokerage Inc., RMJ Securities Corp., and Hilliard Farber & Co. The one exception is Cantor Fitzgerald, which provides its own direct feed.

## II. Comparison with Other Data Sources

All previous work that has studied tax and liquidity effects has done so using dealer quotes, either directly from the dealers or indirectly through the Center for Research in Security Prices. It is worthwhile to examine the origin of the data, their accuracy, and their comparability with the GovPX data.

For much of CRSP history, bond data were taken from the quote sheets of Salomon Brothers. They were also the principal data source used in studies that acquired data directly from a dealer. Salomon Brothers, like Shearson Lehman and other primary dealers, actively traded only a portion of the available government bonds (albeit Salomon was the most active dealer). Thus, the quotes they provided may reflect dealers' opinions about prices rather than actual trades. In 1988, CRSP changed the source of its bond data to the Federal Reserve Bank of New York (Fed). At the time of the change, CRSP replaced the Salomon data with data from the Fed going back to 1962. The Fed surveys five primary dealers selected at random and creates an equally weighted average of the five bid and ask quotes. Although this method of data collection does average out price noise, it uses data from many dealers who may have little knowledge of actual trades for many of the listed issues and little incentive to gather more information. Aware of the shortcomings of this approach, the Fed has recently changed its method of acquiring price data and now records quotes from the electronic feed used in the interdealer market.

Two considerations affect whether dealer quotes are reliable indicators of market clearing prices. First, the information set available to traders will help determine whether their quotes reflect market clearing conditions. Second, the incentive structure will also affect whether traders spend time to estimate quotes that are close to the prices at which the bonds would actually trade.

The technology was such that until the late 1970s, traders received information over the phone from other traders or interdealer brokers. There was little or no systematic recording of data. In the late 1970s and early 1980s, cathode-ray tube monitors were introduced and information came across terminal screens placed on trading desks by the interdealer brokers, one for each broker. This improvement in technology, along with increased trading in Treasuries, dramatically increased the information set available to traders. However, there remained little systematically recorded data. In June 1991, GovPX Inc. was created to supply a consolidated screen for several of the interdealer brokers. This consolidation improved traders' ability to process information. Furthermore, the information could be fed into computers, which allowed for systematic collection. Along with the consolidation of information on Treasury prices, trading volume increased dramatically. The average daily trading volume in January 1970 was \$2.385 billion. It grew to \$17.091 billion in 1980, and by 1990 the daily average was \$117.177 billion. Thus, in recent years all traders are likely to observe current prices.

The accuracy of bid and ask dealer quotes used in previous studies is also dependent on the motivation of traders to supply accurate estimates. Interviews with Salomon Brothers traders of the 1970s and 1980s reveal that during that time they only estimated bid prices.<sup>2</sup> Likewise, interviews with traders at other primary dealers indicate they also estimated only bid prices or the midpoint between the bid and ask. At the end of every day, traders estimated prices for all Treasury securities. These prices were used for internal inventory valuation purposes and were also supplied to their customers as a nonbinding indication of a price range. The traders we interviewed stated that they devoted effort only when estimating the prices of bonds held in their inventory, along with very active issues where dealers were concerned about supplying prices near those at which they might be willing to trade. Prices for illiquid bonds not in their inventory were quickly recorded at rough premiums or discounts to active issues.

What can be learned from this discussion? First, bid-ask spreads used in studies of liquidity were not estimated by traders and were not used by Salomon Brothers when valuing inventory, but instead were clerically added to the data set afterward. Second, illiquid bonds, including those with high and low coupons used in tax studies, were priced by traders—often without observing recent trades. Furthermore, these were also the bonds for which less care was used to estimate prices since they were less likely to be part of each dealer's inventory. Thus, we would expect large estimation errors for these bonds, and that recorded prices reflect what a trader believes is the impact of tax and liquidity on bond prices. The observed variation in dealer estimates lends support to this argument. Sarig and Warga (1989) compare prices found on the quote sheets of two major Treasury dealers, Shearson Lehman and Salomon Brothers (from the Center for Research in Security Prices file). They find that more than 20 percent of the notes and 60 percent of the bonds have prices that differ by more than 20 basis points across the two dealers. Moreover, they show that this inaccuracy is related to variables like liquidity in a way that could seriously bias the results of studies using dealer quotes.

To sum up, the lack of accurate historical price data calls into question the magnitude of liquidity and tax effects found in previous studies. Recent work by Green and Ødegaard (1997) finds evidence of a change in the tax rate faced by the marginal investor when looking at data before and after 1986. This is attributed to changes in tax regulation in 1984 and 1986, but may also be partially explained by differences in the accuracy of the price data across these periods. One of the advantages of the GovPX data set is the availability of daily data, which provides us with many cross sections of

<sup>2</sup> Coleman, Fisher, and Ibbotson (1992) report that until about 1979, prices on dealers' quotations sheets were honored until noon the next day for small transactions. After that, quotes were indicative and although bid prices were used for internal purposes, ask prices were arbitrary. Additionally, they state that during this time period the Fed survey data also used non-binding quotes. The bid price was an average of the surveyed bid quotes, but the ask price was the bid plus a "representative" spread.

accurate data to analyze. Previous studies that utilize CRSP data include observations over time periods in which the price data are less accurate in order to obtain a large number of observations. However, one drawback to the GovPX data is that only recent data are available. Hence we are unable to analyze how markets have changed.

A final consideration is the use of transaction prices versus bid-ask quotes. GovPX contains information on trade prices, whereas CRSP contains bid-ask quotes. Given the increased size of the Treasury market and improvements in the dissemination of price information in the recent past, we would not expect there to be large differences between our trade prices and the quotes contained in CRSP. However, examining trade data does provide a way of screening out stale or model quotes (i.e., quotes for bonds that do not trade each day). On the other hand, trade prices are subject to nonsynchronous trading.<sup>3</sup> Trade prices also are recorded at either the bid or the ask, and thus contain noise attributed to the bid-ask spread.<sup>4</sup>

### III. Proxies For Liquidity

One of the most common proxies for liquidity is the bid-ask spread. The rationale is that dealers require greater compensation for maintaining inventories of illiquid assets, and this results in larger bid-ask spreads for illiquid securities. However, as mentioned previously, the bid-ask spreads listed in the CRSP data are not market data but are merely representative spreads.<sup>5</sup> Thus, the magnitude, characteristics, and determinants of bid-ask spreads in the Treasury market have not been reliably examined before. Table I provides information on the bid-ask spread for the GovPX data. Although we have data for only three days, the bid-ask spread on one day is highly related to the bid-ask spread on the other two days with a simple correlation greater than 0.96. Thus, the bid-ask spread on any one day seems reflective of general conditions, at least over a short period of time. The average bid-ask spread varies from four-tenths of a cent for the lowest decile to 12.5 cents per \$100 for the highest decile, with an average of 5.3 cents.<sup>6</sup>

<sup>3</sup> Balduzzi, Elton, and Green (1997) examine intraday price changes around economic announcements. They find that a considerable portion of daily price changes can be attributed to the release of economic news. Moreover, the impact of the economic news usually occurs within one minute after the announcement and never more than 30 minutes after. Since the last observed trade for each bond is almost always after the last announcement in any day, we would not expect nonsynchronous prices to be an important factor in pricing errors.

<sup>4</sup> Using the spline approach described in Section IV, we compare the pricing errors obtained from fitting the CRSP and GovPX data over the period during which GovPX has existed. Using an identical set of bonds, the correlation of the pricing errors is 0.78. However, fitting the average of the bid-ask quotes in CRSP results in slightly smaller pricing errors.

<sup>5</sup> See Coleman, Fisher, and Ibbotson (1992) and our discussion in the preceding section.

<sup>6</sup> Quote observations are examined if both a bid and ask price are reported. Some of the reported prices for bonds that did not trade are indicative quotes. Removing these observations has little effect on the results.

**Table I**

**Bid-Ask Spreads in the Interdealer Market for Treasury Securities**

Data on bid-ask spreads and trading volume from the interdealer market for Treasury securities are obtained from screen output provided by GovPX Inc. Information on bills and bonds is aggregated over the period from June 11, 1996 through June 13, 1996. Panel A reports the mean and percentiles for the observed bid-ask spreads. Panel B reports the results of regressing bid-ask spreads on security characteristics. Bond is a dummy variable that is 1 if the issue is a note or bond, 0 if it is a bill. Maturity is the number of years left until maturity. Volume is the natural log of the daily trading volume for those securities that traded, and the natural log of the number of days since the security traded for those that did not trade. *p*-values are reported in parentheses below the coefficients.

Panel A: Descriptive Statistics for Bid-Ask Spread		
	Percentile	Sample Mean
10th	0.003906	
20th	0.007813	0.052945
30th	0.015625	
40th	0.031250	
50th	0.062500	
60th	0.062500	
70th	0.062500	
80th	0.078125	
90th	0.125000	
100th	0.125000	
Panel B: Regressions of Bid-Ask Spread on Security Characteristics		
	Traded Securities	Not Traded Securities
Constant	0.0244 (0.0000)	0.0049 (0.1079)
Maturity	0.0044 (0.0000)	0.0039 (0.0000)
Bond	0.0029 (0.4123)	0.0313 (0.0000)
Volume	-0.0046 (0.0000)	0.0014 (0.0107)
Number of obs.	190	397
<i>R</i> <sup>2</sup>	0.8131	0.7944

The existence of bid-ask spreads introduces price error in trade data because observed trade prices can be either buyer or seller initiated. If trades occur randomly at bid or ask prices we would expect the size of the average error to be about 2.75 cents when examining trade data. Using our empirical term structure model that adjusts for both taxes and liquidity, the estimated root mean squared error (RMSE) is about 13.6 cents, so the bid-ask spread accounts for about 20 percent of the RMSE.

Panel B of Table I shows the results of two regressions that examine how bid-ask spread varies with security characteristics. The results are reported separately for securities that trade on the day of the analysis and securities

that do not trade that day. Several variables are used to explore how bid-ask spreads vary across securities. The variable *Bond* is a dummy variable that is 1 if the instrument is a bond and 0 if it is a bill. For bonds and bills that do trade, the variable *Volume* is the natural log of the cumulative trading volume. For bonds and bills that do not trade, *Volume* is the natural log of the number of days since it last traded. These variables along with years to maturity explain about 80 percent of the difference in bid-ask spreads across securities. The bid-ask spread is negatively related to volume and positively related to the length of time since the last trade. Furthermore, the bid-ask spread increases with maturity and is larger for bonds than for bills.<sup>7</sup>

In addition to the bid-ask spread, several other variables are used to measure liquidity. For instance, Amihud and Mendelson (1991) and Kamara (1994) examine Treasuries with less than six months to maturity and use the type of security (bond or bill) as a liquidity proxy.<sup>8</sup> In all cases these proxies are used because volume data are unavailable. The GovPX data set provides us with a robust measure of liquidity, which enables us to examine the reasonableness of other proxies for liquidity. Table II contains volume information for bills and bonds with less than six months to maturity. The columns represent average daily trading volumes over one-day, five-day, and ten-day measurement intervals, as well as the percentage of bonds and bills that did not trade. Over a one-day measurement interval, 85 percent of the different issues of bills traded while 71 percent of the different issues of bonds traded. Over a five-day interval, 99.6 percent of the bills traded while 95 percent of the bonds traded. Thus, bills did trade more frequently. Panel B of Table II shows the volume percentiles of bills and bonds (all numbers are in millions of dollars face value). Over a ten-day interval, the median trade size in the bill market is \$109 million per day and in the bond market is \$17 million per day. However, the relationship is not perfect. The top 10 percent of bonds in trading volume exceeds the lowest 10 percent of bills; thus liquid short term bonds trade more frequently than illiquid bills.<sup>9</sup> Overall, Table II provides evidence that security type is a reasonable liquidity proxy for maturities of less than six months.

Although security type is one of the most often used proxies for liquidity, other variables are used as well. Table III shows the results of a regression of log volume on a series of variables used by others as measures of liquidity. As mentioned above, the bond-bill classification is used by Amihud and Mendelson (1991), Kamara (1994), and Garbade (1996). The age of a security is

<sup>7</sup> The bond dummy variable is not significant in the sample of traded bonds. In the sample of bonds that did not trade, the bond variable may be proxying for volume, thus its importance is unclear.

<sup>8</sup> Amihud and Mendelson use transaction costs as a measure of liquidity. They find that bills have lower transaction costs than notes or bonds and this leads them to use instrument type as an indirect proxy for liquidity.

<sup>9</sup> The data we examine are from the interdealer market. Other investigators have used data in the retail market. Although the volume patterns need not be the same, they should be closely related.

**Table II**  
**Volume Data for Treasury Bills and Bonds**  
**with Less than Six Months to Maturity**

Data on trading volume from the interdealer market for Treasury securities are obtained from GovPX Inc. The reported numbers are for daily volume of all listed noncallable Treasury securities with less than six months to maturity. Panel A reports the percentage of days the securities traded. Panel B reports the percentiles of trading volume. Statistics for the 5- and 10-day intervals are obtained from overlapping observations of 5 and 10 trading days. The sample covers June 17, 1991 through September 29, 1995.

Panel A: Trading Percentages					
Measurement Interval	Bills		Bonds		Percent Traded
	Total Observations	Percent Traded	Total Observations	Percent Traded	
1 Day	30871	85.34	20666	70.80	
5 Days	25766	99.60	19998	94.74	
10 Days	23327	99.97	19162	96.91	

Panel B: Distribution of Volume (\$ Millions)						
Percentile	Bills			Bonds		
	1 Day	5-Day Average	10-Day Average	1 Day	5-Day Average	10-Day Average
10th	0	19.6	25.6	0	1.0	2.2
20th	9	37.8	42.6	0	3.8	5.6
30th	27	57.4	63.4	1	6.8	8.9
40th	51	79.6	86.0	2	10.4	12.4
50th	82	104.4	108.6	6	14.8	16.6
60th	124	133.2	135.6	11	20.0	21.7
70th	186	170.6	168.2	20	26.8	27.7
80th	293	231.8	218.3	34	36.4	35.9
90th	607	399.0	333.2	63	52.4	49.3
95th	1124	752.2	565.3	96	69.2	63.8
100th	8215	2756.2	1776.1	4499	925.4	504.8

utilized by Sarig and Warga (1989), and Warga (1992) proxies liquidity by indicating whether or not an issue is on-the-run (the most recently issued security of a particular maturity). Additionally, since Ederington and Lee (1993) and Harvey and Huang (1993) have results which suggest that volume differs over the week, we include dummy variables for each weekday. The set of variables used by others explains a relatively high proportion of the variation in volume across securities. About 45 percent of the variation in volume is explained by the independent variables, and all variables except the Monday dummy variable are significant. However, there is a fair amount of variation in volume that is not explained by the other measures of liquidity, which suggests that there may be aspects of liquidity not captured by previously used proxies.

**Table III**  
**Regression Results of Volume on Liquidity Parameters**

Data from the interdealer market for Treasury securities are obtained from GovPX Inc. The table reports the results of an ordinary least squares regression of the natural log of daily trading volume on the independent variables.

$$\ln(\text{Vol}) = b_0 + b_1 \text{ Bill} + b_2 \text{ Active} + b_3 \text{ Age} + b_4 \text{ Monday} \\ + b_5 \text{ Tuesday} + b_6 \text{ Wednesday} + b_7 \text{ Thursday} + e.$$

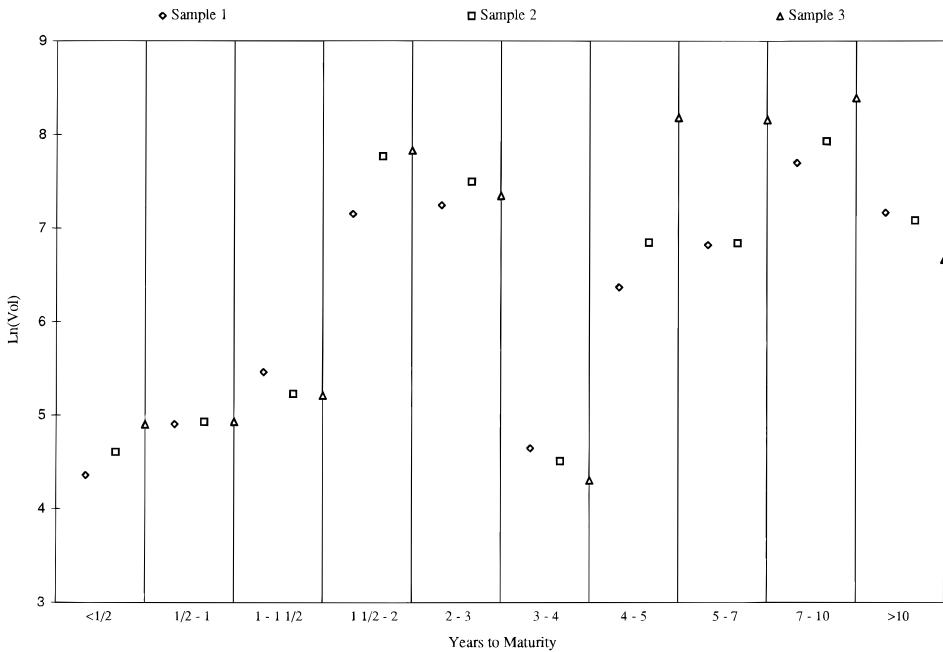
The sample contains information on all noncallable Treasury securities. Bill is 1 if the security is a bill, and 0 otherwise. Active is 1 if the issue is on-the-run, and 0 otherwise. Age is the number of years since issuance. The day-of-the-week dummies are 1 if the observation occurs on that day, and 0 otherwise. Sample 1 covers October 1, 1991 through February 2, 1992, sample 2 covers March 1, 1993 through July 7, 1993, and sample 3 covers May 23, 1995 through September 29, 1995. The results reported are for the combined sample.

	Coefficient	<i>t</i> -Statistic	<i>p</i> -Value
Constant	3.115	175.158	0.000
Bill	0.868	46.315	0.000
Active	3.847	128.928	0.000
Age	-0.182	-68.317	0.000
Monday	0.023	1.006	0.314
Tuesday	0.236	10.994	0.000
Wednesday	0.304	14.053	0.000
Thursday	0.265	12.159	0.000
<i>R</i> <sup>2</sup>	0.453		
Number of obs.	40631		

To provide a better understanding of how liquidity varies across the term structure, Figure 1 shows the relationship between daily trading volume and maturity for bonds. There is not a monotonic relationship over the full maturity range. Trading volume increases with maturity from six months to two years. Beyond two years, volume is roughly constant and the same as that of bonds with two years to maturity. Overall, we find that the liquidity measures used by others are related to volume, but none are highly correlated with volume across all maturities, and using lesser proxies could introduce substantial error.

#### IV. Pricing Errors in Present Values

Although utilizing the GovPX data provides us with an accurate measure of market clearing prices, errors still exist when cash flows are discounted using estimated spot rates. Nonsynchronous trading and the existence of random pricing errors are possible explanations that we will explore again later in this section. However, there are economic influences that could also



**Figure 1. 95th Percentile of the log of daily trading volume grouped by maturity.** The data points in each maturity range represent the 95th percentile of log volume for all noncallable bonds that fall into that maturity range. Sample 1 is from October 1, 1991 through February 11, 1992, sample 2 covers March 1, 1993 through July 7, 1993, and sample 3 covers May 23, 1995 through September 29, 1995.

lead to pricing errors, such as liquidity effects, tax effects, and cross-sectional variation in the demand for assets based on their use as collateral in repurchase agreements.

Theory suggests that illiquid bonds will offer higher returns than similar, more liquid bonds. As Amihud and Mendelson (1991) argue, the bid-ask spread is part of the cost of trading. In order to compensate marketmakers for making a market in illiquid assets, and possibly reflecting a lack of trade information to discern the market clearing price of infrequently traded bonds, bid-ask spreads are larger for illiquid bonds. Thus, to provide the same return after paying transaction costs, illiquid bonds must offer a higher return before transaction costs. Since our pricing formula does not include transaction costs, illiquid bonds should trade at prices below the price estimated using the present value formula.

Taxes may also affect the relative prices of bonds and lead to errors in estimated prices. One way for this to occur is through the presence of tax clienteles. Investors in different tax brackets may desire bonds with different characteristics (see Schaefer (1982)). If the marginal investors for two different bonds are taxed at different rates, the relative prices of these bonds

will be affected. Another way in which taxes can affect bond prices is through tax timing options. Tax timing options are associated with the value of being able to time the sale of a bond to optimize the tax treatment of capital gains or losses (see Constantinides and Ingersoll (1984)). Moreover, it is important to note that even if the ordinary income and capital gains tax rates are the same for all investors, taxes may still enter into the relative prices of bonds. For instance, consider three bonds with different coupons all maturing on the same day. If all three bonds are discount bonds, or all three are premium bonds, then the ratio of bonds one and three necessary to match the cash flows of bond two are the same regardless of whether the cash flows being matched are before or after taxes. However, if bonds one and two are discount bonds and bond three is a premium bond, there may be no combination of bonds one and three that will exactly match the after-tax cash flows of bond two, due to the constant yield method of amortizing the premium of bond three. Thus, if the tax rate of the marginal investor is positive, taxes may have an effect on the relative prices of bonds.

In addition to tax and liquidity effects, there may be shifts in demand or supply for individual bonds that affect their prices relative to other bonds. Duffie (1996) argues that securities that are on special in the repo market (i.e., they have overnight borrowing rates that are below the general collateral rate) will trade at a premium over similar assets that are not on special. Jordan and Jordan (1997) examine repo specials and find that they do significantly impact bond prices. However, their evidence reveals that repo specials alone do not entirely explain the premiums associated with on-the-run issues, suggesting that the high liquidity of these issues has value in itself. Overnight repurchase rates were not reported by GovPX during the time period of our sample, and we are therefore unable to determine which bonds were on special. However, specialness is highly correlated with volume, and may be a partial explanation for any volume effects we find.

In this paper we use two types of tests for understanding the determinants of pricing errors in present values—arbitrage tests and an examination of deviations from a term structure fit. We examine each in turn.

### *A. Arbitrage Tests*

Tests that are based on the principle of no arbitrage are extremely powerful because they do not rely on a valuation model and require only minimal assumptions about preferences. Arbitrage style tests have a long history in examining the determinants of government bond prices (see Litzenberger and Rolfo (1984b), Jordan and Jordan (1991), and Ronn and Shin (1997)). However, these authors examine quite small samples (30 to 40 observations), and thus are constrained to look exclusively at tax effects. Our daily data and access to trading volume allow us to use triplets to examine both tax timing and liquidity effects.

The arbitrage test commonly used to examine tax timing, tax clientele, and tax regime effects involves the use of bond triplets, three bonds with the same maturity but different coupons. Assuming a zero tax rate for the mo-

ment, for each triplet let  $C_i$  and  $P_i$  be the coupon and price of bond  $i$ , where  $i = 1,2,3$  and the bonds are arranged in ascending order by coupon. The law of one price states that

$$P_2 = xP_1 + (1 - x)P_3, \tag{1}$$

where

$$x = \frac{C_2 - C_3}{C_1 - C_3}.$$

Equation (1) must hold because in the proportions shown the cash flows are the same for bond two and the portfolio of bonds one and three. When taxes are present, equation (1) needs to be modified. First, if bondholders are taxed when capital gains and losses are realized, then a tax timing option may be present. If the portfolio always moves exactly as bond two, they would be equally desirable. However, as long as there are states of the world where they move differently, the portfolio of bonds one and three is more valuable than bond two, and a tax timing option exists. This is an application of the principle that a portfolio of options is more valuable than an option on the portfolio. When a tax timing option is present, equation (1) will be an inequality.

To examine the effects of tax timing options, it is necessary to eliminate other tax influences by ensuring that the pretax and posttax cash flows are the same. Since premium and discount bonds are treated differently for tax purposes, the effect of tax timing options is unequivocal only if all three bonds are premium or discount bonds. Because of the lack of a significant number of discount triplet observations, we focus on premium triplets. The amortization of bond premiums also needs to be considered. The Tax Reform Act of 1986 altered the amortization of bonds. Bonds issued before September 28, 1985 (old bonds) may be amortized using the straight-line method, which makes them preferable to bonds issued after that date (new bonds) which must use the constant yield method. Thus, initially we examine only triplets where all three bonds were issued before or after September 28, 1985. Finally, because the tax timing involves controlling the year of the gain or loss, we do not include bonds with less than one year to maturity.

The measure we use to quantify the tax timing and liquidity effects in bond triplet prices is the difference between the price of bond two and the replicating portfolio of bonds one and three. In equation form this difference is:

$$D = P_2 - (xP_1 + (1 - x)P_3). \tag{2}$$

If there is a tax timing option, bond two should be less expensive than the portfolio of bonds one and three and  $D$  should be less than zero. Table IV reports our results.<sup>10</sup> For triplets consisting of new bonds (the first row of

<sup>10</sup> The hypothesis tested is that the percentage of triplet observations with  $D$  less than 0 is equal to 1/2 using the property that  $2(\sin^{-1}\sqrt{p} - \sin^{-1}\sqrt{0.5})/\sqrt{n}$  is distributed standard normal in the limit, where  $p$  is the proportion of observations where  $D$  is greater than 0 and  $n$  is the number of observations (Litzenberger and Rolfo (1984b)).

Table IV

**Evidence of Tax and Liquidity Effects in Bond Triplet Prices**

Data from the interdealer market for Treasury securities are obtained from GovPX Inc. for June 17, 1991 through September 29, 1995. Bond triplets consist of three bonds with differing coupon rates but the same maturity. Tax type S denotes bonds issued before September 28, 1985 (old), for which premiums may be amortized using the straight-line method. Tax type C denotes bonds issued after September 27, 1985 (new), for which the constant yield method must be used. CCS represents a triplet in which the two bonds with smaller coupons are new, and the bond with the highest coupon is old. Volume type H denotes volume observations greater than the median and L denotes observations less than the median. When no volume type is listed, all possible observations of the tax type are examined.  $P_1$ ,  $P_2$ , and  $P_3$  are the prices of the bonds in ascending order of coupon rate.  $x$  is the portfolio weight chosen so that the combination of bonds one and three matches the cash flows of bond two.  $D$  represents the price deviation between the price of bond two and the replicating portfolio of bonds one and three.

$$D = P_2 - (xP_1 + (1-x)P_3).$$

The hypothesis tested is that the percentage of triplet observations with  $D$  less than 0 is equal to 1/2 using the property that:

$$2(\sin^{-1}\sqrt{p} - \sin^{-1}\sqrt{0.5})/\sqrt{n}$$

is distributed standard normal in the limit, where  $p$  is the proportion of observations with  $D$  less than 0 and  $n$  is the total number of observations (Litzenberger and Rolfo (1984b)). "Number Arb. Opp." attempts to capture the number of arbitrage opportunities, which are defined as observations in which the triplet trades are within one half hour of each other and the absolute value of  $D$  is greater than 0.01. When  $D$  is less than (greater than) 0, bid (ask) prices are used for bonds one and three and the ask (bid) price is used for bond two. When these prices are not available, we adjust the observed trade price by a conservative bid-ask spread of 0.05. Panel A reports the statistics for triplets of all new bonds, and Panel B reports the statistics for triplets of all old bonds. Panel C reports the statistics for triplets composed of both old and new bonds, and Panel D reports the statistics when the prices of "old" bonds are adjusted for the average additional value of being able to amortize the premium using the straight-line method over the constant yield method.

Bond Triplet Type		Average	Percent			Number	
Tax Type	Volume Type	$D$	$D < 0$	$t$ -stat	$p$ -val	Arb. Opp.	obs
Panel A: All New Bonds							
CCC		-0.0556	82.82	10.788	0.0000	5	227
CCC	LHL	-0.0142	50.00	0.000	0.5000	0	6
CCC	HLH	-0.0679	86.84	5.107	0.0000	1	38
Panel B: All Old Bonds							
SSS		-0.0285	59.09	0.858	0.1956	0	22
Panel C: Unadjusted Mixture of Old and New Bonds							
CCS		-0.0230	68.88	17.597	0.0000	31	2066
CCS	LHL	-0.0141	63.05	3.763	0.0000	8	203
CCS	HLH	-0.0270	70.91	6.399	0.0000	2	220
Panel D: Adjusted for Tax Regime							
CCS		-0.0218	68.15	16.886	0.0000	31	2066
CCS	LHL	-0.0135	62.56	3.618	0.0002	8	203
CCS	HLH	-0.0257	70.46	6.251	0.0000	2	220

Panel A), the portfolio is more expensive than bond two in 83 percent of the 227 observations, with an average price difference of six cents per \$100 face value. For triplets that include only old bonds, in 60 percent of the 22 observations the portfolio is more costly, with an average difference of three cents. These results are similar to those reported by others. Our access to superior data and a much larger number of observations (others have 30 to 40 observations) does not refute the sign or magnitude of pricing differences between bond triplets.

However, our much larger sample does allow us to explore whether these results could be due to liquidity rather than tax timing effects. In order to look for evidence of liquidity effects, bonds are separated into high and low volume groups based on whether the daily volume for each bond is above or below the median volume for all bonds on that day. Less liquid bonds should have lower prices and offer higher returns. A considerable difference in liquidity between bond two and the bonds in the portfolio should alter the relationship between their prices. When bond two is less liquid than the portfolio (designated by HLH in Table IV), then *ceteris paribus* we would expect bond two to be cheaper and  $D$  to be more negative. On the other hand, when bond two is more liquid than the portfolio (designated by LHL in Table IV), we would expect  $D$  to be less negative or positive if liquidity effects dominate the tax timing effects. Panel A of Table IV shows the results. In both cases, sorting by liquidity affects the relationship in the direction we would theorize. However,  $D$  is always negative, indicating that both tax timing and liquidity effects are present. The difference caused by liquidity is approximately 5 cents per \$100 face value.<sup>11</sup>

By recognizing the different tax treatment of bonds issued before and after September 29, 1985 (old and new bonds), we can dramatically expand our sample size, which is important for distinguishing between the effects of liquidity and taxes. The type of triplet for which we have a substantial number of observations contains two new bonds and one old, with bond three being the old bond. Since old bonds have a tax advantage, examining triplets in which the highest coupon bond is old should result in an increase in the price of bond three and a more negative  $D$ . Panel C in Table IV analyzes this case. We have 2,066 observations. The average difference in price between bond two and the replicating portfolio is approximately three cents, with the portfolio being more expensive 69 percent of the time.

Although the average  $D$  is negative, it is actually closer to zero than in the all old or all new triplets. This is inconsistent with the tax advantage of old bonds being priced. Using a  $t$ -test, we find that the average  $D$  for triplets

<sup>11</sup> At the suggestion of the referee, we also pool the triplet observations together and regress  $D$  on dummy variables for the three cases we consider and a liquidity parameter that is the weighted average of volumes for bonds one and three over the volume for bond two. We find that CCC and CCS are significantly less than zero, and the magnitudes of the coefficients are similar to the average  $D$ 's listed in the table. The liquidity term is not found to be significantly different from zero.

consisting of two new bonds and one old bond is significantly (at the 0.01 level) greater than the average  $D$  for triplets consisting of all new bonds. In other words, we find no evidence that the difference in tax treatment of old and new bonds is reflected in market prices, which is in contrast to the findings of Ronn and Shin (1997). One explanation for this difference is that they examine triplet data from 1985 through 1990, whereas our data are from 1991 through 1995. Moreover, given their time frame, they compare triplets of all old bonds to triplets containing one or more new bonds, whereas we compare triplets of all new bonds to triplets containing one old bond. The lower part of Panel C splits the sample by liquidity to see if the results could be due to liquidity differences. Once again, changes in  $D$  are consistent with liquidity and tax timing effects. When bond two is more liquid than the portfolio,  $D$  is less negative, as we would expect. Likewise, when bond two is less liquid than the portfolio,  $D$  increases but is still negative. Evidence from a  $t$ -test indicates that the average  $D$  for the HLH group is significantly less than the average  $D$  for the LHL group at the 0.01 level.

By adjusting the price of the old bonds by the expected discounted value of its preferential tax treatment, we can compare this adjusted price with the prices of the other two bonds on a common tax basis.<sup>12</sup> The results are shown in Panel *D* of Table IV. As we would expect,  $D$  becomes less negative after decreasing the price of bond three by the value of the tax advantage. None of the previous results change.

The second to last column provides information on the number of potential arbitrage opportunities. The existence of arbitrage opportunities is interesting because it provides evidence that either tax clienteles have an impact on the prices of bonds, or markets are not efficient. Table IV reports the number of observations in which the last trade prices of the three bonds are recorded within a half hour of each other and the difference in price between bond two and the portfolio is greater than 1 cent when using the appropriate bid or ask prices. Specifically, when  $D$  is less than (greater than) zero, the ask (bid) prices are examined for bonds one and three and the bid (ask) price is used for bond two. When necessary the observed trade prices are adjusted by a conservative bid-ask spread of 5 cents to estimate the other quote. The only way for there to be a substantial number of mispricings between bond two and the portfolio is for there to exist tax clienteles who place different values on the bond triplets. The number of violations are sufficiently few that there is little support for the existence of tax clienteles or inefficiency. In summary, the bond triplets provide evidence of a liquidity effect and tax timing options. However, examining bond triplets does not provide evidence that the difference in tax treatment of old and new bonds is reflected in market prices or that tax clienteles affect prices.

<sup>12</sup> The adjustment is made by calculating the amortization schedule for old and new bonds for all maturities and premiums, and discounting these differences by the estimated spot rates. This difference is the added value we would expect to see given the preferential treatment of old bonds, which may or may not be reflected in observed bond prices.

Arbitrage tests depend on having two portfolios with identical cash flows. Bond triplets are one way to construct these portfolios. However, there are many other possibilities. To further explore the effect of liquidity, we use a new approach in which we construct portfolios with more than three bonds. This allows us to create portfolios with more extreme differences in liquidity. Each day two portfolios are created with an equal number of bonds of consecutive maturities. In each portfolio there is a bond that matures every six months, which enables us to match cash flows at each maturity. One portfolio is constructed from one of each high volume bond, the other portfolio contains low volume bonds held in (strictly positive) proportions such that they match the cash flows of the high volume portfolio. The law of one price implies they should have the same price if liquidity is unimportant. If liquidity does have value, then the low volume portfolio should have a lower price. Tax timing should not be an important consideration because the portfolios have roughly the same number of bonds.

In order to obtain a large number of observations, we examine bonds that have cash flows in February and August. Whenever possible, we choose bonds with February 15 and August 15 as the cash flow dates. If more than two bonds of a given maturity trade on the same day, the two that have the greatest difference in volume are selected. In cases where two bonds do not exist with these coupon dates, we include bonds that mature at the end of the month. In sample 1 there are 30 out of 762 bonds that do not pay on the 15th of the month, in sample 2 there are 56 of 608, and in sample 3 there are 189 of 626.<sup>13</sup> When the cash flow dates differ, we adjust the cash flows by the forward rate. If one of the portfolio bonds does not pay on the 15th, its cash flows are adjusted to the 15th using the forward rate for that portfolio. The magnitude of this correction is very small compared to the difference in prices of the two portfolios. Furthermore, the frequency of adjustment is roughly the same for the high and low volume group; thus errors in adjusting cash flows should not affect the results. We include as many periods as possible given that cash flows have to match and no payments can differ by more than 16 days. We require there to be at least five bonds in each portfolio. The maturity of the portfolios varies between 2½ and 5 years, with the median number of years being between 3 and 3½ years.

Table V shows the volume for the high volume portfolio and the low volume portfolio where volume is measured over one day in Panel A and over ten days in Panel B. The average volume difference between the two groups is substantial. To get an idea of the magnitude of this difference, we can consult Table II. Although Table II is restricted to bonds with less than six months to maturity, the high volume shown in Table V would lie in the top decile and the low volume in the lower four deciles. Although there does appear to be a significant volume difference between portfolios, Table V does

<sup>13</sup> Each sample contains 50 days in which at least five bonds are available for each portfolio. Sample 1 is taken from October 2, 1991 through January 8, 1992, sample 2 covers March 3, 1993 through June 14, 1993, and sample 3 covers May 31, 1995 through September 28, 1995.

Table V

### Pricing Differences Between Cash Flow Matching Portfolios Grouped by Volume

Data from the interdealer market for Treasury securities are obtained from GovPX Inc. Each day two portfolios are created from bonds that mature in February and August. The two portfolios contain an equal number of bonds of consecutive maturities; that is, in each portfolio there is a bond that matures on every cash flow date. The portfolios are sorted by volume. In Panel A bonds are separated by daily volume, in Panel B bonds are separated by average volume over the past ten days. Portfolio weights are found for the low volume portfolio to match the cash flows of a high volume portfolio that contains one of each high volume bond. Each sample contains 50 days where at least five bonds were available for each portfolio. The reported statistics are averages over each sample. Low Volume Price is the ratio of the cost of the low volume portfolio over the cost of the high volume portfolio. The  $t$ -statistic and  $p$ -value are for a test of whether this ratio is different from 1. Proportional High and Low Volumes are the weighted averages of the daily trading volumes of each portfolio. High and Low Volume Average Hits are the weighted averages of the number of transaction prices that are recorded at the bid price in each portfolio. Number of Pairs is the average number of bonds in each portfolio. High and Low Volume Coupons are the weighted averages of the coupons in each portfolio. High and Low Volume Time is the weighted average of the number of hours from the last possible trade time (6:00 p.m.) that the trades for each portfolio occurred. High and Low Volume SL is the weighted average of the number of bonds issued before September 28, 1985, which allows any premium to be amortized using the straight-line method. Sample 1 covers October 1, 1991 through January 8, 1992, sample 2 covers March 1, 1993 through June 14, 1993, and sample 3 covers May 31, 1995 through September 28, 1995.

	Low Volume Price	$t$ -Stat	$p$ -Val	Propor. High Volume	Propor. Low Volume	High Volume Avg. Hits	Low Volume Avg. Hits	Number of Pairs	High Volume Coupon	Low Volume Coupon	High Volume Time	Low Volume Time	High Volume SL Bond	Low Volume SL Bond
Panel A: High and Low Volume Portfolios Sorted on Daily Volume														
Sample 1	0.99947	-2.875	0.006	131.534	12.652	0.435	0.480	7.62	7.902	8.596	2.941	3.993	0.055	0.229
Sample 2	1.00009	0.440	0.662	328.844	12.173	0.438	0.485	6.08	6.557	8.077	2.868	4.110	0.079	0.125
Sample 3	1.00047	2.436	0.019	314.778	17.357	0.500	0.519	6.26	6.440	6.524	2.908	4.197	0.016	0.028
Combined	1.00001	0.061	0.951	258.385	14.061	0.458	0.495	6.65	6.966	7.732	2.906	4.100	0.050	0.127
Panel B: High and Low Volume Portfolios Sorted on Ten-Day Average Volume														
Sample 1	0.99935	-4.820	0.000	134.476	13.115	0.428	0.486	7.62	7.732	8.768	3.063	3.872	0.019	0.265
Sample 2	1.00020	0.967	0.338	343.395	13.112	0.468	0.455	6.08	6.216	8.416	2.827	4.153	0.023	0.180
Sample 3	1.00065	3.585	0.000	295.558	19.270	0.500	0.519	6.26	6.251	6.711	3.138	3.967	0.012	0.031
Combined	1.00007	0.354	0.725	257.810	15.166	0.466	0.487	6.65	6.733	7.965	3.009	3.997	0.018	0.159

not support a liquidity effect. The portfolios are normalized so that the high volume portfolio costs \$1. The average cost of the low volume portfolio is shown in column 1. It is significantly different from one in sample 1 and sample 3, but in opposite directions, and overall it is insignificantly different from one.

To compare the value of one portfolio relative to the other, we match pretax cash flows. To test whether tax effects may drive our results, we use coupon as a proxy for tax effects and regress the ratio of prices on the difference between the weighted average coupons for the high and low volume portfolios. Difference in coupon is insignificant in explaining the difference in price between the portfolios for both those sorted by 1-day volume and by 10-day volume.

The results may also be influenced by nonsynchronous trading. Table V provides information on the weighted average of the number of hours before 6:00 p.m. EST that the last trade occurred. As expected, the low volume trades are older on average by about one hour. Potentially, prices could be falling over the day on average and the low volume prices could be an overestimate of the synchronous price. To test for this, on each day we adjusted the earlier price to the later price by using the average return over the day for all bonds adjusted to the appropriate time interval. The prices move up some days and down other days in our sample, and the adjustment results in essentially identical results.

Finally, since we use trade data it is possible that we do not find a significant liquidity effect because our high volume observations are associated with bid prices, and our low volume observations are associated with ask prices. Columns 6 and 7 show the proportion of the trades that are at the bid price. Compared to the high volume portfolio, there is some tendency for the observed trade prices for bonds in the low volume portfolio to occur more often at the bid price. We would expect this to make the low volume portfolio less expensive than the high volume portfolio. However, we find little evidence that the low volume portfolio is less expensive than the high volume portfolio, and there seems to be no relation between the difference in portfolio prices and the difference in proportions of bid trades. Thus, bid-ask spread is not an explanation for the price differences in portfolios.

Overall, the general arbitrage results provide mixed support for liquidity and tax effects in bond prices. We find evidence that tax-timing options have a significant, if economically small, impact on the prices of Treasury securities. However, we do not find evidence that the differential tax treatment of bonds issued before September 28, 1985 is reflected in bond prices. When examining bond triplets, we find that the effects of liquidity are significant but small. On the other hand, we find no strong support for either tax or liquidity effects when we examine cash flow matching portfolios of consecutive maturities.

### *B. Term Structure Tests*

In order to study the effects of taxes and liquidity over the entire spectrum of maturities, it is necessary to first specify a model of the term structure. Once a model is selected, we can fit it to the after-tax cash flows of bonds

and thus infer the tax rate faced by the marginal investor. If the estimated tax rate is significantly different from zero, we can conclude that taxes do affect the prices of bonds. The after-tax term structure is first estimated by McCulloch (1975) assuming a given set of tax rates. Litzenberger and Rolfo (1984a) use a grid search to determine the optimal tax rates implied by the data. More recently, Green and Ødegaard (1997) look for a structural change in the implied taxes before and after the change in tax law in 1986. They find that the tax rate of the marginal investor is positive before 1986, but close to zero afterwards. We use a similar procedure and include an additional parameter to capture the effects of liquidity.

When selecting a model of the term structure, a choice has to be made between two different approaches. One approach estimates the term structure each period using only the information contained in the cross section of bond prices; the other approach constrains the term structure to move with a limited set of state variables, but allows the model to be estimated once over the entire sample. In general, the benefit of cross-sectional models is that they provide a better fit than structural models. The cost is that the model has to be estimated each period, and it is not possible to estimate a single tax rate for the entire sample period. Since we are interested in pricing bonds as accurately as possible, and since there exists no structural model that clearly dominates the flexible form method, we use nonlinear least squares to fit Litzenberger and Rolfo's (1984a) cubic spline to the after-tax cash flows of bonds in each period.<sup>14</sup>

In order to capture the effects of liquidity on the relative prices of bonds, we add a liquidity term (log volume) to the reduced form price equation. There are four pricing scenarios for tax purposes: discount bonds issued before and after July 18, 1984, and premium bonds issued before and after September 28, 1985.<sup>15</sup> Under each tax scenario, we solve for price as a nonlinear function of the tax rate and the parameters of the discount function. To these price functions we add log volume to capture the effects of liquidity on prices. This specification of how liquidity affects prices is ad hoc, but is done with tractability in mind. The reduced form price function under the various tax regulations is a nonlinear function of the tax rate and the parameters of the discount function and can be quite complicated. We want to

<sup>14</sup> There is an abundant literature on splines. See Shea (1984) for a discussion of the issues. Using simulated data, Beim (1992) finds that cubic splines perform as well as or better than other estimation techniques. The methodology we use is along the lines of Litzenberger and Rolfo (1984a). Breakpoints for the spline functions are chosen at 1, 2, and 4 years to maturity. Although it is common to use more breakpoints, we use relatively few to guard against overfitting the data. We use nonlinear regressions to allow for the nonlinear interaction between the tax rate and the parameters of the discount function (see Langetieg and Smoot (1989) for evidence of the advantage of nonlinear methods over the instrumental variable approach used in Litzenberger and Rolfo (1984a)). In order to examine whether the results are sensitive to the particular flexible form chosen, we also fit the data using the Nelson and Siegel methodology and find similar results in terms of magnitude and significance.

<sup>15</sup> See Green and Ødegaard (1997), Ronn and Shin (1997), or Fabozzi and Nirenberg (1991) for the precise treatment of discount and premium bonds under the different tax regulations.

**Table VI**  
**Estimated Tax and Liquidity Parameters**

Data from the interdealer market for Treasury securities are obtained from GovPX Inc. The after-tax term structure is fitted with a cubic spline. Log of volume is added to the reduced form price equation for all bonds to capture the effects of liquidity. The tax and liquidity terms are estimated simultaneously with the spline parameters in a nonlinear regression. Panel A reports the root mean squared errors (pooled over the combined sample) when the tax rate and liquidity terms are estimated freely or constrained to be zero. Panel B reports the mean estimated tax and liquidity parameters for the three samples. Also reported are the number of instances where the parameters are significant at the 0.05 level, using heteroskedasticity-consistent standard errors (Mackinnon and White (1985)). Sample 1 covers October 1, 1991 through February 8, 1992, sample 2 covers March 1, 1993 through July 7, 1993, and sample 3 covers May 23, 1995 through September 29, 1995.

Panel A: Root Mean Squared Errors				
Maturity	Tax = 0 Liquidity = 0	Tax = 0	Liquidity = 0	Both Est. Freely
<1	0.0528	0.0902	0.0694	0.0983
1-3	0.1483	0.1356	0.1471	0.1320
3-5	0.1623	0.1422	0.1476	0.1294
5-10	0.2373	0.2088	0.2132	0.1892
All maturities	0.1566	0.1441	0.1480	0.1363

Panel B: Parameter Means				
	Tax Rate	Number Sig./Sample	Liquidity	Number Sig./Sample
Sample 1	0.1270	69/90	0.0328	62/90
Sample 2	0.0318	11/90	0.0225	55/90
Sample 3	0.0851	18/90	0.0121	43/90
Combined	0.0813	98/270	0.0225	160/270

allow for the presence of liquidity effects without complicating this function further. We examine maturities less than or equal to 10 years. Trade data for long term bonds are relatively sparse, with an average of about five observations a day for maturities greater than 10 years. It has been shown that fitting the curve at long maturities with few observations can lead to spurious results (see Shea (1984)).

Table VI reports the root mean squared errors and the average estimated tax rate and liquidity parameter for three subsamples of 90 days of data as well as for the combined sample.<sup>16</sup> Including tax and liquidity terms improves the fit across all maturities except for those less than one year. The average estimated tax rate over the three subsamples is 8 percent. We find that the tax rate is statistically significant 69 times in the first sample, but

<sup>16</sup> Sample 1 covers October 1, 1991 through February 11, 1992, sample 2 covers March 1, 1993 through July 7, 1993, and sample 3 covers May 23, 1995 through September 29, 1995.

only 11 and 18 times in the second and third samples.<sup>17</sup> This is evident in the pattern of estimated tax rates, as shown in Figure 2. Although the estimated tax rates do exhibit considerable volatility, it is evident that the estimated tax rates are small, especially in samples 2 and 3. This confirms the findings of Green and Ødegaard (1997), who find the estimated tax rate during the 1987–1992 period to be close to zero.

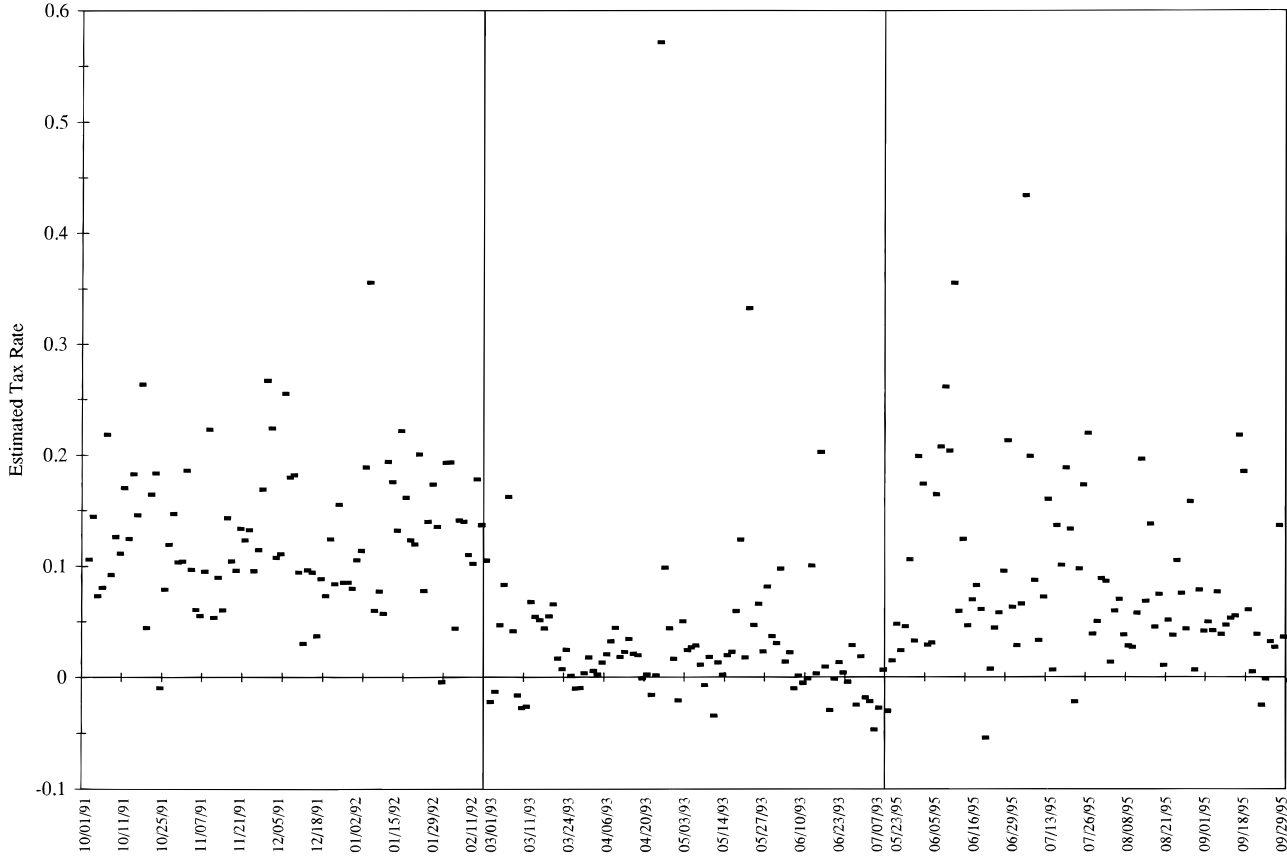
Although we find little evidence of tax effects, it is worth noting that the assumptions commonly made to estimate the after-tax term structure imply that differences between capital gains and ordinary income tax rates have little impact on prices. For instance, it is common to assume that all bonds are held to maturity. Thus, the only capital gains are for bonds selling at a discount, and the only losses come from premium bonds. Moreover, the capital gains tax rate only enters the valuation process for bonds issued before July 18, 1984 and selling at a discount. If they are issued after this date, all discounts are taxed at the ordinary rate. In all three of our subsamples, we observe no bonds issued before July 18, 1984 that traded at a discount. Thus, we have little hope of distinguishing between the tax rate on capital gains and ordinary income. Since the trade-off between these two rates is a major contributor to any tax effects, it is not surprising that we do not find convincing evidence of their existence. In the case of bond triplets, no assumption on the holding period is necessary, and we do find some evidence of tax-timing options, although the magnitude of the effects is quite small.

The liquidity term, on the other hand, appears to carry a higher level of statistical significance. Figure 3 shows that the estimated coefficients are also rather volatile, but the liquidity term is significantly positive in 160 of the 270 regressions.<sup>18</sup> The average liquidity coefficient is 2.25 basis points. This value suggests a range of approximately 13 basis points from the lowest to the highest volume deciles.<sup>19</sup> The magnitude of the liquidity effect is much smaller than that found in previous studies. For example, Warga (1992) finds a 40–100 basis point difference in returns between active issues and duration matching portfolios, and Amihud and Mendelson (1991) report a 40 basis point difference in yield on similar notes and bills. Although liquidity does appear to be important, its value is not nearly as substantial as previously reported. The large liquidity effects found in earlier work may be due to inaccurate liquidity measures and the lack of precise price data. On the other hand, the increased size of the Treasury market and the widespread use of empirical term structure models, along with a maturing strips market, may also have led to a smaller economic role for liquidity. Whether

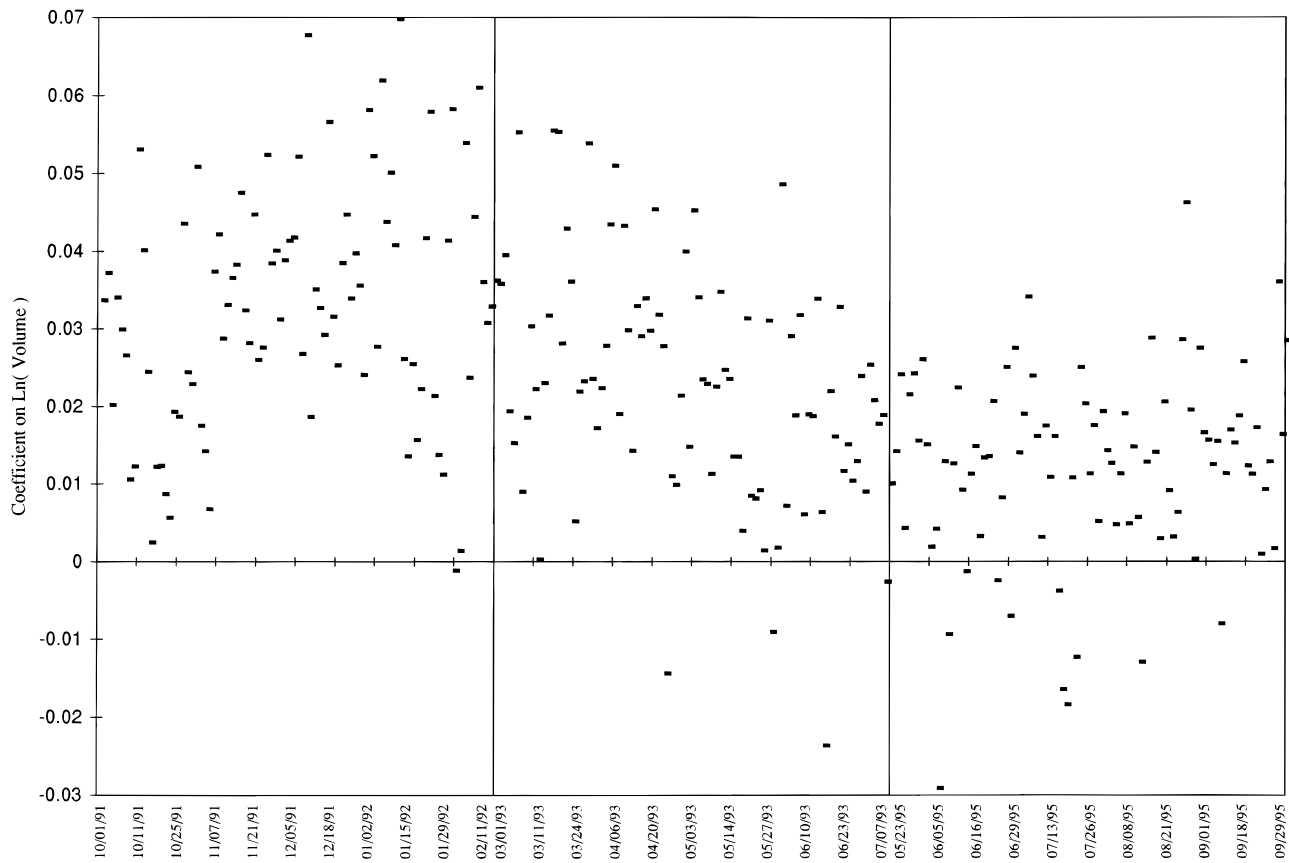
<sup>17</sup> Standard errors are derived from a heteroskedasticity-consistent estimate of the covariance matrix (see MacKinnon and White (1985)).

<sup>18</sup> Assuming the liquidity parameters are independent draws from a normal distribution, the mean liquidity parameter is significantly different from zero at any level of significance. The level of autocorrelation in the liquidity estimates is small, and does not materially alter this result.

<sup>19</sup> Daily trading volume (in logs) for notes and bonds with less than ten years to maturity ranges from 0 for the bottom decile to 5.8 for the 95th percentile.



**Figure 2. Estimated tax rates.** A cubic spline with a liquidity term is fit to the after-tax term structure. The chart reports the estimated tax rates for three samples. Sample 1 covers October 10, 1991 through February 11, 1992, sample 2 covers March 1, 1993 through July 7, 1993, and sample 3 covers May 23, 1995 through September 29, 1995.



**Figure 3. Estimated liquidity parameter.** A cubic spline with a liquidity term is fit to the after-tax term structure. The chart reports the estimated liquidity parameter for three samples. Sample 1 covers October 1, 1991 through February 11, 1992, sample 2 covers March 1, 1993 through July 7, 1993, and sample 3 covers May 23, 1995 through September 29, 1995.

the diminished role of liquidity we observe is due to the use of more precise data or is a result of the increased efficiency of the Treasury market is unclear. In either case, our overall finding is that the current economic role of liquidity in the relative prices of Treasury securities is quite small.

The inconsequential tax and liquidity effects that we find suggest that more bonds can be included in term structure estimation. Market participants commonly narrow the pool of bonds they consider when estimating the term structure in order to use bonds they believe are unaffected by tax and liquidity effects. Our evidence implies that a larger sample of bonds can be used to estimate the term structure, which could lead to smaller estimation error. The small liquidity effects we observe also have relevance for the trade-off between the larger bid-ask spreads and higher expected returns of illiquid bonds, which is relevant to investors who must decide whether to hold illiquid bonds.

Table VI and Figure 1 may provide some explanation as to why we do not find convincing evidence of liquidity effects in the arbitrage tests, yet we do observe liquidity effects in the term structure estimation. The arbitrage tests utilize bonds near the short end of the maturity spectrum in order to come up with enough issues to match cash flows. Neither the bond triplet approach nor the cash flow matching approach utilize bonds with more than five years to maturity. However, Figure 1 shows that many of the highly liquid bonds have maturities greater than five years. Moreover, Table VI shows that the reduction in pricing errors by including the liquidity term is strongest for longer maturity bonds. Thus, the term structure approach is more robust in that it allows us to examine liquidity over a broader range of maturities.

The average root mean squared error is 0.1363 per \$100 face value, or about 14 basis points. As discussed previously, the average bid-ask spread for notes and bonds is 0.053. Thus, roughly 20 percent of the root mean squared error can be attributed to the bid-ask spread since we fit the curve to trade data. To examine whether remaining errors are systematically related to certain bond attributes, we regress pricing errors on a series of bond characteristics. The results are reported in Table VII. The left column reports the regression results when the tax and liquidity parameter are estimated freely. The right column reports the results when the two terms are constrained to be zero. In order to capture the effects of tax timing options, which are not explicitly modeled in the after-tax cash flows, we include the dollar premium or discount. We also use an age variable to examine whether there are remaining liquidity effects not captured by our liquidity term. We also include a measure of nonsynchronicity. The nonsynchronous variable is the daily price change in the nearest active issue times the fraction of the day between the last trade and 6:00 p.m. EST. A tax regime dummy variable is utilized that equals 1 if the bond trades at a premium and is issued before September 28, 1985; this is meant to capture the preferential amortization rules for old bonds. Another dummy variable set equal to 1 is used if the issue is on-the-run; this is designed to capture the effects of repo special-

Table VII

**Results of Regressions of Spline Errors on Bond Characteristics**

Data from the interdealer market for Treasury securities are obtained from GovPX Inc. Each day a cubic spline is used to estimate the after-tax term structure for noncallable bonds with fewer than ten years to maturity. When estimating the spline, the log of trading volume is included to capture the importance of liquidity. The resulting pricing errors are regressed on various bond characteristics. *t*-statistics are created using a heteroskedasticity- and autocorrelation-consistent (HAC) estimate of the covariance matrix. Discount (Premium) is the dollar amount of discount (premium), zero otherwise, for each bond assuming a face value of \$100. Age is the number of years since issuance. Nonsync is an adjustment for nonsynchronous trading in which the daily price changes of active issues are assumed to occur in a linear fashion throughout the day. For each bond the change in the nearest active issue is used to approximate the change in price that occurred from the last trade to the close of trading. Tax Reg. is a dummy variable that is 1 if the bond was issued before September 28, 1985, the cutoff date for the Tax Reform Act of 1986. Bid is 1 if the trade occurred at the bid price, 0 if at the ask. Sample 1 contains daily observations from October 1, 1991 through February 11, 1992, sample 2 covers March 1, 1993 through July 7, 1993, and sample 3 covers May 23, 1995 through September 29, 1995. The results reported are for the combined sample.

	Tax and Liquidity Parameters Freely Estimated			Tax and Liquidity Parameters Constrained to be Zero		
	Coeff.	<i>t</i> -Stat.	<i>p</i> -val.	Coeff.	<i>t</i> -Stat.	<i>p</i> -val.
Constant	0.0171	3.811	0.000	0.0356	7.455	0.000
Premium	-0.0035	-4.406	0.000	-0.0051	-6.801	0.000
Discount	-0.0057	-1.323	0.186	-0.0026	-0.529	0.597
Age	-0.0002	-0.135	0.893	-0.0036	-2.510	0.012
Nonsync.	-0.3900	-10.435	0.000	-0.5826	-13.439	0.000
Tax Reg.	-0.0114	-0.402	0.688	-0.0464	-1.722	0.085
Bid	-0.0102	-5.719	0.000	-0.0114	-6.013	0.000
Active	0.1071	7.034	0.000	0.2011	10.842	0.000
Number of obs.		31696			31696	
Mean error		-0.0019			0.0015	
$R^2$		0.0480			0.1210	

ness. Finally, to measure the impact of the bid-ask spread, we add a bid dummy variable that equals 1 if the trade occurs at the bid price, and 0 if at the ask. Since we pool the errors across maturities and time, it is highly likely that the errors will be heteroskedastic as well as autocorrelated. In order to obtain robust statistical measures, we need a heteroskedasticity and autocorrelation consistent (HAC) estimate of the covariance matrix. The method we employ is a variant of the Newey–West correction that is discussed in Green and Ødegaard (1997).<sup>20</sup>

We will first look for the presence of tax effects not captured by our estimated tax rate. The tax regime coefficient is insignificant in the combined sample. Since we account for the tax regime when discounting after tax cash

<sup>20</sup> The approach uses errors lagged across time and across the term structure. For a discussion of HAC estimators, see chapter 17 of Davidson and MacKinnon (1993).

flows, we would not expect the tax regime coefficient to be significantly different from zero. It appears for the most part that investors do take advantage of the preferential amortization treatment of bonds issued before September 28, 1985. The coefficient for the premium variable is negative and significant in the combined sample. The coefficient for the discount variable is mixed; it is negative and insignificant in the combined sample. The negative estimated coefficients are not consistent with tax-timing options, which we would expect to be more valuable with a larger discount or premium. Nor are the estimated coefficients associated with a misestimated tax rate, which should result in different signs for the premium and discount variables. Instead, our evidence generally suggests that a significant group of investors is adverse to holding both high discount and high premium bonds. One possible explanation that is consistent with the evidence is the behavior of fiduciaries. Fiduciaries who hold bonds in trust funds face a trade-off between interest income and capital gains. The higher the coupon, the larger the amount of interest income that accrues to the current beneficiary and the smaller the capital gains that go to the heirs. Likewise, the lower the coupon, the smaller the amount of interest income that accrues to the current beneficiary and the larger the amount of capital gains for the heirs. Trustees avoid bonds with large discounts or premiums because they would be vulnerable to accusations of favoring one beneficiary over another, which could lead to legal action. We find that large premium and large discount bonds sell at prices lower than we would expect when discounting their cash flows by estimated spot rates, which supports this explanation.<sup>21</sup> The effect of premiums and discounts on pricing errors is only important for the most extreme 5 percent of the observations. For these observations the effect varies from 4 to 60 cents per \$100 face value. When we constrain the tax rate to be zero, the tax terms included in the error regressions are still not significant, although the coefficients all change in the expected direction. This is further evidence that tax effects, if any, are not large.

The age variable is not significant, which is not surprising given the inclusion of the liquidity term (log of volume) when estimating the discount function. When the liquidity coefficient is constrained to be zero, the age coefficient is negative and significant in the combined sample, suggesting that the age variable proxies for liquidity in this case. The nonsynchronous adjustment variable is negative in every sample. If prices increase steadily throughout the day, and a bond's last trade occurred several hours before the close, we would expect the observed price to be below its fair closing value and therefore we would observe a negative pricing error. The negative coefficient along with a positive nonsynchronous adjustment variable would result in a negative prediction. Furthermore, if prices decrease over the day, the pricing error should be positive, but since the variable is negative, a negative coefficient predicts the positive error. Thus, we expect the sign on the coefficient for our nonsynchronous adjustment variable to be negative

<sup>21</sup> The authors thank Ken Garbade for this insight.

regardless of how prices move. The estimated nonsynchronous adjustment variable is negative, but it is sizable for only about 20 percent of the observations. For this part of the sample the adjustment for nonsynchronous trading is about 2 cents per \$100 face value.

The on-the-run dummy variable is positive and significant in two of the three subsamples. Since we have already adjusted for the effects of liquidity, we suggest this variable proxies for the effects of specialization in the overnight repurchase market.<sup>22</sup> It is common for on-the-run issues to be associated with overnight borrowing rates that are lower than the general collateral rate (see Duffie (1996)). The size of the coefficient in the combined sample is 0.1071, which translates to an effect of about 10 basis points for issues on special, which is much smaller than the 20–40 basis point effects reported in Jordan and Jordan (1996). However, the subsample closest to their time period has the largest coefficient, with on-the-run issues commanding premiums of 17 basis points. The average pricing error for on-the-run issues is about 22 cents when the liquidity term is constrained to be zero. The average pricing error drops to 11 cents when the liquidity term is included. The coefficient for the on-the-run variable, and the reduction in pricing error when freely estimating the liquidity term, suggests that liquidity and repo specials each explain roughly half of the premium associated with on-the-run issues.

Finally, the bid dummy variable is negative and significant in every subsample. The combined coefficient is  $-0.01$ , which suggests the pricing error due to bid-ask spreads is significant, but quite small.

## V. Conclusion

This paper looks for evidence of tax and liquidity effects in the relative prices of Treasury securities. We utilize two types of arbitrage tests in addition to jointly estimating the tax rate and a liquidity term when fitting a cubic spline to the term structure. We examine trade data from the interdealer market. Our data set provides us with prices that more accurately reflect market clearing conditions than dealer estimates used in previous studies. We also have access to a more robust measure of liquidity than previously available. Using these more accurate prices, we still find evidence of tax timing options and liquidity effects, although the effects of liquidity are much smaller than previously reported. Our evidence suggests that part of the premium associated with the most liquid bonds, on-the-run issues, is due to specialization in the overnight repurchase market.

We find evidence that large premium and discount bonds sell at a discount, which is consistent with fiduciaries avoiding these bonds. However, when trade data are used, all pricing errors are small and the effects of tax timing options, tax regime shifts, and differences in liquidity are measured

<sup>22</sup> Alternatively, the significance of the on-the-run dummy can be associated with some aspect of liquidity not captured by the volume proxy.

in pennies. Thus, a significant portion of the liquidity and tax effects found by previous authors appears to be no longer relevant, either because of increased efficiencies in the Treasury market, or perhaps because data problems influenced the calculation of the original estimates.

The lack of substantial tax and liquidity effects in the relative prices of bonds has important implications for investors deciding when to select bonds and for practitioners deciding which bonds to include in their term structure estimations for use by traders.

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