

The Cash Flow Sensitivity of Cash

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*Almeida is at New York University. Campello and Weisbach are at the University of Illinois. We thank an anonymous referee, Rick Green (the editor), Viral Acharya, Dan Bernhardt, Matt Billett, Long Chen, Ted Fee, Jon Garfinkel, John Graham, Charlie Hadlock, Jarrad Harford, Harrison Hong, George Pennacchi, Eric Rasmussen, René Stulz, Toni Whited, and Jeffrey Wurgler for their very helpful suggestions. Comments from seminar participants at the November 2002 NBER Corporate Finance Meeting, Duke University, Georgia State University, Indiana University, Louisiana State University, New York University, New York Federal Reserve Bank, University of Illinois, and University of Iowa are also appreciated.

ABSTRACT

We model a firm's demand for liquidity to develop a new test of the effect of financial constraints on corporate policies. The effect of financial constraints is captured by the firm's propensity to save cash out of cash flows (the *cash flow sensitivity of cash*). We hypothesize that constrained firms should have a positive cash flow sensitivity of cash, while unconstrained firms' cash savings should not be systematically related to cash flows. We empirically estimate the cash flow sensitivity of cash using a large sample of manufacturing firms over the 1971 to 2000 period and find robust support for our theory.

Two important areas of research in corporate finance are the effects of financial constraints on firm behavior and the manner in which firms perform financial management. These two issues, although often studied separately, are fundamentally linked. As originally proposed by Keynes (1936), a major advantage of a liquid balance sheet is that it allows firms to undertake valuable projects when they arise. However, Keynes also argued that the importance of balance sheet liquidity is influenced by the extent to which firms have access to external capital markets (p. 196). If a firm has unrestricted access to external capital — that is, if a firm is financially unconstrained — there is no need to safeguard against future investment needs and corporate liquidity becomes irrelevant. In contrast, when the firm faces financing frictions, liquidity management may become a key issue for corporate policy.

Despite the link between financial constraints and corporate liquidity demand, the literature that examines the effects of financial constraints on firm behavior has traditionally focused on corporate *investment* demand.¹ In an influential paper, Fazzari, Hubbard, and Petersen (1988) propose that when firms face financing constraints, investment spending will vary with the availability of internal funds, rather than just with the availability of positive net present value (NPV) projects. Accordingly, one should be able to examine the influence of financing frictions on corporate investment by comparing the empirical sensitivity of investment to cash flow across groups of firms sorted according to a proxy for financial constraints. Recent research, however, has identified several problems with that strategy. The robustness of the implications proposed by Fazzari, Hubbard, and Petersen has been challenged on theoretical grounds by Kaplan and Zingales (1997), Povel and Raith (2001), and Almeida and Campello (2002), while the robustness of cross-sectional patterns presented in their empirical work (and in the subsequent literature) has been questioned by Kaplan and Zingales, Cleary (1999), and Erickson and Whited (2000). Alti (2003) further demonstrates that because cash flows contain valuable information about a firm's investment opportunities, the cross-sectional patterns reported by Fazzari, Hubbard, and Petersen can be consistent with a model with no financing frictions (see also Gomes (2001)). This argument casts doubt on the very meaning of the empirical cash flow sensitivities of investment reported in the literature.

In this paper, we argue that the link between financial constraints and a firm's demand for liquidity can help us identify whether financial constraints are an important determinant of firm behavior. We first present a model of a firm's liquidity demand that formalizes Keynes' intuition. In it, firms anticipating financing constraints in the future respond to those potential constraints by hoarding cash today. Holding cash is costly, nonetheless, since higher cash savings require reductions in current, valuable investments. Constrained firms thus choose their optimal cash policy to balance the profitability of current and future investments. This policy is in contrast to that of firms that are able to fund all of their positive NPV investments: Financially unconstrained firms have no use for cash, but also face no cost of holding cash (i.e., their cash policies are indeterminate).

The stark difference in the implied cash policies of constrained and unconstrained firms allows us to formulate an empirical prediction about the effect of financial constraints on firms' financial policies. Our model suggests that financial constraints should be related to a firm's propensity to save cash out of cash inflows, which we refer to as the *cash flow sensitivity of cash*. In particular, financially unconstrained firms should not display a systematic propensity to save cash, while firms that are constrained should have a positive cash flow sensitivity of cash. As such, the cash flow sensitivity of cash provides a theoretically justified, empirically implementable measure of the importance of financial constraints.

The use of cash flow sensitivities of cash to test for financial constraints avoids some of the problems associated with the investment–cash flow literature. In particular, because cash is a financial (as opposed to a real) variable, it is difficult to argue that the explanatory power of cash flows over cash policies could be ascribed to its ability to forecast future business conditions (investment demand). For unconstrained firms, changes in cash holdings should depend neither on current cash flows nor on future investment opportunities, so in the absence of financial constraints, one should expect no systematic patterns in cash policies. Evidence that the sensitivity of cash holdings to cash flow varies systematically with proxies for financing frictions is therefore more powerful and less ambiguous evidence of the role of financial constraints than what investment–cash flow sensitivities can provide.

We evaluate the extent to which the cash flow sensitivity of cash provides an empirically useful measure of financial constraints using a large sample of manufacturing firms between 1971 and 2000. We estimate that sensitivity for various firm subsamples, partitioned on the basis of the likelihood that firms have constrained access to external capital. In doing so, we use five alternative approaches suggested by the literature to partition the sample into unconstrained and constrained subsamples: Payout policy, asset size, bond ratings, commercial paper ratings, and an index measure derived from results in Kaplan and Zingales (1997) (the “KZ index”). We find that under each of the first four classification schemes, the cash flow sensitivity of cash is close to and not statistically different from zero for the unconstrained firms, but positive and significantly different from zero for the constrained firms. The KZ index generates constrained/unconstrained firm assignments that are mostly negatively correlated with those of the other four classification criteria. Not surprisingly, we obtain the very opposite results for our estimates of the cash flow sensitivity of cash that use the KZ index. All of the patterns we observe in our basic tests remain after we subject our estimations to various robustness checks involving changes in empirical specifications, sampling restrictions, and econometric methodologies. Our findings are fully consistent with the implications of our model of corporate liquidity.

We further test the intuition of our argument by investigating firms’ propensity to save cash out of cash inflows over the business cycle. Our model implies changes in corporate liquidity demand over the business cycle, because aggregate demand fluctuations work as exogenous shocks affecting *both* the size of current cash flows as well as the relative attractiveness of current investments vis-à-vis future ones. In a recession, financially constrained firms should save a greater proportion of their cash flows, while unconstrained firms’ cash policies should not show any systematic changes. We find that for constrained firms, cash–cash flow sensitivities appear to be negatively associated with shocks to aggregate demand (i.e., on the margin, constrained firms save more in recessions), while unconstrained firms display no change in their cash–cash flow sensitivities in response to macroeconomic shocks. Once again, these results hold for four of our proxies for financial constraints (payout policy, size, bond ratings, and commercial paper ratings), but not for the KZ index. The

macro-level tests provide additional support for our argument for two different reasons. They confirm that a natural extension of our model is consistent with the data, and they help sidestep the usual concerns with estimation biases that arise in standard regression analysis involving firm-level data.

While the literature has examined the effects of financial constraints on corporate policies such as fixed investment (e.g., Fazzari, Hubbard, and Petersen (1988) and Almeida and Campello (2003)), working capital (Fazzari and Petersen (1993) and Calomiris, Himmelberg, and Wachtel (1995)), and inventory demand (Carpenter, Fazzari, and Petersen (1994) and Kashyap, Lamont, and Stein (1994)), it has not explicitly considered the relationship between financial constraints and a firm's liquidity demand.² A number of recent empirical studies do examine the cross-section of cash reserves and the factors that appear to be associated with higher holdings of cash.³ Among other results, these papers find that the levels of cash tend to be positively associated with future investment opportunities, business risk, and negatively associated with proxies for the level of protection of outside investors. However, while these studies focus on differences in the *level* of cash across firms, our paper examines differences in the *sensitivity* of cash holdings to cash flow and the extent to which they are affected by financial constraints. We do so because our theory has much clearer predictions about firms' marginal propensity to save/disburse funds out of cash flow innovations than about the amount of cash in their balance sheets. To our knowledge, our paper is the first to pursue this approach in dealing with the issue of corporate liquidity.

The remainder of the paper proceeds as follows. Section I introduces a theory of corporate liquidity demand and derives our main empirical implications. Section II presents the empirical tests of these implications. Section III concludes.

I. A Model of Liquidity Demand

The first step of our analysis is to model corporate demand for liquid assets as a means of ensuring the firm's ability to invest in an imperfect capital market. Our basic model is a simple representation of a dynamic problem in which the firm has both present and future investment

opportunities, and in which cash flows from assets in place might not be sufficient to fund all positive NPV projects. Depending on the firm’s capacity for external finance, hoarding cash may facilitate future investments. Another way the firm can plan for the funding of future investments is by hedging against future earnings. In all, our framework considers four components of financial policy: Cash management, hedging, dividend payouts, and borrowing.

A. Structure

The model has three dates, 0, 1, and 2. At time 0, the firm is an ongoing concern whose cash flow from current operations is c_0 .⁴ At that date, the firm has the option to invest in a long-term project that requires I_0 today and pays off $F(I_0)$ at time 2. Additionally, the firm expects to have access to another investment opportunity at time 1. If the firm invests I_1 at time 1, the technology produces $G(I_1)$ at time 2. The production functions $F(\cdot)$ and $G(\cdot)$ have standard properties; i.e., they are increasing, concave, and continuously differentiable. The firm has existing assets that produce a cash flow equal to c_1 at time 1. With probability p , the time 1 cash flow is high, equal to c_1^H , and with probability $(1 - p)$, equal to $c_1^L < c_1^H$.⁵ We assume that the discount factor is 1, that everyone is risk neutral, and the cost of investment goods at dates 0 and 1 is equal to 1. Finally, the investments I_0 and I_1 can be liquidated at the final date, generating a payoff equal to $q(I_0 + I_1)$, where $q \leq 1$ and $I_0, I_1 > 0$. Define total cash flows from investments as $f(I_0) \equiv F(I_0) + qI_0$, and $g(I_1) \equiv G(I_1) + qI_1$.

We suppose that the cash flows $F(I_0)$ and $G(I_1)$ are not verifiable and thus cannot be contracted upon. While the firm cannot pledge those cash flows to outside investors, it can raise external finance by pledging the underlying productive assets as collateral. Following Hart and Moore (1994), the idea is that the liquidation value of “hard” assets is verifiable by a court, and if the firm reneges on its debt, creditors will seize those assets. We assume that the liquidation value of assets that can be captured by creditors is given by $(1 - \tau)qI$. The parameter $\tau \in (0, 1)$ is a function of factors such as the tangibility of a firm’s assets and of the legal environment that dictates relations between debtors and creditors (see Myers and Rajan (1998)). For a high enough τ , the firm may pass up

positive NPV projects for lack of external financing and may thus become financially constrained.

In our setup, the firm is concerned only about whether or not to store cash from time 0 until time 1; there are no new investment opportunities to fund at time 2. We denote by C the amount of cash the firm chooses to carry from time 0 until time 1. We also assume that the firm can fully hedge future earnings at a fair cost. As argued by Froot, Scharfstein, and Stein (1993), a binding financial constraint creates a motive for hedging future cash flows.⁶

Two comments are in order before we analyze demand for liquidity in our proposed setup. First, we note that although the optimal contract in our Hart-Moore-type framework is most easily interpreted as collateralized debt, none of our conclusions hinge on this strict interpretation. The crucial feature for our theory is that some firms have limitations in their capacity to raise external finance, and that such limitations may cause those firms to invest below first-best levels. In particular, the model's intuition is unchanged if we allow for uncollateralized debt or equity issues, so long as constrained firms have to pay a premium over and above the fair cost of uncollateralized debt and/or equity, or so long as there is a maximum amount of funds that constrained firms can raise using those instruments in the capital markets.⁷ Second, notice that the model does not imply a one-to-one correspondence between asset liquidity (τ) and financial constraints. While some degree of imperfection is necessary for a firm to be constrained — i.e., we need some degree of illiquidity, or a high enough τ — whether or not a particular firm is constrained also depends on the size of cash flows from existing assets relative to the magnitude of capital expenditures associated with the new investment opportunities.

B. Analysis

The firm's objective is to maximize the expected lifetime sum of all dividends subject to various budget and financial constraints. This problem can be written as:

$$\max_{C,h,I} (d_0 + pd_1^H + (1-p)d_1^L + pd_2^H + (1-p)d_2^L) \quad s.t. \quad (1)$$

$$\begin{aligned}
d_0 &= c_0 + B_0 - I_0 - C \geq 0 \\
d_1^S &= c_1^S + h^S + B_1^S - I_1^S + C \geq 0, \text{ for } S = H, L \\
d_2^S &= f(I_0) + g(I_1^S) - B_0 - B_1^S, \text{ for } S = H, L \\
B_0 &\leq (1 - \tau)qI_0 \\
B_1^S &\leq (1 - \tau)qI_1^S, \text{ for } S = H, L \\
ph^H + (1 - p)h^L &= 0.
\end{aligned}$$

The first two constraints restrict dividends (d) to be non-negative in times 0 and 1. The terms B_0 and B_1 are the borrowing amounts, which have to be lower than the collateral value generated by the new investments. Debt obligations are repaid at the time when the assets they help finance generate cash flows. Hedging payments in states H and L are denoted by h^H and h^L , respectively. The hedging strategies we focus on typically give $h^H < 0$ and $h^L > 0$. If the firm uses futures contracts, for example, we should think of $c_1^S + h^S$ as the futures payoff in state S . The firm sells futures at a price equal to the expected future spot value, and thus increases cash flows in state L at the expense of reducing cash flows in state H . Finally, note that the fair hedging constraint defines h^H as a function of h^L ($h^H = -\frac{(1-p)}{p}h^L$).

B.1. First-Best Solution

The firm is *financially unconstrained* if it is able to invest at the first-best levels at times 0 and 1, which are defined as

$$\begin{aligned}
f'(I_0^{FB}) &= 1 \\
g'(I_1^{FB,S}) &= 1, \text{ for } S = H, L.
\end{aligned}$$

Since the productivity of investment does not vary across states, we have $I_1^{FB,H} = I_1^{FB,L} \equiv I_1^{FB}$.

When the firm is unconstrained, its investment policy satisfies all the dividend, hedging, and borrowing constraints above for some financial policy (B_0, B_1^S, C, h^H) . More explicitly, the condition

for the firm to be unconstrained is that there exists a financial policy (B_0, B_1^S, C, h^H) such that

$$\begin{aligned} I_0^{FB} &\leq c_0 + B_0 - C \\ I_1^{FB} &\leq c_1^H - \frac{1-p}{p}h^L + B_1^H + C \\ I_1^{FB} &\leq c_1^L + h^L + B_1^L + C, \end{aligned} \tag{2}$$

for amounts B_0 and B_1^S that are less than or equal to the collateral value created by the first-best investments.

The exogenous parameters that determine whether a firm is unconstrained are the cash flows from existing assets, the liquidity of a firm's assets, and the first-best investment levels. Unconstrained firms are thus either those that have low τ (high capacity for external finance) or those that have sufficient internal funds (high c_0 and c_1) relative to the size of I_0^{FB} and I_1^{FB} .⁸

Except for the case when the constraints above are exactly binding at the first-best solution, the financial policy of an unconstrained firm is indeterminate. In particular, if a firm j is financially unconstrained, then its financial policy $(B_{0j}, B_{1j}, C_j, h_j^H)$ can be replaced by an entirely different financial policy $(\hat{B}_{0j}, \hat{B}_{1j}, \hat{C}_j, \hat{h}_j^H)$ with no implications for firm value. Consequently, there is no unique optimal cash policy for a financially unconstrained firm. To see the intuition, suppose the firm increases its cash holdings by a small amount, ΔC . Would that policy entail any costs? The answer is no. The firm can compensate for ΔC by paying a smaller dividend today (or by borrowing more). Are there benefits to the increase in cash holdings? The answer is also no. The firm is already investing at the first-best level at time 1, and an increase in cash is a zero NPV project, since the firm foregoes paying a dividend today for a dividend tomorrow that is discounted at the market rate of return.

For our purposes, the main implication of this “irrelevance of liquidity” result is that for an unconstrained firm, there should be no systematic relationship between changes in cash holdings and current cash flows. Given an extra dollar of excess cash flows, the firm will be indifferent between paying out this dollar to investors and retaining this dollar in the balance sheet as cash.

B.2. Constrained Solution

The firm is *financially constrained* if its investment policy is distorted from the first-best level — i.e., $(I_0^*, I_1^*) < (I_0^{FB}, I_1^{FB})$ — because of capital market frictions. For a financially constrained firm, holding cash entails *both* costs and benefits. A constrained firm cannot undertake all of its positive NPV projects, so holding cash is costly because it requires sacrificing some valuable investment projects today. The benefit of cash is the increase in the firm’s ability to finance future projects that might become available. Optimal cash policies arise as a trade-off between these costs and benefits, both of which are generated by the same underlying capital market imperfection. Financial constraints lead to an optimal cash policy C^* , in stark contrast with the “irrelevance of liquidity” result that holds for financially unconstrained firms.

Notice that since foregoing a dividend payment or borrowing an additional unit is a zero NPV project, it will not be optimal for a constrained firm to pay any dividends at times 0 and 1; in addition, borrowing capacity will be exhausted in both periods and in both states at time 1. Using these facts, we can write the firm’s problem as:

$$\max_{C, h^L} f\left(\frac{c_0 - C}{1 - q + \tau q}\right) + pg\left(\frac{c_1^H - \frac{1-p}{p}h^L + C}{1 - q + \tau q}\right) + (1 - p)g\left(\frac{c_1^L + h^L + C}{1 - q + \tau q}\right). \quad (3)$$

To economize on notation, define $\lambda \equiv 1 - q + \tau q$.

Because hedging is fairly priced, the firm can eliminate its cash flow risk. This implies that the optimal amount of hedging is given by $h^L = p(c_1^H - c_1^L)$, leading to equal cash flows in both states (equal to $E_0[c_1]$).⁹ Given optimal hedging, the optimal cash policy C^* is determined by:

$$f'\left(\frac{c_0 - C^*}{\lambda}\right) = g'\left(\frac{E_0[c_1] + C^*}{\lambda}\right). \quad (4)$$

The left-hand side of equation (4) is the marginal *cost* of increasing cash holdings. When the firm holds incremental cash, it sacrifices valuable (positive NPV) current investment opportunities. The right-hand side of equation (4) is the marginal *benefit* of hoarding cash under financial constraints. By holding more cash the firm is able to relax the constraints on its ability to invest in the future.

How much of its current cash flow will a constrained firm save? This can be calculated from the derivative $\frac{\partial C^*}{\partial c_0}$, which we define as the *cash flow sensitivity of cash*. As we illustrate below, the

cash flow sensitivity of cash reveals a dimension of corporate liquidity policy that is suitable for empirical analysis of the effects of financial constraints. In the presence of financial constraints, the cash flow sensitivity of cash is given by:

$$\frac{\partial C^*}{\partial c_0} = \frac{f''(I_0^*)}{f''(I_0^*) + g''(I_1^*)}. \quad (5)$$

This sensitivity is positive, indicating that if a financially constrained firm gets a positive cash flow innovation this period, it will optimally allocate the extra cash across time, saving a fraction of those resources to fund future investments.

A simple example can show in a more intuitive way the testable implications of our model. Consider parameterizing the production functions $F(\cdot)$ and $G(\cdot)$ as follows:

$$F(x) = A \ln(x) \quad \text{and} \quad G(y) = B \ln(y) \quad (6)$$

This parameterization assumes that while the concavity of the production function is the same in periods 0 and 1, the marginal productivity of investment may change over time.¹⁰ With these restrictions, it is possible to derive an explicit formula for C^* :

$$C^* = \frac{\delta c_0 - E_0 [c_1]}{1 + \delta}, \quad (7)$$

where $\delta \equiv \frac{B}{A} > 0$. The cash flow sensitivity of cash is given by $\frac{\delta}{1+\delta}$, where the parameter δ can be interpreted as a measure of the importance of future growth opportunities vis-à-vis current opportunities. Equation (7) shows that C^* is increasing in δ (i.e., $\frac{\partial C^*}{\partial \delta} > 0$), which agrees with the intuition that a financially constrained firm will hoard more cash today if future investment opportunities are more profitable.

Notice that the cash flow sensitivity of cash of constrained firms is not a direct function of the *degree* of the financial constraint. In our model, the degree of the financial constraint depends on borrowing capacity and on the size of the firm's cash flows relative to its investment opportunities. The higher the borrowing capacity, or the higher the firm's cash flows, the lower the investment distortion relative to the first-best (I_0^* and I_1^* approach I_0^{FB} and I_1^{FB}). While a change in the degree of financial constraints is generally relevant for the firm's policies, it has no systematic, first-order

effect on the cash flow sensitivity of cash. The reason why the degree of financial constraints does not affect cash levels is that varying the degree of constraints affects *both* the benefits and the costs of holding cash in an offsetting manner, so a relatively “more constrained” firm will not necessarily save any more or less cash than a “less constrained” one.¹¹

C. Implications

We state the main implications of our model in the form of a proposition.

PROPOSITION: *The cash flow sensitivity of cash, $\frac{\partial C}{\partial c_0}$, has the following properties:*

- i) $\frac{\partial C}{\partial c_0}$ is positive for financially constrained firms
- ii) $\frac{\partial C}{\partial c_0}$ is indeterminate for financially unconstrained firms.

In empirical terms, Proposition 1 implies that firms should increase their stocks of liquid assets in response to positive cash flow innovations if they face financial constraints ($\frac{\partial C}{\partial c_0} > 0$). In contrast, unconstrained firms should display no such systematic behavior in managing liquidity; i.e., their estimate of the cash flow sensitivity of cash should not be statistically different from zero ($\frac{\partial C}{\partial c_0} \simeq 0$). As we have argued, the cash flow sensitivity of cash for constrained firms should bear no obvious relationship to the “degree” of financial constraints. Our theoretical result is driven by a comparison between constrained and unconstrained firms and our empirical analysis will revolve around this type of contrast. In the tests that follow, we borrow a set of different financial constraint measures from the extant literature.

II. Empirical Tests

We now test our model’s main predictions about a firm’s propensity to save cash out of cash flows and its relation to financial constraints. To do so, we consider the sample of all manufacturing firms (SICs 2000 to 3999) over the 1971 to 2000 period with data available from COMPUSTAT’s P/S/T, Full Coverage, and Research tapes on total assets, sales, market capitalization, capital expenditures, and holdings of cash and marketable securities. All data are CPI-adjusted. We

eliminate firm-years for which cash holdings exceeded the value of total assets, those for which market capitalization was less than \$10 million (in 1971 dollars), and those displaying asset or sales growth exceeding 100%.¹² Our final sample consists of 29,954 firm-years.

A. *Measuring the Cash Flow Sensitivity of Cash and Financial Constraints*

According to our theory, we should expect to find a strong positive relation between cash flow and changes in cash holdings when we look at data from financially constrained firms. Unconstrained firms, in contrast, should display no such relation. In order to implement a test of this argument, we need to specify an empirical model relating changes in cash holdings to cash flows, and also to distinguish between financially constrained and unconstrained firms. We tackle these two issues in turn.

A.1. *Empirical Models of Cash Flow Retention*

We use two alternative specifications to empirically model the cash flow sensitivity of cash. The first model is a parsimonious one, and in addition to firm size, only includes proxies for variables that we believe would capture information related to the primitives of the model: Cash flow innovations and investment opportunities. Define *CashHoldings* as the ratio of holdings of cash and marketable securities to total assets, *CashFlow* as the ratio of earnings before extraordinary items and depreciation (minus dividends) to total assets, and *Q* as the market value divided by the book value of assets. Our baseline empirical model can be written as:

$$\Delta \text{CashHoldings}_{i,t} = \alpha_0 + \alpha_1 \text{CashFlow}_{i,t} + \alpha_2 Q_{i,t} + \alpha_3 \text{Size}_{i,t} + \varepsilon_{i,t}, \quad (8)$$

where *Size* is the natural log of assets. We control for size because of standard arguments of economies of scale in cash management. Our theory's predictions concern the change in cash holdings in response to a shock to cash flows, captured by α_1 in equation (8). The theory also suggests that a constrained firm's cash policy should be influenced by the attractiveness of future

investment opportunities. These opportunities are clearly difficult to measure, so we include Q to capture otherwise unobservable information about the value of long-term growth options that are available to the firm. In principle, we expect α_2 to be positive for constrained firms and unsigned for unconstrained firms. But we recognize that the estimate returned for α_2 might give less useful information about the effect of financial constraints on cash policies than the estimate of α_1 .

One issue we have to consider is whether including Q in our regressions will bias the inferences that we can make about α_1 . Such concerns have become a major issue in the related investment–cash flow literature, as evidence of higher cash flow sensitivities of constrained firms has been ascribed to measurement problems with Q (see, e.g., Erickson and Whited (2000), Gomes (2001), and Alti (2003)).¹³ Fortunately, such problems are unlikely to bias our inferences as we use a *financial* (as opposed to a real) variable in the left-hand side of our empirical specification. In the absence of financial constraints, we expect no systematic patterns in cash policies because changes in cash holdings for unconstrained firms should depend neither on current cash flows nor on future investment opportunities. It is therefore highly unlikely that a positive correlation between cash flows and changes in cash holdings for constrained firms — that is, a positive α_1 in equation (8) — would be simply reflecting a relationship between cash policies and investment opportunities that would obtain even in the absence of financing frictions. Accordingly, the cash flow sensitivity of cash can provide for less ambiguous evidence on the role of financial constraints than standard investment–cash flow sensitivities.

An alternative measure of the empirical cash flow sensitivity of cash is estimated from a specification in which a firm’s decision to change its cash holdings is modeled as a function of a number of sources and (competing) uses of funds. Here, we borrow insights from the literature on investment demand (e.g., Fazzari, Hubbard, and Petersen (1988), Fazzari and Petersen (1993), and Calomiris, Himmelberg, and Wachtel (1995)) and on cash management (Kim, Mauer, and Sherman (1998), Opler, Pinkowitz, Stulz, and Williamson (1999), and Harford (1999)), modeling the annual change in a firm’s cash to total assets also as a function of capital expenditures (*Expenditures*), acquisitions (*Acquisitions*), changes in non-cash net working capital (*NWC*), and changes in short-term debt

(*ShortDebt*), where all of these four additional variables are scaled by assets:

$$\begin{aligned} \Delta CashHoldings_{i,t} = & \alpha_0 + \alpha_1 CashFlow_{i,t} + \alpha_2 Q_{i,t} + \alpha_3 Size_{i,t} \\ & + \alpha_4 Expenditures_{i,t} + \alpha_5 Acquisitions_{i,t} \\ & + \alpha_6 \Delta NWC_{i,t} + \alpha_7 \Delta ShortDebt_{i,t} + \varepsilon_{i,t}. \end{aligned} \tag{9}$$

We control for investment expenditures and acquisitions because firms can draw down on cash reserves in a given year in order to pay for investments and acquisitions. We thus expect α_4 and α_5 to be negative. We control for the change in net working capital because working capital can be a substitute for cash (Opler, Pinkowitz, Stulz, and Williamson (1999)), or it may compete for the available pool of resources (Fazzari and Petersen (1993)). Finally, we add changes in the ratio of short-term debt to total assets because (similarly to net working capital) changes in short-term debt could be a substitute for cash (“cash is negative debt”), or because firms may use short-term debt to build cash reserves.

Notice that one should expect a larger estimate for α_1 to be returned from the augmented equation (9) relative to that from equation (8) if cash flows indeed drive cash savings. The reason is that as we explicitly add controls for alternative uses of funds to a model of savings, we approach an accounting identity in which each new dollar that is not spent must be credited to the “savings account.” Notice, though, that equation (9) is not a perfect identity, and if a firm is financially unconstrained we still expect the coefficient α_1 to be insignificantly different from zero. The empirical tests that use equation (9) are a stronger check of our hypothesis that there should be no systematic relationship between cash flow and cash savings for unconstrained firms.

In estimating equation (9), we explicitly recognize the endogeneity of financial and investment decisions and use an instrumental variables (IV) approach. Clearly, the selection of appropriate instruments is not an obvious task. Our approach follows roughly the rationale proposed by Fazzari and Petersen (1993), which suggests that investment in a specific asset category should depend negatively on the initial stock of that asset due to decreasing marginal valuation associated with stock levels.¹⁴ Our set of instruments includes two lags of the *level* of fixed capital (net plant,

property, and investment to total assets), lagged acquisitions, lagged net working capital, and lagged short-term debt, as well as two-digit SIC industry indicators and twice-lagged sales growth. Finally, in all of our estimations we explicitly control for possible simultaneity biases stemming from unobserved individual heterogeneity by including firm-fixed effects. Because it is possible that unspecified time effects could influence our estimations, we allow the residuals to be correlated within years (across firms) using the “sandwich” (or Huber-White) variance/covariance matrix estimator.

A.2. Financial Constraints Criteria

Testing the implications of our model requires separating firms according to *a priori* measures of the financing frictions they face. Which particular measures to use is a matter of debate in the literature. There are a number of plausible approaches to sorting firms into financially constrained and unconstrained categories. Since we do not have strong priors about which approach is best, we use five alternative schemes to partition our sample:

- Scheme #1: In every year over the 1971 to 2000 period, we rank firms based on their payout ratio and assign to the financially constrained (unconstrained) group those firms in the bottom (top) three deciles of the annual payout distribution. We compute the payout ratio as the ratio of total distributions (dividends plus stock repurchases) to operating income. The intuition that financially constrained firms have significantly lower payout ratios follows from Fazzari, Hubbard, and Petersen (1988), among others.^{15,16}
- Scheme #2: We rank firms based on their asset size over the 1971 to 2000 period, and assign to the financially constrained (unconstrained) group those firms in the bottom (top) three deciles of the size distribution. The rankings are again performed on an annual basis. This approach resembles that of Gilchrist and Himmelberg (1995), who also distinguish between groups of financially constrained and unconstrained firms on the basis of size. The argument

for size as a good observable measure of financial constraints is that small firms are typically young, less well known, and thus more vulnerable to capital market imperfections.

- Scheme #3: We retrieve data on firms’ bond ratings and categorize those firms that never had their public debt rated during our sample period as financially constrained.¹⁷ Observations from those firms are only assigned to the constrained subsample in years when the firms report positive debt.¹⁸ Financially unconstrained firms are those whose bonds have been rated during the sample period. Related approaches for characterizing financial constraints are used by Whited (1992), Kashyap, Lamont, and Stein (1994), and Gilchrist and Himmelberg (1995). The advantage of this measure over the former two is that it gauges the *market’s* assessment of a firm’s credit quality. The same rationale applies to the next proxy.
- Scheme #4: We retrieve data on firms’ commercial paper ratings and assign to the financially constrained group those firms that never had their issues rated during our sample period. Observations from those firms are only assigned to a financially constrained subsample when the firms report positive debt. Firms that issued commercial papers receiving ratings at some point during the sample period are considered unconstrained. This approach follows from the work of Calomiris, Himmelberg, and Wachtel (1995) on the characteristics of commercial paper issuers.
- Scheme #5: We construct an index of firm financial constraints based on results in Kaplan and Zingales (1997) and separate firms according to this measure (which we call the “KZ index”). Following Lamont, Polk, and Saá-Requejo (2001), we first construct an index of the likelihood that a firm faces financial constraints by applying the following linearization to the data:¹⁹

$$\begin{aligned}
 KZindex = & -1.002 \times CashFlow + 0.283 \times Q + 3.139 \times Leverage & (10) \\
 & -39.368 \times Dividends - 1.315 \times CashHoldings.
 \end{aligned}$$

Firms in the bottom (top) three deciles of the KZ index ranking are considered financially unconstrained (constrained). We again allow firms to change their status over our sample

period by ranking firms on an annual basis. Baker, Stein, and Wurgler (2003) use a similar approach to measure financial constraints.

Table I reports the number of firm-years under each of the ten financial constraints categories used in our analysis. According to the payout scheme, for example, there are 9,010 financially constrained firm-years and 8,821 financially unconstrained firm-years. Table I also displays the association among the various classification schemes, illustrating the differences in sampling across the different criteria. For instance, out of the 9,010 firm-years considered constrained according to payout, 4,010 are also constrained according to size, while 1,599 are considered unconstrained. The remaining firm-years represent payout-constrained firms that are neither constrained nor unconstrained according to the size classification.

— insert Table I here —

In general, there is a positive (but less than perfect) association among the sample splits generated by the first four measures of financial constraints. For example, most small (large) firms lack (have) bond ratings. Also, most small (large) firms have low (high) payouts. The noticeable exception is the financial constraint categorization provided by the KZ index. For example, out of the 7,208 KZ-unconstrained firms, only 1,817 (or 25%) are considered size-unconstrained, while 2,578 (or 36%) are size-constrained. Also, out of the 14,149 firm-years classified as unconstrained because of their bond ratings, only 2,706 (or 19%) are also unconstrained according to the KZ index, while a much larger number (4,295) are in fact classified as KZ-constrained. This marked difference in the sample splits generated by the KZ index and the other empirical measures is not surprising in light of the ongoing debate in the literature regarding the set of features one should use to characterize firm financial constraint status (see Fazzari, Hubbard, and Petersen (2000) and Kaplan and Zingales (1997, 2000)). The results we report below will give further evidence that the KZ index and the other measures previously used to proxy for the presence of financial constraints are picking up firms with much different characteristics and behaviors.

B. Results

Table II presents summary statistics on the level of cash holdings of firms in our sample after we classify them into constrained and unconstrained categories. According to the payout ratio, size, and the ratings criteria, unconstrained firms hold on average 8 to 9% of their total assets in the form of cash and marketable securities. These figures resemble those of Kim, Mauer, and Sherman (1998), who report average holdings of 8.1%. Constrained firms, on the other hand, hold far more cash in their balance sheets; on average, some 15% of total assets. Mean and median tests reject equality in the level of cash holdings across groups in all cases. The one classification scheme that yields figures substantially different from the others is the KZ index, which classifies firms that hold more cash as unconstrained. This discrepancy should be expected, given the negative associations between the classifications reported in Table I and the way in which that index is computed. If we think of a firm's cash position as an endogenous variable that is affected by financial constraints, it is not surprising to find that *KZ-unconstrained* firms behave as *payout-*, *size-*, and *ratings-constrained* firms with respect to cash holdings.

– insert Table II here –

Table III presents the results obtained from the estimation of our baseline regression model (equation(8)) within each of the above sample partitions. A total of 10 estimated equations are reported in the table (5 constraints criteria \times 2 constraints categories). Overall, the set of constrained firms displays significantly positive sensitivities of cash to cash flow, while unconstrained firms show insignificant cash–cash flow sensitivities. The sensitivity estimates for constrained firms vary between 0.051 and 0.062 and are all statistically significant at better than the 1% level (excluding the KZ index). These estimates suggest that for each dollar of additional cash flow (normalized by assets), a constrained firm will save around 5 to 6 cents, while unconstrained firms do nothing. The difference in sensitivities between constrained and unconstrained firms is significant at better than the 5% level for the payout, size, bond ratings, and KZ index (with the direction reversed) measures, and at a 10% level for the commercial paper rating measure. These results are consistent

with the predictions of our model.

— insert Table III here —

The fact that we obtain reversed results for the KZ index partitions — that is, significantly positive cash flow sensitivity of cash for unconstrained firms and no systematic sensitivity for the constrained firms — is not surprising given that the sample splits generated by the KZ index are negatively correlated with those generated by most of the other measures, as shown in Table I. Table III simply gives further evidence that KZ-constrained firms seem to behave similarly to firms that are classified as unconstrained according to the other measures with respect to cash policies.

The Q -sensitivity of cash is always positive and it is significant in most of the constrained-sample regressions (payout, bonds, and commercial paper ratings). This result is also consistent with our prior that future investment opportunities should only matter in the constrained sample. Finally, the coefficient for firm size shows wide variation across estimations.

Table IV reports the results we obtain by fitting equation (9) to the data. The model is estimated via IV with firm fixed-effects and robust standard errors. The cash flow sensitivity of cash estimates reveal the same patterns reported in Table III. The sensitivity estimates are all positive and mostly highly significant for constrained firms but insignificant for the unconstrained ones, with the usual exception of the KZ index regressions. One noticeable feature is the magnitude of the reported sensitivities. The median estimate of the constrained firms' sensitivity (taken from the commercial paper ratings regression) is nearly 5 times larger than the corresponding estimate from Table III. This increase in estimated cash flow sensitivities of cash is expected, given that the estimates in Table IV control for additional sources and uses of funds. At the same time, however, the cash flow sensitivities of unconstrained firms remain insignificant, even after controlling for these additional variables. This finding is again consistent with the view that there are systematic differences between constrained and unconstrained firms in the way they conduct their cash policies, and that these differences are manifested along the lines suggested by our theory. Most of the coefficients

for the other regressors attract the expected signs.

— insert Table IV here —

C. *Robustness*

We subject our estimates to a number of robustness checks in order to address potential concerns about model specification and other estimation issues. In Table V, we present estimates of the cash flow sensitivity of cash using four alternative sampling/specifications. To save space, we only report the results for the estimated cash flow coefficient that are returned after we impose changes to our baseline model, equation (8).

— insert Table V here —

At the top of Table V (see row 1) we report cash flow sensitivities from a sample of firm-years for which cash flows are strictly larger than required investment outlays (i.e., firm-years with positive “free cash flow”). One could argue that the positive cash flow sensitivity of cash we have observed is driven by a simple mechanical relation that dictates that cash holdings ought to decline when required investments exceed operating income (i.e., when cash flows are “too low”). Of course, the level of investment is (to an extent) set at the discretion of the firm, and to test the argument that the firm will draw down reserves to make necessary investments, we would need a measure of nondiscretionary investment. We use the ratio of depreciation over assets as a proxy for nondiscretionary (required) investment outlays and define free cash flow as the difference between cash flows and depreciation. After eliminating those 4,416 firm-years for which cash holdings and cash flows could be “hard-wired” by a simple financing deficit, we still find the same patterns: Only constrained firms display significant cash–cash flow sensitivities (except when using the KZ index).

A different specification we experiment with replaces Q with an alternative proxy for the firm’s investment opportunity set. As suggested by Altı (2003), Q contains useful information about a firm’s growth options and thus about a firm’s future investment opportunities. However, to

the extent that Q also contains information about current investment opportunities, it will be a noisy measure of the variable that the model suggests should drive cash policies. One way to check directly whether the use of the standard Q measure in the basic regressions is somehow contributing to our empirical findings is to replace it with another proxy relating future and current investment opportunities. We do so by replacing Q with the actual ratio of future investment to current investment in our baseline regression.²⁰ The cash flow sensitivities that are returned after substituting the proxy for future investment opportunities are reported in row 2 of Table V. There is virtually no change in our estimates of the cash flow sensitivity of cash.

In the third row of Table V, we adopt a novel approach to mitigate the concern that Q is poorly measured. Following Cummins, Hasset, and Oliner (1999) and Abel and Eberly (2002), we use financial analysts' forecasts of earnings as an instrument for Q in an IV estimation of our baseline model. As in Polk and Sapienza (2002), we employ the median forecast of the two-year ahead earnings scaled by lagged total assets to construct the earnings forecast measure. The earnings data come from IBES, where extensive coverage only starts in 1984. Although only some 58% of the firm-years in our original sample provide valid observations for earnings forecasts, our basic results remain nearly intact, with the exception of the firm size split.²¹

Recall that our model does not formally attempt to distinguish between firms that are *relatively* more or less financially constrained at a given point in time. Specifically, the model does not predict that relatively more constrained firms should have higher (or lower) cash flow sensitivities of cash. Nonetheless, using a measure of the degree of financial constraint faced by a firm as a control variable is a worthwhile exercise from an empirical point of view.²² We therefore add to the baseline model the twice-lagged level of firm cash holdings, as well as its interaction with the cash flow variable.²³ We report the results of this model estimation in row 4 of Table V. While the coefficients on lagged cash are significantly negative (indicating that higher lagged cash reduces the *level* of additional savings), the coefficients on the interaction terms are indistinguishable from zero in all estimations performed (coefficients omitted). More importantly, the estimates for the cash flow sensitivity of cash are not significantly affected by the inclusion of the proposed controls.

We also perform a number of further robustness checks. First, we move the lagged level of cash/assets from the left-hand side to the right-hand side of our baseline model, effectively removing the constraint that lag cash/assets should have a coefficient of 1. This approach, which resembles more closely that of Opler, Pinkowitz, Stulz, and Williamson (1999), yields no qualitative changes in our estimates. Second, we notice that a fraction of the firms in our manufacturing sample are part of large conglomerates controlling financial subsidiaries with sizeable contributions to conglomerate operations.²⁴ Since those firms’ cash policies could be influenced by the presence of “financial arms” in their conglomerates in ways that differentiate their liquidity management behavior, we re-estimate our tests after deleting those firms from the sample.²⁵ Our results are unaffected by this sampling restriction. Finally, because of the possibility of extreme outliers having undue influence on our results, we re-estimate our models using trimmed data and (alternatively) via quantile regressions. Doing so does not affect our conclusions.

D. Dynamics of Liquidity Management: Responses to Macroeconomic Shocks

A potential objection to the results presented above arises from concerns about estimation biases that are likely to be present in any regression in which unobserved characteristics or measurement problems could play a role. In this case, it is possible that measurement error in Q or one of our other variables could cause the *levels* of the cash–cash flow sensitivity estimates presented in Tables III through V to be biased in a way that confirms our hypothesis. One way of providing independent confirmation of the interpretation we propose is to take the empirics to the next logical step of our theory. Recall that our theory is about the role of financial constraints in determining cash policies, where these policies (when they exist) are meant to balance a firm’s ability to generate resources and make optimal investment decisions over time. A natural question to ask is: How do cash policies change in response to events affecting *both* the firms’ ability to generate cash flows as well as the shadow cost of new investment?

While we have an empirical model of cash flow retention, we need to identify some type of natural event or shock that can help us answer this question. Of course, such an event should be exogenous

to the firm policy set and preferably economy-wide, simultaneously affecting all firms in the sample at a given point in time, thus providing for cross-sectional contrasts. As it turns out, examining the path of the cash flow sensitivity of cash holdings over the business cycle allows for an alternative test of the idea that financial constraints drive significant differences in corporate cash policies. To wit, if our conjecture about those policies is correct, then we should see financially constrained firms saving an even greater proportion of their cash flows during downturns. This should happen because these periods are characterized both by an increase in the marginal attractiveness of future investments (when compared to current ones), as well as by a decline in current income flows.²⁶ The cash policy of financially unconstrained firms, on the other hand, should not display such pronounced patterns. In other words, the *responses* of cash flow sensitivities of cash to shocks to aggregate demand should be stronger (i.e., more countercyclical) for financially constrained firms. This should happen regardless of the *levels* of those sensitivity estimates. Our macro-level test will thus sidestep concerns with biases in the baseline equation.

To implement this test, we use a two-step approach similar to that used by Kashyap and Stein (2000) and Campello (2003). The idea is to relate the sensitivity of cash to cash flow and aggregate demand conditions by combining cross-sectional and times series regressions. The first step of the procedure consists of estimating the baseline regression model (equation (8)) every year separately for groups of financially constrained and unconstrained firms. From each sequence of cross-sectional regressions, we collect the coefficients returned for cash flow (i.e., α_1) and “stack” them into the vector Ψ_t , which is then used as the dependent variable in the following (second-stage) time series regression:

$$\Psi_t = \eta + \phi \Delta Activity_t + \rho Trend_t + u_t, \tag{11}$$

where the term $\Delta Activity$ represents innovations to aggregate activity. These innovations are computed from the residual of an autoregression of log real GDP on three lags of itself, with the error structure following a moving average process.²⁷ The impact of unforecasted shocks to aggregate activity on the sensitivity of cash to cash flow can be gauged from ϕ . A time trend ($Trend$) is included to capture secular changes in cash policies. Finally, because movements in

aggregate demand and other macroeconomic variables often coincide, in “multivariate” versions of equation (11) we also include changes in inflation (log CPI) and changes in basic interest rates (Fed funds rate) to ensure that our findings are not driven by contemporaneous innovations affecting the cost of money. These series are gathered from the Bureau of Labor Statistics and from the Federal Reserve (*Statistical Release H.15*).

The results from the two-stage estimator are summarized in Table VI. The table reports the coefficients for ϕ from equation (11), along with the associated p -values (calculated via Newey-West). Row 1 collects the results for financially constrained firms and row 2 reports results for unconstrained firms. Additional tests for differences between coefficients across groups are reported at the bottom of the table (row 3). Standard errors for the “difference” coefficients are estimated via a SUR system that combines the two constraint categories (p -values reported).

– insert Table VI here –

The GDP-response coefficients for the constrained firms reported in row 1 are negative and statistically significant, suggesting that constrained firms’ cash policies respond to shocks affecting cash flows and the intertemporal attractiveness of investment along the lines suggested by our theory. In contrast, the response coefficients for the unconstrained firms (row 2) are uniformly positive and typically indistinguishable from zero. The only exception is the KZ index-based sensitivity series, which once again displays the exact opposite pattern. The differences between those sets of coefficients (in row 3) suggest that the cash flow sensitivities of cash for financially constrained and unconstrained firms follow markedly different paths in the aftermath of negative shocks to macroeconomic conditions: Constrained firms save significantly larger fractions of their cash flows than unconstrained firms do following those shocks. These patterns should be expected in the context of the theory of liquidity management we propose.

III. Concluding Remarks

We model the link between financial constraints and corporate liquidity demand and propose a new empirical test of the impact of financial constraints on firm policies. Because only those firms whose investments are constrained by capital market imperfections manage liquidity to maximize value, financial constraints can be captured by a firm's *cash flow sensitivity of cash*. Empirically, financially constrained firms' holdings of liquid assets should increase when cash flows are higher, and thus their cash flow sensitivity of cash should be positive. In contrast, unconstrained firms' cash flow sensitivity of cash should display no systematic patterns. The use of cash–cash flow sensitivities to test for financial constraints sidesteps some of the criticisms that have plagued the interpretation of tests of financial constraints that use investment–cash flow sensitivities (see Gomes (2001) and Alti (2003)).

To examine our model's implication, we estimate cash–cash flow sensitivities using a sample of publicly traded manufacturing firms between 1971 and 2000. First, we classify firms according to empirical proxies for the likelihood that they face financial constraints using five alternative approaches suggested by the literature: Firm payout policy, asset size, bond ratings, commercial paper ratings, and an index measure derived from results in Kaplan and Zingales (1997) (KZ index). We then test the hypothesis that the propensity to save from cash inflows is positive for the constrained firms, but is indistinguishable from zero for the unconstrained ones. Our empirical results are consistent with our theoretical priors. For of four of our classification schemes, we find that constrained firms display significantly positive cash–cash flow sensitivities, while unconstrained firms do not. The exact opposite results obtain for the KZ index, which, as we document, generates constrained/unconstrained firm assignments that correlate mostly negatively with those of the other four classification criteria.

Our theory also implies that cash holding patterns should vary over the business cycle. In particular, financially constrained firms should increase their propensity to retain cash following negative macroeconomic shocks, while unconstrained firms should not. We examine this hypothesis empirically and find that it holds for four of our classification criteria, but again not for the KZ

index.

Gauging the impact of financial constraints on observed firm behavior has become a challenging issue in corporate finance. Our analysis suggests that the cash flow sensitivity of cash is a theoretically justified and empirically useful variable that is correlated with a firm's ability to access capital markets. We hope future researchers find our testing strategy to be a valuable tool in addressing questions in which financial constraints are likely to play a role, such as efficiency of internal capital markets (Lamont (1997) and Shin and Stulz (1998)), the effect of agency on firm policies (Hadlock (1998)), and the influence of managerial characteristics on firm behavior (Malmendier and Tate (2003)).

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Notes

¹See Hubbard (1998) for a comprehensive survey. Some representative references are Hoshi, Kashyap, and Stein (1991), Whited (1992), Calomiris and Hubbard (1994), Gilchrist and Himmelberg (1995), and Lamont (1997).

²This has happened in spite of ample evidence suggesting that CFOs consider financial flexibility (i.e., having enough internal funds to finance future investments) to be the primary determinant of their policy decisions (see the recent surveys by Graham and Harvey (2001) and Bancel and Mittoo (2002)).

³An incomplete list of papers includes Kim, Mauer, and Sherman (1998), Opler, Pinkowitz, Stulz, and Williamson (1999), Pinkowitz and Williamson (2001), Billett and Garfinkel (2002), Faulkender (2003), Ozkan and Ozkan (2002), Mikkelsen and Partch (2003), and Dittmar, Mahrt-Smith, and Servaes (2003). Opler, Pinkowitz, Stulz, and Williamson further examine the persistence of cash holdings, and characterize what firms do with “excess” cash. Other related papers are John (1993), who studies the link between liquidity and financial distress costs, and Acharya, Huang, Subrahmanyam, and Sundaram (2002), who consider the effect of optimal cash policies on corporate credit spreads.

⁴We implicitly assume that the firm’s existing cash stock is equal to zero. Alternatively, one could see the parameter c_0 as the sum of the existing cash stock and time 0 cash flow.

⁵We simplify the analysis by assuming that the time 1 cash flow from existing assets is unrelated to new investment at time 0. More weakly, what we need is that the firm might need to make the capital expenditure I_1 before the investment I_0 pays off fully — i.e., an intertemporal mismatch between investment outlays and payoffs.

⁶More generally, one might think that the underlying source of incomplete contractibility will also cap the firm’s ability to transfer resources across states. In a previous version of this paper we analyze the effect of constrained hedging in the model. It turns out that the possibility of constrained hedging, while increasing the potential value of cash holdings, does not change our main conclusions.

⁷An alternative theoretical framework that yields a quantity constraint on the amount of equity finance that the firm can raise is the moral hazard model of Holmstrom and Tirole

(1998). In that setup, it is not optimal for firms to issue equity beyond a certain threshold due to private benefits of control. Similarly to Almeida and Campello (2002), our analysis could be alternatively conducted using the Holmstrom-Tirole framework.

⁸Under the alternative interpretation that the initial cash flow c_0 includes the existing cash stock (see footnote 4), the firm would also be more likely to be unconstrained when it “enters the model” with a large cash stock. Of course, another possibility is that a large cash stock is indicative that the firm anticipates facing financing constraints in the future (as opposed to implying that the firm faces no such constraints).

⁹This is just a traditional “full-insurance” result. Full insurance is optimal because the productivity of investment is the same in both states (see Froot, Scharfstein, and Stein (1993)).

¹⁰Similar results hold for a more general Cobb-Douglas specification for the production function, namely $F(x) = Ax^\alpha$ and $G(x) = Bx^\alpha$. The important assumption is that the degree of concavity of the functions $F(\cdot)$ and $G(\cdot)$ is the same. Given this, the particular value of α is immaterial.

¹¹While the result that the cash flow sensitivity is completely unrelated to the degree of the financial constraint holds strictly only for Cobb-Douglas production functions in which $f(\cdot)$ and $g(\cdot)$ have similar concavities, it is still true that there is no obvious relationship between the degree of the financial constraint and the cash flow sensitivity of cash for more general production functions.

¹²This last screen eliminates from the sample those firm-years registering large jumps in business fundamentals (size and sales); these are typically indicative of mergers, reorganizations, and other major corporate events.

¹³The fundamental reason why there are inference problems in the investment–cash flow literature is that even in the absence of financial constraints, we should still expect investment and cash flows to be positively correlated if cash flows contain information about the relationship between real investment demand and investment opportunities.

¹⁴For example, this year’s investment in working capital should be negatively correlated with the beginning-of-period level of working capital.

¹⁵Fazzari, Hubbard, and Petersen (1988) only retain those firms with positive real sales growth in their tests. Since our sample period encompasses data from a number of recessions, and because we explore these cycle movements to test our model, we relax the sales growth restriction to avoid data selection biases. We note, though, that our results are even stronger when we impose this extra restriction.

¹⁶The deciles are set according to the distribution of the actual ratio of the dividend payout reported by the firms and thus generate an unequal number of observations being assigned to each of our groups. Our approach ensures that we do not assign firms with low payouts to the financially unconstrained group, and that firms with the same payout are always assigned to the same group. For instance, even when more than 30% of the firms have a zero payout in a given year, all zero payout firms are assigned to the constrained group. The minimum payout for the firms in the top three deciles of the payout ratio is 0.42 (across all years), whereas the maximum payout of the low three decile firms is 0.27.

¹⁷Comprehensive coverage of bond ratings by COMPUSTAT only starts in the mid-1980s.

¹⁸Firms with no bond rating and no debt are considered unconstrained. Our results are not affected if we treat these firms as neither constrained nor unconstrained. We use the same criterion for firms with no commercial paper rating and no debt in scheme #4.

¹⁹To compute the KZ index we use the original variable definitions of Kaplan and Zingales (1997).

²⁰For a given firm in year 0 this ratio is computed as $(I_1 + I_2) / 2I_0$. Our results are robust to the use of alternative time horizons to measure future investment.

²¹We have also experimented with using an industry-level measure of Q , and with removing Q from the benchmark specification altogether. Our conclusions about cash–cash flow sensitivities are insensitive to both of these changes.

²²We thank an anonymous referee for making this suggestion.

²³We use the second lag to avoid a spurious correlation between lagged cash and our dependent variable.

²⁴Examples found in our sample are John Deere, Ford, GM, Whirlpool, and Xerox. There

are 401 firm-years in this category.

²⁵For example, concerns with “low” cash positions could reflect particularly negatively on the financial subsidiaries. Alternatively, those subsidiaries’ closer ties to the capital markets could alter liquidity management in the presence of active internal capital markets in their conglomerates.

²⁶Our model suggests that if there is an exogenous increase in future investment opportunities relative to current ones — an increase in the parameter δ in equation (7) — the cash flow sensitivity of cash should increase.

²⁷Although the macro innovation proxy is a generated regressor, the coefficient estimates of equation (11) are consistent (see Pagan (1984)).

Table I
Cross-Classification of Constraint Types

This table displays firm-year cross-classification for the various criteria used to categorize firm-years as either financially constrained or unconstrained. To facilitate visualization, we assign the letter (A) for constrained firms and (B) for unconstrained firms in each row/column. The sampled firms include only manufacturers (SICs 2000 to 3999) in the COMPUSTAT annual industrial tapes. The sample period is 1971 through 2000.

Financial Constraints Criteria	Payout Ratio		Firm Size		Bond Ratings		CP Ratings		KZ Index	
	(A)	(B)	(A)	(B)	(A)	(B)	(A)	(B)	(A)	(B)
1. Payout Ratio										
Constrained firms (A)	9,010									
Unconstrained firms (B)		8,821								
2. Firm Size										
Constrained firms (A)	4,010	1,753	9,002							
Unconstrained firms (B)	1,599	3,752		9,272						
3. Bond Ratings										
Constrained firms (A)	5,393	3,930	7,759	1,712	15,805					
Unconstrained firms (B)	3,617	4,891	1,243	7,560		14,149				
4. Comm. Paper Ratings										
Constrained firms (A)	7,751	5,453	8,789	3,457	15,474	6,457	21,931			
Unconstrained firms (B)	1,259	3,368	213	5,815	331	7,692		8,023		
5. Kaplan-Zingales Index										
Constrained firms (A)	3,540	1,559	1,463	2,666	3,126	4,295	5,625	1,796	7,421	
Unconstrained firms (B)	992	3,064	2,578	1,817	4,502	2,706	5,144	2,064		7,208

Table II**Summary Statistics of Cash Holdings**

This table displays summary statistics for cash holdings across groups of financially constrained and unconstrained firms. We assign the letter (A) for constrained firms and (B) for unconstrained firms in each row. All data are from the annual COMPUSTAT industrial tapes. The sampled firms include only manufacturers (SICs 2000 to 3999) and the sample period is 1971 through 2000.

Cash Holdings	Mean	Median	Std. Dev.	N. Obs
Financial Constraints Criteria				
1. Payout Ratio				
Constrained firms (A)	0.145	0.074	0.177	9,010
Unconstrained firms (B)	0.090	0.051	0.106	8,821
<i>p</i> -value (A–B≠0)	0.00	0.00		
2. Firm Size				
Constrained firms (A)	0.178	0.110	0.191	9,002
Unconstrained firms (B)	0.079	0.051	0.082	9,272
<i>p</i> -value (A–B≠0)	0.00	0.00		
3. Bond Ratings				
Constrained firms (A)	0.146	0.085	0.167	15,805
Unconstrained firms (B)	0.081	0.049	0.092	14,149
<i>p</i> -value (A–B≠0)	0.00	0.00		
4. Comm. Paper Ratings				
Constrained firms (A)	0.129	0.070	0.155	21,931
Unconstrained firms (B)	0.076	0.051	0.076	8,023
<i>p</i> -value (A–B≠0)	0.00	0.00		
5. Kaplan-Zingales Index				
Constrained firms (A)	0.055	0.030	0.076	7,421
Unconstrained firms (B)	0.179	0.134	0.166	7,208
<i>p</i> -value (A–B≠0)	0.00	0.00		

Table III**The Baseline Regression Model**

This table displays results for OLS estimations of the baseline regression model (equation (8) in the text). All data are from the annual COMPUSTAT industrial tapes. The sampled firms include only manufacturers (SICs 2000 to 3999) and the sample period is 1971 through 2000. The estimations include firm effects and correct the error structure for heteroskedasticity and for within-period error correlation using the White-Huber estimator. The associated t -statistics are reported in parentheses.

Dependent Variable	Independent Variables			R^2
$\Delta CashHoldings$	$CashFlow$	Q	$Size$	
Financial Constraints Criteria				
1. Payout Ratio				
Constrained firms	0.0593 (4.53)*	0.0029 (2.41)**	0.0019 (0.61)	0.28
Unconstrained firms	-0.0074 (-0.28)	0.0001 (0.01)	0.0001 (0.05)	0.28
2. Firm Size				
Constrained firms	0.0620 (4.12)*	0.0016 (1.65)	-0.0014 (-0.28)	0.26
Unconstrained firms	0.0099 (0.47)	0.0015 (1.52)	-0.0035 (-1.55)	0.17
3. Bond Ratings				
Constrained firms	0.0580 (4.80)*	0.0016 (2.31)**	-0.0029 (-1.07)	0.20
Unconstrained firms	0.0179 (1.35)	0.0025 (1.91)	-0.0022 (-0.92)	0.15
4. Comm. Paper Ratings				
Constrained firms	0.0505 (4.83)*	0.0017 (2.93)*	-0.0031 (-1.37)	0.17
Unconstrained firms	0.0108 (0.49)	0.0022 (1.78)	-0.0014 (-0.46)	0.12
5. Kaplan-Zingales Index				
Constrained firms	0.0045 (0.26)	0.0061 (4.43)*	0.0015 (0.71)	0.33
Unconstrained firms	0.1066 (3.01)*	0.0003 (0.26)	-0.0026 (-0.61)	0.22

* and ** indicate statistical significance at the 1% and 5% (two-tail) test levels, respectively.

Table IV
The Augmented Regression Model

This table displays results for IV estimations of the augmented regression model (equation (9) in the text). All data are from the annual COMPUSTAT industrial tapes. The sampled firms include only manufacturers (SICs 2000 to 3999) and the sample period is 1971 through 2000. The estimations include firm effects and correct the error structure for heteroskedasticity and for within-period error correlation using the White-Huber estimator. The associated *t*-statistics are reported in parentheses.

Dependent Variable	Independent Variables							R^2
$\Delta CashHoldings$	$CashFlow$	Q	$Size$	$Expenditures$	$Acquisitions$	ΔNWC	$\Delta ShortDebt$	
Financial Constraints Criteria								
1. Payout Ratio								
Constrained firms	0.1488 (1.86)	0.0103 (1.65)	0.0123 (1.11)	-0.6471 (-0.92)	-0.5061 (-0.53)	-0.0013 (-4.35)*	0.1196 (1.05)	0.11
Unconstrained firms	0.0240 (0.44)	-0.0015 (-0.39)	0.0608 (2.75)*	-1.6137 (-1.89)	-3.1656 (-2.72)*	-0.0004 (-1.92)	0.5432 (2.39)**	0.07
2. Firm Size								
Constrained firms	0.3836 (12.24)*	0.0118 (5.22)*	0.0539 (5.05)*	-1.2773 (-4.78)*	-0.1763 (-0.70)	-0.0223 (-14.27)*	0.3735 (4.78)*	0.15
Unconstrained firms	0.0364 (1.08)	0.0010 (0.62)	0.0179 (3.12)*	0.0626 (0.25)	-0.7855 (-2.74)*	-0.0001 (-2.82)*	0.2435 (2.32)**	0.12
3. Bond Ratings								
Constrained firms	0.2793 (3.17)*	0.0073 (1.23)	0.0291 (1.26)	-1.1936 (-1.63)	0.1016 (0.13)	-0.0051 (-4.41)*	0.2735 (1.58)	0.11
Unconstrained firms	-0.0975 (-1.81)	0.0109 (3.31)*	0.0358 (3.92)*	-0.2179 (-0.64)	-2.4192 (-3.68)*	-0.0001 (-0.63)	0.1575 (1.41)	0.13
4. Comm. Paper Ratings								
Constrained firms	0.3374 (5.13)*	0.0127 (3.51)*	0.0265 (2.16)**	-1.6352 (-3.17)*	-0.2931 (-0.53)	-0.0049 (-6.11)*	0.2443 (2.42)**	0.12
Unconstrained firms	-0.2777 (-2.13)**	0.0050 (1.93)	0.0330 (3.06)*	0.6430 (2.08)**	-1.6936 (-2.43)**	0.0001 (0.49)	0.0093 (0.09)	0.12
5. Kaplan-Zingales Index								
Constrained firms	0.1621 (1.97)**	0.0080 (1.60)	-0.0097 (-0.88)	-0.9989 (-2.52)**	0.0552 (0.13)	-0.0003 (-3.02)*	0.2937 (2.28)**	0.10
Unconstrained firms	0.3053 (3.40)*	0.0216 (2.27)**	0.0398 (1.32)	-2.1115 (-1.33)	0.2851 (0.28)	-0.0053 (-4.31)*	0.7159 (2.29)**	0.09

* and ** indicate statistical significance at the 1% and 5% (two-tail) test levels, respectively.

Table V
Robustness Checks

This table shows results for alternative versions of the baseline model (equation (8)) and alternative sampling. Only the coefficients returned for *CashFlow* are reported. All data are from the annual COMPUSTAT industrial tapes. The sampled firms include only manufacturers (SICs 2000 to 3999) from 1971 through 2000. The estimations include firm effects and correct the error structure for heteroskedasticity and for within-period error correlation using the White-Huber estimator. The associated *t*-statistics are reported in parentheses.

Dependent Variable <i>ΔCashHoldings</i>	Financial Constraints Criteria				
	Payout Ratio	Firm Size	Bond Ratings	CP Ratings	KZ Index
1. Sample restricted to firms with positive free cash flow					
Constrained firms	0.0789 (1.89)	0.0889 (2.40)**	0.0954 (3.58)*	0.0966 (4.39)*	0.0157 (0.43)
Unconstrained firms	−0.1052 (−2.32)**	0.0520 (1.05)	0.0657 (1.18)	0.0062 (0.11)	0.1815 (2.89)*
2. <i>Q</i> is replaced by the ratio of (realized) future investment to current investment					
Constrained firms	0.0450 (2.35)**	0.0729 (3.17)*	0.0614 (3.22)*	0.0498 (3.57)*	0.0020 (0.09)
Unconstrained firms	0.0068 (0.22)	0.0090 (0.58)	0.0120 (0.86)	0.0044 (0.27)	0.1118 (2.70)*
3. <i>Q</i> is instrumented with financial analysts' earnings forecasts					
Constrained firms	0.0447 (3.50)*	0.0144 (0.71)	0.0568 (2.76)*	0.0867 (2.06)**	0.0496 (1.30)
Unconstrained firms	−0.0091 (−0.68)	0.0181 (0.98)	0.0255 (1.06)	0.0119 (0.72)	0.0950 (4.94)*
4. Lagged <i>Cash/Assets</i> and its interaction with <i>CashFlow</i> are added to the r.h.s. of the model					
Constrained firms	0.0621 (2.63)*	0.1243 (3.80)*	0.0431 (1.98)**	0.0415 (2.96)*	0.0021 (0.07)
Unconstrained firms	−0.0127 (−0.40)	0.0178 (1.01)	0.0204 (1.26)	−0.0209 (−0.88)	0.0597 (2.26)**

* and ** indicate statistical significance at the 1% and 5% (two-tail) test levels, respectively.

Table VI
Macroeconomic Dynamics

The dependent variable is the estimated sensitivity of cash holdings to cash flow. In each estimation, the dependent variable is regressed on the residual of an autoregression of the log real GDP on three of its own lags ($\Delta Activity$). All regressions include a constant and a time trend. Only the coefficients returned for $\Delta Activity$ are reported in the table. In the multivariate regressions, changes in inflation (log CPI) and changes in basic interest rates (Fed funds rate) are also added. The sampled firms include only manufacturers (SICs 2000 to 3999) and the sample period is 1971 through 2000. Heteroskedasticity- and autocorrelation-consistent errors are computed with a Newey-West lag window of size four. The standard errors for cross-equation differences in the GDP-innovation coefficients are computed via a SUR system that estimates the group regressions jointly. The associated p -values are reported in parentheses.

	Financial Constraints Criteria				
	Payout Ratio	Firm Size	Bond Ratings	CP Ratings	KZ Index
1. Constrained Firms					
Univariate	-1.841 (0.01)	-1.576 (0.08)	-1.898 (0.00)	-1.333 (0.00)	0.156 (0.31)
Multivariate	-2.233 (0.02)	-2.283 (0.08)	-2.268 (0.01)	-1.491 (0.01)	-0.255 (0.70)
2. Unconstrained Firms					
Univariate	1.867 (0.31)	1.509 (0.03)	0.212 (0.81)	0.345 (0.74)	-3.547 (0.00)
Multivariate	1.769 (0.25)	1.675 (0.01)	0.426 (0.67)	0.292 (0.82)	-3.876 (0.00)
3. Difference Constrained–Unconstrained					
Univariate	-3.708 (0.05)	-3.086 (0.02)	-2.110 (0.02)	-1.678 (0.10)	3.703 (0.01)
Multivariate	-4.002 (0.03)	-3.958 (0.00)	-2.693 (0.01)	-1.783 (0.12)	3.621 (0.02)