Capital Structure and Hedging Demand with Incomplete Markets*

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Abstract

We develop a general equilibrium model with production and incomplete markets. Firms optimally design their capital structure to cater to investors' hedging needs. Depending on the heterogeneity of such needs, equilibrium may feature either complete financial market segmentation or only partial segmentation. Firms respond to greater hedging needs by issuing more debt and allocating most of the proceeds to shareholders. How much more debt, depends on the availability of competing risk-sharing instruments. When the capital structure is jointly shaped by hedging demand and agency (asset substitution), the greater risk induced by asymmetric information has countervailing effects on debt: Debt is reduced to nudge shareholders into choosing lower risk; however, the greater risk in production affects the state prices and calls for more debt.

Key words: Leverage, heterogeneous agents, risk-sharing, agency, asset substitution.

JEL Codes: D51, D52, D53, G32.

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1 Introduction

In this paper, we introduce a theoretical framework where non-financial firms issue liabilities with the intent of catering to investors' hedging needs. The corporate finance literature has established that capital structure choices are likely shaped by tax considerations, costly default, agency, and asymmetric information. We refer to these as *supply considerations*, as they affect the relative costs of issuing one versus another type of securities. However, as documented in the empirical literature, firms' capital structure choices also depend on the prices of their liabilities – equity and bonds, in particular – which are in turn affected by *hedging demand considerations*. These are the factors leading investors to develop a differential appetite for corporate securities with different risk properties.[1]

While a large body of literature has investigated in detail the implications of supply considerations, much less attention has been paid to the role played by hedging demand. This may be due in part to the fact that capital structure responds to variations in hedging demand only in incomplete-market general equilibrium economies, whose analysis is often involved.[2]

Here, we analyze such an economy. We consider first a scenario free of any supply considerations. There is neither a tax advantage of debt, nor default costs, nor agency problems. It is only the cross–sectional variation in investors' hedging needs, via its effect on asset prices, that drives firms' capital structure.

Our model features a continuum of ex-ante identical firms, equipped with a decreasing-return-to-scale production function subject to an aggregate productivity shock – the only aggregate factor. Markets are incomplete, in the sense that households can only invest in firm-issued bonds and stocks. Investors differ in the correlation of their endowment with the aggregate factor. Riskier investors – those with relatively higher correlation – display a higher willingness to pay for corporate bonds, which allows for better risk sharing. Although not default-free, corporate debt provides hedging services and therefore commands a safety premium over equity at equilibrium.

This safety premium depends negatively on the firm's default risk, hence on its leverage. In our economy with incomplete markets, it is the trade-off between the quantity of bonds issued – the firm's leverage – and their unit price, reflecting the safety premium,

[1] We are wary of a potential terminological confusion. Investors' demand for hedging instruments issued by firms can also be envisioned as their supply of capital to firms. Indeed, the latter convention is adopted by part of the literature. For example, see Baker (2009).

[2] Under complete markets, hedging demand can affect the capital structure only indirectly, by influencing the trade-offs that commonly determine financing in partial equilibrium. We discuss this literature in Section 1.1.

that pins down the capital structure.

As the hedging demand by riskier investors rises, firms cater to their needs by increasing both their capital stock and their debt. The debt rises to allow investors to better allocate consumption across states of nature. The capital stock increases to attenuate the effect of higher debt on default risk.

Equilibria characterized by greater hedging demand are associated with higher debt, leverage, and price of debt, therefore rationalizing the finding that securities' issuance responds to market valuation. Ritter (1991), Baker and Wurgler (2000) and Ma (2019) among others, document that firms systematically react to hikes in security prices by increasing their issuance. Because of the Modigliani-Miller indeterminacy result, it is impossible to make sense of this evidence under complete markets.[3]

Market leverage increases when hedging demand increases, because the proceeds from debt issuance grow faster than investment. The difference is passed on to shareholders as greater dividends. This result further highlights that debt is issued by virtue of its role as hedging instrument, independently of investment-driven financing needs. It is consistent with Ma (2019) and Mota (2023), who document that when firms issue debt in response to higher bond prices, the proceeds tend to be passed to shareholders rather than used to finance investment.[4]

In our framework, an increase in hedging demand may also result from higher aggregate risk. This implies a positive association between risk and leverage. In spite of the large body of work on the time-variation of aggregate risk since Bloom (2009)'s seminal contribution, its relation with capital structure is not well understood. However, since risk is found to be greater in recessions, and even though in our analysis an increase in risk is not accompanied by a decline in aggregate productivity, our results can be interpreted as rationalizing Halling et al. (2016)'s finding that leverage is countercyclical.

We are aware that corporate bonds are not the primary means of pursuing insurance against consumption risk. The literature has long recognized that so-called safe assets such as the sovereign bonds issued in a restricted set of developed economies play an outsize role in this arena. Our analysis identifies conditions under which low-risk corporate bonds are imperfect substitutes of such sovereigns, and firms issue them to cater to investors' hedging needs.

In an extension of our baseline model, we allow for the exogenous supply of a perfectly

[3] We remind the reader to Section 1.1 for a discussion of alternative rationalizations of this evidence.

[4] Kubitza (2024) also finds that when insurers' demand for corporate bonds raises bond prices, firms respond by issuing more debt. However, the proceeds are funneled towards investment rather than equity payouts.

safe asset – a risk-free bond. In that framework, a rise in hedging needs that is not matched by a proportionate rise in the supply of perfectly safe asset leads firms to increase their debt issuance, and to destine a fraction of the greater proceeds to equity payout.

This prediction aligns well with the evidence on *safe-asset shortage*. Caballero et al. (2008), Gorton et al. (2012), and Caballero et al. (2017), among others, have argued convincingly that the protracted decline in the level of interest rates that we witnessed over the last three decades finds its root cause in a sustained increase in worldwide precautionary saving in the face of a stable supply of safe asset. Mota (2023) finds that in the period since the 2008 financial crisis, when the excess demand for safe asset likely intensified, non-financial corporations almost doubled their debt. At the same time, as documented by Covarrubias et al. (2020) and Crouzet and Eberly (2018), aggregate investment was weak.

Modern financial markets can also accommodate a rise in the demand for safety by increasing the supply of derivative assets. In order to understand the effect of such channel on firms' capital structure decisions, we relax the short-selling constraint and allow financial intermediaries to sell short positions on corporate debt. We show that a decline in transaction costs leading to an expansion of the derivative market crowds out the underlying asset, leading to a drop in corporate debt and leverage.

We further show that when firms are allowed to choose among technologies of different risk, an increase in hedging demand is met by a rise in the fraction of firms opting for production plans yielding safer cash flows. The securities issued by relatively safer firms crowd out the debt of riskier firms, leading the latter to reduce their leverage.

Given the emphasis the capital structure literature has placed on supply considerations in the past, it is interesting to understand how they interact with hedging demand in shaping financing decisions in our incomplete-market model. In the spirit of the asset substitution literature, we assume that it is up to shareholders alone to choose the risk profile of the firm's cash flows and that such profile is unobservable to other investors. Compared to the symmetric-information benchmark, the risk chosen by shareholders is higher, as it is commonly the case in partial equilibrium. The effect of agency on the debt choice, however, is ambiguous. It is still the case that, as in the textbook asset substitution scenario, initial shareholders have the incentive to reduce leverage in order to nudge future shareholders into choosing less risk. In general equilibrium, however, the larger risk associated with agency leads to a change in state prices and calls for more debt. Beyond uncovering a clear role for hedging demand as driver of capital structure choices, allowing for financial market incompleteness suggests that in general equilibrium the strength of supply considerations such as agency depends on the risk-sharing opportunities available to investors.

The analysis of economies with production and incomplete markets requires to rigorously address the issue of what is the appropriate objective function of the firm. Generally, when markets are incomplete, the value of a firm in correspondence of any possible production and financing plan is not guaranteed to be the same when determined using the stochastic discount factor of different investors. The reason is that the intertemporal marginal rates of substitutions are no longer equalized across agents. Following Makowski (1983a,b), we show that when firms' conjectures about the prices associated to all their feasible plans are rational, value maximization is unanimously supported by shareholders. The rationality of the conjectures requires that firms' beliefs about their market value, for any production and capital structure plan they may choose to undertake, are determined by the highest marginal valuation across all consumers. Provided the economy is not plagued by pecuniary externalities, value maximization induces constrained-efficient allocations.

The remainder of the paper is organized as follows. In Section 1.1, we relate our work to the extant literature. The model is introduced in Section 2. In Section 3, we describe the equilibrium firms' investment and financing choices, as well as the investors' portfolios, and illustrate how they are shaped by hedging demand. We also show how firms' financing choices respond to the provision of a risk-free asset, to a relaxation of the short-selling constraints, and to the availability of less risky technologies, respectively. In Section 4, we bring together demand and supply consideration and describe how asset substitution and hedging demand jointly shape capital structure. Section 5 concludes. The proofs of the theoretical results are relegated to the Appendix.

1.1 Related Literature

Our main contribution is the analysis of the role played by investors' hedging demand in shaping firms' capital structure. Most of the theoretical and empirical literature centers instead on what we labeled supply determinants of the capital structure.

A large fraction of these studies are conducted in partial equilibrium. Typically, they assume a perfectly elastic provision of funds by investors at a given price, so that firms choose their capital structure by trading off the benefits versus the cost of issuing bonds. A celebrated version of such trade-off is that between the tax advantage of debt and the cost of default, an advanced treatment of which can be found in Hennessy and Whited (2005). An alternative approach, pioneered by Jensen and Meckling (1976), relies on asymmetric information between claim-holders.

This approach has a hard time accounting for the time series of capital structure. Ritter (1991), Baker and Wurgler (2000) and Ma (2019), among others, document that securities issuance responds strongly to market valuation. This evidence prompted the development of what became to be known as capital-market driven corporate finance. Early contributions carved out a role for market conditions as capital structure driver in partial equilibrium models, by assuming a departure from rational expectations and/or frictions in financial intermediation. Baker (2009)'s survey shows how non-fundamental investor demand, due to investor inertia or overconfidence, as well as limited arbitrage and imperfectly competitive intermediaries, may lead asset prices to deviate from their fundamental values and shape the capital structure of optimizing firms.

Other contributions, closer to ours in spirit, show how aggregate shocks may affect well-understood trade-offs and therefore impact capital structure decisions in dynamic partial-equilibrium environments. Hackbarth et al. (2006) show how aggregate productivity shocks impact capital structure via their effects on both the tax benefit of debt and bankruptcy costs. In Chen and Manso (2017)'s treatment of debt overhang, agency costs are higher in recession, since transfers from shareholders to bondholders are larger in downturns, when the former's stochastic discount factor is higher. Because of the partial equilibrium nature of these analyses, there is no feedback from firms' choices on investors' consumption processes and therefore on state prices.

Bhamra et al. (2010), Chen (2010) and Gomes and Schmid (2021) go one step further, by embedding the classical trade-off between tax advantage of debt and bankruptcy cost in general equilibrium economies with complete markets. Hedging demand contributes to shaping capital structure, since the equilibrium state prices affect the terms of the trade-off. However, because of the complete market assumption, the firms' debt choice has no effect on hedging opportunities available to investors. The pricing kernel, i.e. the intertemporal marginal rate of substitution of the representative investor, is fully determined by aggregate consumption. Without the trade-off between tax advantage of debt and default costs, capital structure would be indeterminate – a direct implication of the Modigliani-Miller theorem.

Absent that trade-off, for capital structure to be determined in a complete market economy, the literature has resorted to assuming either mis-valuation, as described in Baker (2009), or exogenous external debt demand, as in Corbae and Quintin (2021), or safety services in the utility function, as in Krishnamurthy and Vissing-Jorgensen (2012) and Mota (2023).[5]

In our incomplete-market economy, hedging needs drive investment and capital structure decisions, because such choices directly affect investors' hedging opportunities. Our theoretical framework builds on the pathbreaking work by Allen and Gale (Allen and Gale, 1988, 1991), who adopted Makowski's rational conjectures to study optimal security design – the optimal split of cash-flows across securities – in an endowment economy. The goals of our analysis are different.

We innovate with respect to Allen and Gale's work in several dimensions. Most important, ours is a production economy. The joint consideration of investment and capital structure decisions leads to non-trivial insights, as the two choices are interdependent.

Our structure accommodates the provision of derivatives, heterogeneity in risk-taking, and classical agency. We provide a thorough characterization of how investment and capital structure decisions as well as asset prices are shaped by each of these elements. To our knowledge, the analysis of economies with production and agency frictions had been previously confined to complete markets environments (Prescott and Townsend (1984), Prescott and Townsend (2006), and Zame (2007)).

Lastly, we contribute to the academic debate on corporate governance. It has been extensively argued that, since shareholders value firms' production plans differently under incomplete markets, such economies will feature lack of shareholders' unanimity on the firms' decisions.[6] The objective functions that have been proposed to address this issue[7] have led to the conclusion that equilibria may be constrained-inefficient, carving a normative role for some form of stakeholder governance.[8]

Our analysis does not support this stance. Even though markets are incomplete in our environment - and hence shareholders value firms' production plans differently at equilibrium - value maximization is unanimously supported. Since equilibria are constrained-efficient, stakeholder governance is not warranted by efficiency considerations.

[5]In Mota (2023), a corporate bond's safety premium is proxied by the excess-yield (with respect to Treasuries) on a position consisting of the bond and its associated maturity-matched credit default swap (CDS). In principle, such measure should reflect a purely idiosyncratic safety element. A residual aggregate component would be present if, due to counter-party risk and/or the risk of changes in the price and collateral requirements of CDS contracts, CDS did not fully insure against aggregate risk.

[6]The complete market analysis of firms' decisions can be directly extended to incomplete market economies only in rather special environments; that is, when (i) firms' production and capital structure decisions do not affect investors' hedging opportunities, as in Diamond (1967) and Carceles-Poveda and Coen Pirani (2009), or (ii) firms operate with a backyard technology and are managed by households, so that equity is not traded, as in Heathcoate et al. (2009) among others. In all these environments, however, hedging demand has no effects on capital structure by construction.

[7]See Dreze (1974), Grossman and Hart (1979) and Duffie and Shafer (1986). Caramp et al. (2024) study investment in a multi-period model where firms adopt the Grossman-Hart's criterion.

[8]See Hart and Zingales (2017), Bebchuk and Tallarita (2020, 2022), and Bebchuk et al. (2021).

2 Benchmark model

There are two dates, indexed by $t = 0, 1$. The economy is perfectly competitive, and all agents are price-takers. Only one good is available for both production and consumption.

There is a continuum of ex-ante identical firms, of unit mass, which produce according to the production function $y = e^{\varepsilon} A k^{\alpha}$, where $k > 0$ denotes investment at $t = 0$, y is output at $t = 1$, $\alpha \in (0, 1)$, and ε is a random variable distributed as $N(\mu, \sigma^2)$ with $\sigma > 0$. Each firm faces a production and a capital structure decision. That is, it chooses the investment level k and the notional amount $B \geq 0$ of debt it issues. When $e^{\varepsilon} A k^{\alpha} < B$, the firm defaults on its debt obligations and output is divided pro–rata among all bondholders. It follows that the conditional payoffs to equity and debt at date 1 associated to any choice (k, B) are

$$d^e(k, B; \varepsilon) = \max\{e^{\varepsilon} A k^{\alpha} - B, 0\},$$
$$d^b(k, B; \varepsilon) = \min\{1, e^{\varepsilon} A k^{\alpha}/B\},$$

respectively.

The economy is populated by a continuum of households of two different types, $i = 1, 2$, each of unit mass. Each type is endowed with the same amount $w_0 \geq 0$ of commodity at $t = 0$ and initial equity ownership is uniformly distributed across households, i.e. $\theta_0^i = 1/2$, $i = 1, 2$. Household types only differ in their endowment at $t = 1$, given by the random variable

$$w_1^i(\varepsilon) = e^{-\chi_i \mu - \frac{1}{2}\chi_i^2 \sigma^2 + \chi_i \varepsilon}, \; i = 1, 2,$$

with $\chi_i \geq 0$ and $\chi_1 \neq \chi_2$.

For simplicity, we focus on the case where the endowment of type-1 households is riskless, i.e. $\chi_1 = 0$, while that of type-2 is risky: $\chi_2 > 0$. Our assumptions imply that w_1^2 is log-normally distributed with mean parameter $-\frac{1}{2}\chi_2^2\sigma^2$ and variance parameter $\chi_2^2\sigma^2$.

A higher value of χ_2 is associated with strictly greater variance and skewness of w_1^2, as well as greater covariance with the risk factor, but no change in expected value, as $\mathbb{E}\left(w_1^2\right) = 1$.[9] Hence, households differ in the exposure of their future income to aggregate risk: Their endowment growth exhibits different levels of correlation with the aggregate factor and so with the returns of corporate securities.[10] See Figure 1 for an illustration of

[9]It is immediate to verify that $var(w_1^2) = e^{\chi_2^2\sigma^2} - 1$ and $cov(w_1^2, e^{\varepsilon}) = e^{\mu + \frac{1}{2}\sigma^2}(e^{\chi_2\sigma^2} - 1)$.

[10]Whether households' endowments have a positive loading on the aggregate factor is the subject of debate in the empirical literature. See for example Storesletten et al. (2004), Cocco et al. (2005), Benzoni et al. (2007), Lynch and Tan (2011), Coeurdacier and Gourinchas (2016), Catherine (2022). See below for a brief discussion of how our results would differ for $\chi_2 < 0$.

how endowments and output vary with the aggregate shock.

Households' preferences are described by a common strictly increasing, strictly quasi-concave, Von Neumann-Morgerstern utility function over random consumption sequences $\{c_0^i, c_1^i(\varepsilon)\}$:

$$U(c_0^i, c_1^i) \equiv u(c_0^i) + \beta \mathbb{E}\left[u(c_1^i)\right], \quad \beta > 0,$$

with $u(c) = \frac{c^{1-\psi}}{1-\psi}$, $\psi > 0$, and $\psi \neq 1$.

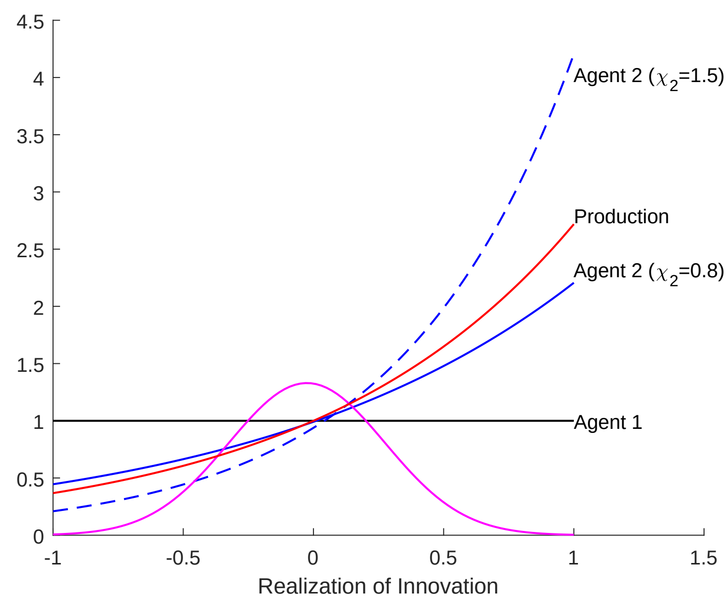

Figure 1: Households' Endowments and Firms' Output.

Trade in financial assets occurs at $t = 0$. Markets are incomplete, as the only financial assets are firm-issued equity and debt. Let $d^b(\varepsilon)$ and $d^e(\varepsilon)$ denote the payoffs of bonds and equity, respectively, implied by the values of k, B chosen by firms.[11] Consumers can purchase corporate bonds at the price p and rearrange their stock-holdings by either buying or selling equity at the price q.

Letting V denote the cum-dividend market value of the firm and using θ^i and b^i to denote type i's post-trade holdings of equity and bonds, respectively, the household's

[11] Below it will become clear that the choice problem of the firm is not guaranteed to be convex. While not an issue for existence, the lack of convexity implies that identical firms might differentiate their behavior at equilibrium. We will illustrate one such instance in Section 3. For notational simplicity, we do not explicitly allow for this possibility when describing the securities' payoffs and when defining the competitive equilibrium.

optimization problem writes as

$$\max_{c_0^i,\theta^i,b^i,c_1^i(\varepsilon)} u(c_0^i) + \beta\mathbb{E}\left[u(c_1^i)\right] \tag{1}$$

$$\text{s.t.} \quad c_0^i = w_0 + \theta_0^i V - q\theta^i - pb^i, \tag{2}$$

$$c_1^i(\varepsilon) = w_1^i(\varepsilon) + \theta^i d^e(\varepsilon) + b^i d^b(\varepsilon) \;\; \forall\, \varepsilon,$$

$$\theta^i \geq 0, \;\; b^i \geq 0.$$

The two inequality constraints rule out short-selling.

Notice that, since $V = q - k + pB$, equation (2) can be rewritten as $c_0^i = w_0 + \theta_0^i(-k + pB) + q(\theta_0^i - \theta^i) - pb^i$. Whenever $k < pB$, firms pay out dividends to initial shareholders as of $t = 0$. Conversely, when $k > pB$, initial shareholders are asked to contribute to the firm's investment.

The firm's problem consists in choosing the pair $\{k, B\}$ that maximizes its market value, i.e.

$$\max_{k,B} -k + q(k, B) + p(k, B)B, \tag{3}$$

where $q(k, B)$ and $p(k, B)$ are the price conjectures of equity and debt associated to any possible plan $\{k, B\}$.

With complete markets, intertemporal marginal rates of substitution are equalized across investors. It follows that firms' financing and production plans are evaluated using the state prices common to all initial shareholders. With incomplete markets, marginal rates of substitution are not equalized in equilibrium. This requires the researcher to specify at what prices firms will value equity and bonds for any pair $\{k, B\}$.

Following Makowski (1980, 1983a), we require the pricing conjectures to be *rational*. This means that, for any choice of capital and debt, the conjectured price of each security equals the highest marginal valuation of its payoffs across investors. For any $\{k, B\}$,

$$q(k, B) = \max_i \; \mathbb{E}\left[m^i d^e(k, B)\right] \tag{4}$$

$$p(k, B) = \max_i \; \mathbb{E}\left[m^i d^b(k, B)\right], \tag{5}$$

where $m^i \equiv \beta \frac{u_1'^i}{u_0'^i}$ is agent i's intertemporal marginal rate of substitution evaluated at equilibrium consumption.

Notice that the stochastic discount factor pricing equity may be different from that pricing bonds, and both may vary across different plans $\{k, B\}$. To gain intuition, think of the firms' initial shareholders contemplating the possibility of selling the firm. For any $\{k, B\}$ they value equity and debt at the price that they would fetch by selling the securities to the investors that value them the most.

When rational conjectures are evaluated at the firms' equilibrium choices, they are also *consistent*, i.e. they equal equilibrium market prices. This follows immediately from observing that, at such choices, the right-hand-sides of (4)-(5) satisfy the Euler equations of agents long on equity and debt, respectively. In equilibrium, firms price each security by means of the stochastic discount factor of the investors that purchase it. Rationality requires that those conditions hold true for all feasible pairs (k, B), including all out-of-equilibrium ones.

It is worth emphasizing that price conjectures are determined taking the households' marginal rates of substitution as given, in line with the assumption of competitive financial markets. Given the large number of firms and the no short-sale condition, each firm is "small" relative to the market and therefore a price taker.

A competitive equilibrium consists of firms' choices k, B, cum-dividend value V, asset prices q, p, price conjectures $p(k, B)$, $q(k, B)$ for all possible k, B, as well as consumption choices $\left(c_0^i, c_1^i(\varepsilon)\right)$ and portfolio choices $\left(\theta^i, b^i\right)$ for each type $i = 1, 2$, such that (i) k, B attain the maximum in problem (3), (ii) V is the value of problem (3), (iii) $\left(c_0^i, c_1^i(\varepsilon)\right)$ and $\left(\theta^i, b^i\right)$ solve problem (1) for each type $i = 1, 2$, (iv) $p(k, B)$ and $q(k, B)$ are rational, i.e. they satisfy (4)-(5), (v) price conjectures and asset payoffs at the equilibrium choices k, B are consistent, i.e.

$$q = q(k, B), \;\; p = p(k, B), \;\; d^e = d^e(k, B), \;\; d^b = d^b(k, B), \tag{6}$$

and (vi) markets clear:

$$\sum_i b^i \leq B, \;\; \sum_i \theta^i \leq 1. \tag{7}$$

Competitive equilibria have appealing properties. Adapting arguments due to Louis Makowski, one can show that shareholders *unanimously support* the decisions which maximize firms' market value. That is, every agent i holding a stake $\theta_0^i > 0$ in a firm's equity will be made weakly worse off by any possible pair (k', B') other than the equilibrium's value maximizing choice. In this sense, the firms' objective we consider is well-defined.

The rationality of firms' conjectures is also intimately tied to the welfare properties of equilibrium allocations. Competitive equilibria are *constrained-efficient*: There does not exist a feasible allocation – i.e., one that satisfies resource feasibility and is attainable by trading in equity and debt – which Pareto-dominates the one arising in competitive equilibrium. In the Appendix, we formally establish these properties in the context of a more general framework.

3 Capital structure and equilibrium configurations

In this section, we illustrate the various financial markets' configurations that can obtain in equilibrium by means of a numerical example.[12] We characterize in detail the firms' capital structure and investment choices as well as the investors' portfolios.

The firm's problem (3) can be rewritten as the choice of k and B that maximize

$$-k + \max_{i=1,2} \int_{\varepsilon^*}^{+\infty} (Ake^{\varepsilon} - B) m^i(\varepsilon) g(\varepsilon) d\varepsilon + \\ + B \max_{i=1,2} \left[\int_{\varepsilon^*}^{\infty} m^i(\varepsilon) g(\varepsilon) d\varepsilon + \int_{-\infty}^{\varepsilon^*} \frac{Ake^{\varepsilon}}{B} m^i(\varepsilon) g(\varepsilon) d\varepsilon \right], \tag{8}$$

where $g(.)$ denotes the density of the normal distribution and $\varepsilon^* \equiv \log\left(\frac{B}{Ak^{\alpha}}\right)$ is the lowest among the realizations of ε consistent with solvency of the firm.

Since type-1 agents have a riskless endowment, hedging demand is well captured by χ_2, which parameterizes the exposure of type-2 agents' endowment to the aggregate shock. We will show that χ_2 is a key driver of both investors' and households' decisions.

Type-2 agents wish to shift consumption from high- to low-realization states. However, since their income is positively correlated with firms' cash-flows, neither debt nor equity are ideal hedging instruments. This is particularly true for equity, which pays a larger share of the firms' cash-flow in states where the agents' endowment realization is relatively high. In contrast, debt pays the largest share of the firms'cash-flow when the endowment realization is relatively low.

Depending on the magnitude of χ_2, we identify three different equilibrium configurations. Two of them are in pure strategies, i.e. all firms select the same investment and leverage. In Figure 2, we label such choices as strategy X. In the remaining scenario, firms play mixed strategies: A fraction of them make choices that are qualitatively similar to those expressed by X, while the remainder choose higher debt. We label the latter strategy Z.

When χ_2 is relatively high ($\chi_2 > 0.75$), type-2 agents' hedging needs are so strong that they value debt strictly more than type-1s and they value equity strictly less:

$$\mathbb{E}\left[m^2 d^b(k,B)\right] > \mathbb{E}\left[m^1 d^b(k,B)\right], \quad \mathbb{E}\left[m^1 d^e(k,B)\right] > \mathbb{E}\left[m^2 d^e(k,B)\right].$$

[12]Due to the stylized nature of the model, parameters are chosen to facilitate the illustration of our results, without any ambition to reproduce empirical regularities. The relative risk aversion coefficient is $\psi = 3$, the discount factor $\beta = 0.96$, and the initial endowment $w_0 = 1$. The span of control parameter is $\alpha = 0.6$, while the scale factor is $A = 2.5$. The distribution of the productivity shock is fully characterized by $\sigma = 0.3$ and $\mu = -0.025$.

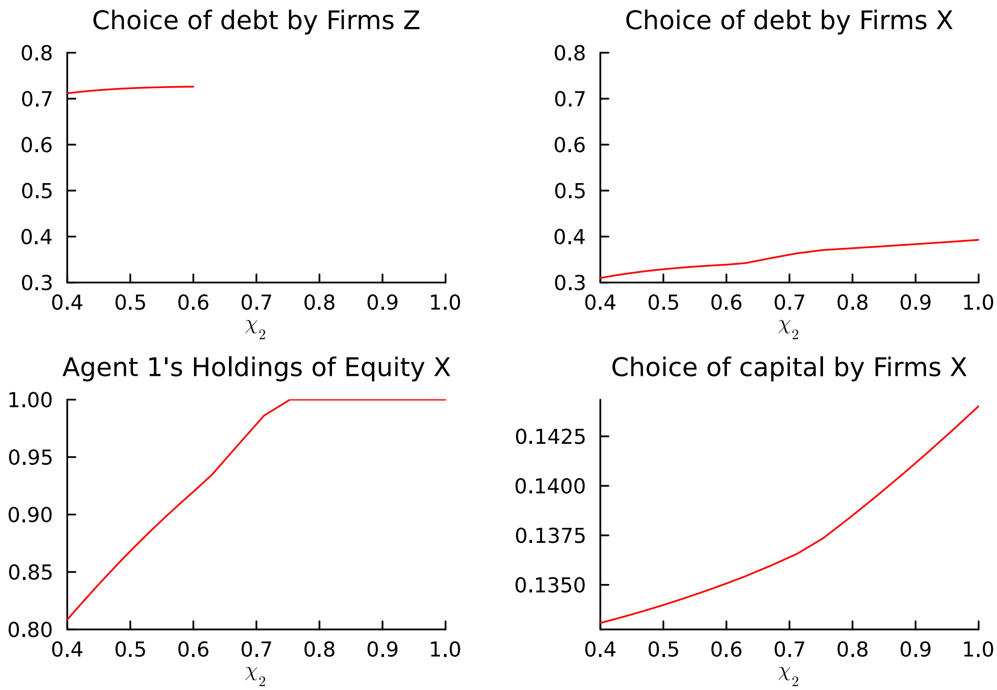

Figure 2: Equilibrium Configurations.

The competitive equilibrium features complete market segmentation: All equity is held by type-1 agents, while all debt is purchased by type-2's.

It follows that rational price conjectures are locally evaluated by type-1 agents' kernel m^1 for equity and by type-2's kernel m^2 for debt. The first-order conditions for firm value maximization with respect to capital and debt are, respectively,

$$\alpha A k^{\alpha-1}\left[\int_{\varepsilon^*(k,B)}^{+\infty} m^1(\varepsilon)e^{\varepsilon}g(\varepsilon)d\varepsilon + \int_{-\infty}^{\varepsilon^*(k,B)} m^2(\varepsilon)e^{\varepsilon}g(\varepsilon)d\varepsilon\right] = 1 \quad (9)$$

and

$$\int_{\varepsilon^*(k,B)}^{+\infty} m^1(\varepsilon)g(\varepsilon)d\varepsilon = \int_{\varepsilon^*(k,B)}^{+\infty} m^2(\varepsilon)g(\varepsilon)d\varepsilon. \quad (10)$$

Equation (9) requires that the marginal benefit of adding capital equals 1, its marginal cost. The marginal unit of capital benefits shareholders (type-1 households) in the solvency states and bondholders (type-2 households) over the default region. Similarly, (10) requires that the marginal benefit of increasing debt equals its cost. At the margin, raising debt shifts resources from shareholders to bondholders over the solvency states. Condition (10) then equates the bondholders' marginal benefit of issuing one more unit of debt to the shareholders' marginal loss.[13]

[13]Since problem (8) is not necessarily convex, we need to verify that the second order conditions for an

Since $m^1 \neq m^2$, the capital structure of any individual firm is determinate. In contrast, if markets were complete, the marginal rates of substitution would be equalized. Condition (10) would hold as an identity, leaving debt undetermined as per the Modigliani-Miller result.

Since marginal utility is convex, type-2 agents respond to increases in their exposure to the aggregate shock by raising their mean consumption growth – the classical precautionary saving's motive. Figure 2 illustrates that, for $\chi_2 > 0.75$, equilibrium debt and investment increase with χ_2 – and so does the cash-flow paid to type-2 investors at $t = 1$.

For lower levels of risk heterogeneity – $0.63 < \chi_2 < 0.75$ in our case – the equilibrium features only partial financial market segmentation: Type-2 agents still hold all the bonds, but both types of investor are long on equity. This also means that the households' marginal valuations for equity coincide:

$$\mathbb{E}\left[m^2 d^b(k,B)\right] > \mathbb{E}\left[m^1 d^b(k,B)\right], \quad \mathbb{E}\left[m^1 d^e(k,B)\right] = \mathbb{E}\left[m^2 d^e(k,B)\right].$$

Due to the rationality of prices conjectures, it is not immediate that firm value (8) is differentiable at equilibrium. In Appendix A.2, we show that indeed it is and that the first-order conditions for firm's optimality are still given by (9)-(10).

Those conditions, however, are not sufficient. It turns out that for a pair of investment and debt choices satisfying the first-order condition to be a maximum, type-2 agents must have a strictly higher valuation for equity for local deviations in B and k. See Appendix A.2 for a proof of this claim.

Notice that, since type-2 agents have a strictly greater valuation for debt, they end up pricing the entire cash-flow of the firm. Thus, at an equilibrium with partial segmentation, the value V of any individual firm is invariant with respect to changes in B. In this scenario, the failure of the Modigliani-Miller theorem is rather subtle: Aggregate debt and aggregate capital are jointly determined by (9)-(10). Yet, the capital structure choice of each infinitesimal firm is indeterminate!

For even lower levels of exposure ($\chi_2 < 0.63$), the hedging needs of type-2 investors become rather small and the income profiles of the two types of investors very similar. In this case, the competitive equilibrium is asymmetric: Ex-ante identical firms make different production and financing choices.

The majority of firms still make choices that are in continuity with the decisions made by firms in the two equilibrium configurations examined above. Their equity is held by both agents, while their debt is still in the hands of type-2 agents only. A minority of

optimum are satisfied. See Appendix A.2 for a detailed discussion.

firms, which we label Z, end up choosing a similar level of capital, but substantially higher leverage. Their equity is held by type-2 agents and their debt by type-1's.[14]

As intuitive as these features of the equilibrium allocation may be, they could never arise in the familiar complete market environment. In that scenario, all investors have the same valuations for all financial assets and, most important, firms cannot improve their hedging opportunities.[15]

3.1 Comparative statics: Hedging demand

We have shown that, with incomplete markets, firms' production and financing decisions respond to investors' hedging needs. In this section, we carry out a detailed comparative statics analysis of the effects of an increase in χ_2 on the equilibrium values of capital, leverage, consumption, and financial assets' returns. For simplicity, we will study the equilibria that obtain for $\chi_2 \in [0.8, 5]$, thereby restricting our attention to scenarios featuring perfect segmentation.

3.1.1 Investment and leverage

Recall that raising χ_2 induces a mean–preserving spread of type-2's endowment. Hence, the greater is χ_2, the larger are those consumers' hedging needs. Because of convexity in marginal utility, all else equal this implies a rise in the right-hand-side term of condition (10) – the firm's marginal benefit from issuing debt. Firms cater to the increase in consumers' hedging demand by issuing more bonds. This happens even though a higher χ_2 also increases the covariance between type 2's consumption growth and bond returns over the default region, thus making corporate bonds a worse hedging instrument.

A higher χ_2 also induces firms to increase k. To see why, rewrite the terms between square brackets in condition (9) as

$$\begin{aligned} & [1 - G(\varepsilon^*)] \; cov \left[m^1(\varepsilon), e^{\varepsilon} | \varepsilon > \varepsilon^*\right] + G(\varepsilon^*) \; cov \left[m^2(\varepsilon), e^{\varepsilon} | \varepsilon < \varepsilon^*\right] + \\ & + [1 - G(\varepsilon^*)] \, \mathbb{E}\left[m^1(\varepsilon) | \varepsilon > \varepsilon^*\right] \mathbb{E}\left(e^{\varepsilon} \mid \varepsilon > \varepsilon^*\right) + \\ & + G(\varepsilon^*) \, \mathbb{E}\left[m^2(\varepsilon) | \varepsilon < \varepsilon^*\right] \mathbb{E}\left(e^{\varepsilon} | \varepsilon < \varepsilon^*\right), \end{aligned} \tag{11}$$

[14]If the endowment of type 2 agents had a negative loading on the aggregate shock, i.e. $\chi_2 < 0$, the hedging properties of the securities issued by the firm would change. Equity would be a better hedge. We thus conjecture that, for χ_2 sufficiently negative, the equilibrium would feature again perfect segmentation, with type-2 agents holding only equity and type-1 only debt.

[15]In Aiyagari (1994)'s incomplete market economy, equity is not traded. If it were, due to the absence of aggregate uncertainty, it would boil down to a riskless asset. A fortiori, the capital structure has no role in that model. Adding aggregate productivity shocks to it would generate a correlation between the firm's cash flows and the consumers' income process, via variation in wages. In such a framework, capital structure choices would be driven by hedging considerations, as in our model.

where G is the CDF of the normal distribution. Expression (11) is the generalization to the incomplete-market scenario of terms arising in the analysis of complete-markets asset pricing models. The marginal unit of capital adds to equity-holders' payoff in solvency states and to bondholders' payoff in default states, respectively.

In the first line are conditional covariances between the investors' marginal rates of substitution and the productivity innovation. Both terms are negative, reflecting the adverse impact of the marginal unit of investment on investors' utility deriving from the positive conditional correlation between assets' payoffs and investors' consumption growth. An increase in χ_2 is associated with a rise in the absolute value of these terms, therefore discouraging investment.

The remaining terms are positive, and their values reflect the precautionary saving motive. Consider the last addendum. As noted above, due to convexity of marginal utility, a higher χ_2 leads to an increase in debt-holders' conditional expected marginal rate of substitution. In other words, it leads to a higher demand for consumption in default states. This encourages investment. The latter effect prevails for all values of χ_2 we have considered.[16] Hence, an increase in the demand for hedging induces an increase in investment.

We illustrate our comparative statics in Figure 3. The red solid curves describe the values obtained at the equilibrium allocations of our incomplete market economy. We benchmark these values against those arising under two alternative financial market arrangements. In one (dashed, black curves), markets are complete. Equilibria feature the equalization of marginal rates of substitution. In the other (dash-dot, blue lines), equity is the only asset available to investors.

A higher χ_2 is associated with greater values for both B and k, i.e. a greater supply of hedging instruments for the benefit of type-2 agents. At the margin, more debt is appealing to them because it allows for a better distribution of consumption over the solvency region. More capital gives them the opportunity to improve upon their consumption allocation over default states.

Our theory also features a non-trivial complementarity between the two firm's choices. For given k, increasing B has a negative impact on type-1 agents, since they only hold equity. A higher k has the effect of reducing the magnitude of such impact, thereby allowing for an even greater increase in debt. To see this, simply substitute (10) into (11).

[16] In the absence of a precautionary motive, the magnitude of this second effect would be smaller. Because of risk aversion, the demand for consumption would still increase in the default states, but not the demand for riskless debt. As a consequence, an increase in the demand for hedging may induce a decrease in investment.

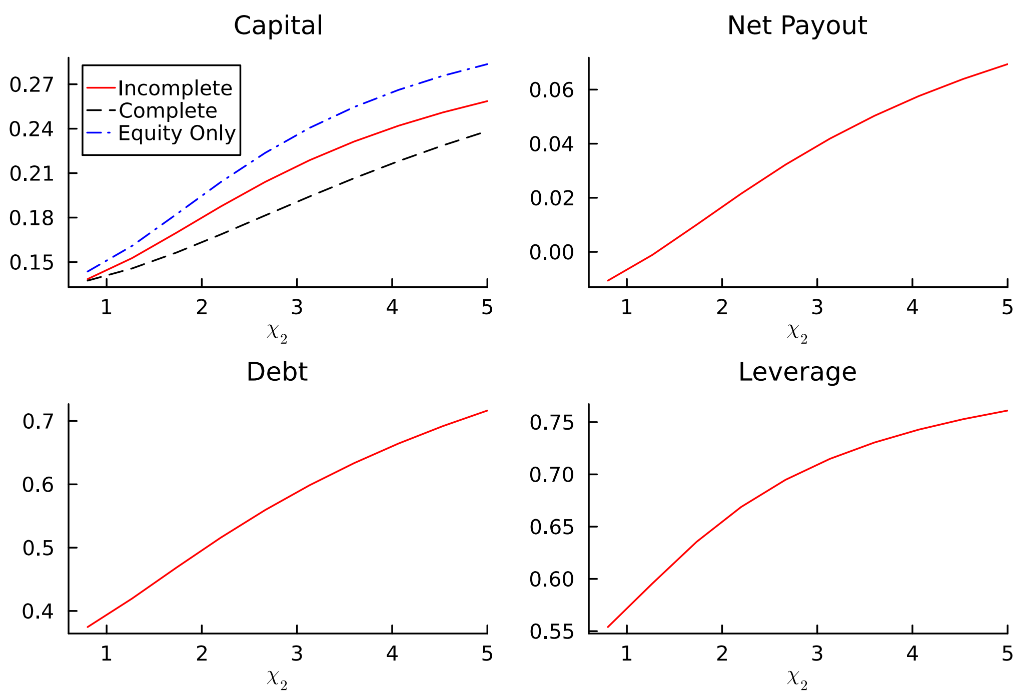

Figure 3: Firms' Choices.

The increase in debt issuance is associated with an increase in market leverage $pB/(pB+q)$. To a large extent, this is due to the relative increase in B and k. The top-right panel of Figure 3 illustrates $pB-k$, i.e. the dividend paid to initial shareholders at $t=0$. For low values of χ_2, dividends are negative, indicating that initial shareholders are called to fund investment. However, as χ_2 rises, dividends eventually become positive and progressively larger. Not only firms respond to the increase in hedging demand by issuing more debt. They increase debt at a higher rate than investment, transferring a larger fraction of debt issuance proceeds to shareholders. Firms determine their capital structure not just based on their investment financing needs. Rather, they generate value for their shareholders by catering to investors' hedging needs. This is a novel factor driving capital structure choice, which can only arise in an incomplete market economy.

Our theory rationalizes recent evidence that other approaches to capital structure choice have a hard time accounting for. Using cross-sectional data for US non-financial firms, Mota (2023) constructs estimates of corporate bonds' safety premia. It then documents that while a high premium forecasts debt issuance, it has little or no predictive power for investment or acquisitions. In other words, companies with relatively high safety premia issue more debt and distribute most of the issuance proceeds to their shareholders.

Neither dividends, nor debt, nor leverage are reported for the alternative market arrangements, since debt is not available in one scenario and is indeterminate in the other. Investment and firm value are uniformly higher when equity is the only asset, since the more limited insurance possibilities afforded by agents when firms cannot issue debt imply a stronger type-2's precautionary motive.

With complete markets, a full set of contingent claims allow the agents to efficiently share risk. The portion that is left, monotonically increasing in χ_2, is aggregate in nature. As χ_2 rises, firms cater to the larger precautionary motive by providing investors with more of the only means to move resources intertemporally.

3.1.2 Consumption and asset returns

An increase in the demand for hedging also affects equilibrium asset prices and households' consumption. In Figure 4, we plot the mean and variance of consumption growth for both agents. The variance of agent-2's consumption growth increases monotonically with χ_2.

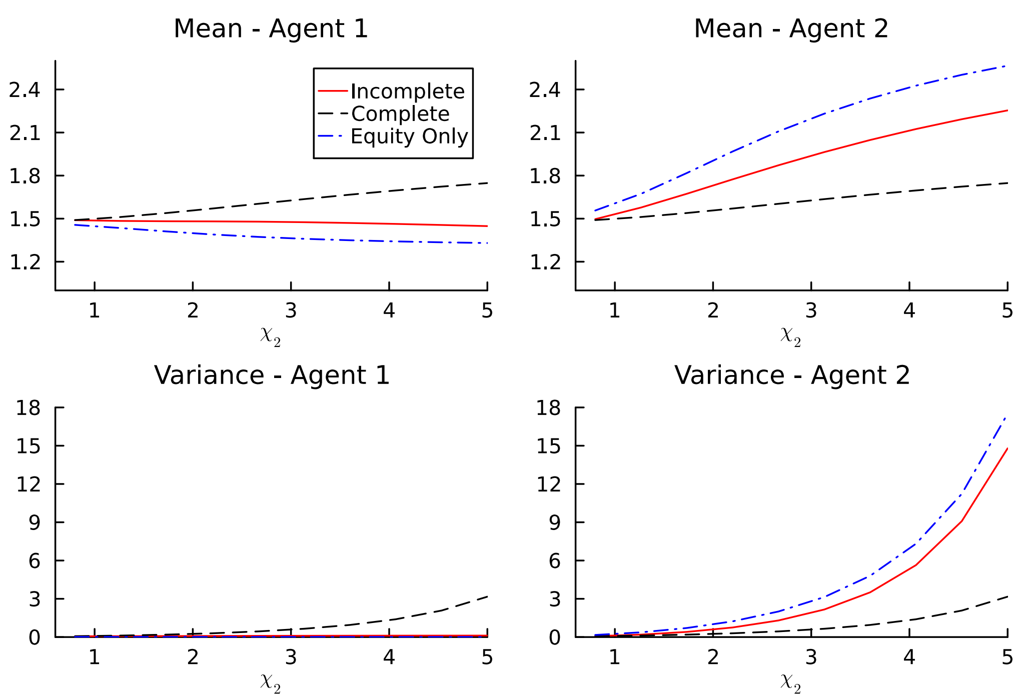

Figure 4: Consumption growth.

However, this is also the case with complete markets, since the aggregate endowment's risk is monotonically increasing in χ_2.

What is most relevant is that the difference between the variances that obtain in the incomplete and complete markets setups, respectively, also increases with χ_2. The reason is that with incomplete markets type-2 agents only rely on bonds, an imperfect hedging instrument, to satisfy their larger hedging needs.

Type-2 agents' mean consumption growth increases with χ_2, since they purchase the entire supply of corporate debt. Type-1 agents' mean consumption growth declines, because firms distribute most of the proceeds from larger debt issuance to shareholders at t=0, rather than invest them in capital and increase shareholders' payoff at t=1.

We turn next to asset returns. Since a risk-free bond is not available for trade, the risk-free rate displayed in Figure 5 is the inverse of the shadow price of an asset with riskless unit payoff. Since type-1 agents would always value such asset strictly less than type-2 agents, the price coincides with the latter's marginal valuation and is an indicator of their hedging needs.

The monotonicity of the risk-free rate in χ_2 reflects type-2 agents' consumption growth process. On the one hand, higher mean growth is associated with a higher risk-free rate. On the other hand, higher variance of growth tends to reduce it. The latter effect dominates.

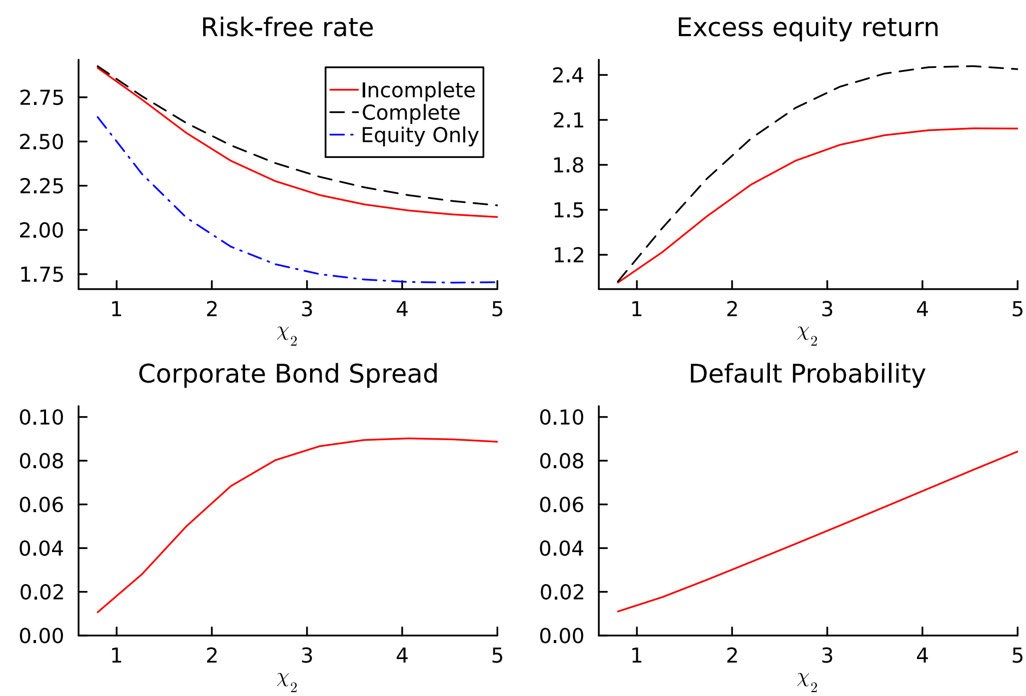

Figure 5: Asset returns

The corporate bond spread is the difference between the expected return on the bond – its expected payoff divided by the equilibrium price – and the risk-free rate. One can easily show that

$$\mathbb{E}(R^b) - R^f = -cor(m^2, R^b)\frac{\sigma\left(m^2\right)}{\mathbb{E}\left(m^2\right)}\sigma\left(R^b\right).$$

As χ_2 increases, both the standard deviation of returns $\sigma\left(R^b\right)$ and the price of risk – measured as $\sigma\left(m^2\right)/\mathbb{E}\left(m^2\right)$ – rise. Since $cor(m^2, R^b) < 0$, the corporate bond spread also rises with χ_2.

The excess return on equity is also monotonically increasing in χ_2. To investigate why, let's express it as

$$\mathbb{E}(R^e) - R^f = \left[\frac{1}{\mathbb{E}\left(m^1\right)} - \frac{1}{\mathbb{E}\left(m^2\right)}\right] - cor(m^1, R^e)\frac{\sigma\left(m^1\right)}{\mathbb{E}\left(m^1\right)}\sigma\left(R^e\right). \tag{12}$$

The term in square brackets, which is specific to the incomplete market environment, reflects the fact that the pricing kernels of equity and the risk–free bond are different. Since type-1 agents value the risk-free asset strictly less than type-2s, that term is strictly positive.

The pattern of the excess equity return is mainly the result of two forces. The rise in χ_2 triggers an increase in leverage and in the risk of type-2 agents' consumption growth. The former drives the variance of equity returns $\sigma\left(R^e\right)$ higher. The latter leads to a decline in the risk–free rate $1/\mathbb{E}(m^2)$.

The risk-free rate lies between the values that obtain with complete markets and with equity only, respectively. The pattern is driven by the variance of consumption growth of type-2 agents, who price the bonds in all three scenarios. As shown in Figure 4, the variance is largest with equity only and smallest with complete markets.

In the right-top panel of Figure 5, debt in the complete-market scenario was chosen so that the default probability is the same as in the incomplete-market case. This choice eliminates any effect of leverage on the difference in expected excess equity returns via its impact on $\sigma(R^e)$. Refer to equation (12). When markets are complete, the term in square brackets is identically zero. However, the market price of risk is higher, because the complete-market variance of consumption growth is higher than type-1's variance under incomplete markets. In our example, this second consideration dominates, leading to a higher excess equity return under complete markets.

3.2 Comparative statics: Aggregate risk

We now turn to the study of how key features of equilibrium allocations vary with respect to changes in the variance of the aggregate shocks. The goal is to gain some insight into the properties of more general versions of our setup, featuring time-varying aggregate risk.

In Figures 6 and 7 we illustrate how the firms' equilibrium choices and asset returns vary with σ for $\chi_2 = 0.8$ and $\chi_2 = 3.0$, respectively. The parameter μ is adjusted so that $\mathbb{E}(e^{\varepsilon}) = e^{0.02}$ always.

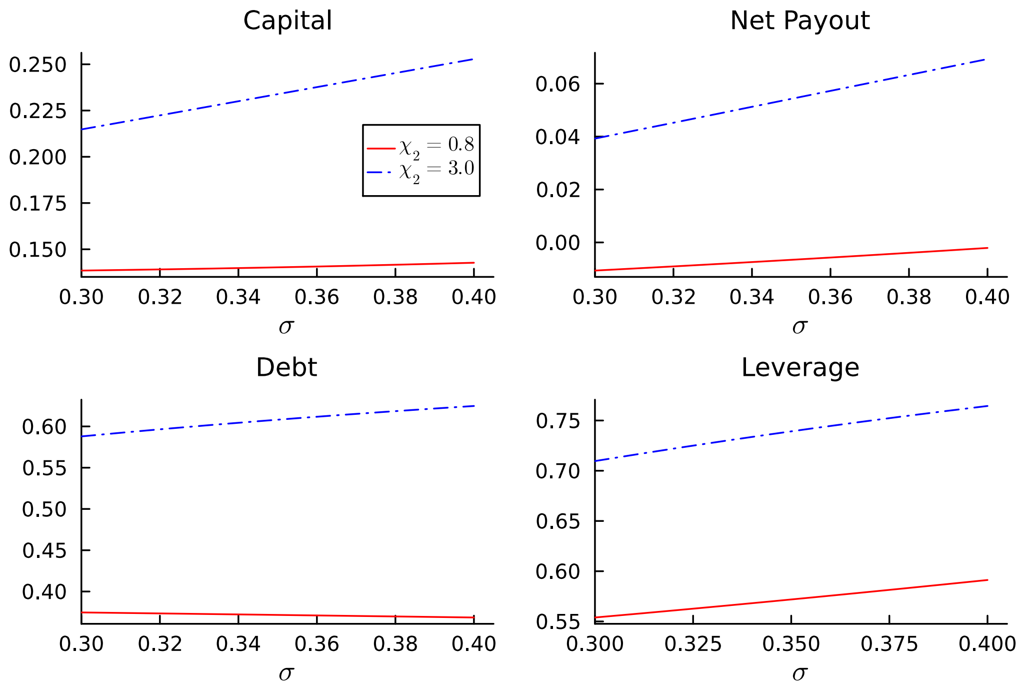

Figure 6: Capital Structure and Aggregate Risk

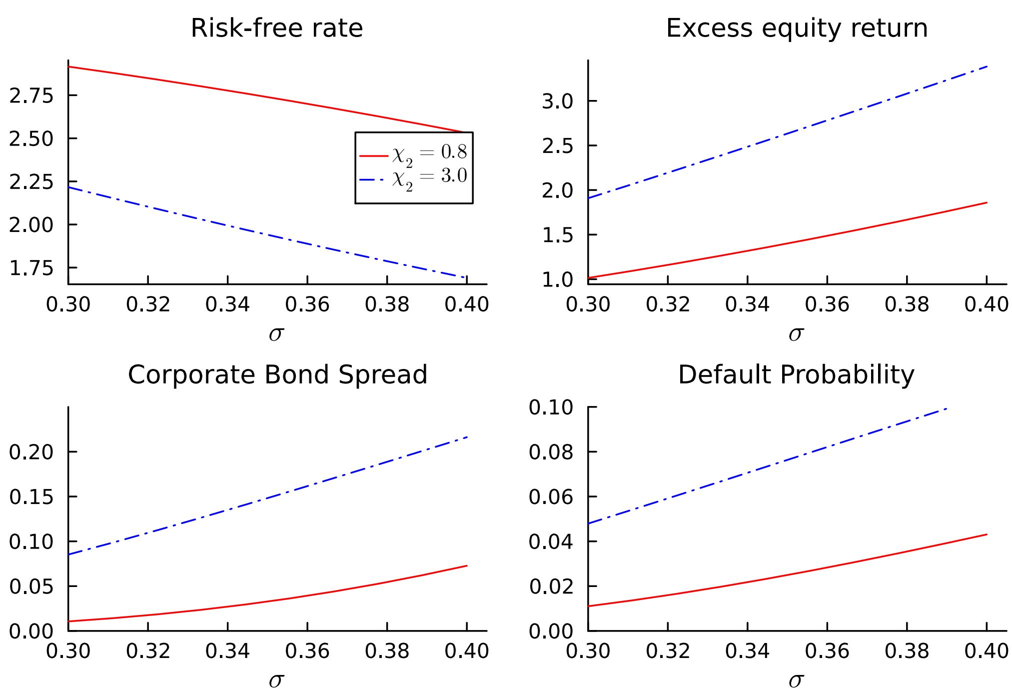

Figure 7: Asset returns and Aggregate Risk

An increase in σ has countervailing effects on firms' incentives to issue debt. Refer once again to condition (10). On the one hand, the variance of type–2 households' endowment rises, leading to greater hedging demand. On the other hand, the variance of firms' cash-flows also rises, leading to a higher variance of consumption growth for type-1 agents. In turn, this means a higher firm's marginal loss of increasing debt.

Figure 6 suggests that the net effect on debt depends on other parameters, among which the loading of type-2 agents on the aggregate shock. When χ_2 is relatively low ($\chi_2 = 0.8$), corporate debt declines with σ, while the opposite occurs for higher χ_2.

Leverage, however, is monotone increasing in σ. Since empirical proxies for aggregate risk covary negatively with output, our finding provides a rationalization for the recent finding – see Halling et al. (2016) – that US firms' leverage is countercyclical.

Beyond the obvious impact of debt, the comparative statics of market leverage $pB/(pB+q)$ depends on the effects of σ on the price of debt and on investment. The top-right panel in Figure 6 shows that dividends $pB - k$ increase with σ. This is in part due to the fact that in equilibrium a larger demand for bonds leads to a higher price p.

The above discussion clarifies that the forces shaping the response of firms' capital structure to aggregate shocks are different from those at work in the partial equilibrium model of Hackbarth et al. (2006) and in the complete market model of Bhamra et al. (2010). In those frameworks, fluctuations in the economy's stochastic discount factor induced by time-varying aggregate risk affect the trade-off between the tax advantage of debt and the cost of bankruptcy.

3.3 Comparative statics: Supply of public debt

We have so far illustrated the hedging services provided by corporate debt in a scenario where equity is the only alternative asset. In reality, investors with insurance needs also rely on other financial instruments, among which sovereign bonds and derivatives.

We now turn to investigating how the equilibria described above change when households can also purchase risk-free debt, available in fixed and exogenous supply. We will refer to such asset as public debt. In the next section we will explicitly model derivatives.

Figures 8 and 9 illustrate equilibria in scenarios that only differ in the provision of public debt. The entire supply of this new asset is purchased by type–2 agents, reducing the demand for firms' hedging services and leading to lower capital and corporate debt. Notice however that the crowding out of privately-provided hedging instruments is only partial – both aggregate savings and average consumption growth increase.

As a result of the improved risk–sharing opportunities afforded by type-2 agents, the risk–free rate increases. The excess return on equity declines, mostly because of the lower volatility of equity's payoff implied by lower leverage.

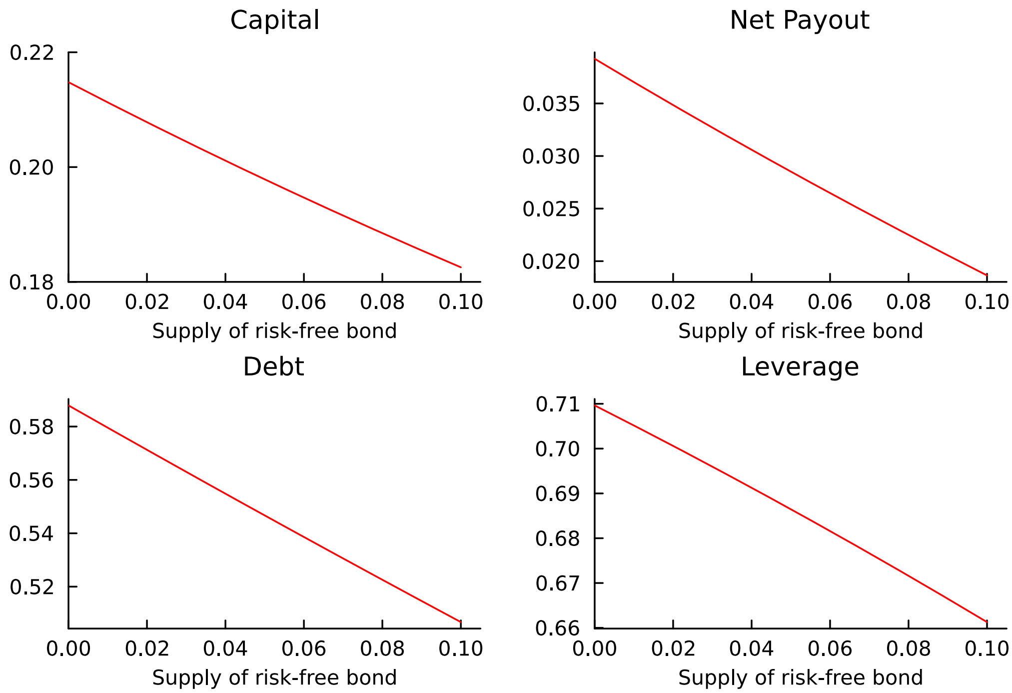

Figure 8: Capital structure choices with risk–free public debt

In their study of US non-financial firms, Graham et al. (2014) find that government debt is negatively correlated with corporate debt and investment, and strongly so for relatively safer issuers. Demirci et al. (2019) find similar results in a large cross-section of countries. Our finding provides a firm theoretical grounding for Graham et al. (2014)'s suggestion that "financially healthy corporations act as liquidity providers by supplying relatively safe securities to investors when alternatives are in short supply, and that this financial strategy influences firms' capital structures and investment policies."

3.4 Comparative statics: Short-selling costs

In all the equilibria considered above, the short–sale constraint on debt is binding for type-1 agents. We now relax this constraint, by allowing for intermediated short-sales.[17]

We introduce financial intermediaries who issue derivatives corresponding to short positions on firms' debt for the benefit of type-1 agents and long positions on the same underlying security, for the benefit of type-2 investors.

[17]The analysis of this extension is important from a theoretical standpoint. See Appendix A.4 for a discussion of the general properties of its equilibria.

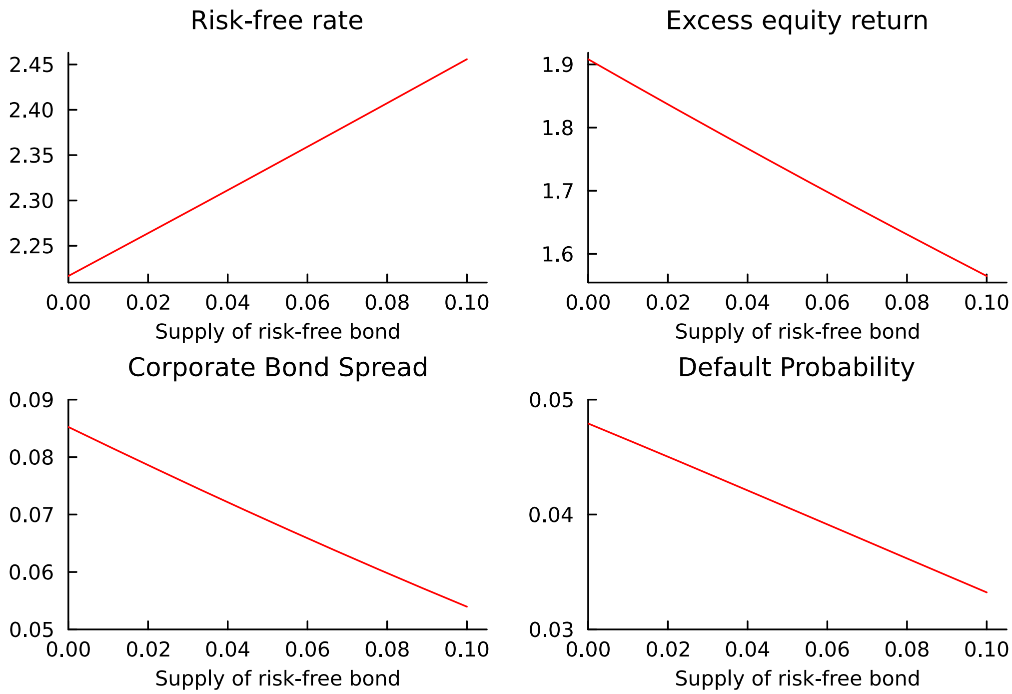

Figure 9: Asset returns with risk–free public debt

This exercise has clear analogies to the addition of public debt considered above. In common with that scenario, the supply of hedging instruments available to type-2 consumers increases. It differs however in two key dimensions. To start with, the supply of derivatives is endogenous, i.e. it depends upon type–1 households' appetite for short positions and on intermediation costs. Secondly, the net supply of assets still equals the sum of equity and debt issued by firms.

We assume that consumers taking a short position on firm's debt partly default. Default is costly, in the sense that a portion of the repayment from short-positions' holders does not reach the intermediaries. For simplicity, we posit that the deadweight loss is a fraction $\delta \in (0,1)$ of the amount due.

To ensure its ability to meet its future obligations in the presence of default risk, an intermediary who issues H long and short positions, respectively, will hold corporate debt in an amount γ such that

$$H \leq H(1-\delta) + \gamma. \tag{13}$$

Hence, originating short positions involves a linear cost, given by the face value of the debt – δH – needed to fully cover the shortfall in proceeds due to investors' default. To cover such cost, a spread will arise between the price of long positions p^+ and the price of short positions p^-.

Let p still denote the market price of debt. The intermediary chooses the number of positions H and the quantity γ of debt held (collateral) to maximize its profits at $t = 0$, given by $(p^+ - p^-)H - p\gamma$, subject to the solvency constraint (13).

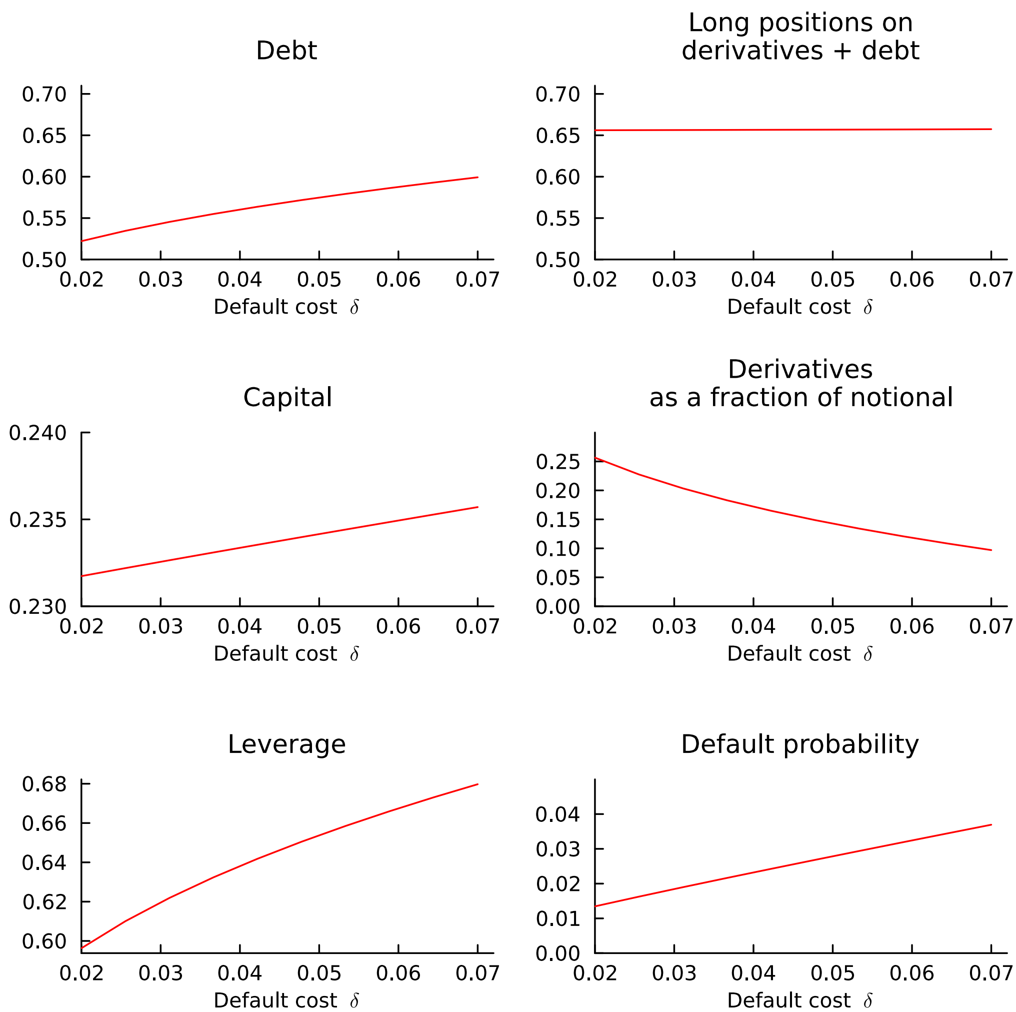

Figure 10: Short Sales and Capital Structure.

A solution to the intermediary's problem exists and features a strictly positive level of intermediation, provided that the spread between the price of long and short positions satisfies the no-arbitrage condition $p^+ - p^- = \delta p$. In such case, the spread allows to fully recoup the default cost of intermediation. Intermediaries make zero profits and purchase an amount δ of corporate debt per unit of derivative issued – just enough to cover the shortfall due to default. It follows that overall intermediation activity is limited by the amount of the firm's outstanding debt, B.

When the volume of intermediation is not constrained, i.e. $\delta H < B$, a portion of the outstanding debt is directly held by consumers. In this case, debt and long positions trade at the same price, i.e. $p = p^+$. If instead $\delta H = B$, the firm's debt is entirely held by intermediaries. In such scenario, debt sells at a premium over the long positions, due to its additional role as collateral. That is, $p > p^+$.

In Figure 10 we compare equilibria that differ only in the default rate δ, which is also the unit cost of intermediation.[18] For our parameter values, intermediation is never constrained. Type–1 investors acquire the short positions on debt, while type–2 households purchase all long positions.

When intermediation costs are relatively high, the volume of intermediation is negligible. As the cost declines, intermediation activity rises and the availability of derivatives increases the supply of hedging opportunities available to type-2 households. As a consequence, firms optimally choose to lower their investment and leverage. These features are reminiscent of those arising when an inelastic supply of risk-free debt is introduced in the economy.

Differently from the scenario of Section 3.3, however, the net supply of assets only changes due to the firms' equilibrium response. The improved hedging services available to type-2 agents are provided by type–1 investors, who purchase the short positions. In turn, this means that the implications for households' consumption processes are different. In particular, as intermediation increases, mean consumption growth declines for both types. Furthermore, since the long positions are perfect substitutes for corporate bonds in the eyes of type-2 agents, the crowding out of corporate bonds is essentially complete. The top-right panel of Figure 10, which reproduces the sum of long positions on derivatives and debt holdings, reveals that the increase in the supply of derivatives is accompanied by an equivalent decline in corporate debt.

The collateralizability of corporate debt generates an additional demand for debt, coming from intermediaries. Pelizzon et al. (2024) document the impact of this channel on capital structure in the case of firms whose bonds become eligible for collateral under the ECB framework. The paper shows that in the four quarters following the announcement that firms' corporate bonds become eligible, firms increase their public debt and their borrowing altogether. In our environment, this effect is dominated by the decline in demand driven by the fact that intermediation produces substitutes for the corporate debt in households' portfolios.

[18] Equilibria illustrated in Figure 10 are for $\chi_2 = 4$.

3.5 Comparative statics: Technology choice

In this section we generalize our model to show how the supply of hedging instruments may depend on firms' technology choices. We expand the production possibilities allowing firms to choose between the risky technology $e^{\varepsilon}Ak^{\alpha}$ and an alternative, safer technology. For simplicity, let the latter be entirely deterministic, i.e. $A_w k^{\alpha}$, with $A_w < A\mathbb{E}(e^{\varepsilon})$.[19] The production function becomes

$$F(k, \phi; \varepsilon) = \phi e^{\varepsilon}Ak^{\alpha} + (1-\phi)A_w k^{\alpha}, \quad \phi \in \{0, 1\}.$$

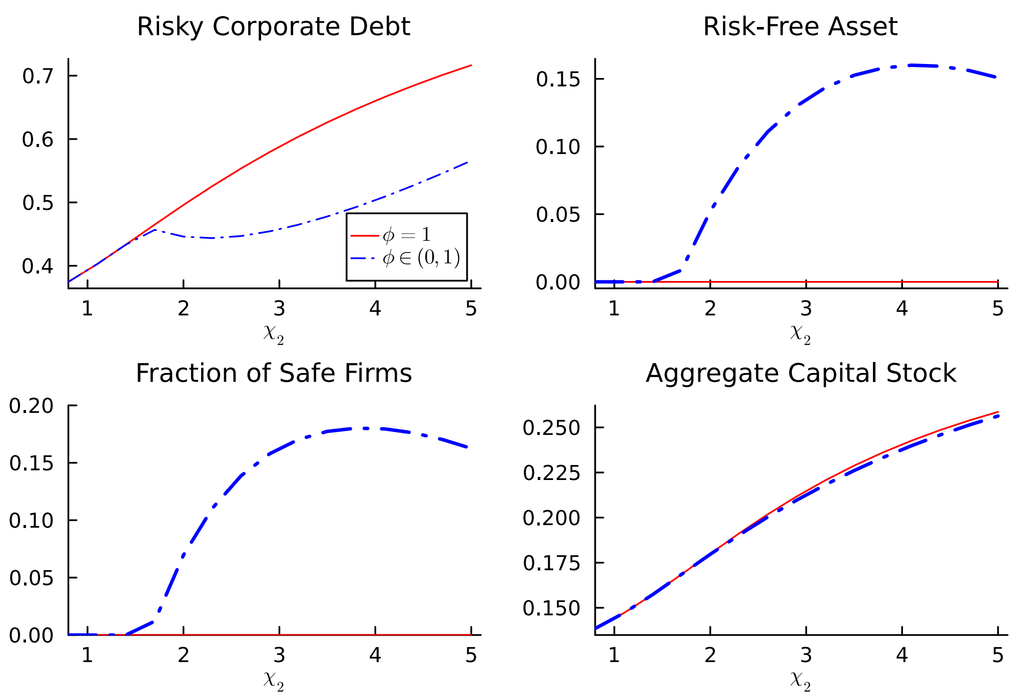

Figure 11: Specialization

We look for conditions under which ex-ante identical firms specialize, with a positive measure of them opting for the safe technology, i.e. setting $\phi = 0$, and the remainder still operating the risky technology ($\phi = 1$).[20] Figures 11, 12 and 13 illustrate equilibria indexed by $\chi_2 \in [0.8, 5]$. Red solid lines describe equilibrium values arising in the scenario above, when $\phi = 1$ by assumption for all firms. The blue (dash-dot) lines illustrate instead

[19] In the numerical example, $A_w = 2.1 < A\mathbb{E}(e^{\varepsilon}) = 2.5e^{0.02}$.

[20] The analysis conducted in Section 3 reveals that non-convexity in the choice of technology is not needed to generate specialization. When markets are incomplete, rationality of the price conjectures implies that the firm's choice problem is not convex. Thus, equilibria where ex-ante identical firms specialize in their production or financing choices may arise in the absence of any further assumption. We make this further assumption with the only purpose of sharpening our exposition.

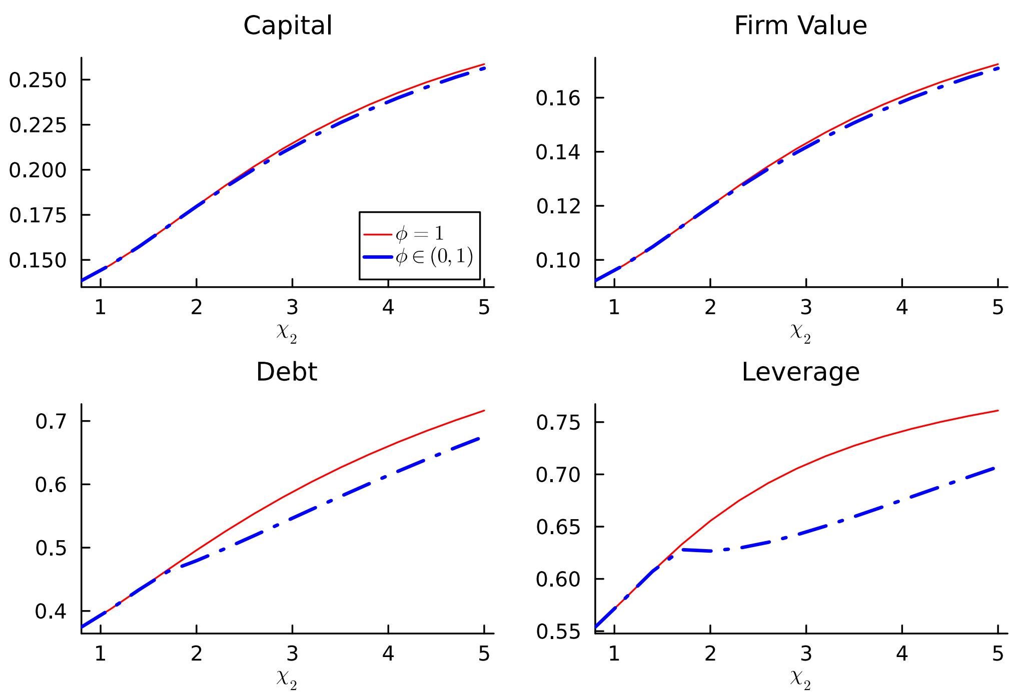

Figure 12: Choices of risky firms with specialization

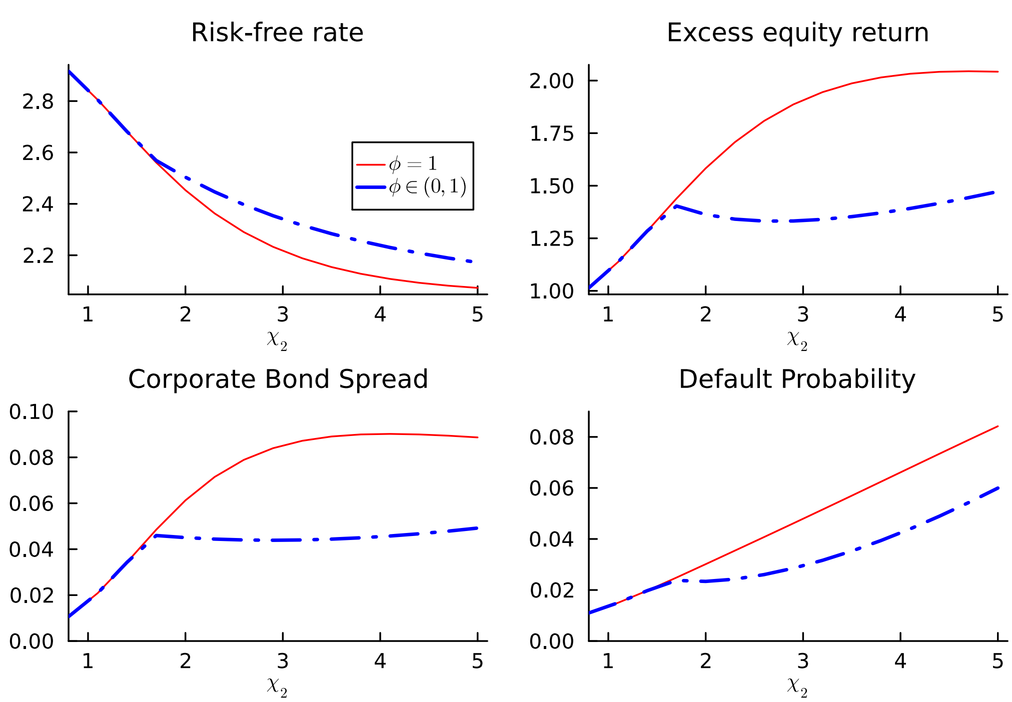

Figure 13: Specialization and asset returns.

equilibria when firms are free to randomize over the feasible choices of $\phi = 0, 1$.

The lower-left panel in Figure 11 shows that when χ_2 is relatively low, all firms choose the risky, more productive technology. As the endowment of type–2 households becomes riskier, specialization arises. Equilibria feature a non-zero fraction of firms choosing the safe technology.

Since the output of firms selecting $\phi = 0$ is risk-free, type-2 agents always value their equity strictly more than type-1's. Thus, the conjectured equilibrium value of operating the risk-free technology is

$$\max_k \ -k + A_w k^\alpha \int_{-\infty}^{+\infty} m^2(\varepsilon) g(\varepsilon) d\varepsilon = \max_k \ -k + \frac{A_w k^\alpha}{R^f}. \tag{14}$$

For relatively low χ_2, the risk-free rate – which proxies for the hedging value of the riskless asset – is not small enough to compensate for the lower productivity of the safe technology. The value of problem (14) is strictly lower than that guaranteed by the risky technology. As a result, the lottery over ϕ is degenerate. All firms choose risk.

As χ_2 grows larger, so do type-2 agents' hedging needs and their valuation of the risk-free asset. For χ_2 greater than a certain threshold, a non–zero measure of firms respond to the higher hedging demand and associated lower value of R^f by selecting the safe technology.

Because of firms' specialization, a new asset becomes available to households, making markets endogenously more complete. In addition to the corporate debt issued by firms selecting the risky technology, type–2 agents purchase the riskless asset issued by safe firms. Due to the relatively low default probability of risky firms, their debt and the safe asset are close substitute. This is why the risk-free equity crowds out risky corporate debt almost perfectly.

This outcome bears obvious analogies with those reached in the previous section. There, the increase in hedging instruments was due to financial intermediaries that found it optimal to increase their production of derivatives on corporate debt. Here, the novel asset is directly produced by non-financial firms who alter their technology choice and hence the risk of their liabilities.

The positive association between hedging demand and the supply of assets by safer firms is in line with Mota (2023), who finds that debt issuance by safer firms is larger and more responsive to increases in the aggregate safety premium.

Figure 12 shows that in our example, capital and market value of the risky firms are both lower when specialization takes place. Recall from our discussion in Section 3 that capital investment reduces the risk imposed by debt on equityholders. Since with specialization risky debt is lower, so is capital. Figure 11 shows that aggregate capital is also lower.

Figure 13 illustrates the impact of specialization on returns. The supply of risk-free assets by firms choosing the safe technology reduces the variance of type–2 agents' consumption growth. As a result, the risk–free rate increases. Lower leverage by risky firms leads then to lower excess equity return, default probability, and corporate bond spreads. These changes mirror those observed in Sections 3.3 and 3.4. In all such scenarios, their root cause lies in the greater supply of hedging instruments.

4 Agency

We generalize our environment by introducing an agency friction akin to the standard asset substitution problem pioneered by Jensen and Meckling (1976). The production function is the same as in Section 3.5 except for the assumption that firms can now choose any combination of the safe and the risky technology. The production possibility frontier is

$$F(k, \phi; \varepsilon) = \phi e^{\varepsilon} A k^{\alpha} + (1 - \phi) A_w k^{\alpha}, \quad \phi \in [0, 1].$$

For given k, a larger ϕ is associated with greater output volatility and higher expected output. Figure 14 provides a graphical rendition of how – everything else equal – changes in the loading ϕ affect the output of production as well as the payoffs of equity and debt. The production functions cross for $\varepsilon = \log(A_w/A)$. As ϕ rises, the yield of equity tends to increase, while the yield of debt tends to decrease.

The choices of k and B are taken by initial shareholders to maximize firm value. Both are observable by outside investors and even contractible upon. The choice of ϕ, instead, is not observable by outside investors and is taken by end-of-period shareholders to maximize their benefits from holding equity alone, hence the agency friction.

The conflict between bondholders and end-of-period shareholders arises because, for given k and B, the shareholders' valuation of equity will be maximal for a loading ϕ greater than the level that maximizes firm value. This follows from the effect of ϕ on the payoffs of equity and debt described above.

We posit that increasing the expected value of the firm's production as well as its risk to levels implied by ϕ requires end-of-period shareholders to incur a cost $C(\phi)$, where C is a twice continuously differentiable, strictly increasing, and strictly convex function of ϕ. Since the cost is borne by end-of-period shareholders, acquiring one unit of equity at $t = 0$ requires from them an overall disbursement of $q + C(\phi)$, where the second addendum is

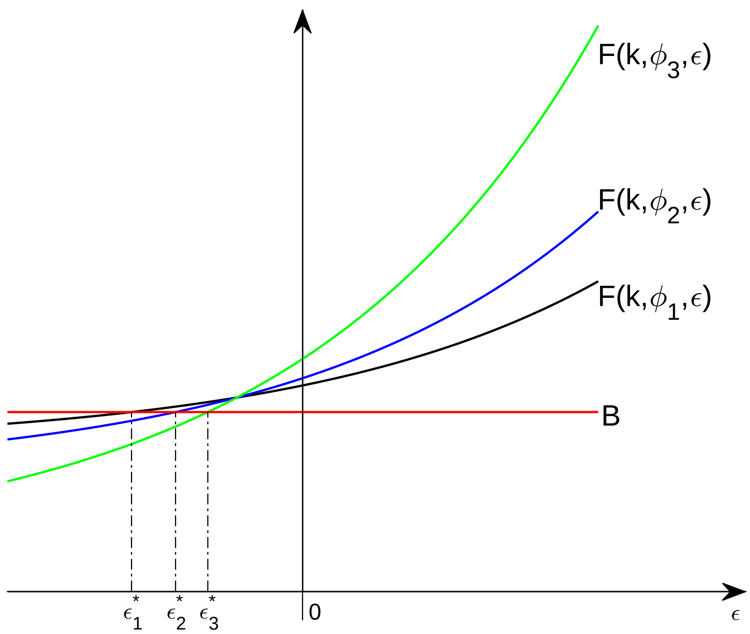

Figure 14: The effect of ϕ on the payoffs of debt and equity.

the cost function evaluated at the equilibrium ϕ.

We introduce the function C with the only purpose of avoiding the uninteresting scenario in which shareholders always opt for the corner solution $\phi = 1$. In the numerical exercises to follow, we will assume $C(\phi) = c/(1-\phi)$, for $c > 0$.

The specification of the *rational* price conjectures associated to any triplet $\{k, B, \phi\}$ is the natural extension of that in (4)-(5) to the scenario where the payoffs of equity and debt are also a function of ϕ. For any choice $\{k, B\}$, the anticipated risk loading is the value of ϕ that maximizes the value of equity

$$q(k,\phi,B) = \int_{\varepsilon^*(k,B,\phi)}^{+\infty} m^1(\varepsilon)\left[\left(\phi A e^{\varepsilon} + (1-\phi)A_w\right)k^{\alpha} - B\right] g(\varepsilon) d\varepsilon - C(\phi), \tag{15}$$

where $\varepsilon^*(k,B,\phi) = \log\left[\frac{Bk^{-\alpha}-(1-\phi)A_w}{\phi A}\right]$ is the default threshold. Equation (15) results from observing that in equilibrium type-1 households have the highest valuation for equity.

The firm's optimization problem then writes as

$$\max_{k,B} \; -k + q(k,B,\phi) + p(k,B,\phi)B \tag{16}$$

$$\text{s.t.} \;\; \phi \in \arg\max \; q(k,B,\phi), \tag{17}$$

where (17) is the end-of-period shareholders' incentive compatibility constraint. Consisting of a simple modification of that in Section 2, the definition of equilibrium is omitted for brevity.

Provided that the second derivative of the cost function is large enough, ϕ obtains as the unique solution to the first-order condition

$$k^{\alpha}\left[\int_{\varepsilon^*(k,B,\phi)}^{+\infty} m^1(\varepsilon)[Ae^{\varepsilon}-A_w]g(\varepsilon)d\varepsilon\right]-C'(\phi)=0. \tag{18}$$

In the benchmark without agency, the choice of ϕ is observable and is taken by equity-holders to maximize firm's value. In that scenario, the first order condition is

$$k^{\alpha}\left[\int_{-\infty}^{\varepsilon^*(k,B,\phi)} m^2(\varepsilon)[Ae^{\varepsilon}-A_w]g(\varepsilon)d\varepsilon+\int_{\varepsilon^*(k,B,\phi)}^{+\infty} m^1(\varepsilon)[Ae^{\varepsilon}-A_w]g(\varepsilon)d\varepsilon\right]-C'(\phi)=0. \tag{19}$$

The additional term appearing in (19) reflects the fact that, without agency, the shareholders internalize the effect of ϕ on bondholders' value. Provided that $B < A_w k^{\alpha}$, so that $\varepsilon^* < \log(A_w/A)$, such term is going to be negative. In turn, this implies that – everything else equal – agency induces shareholders to choose an inefficiently high risk loading.

We turn next to the effects of agency on the initial shareholders' choice problem and on the equilibrium values of k, B. Let $\phi(k, B)$ denote the map defined by (18) for any given k, B. Such relation describes the risk loading ϕ that is anticipated by outside investors for any k, B. Initial shareholders will take such mapping as given when choosing capital and debt. In other words, requiring $\phi = \phi(k, B)$ implements the incentive constraint (17). Necessary conditions for a solution of the initial shareholders' problem are

$$\begin{aligned}\phi\alpha Ak^{\alpha-1}&\left[\int_{\varepsilon^*}^{+\infty} m^1(\varepsilon)e^{\varepsilon}g(\varepsilon)d\varepsilon+\int_{-\infty}^{\varepsilon^*} m^2(\varepsilon)e^{\varepsilon}g(\varepsilon)d\varepsilon\right]+\\ +(1-\phi)\alpha A_w k^{\alpha-1}&\left[\int_{\varepsilon^*}^{+\infty} m^1(\varepsilon)g(\varepsilon)d\varepsilon+\int_{-\infty}^{\varepsilon^*} m^2(\varepsilon)g(\varepsilon)d\varepsilon\right]+\\ +\frac{\partial\phi}{\partial k}k^{\alpha}&\int_{-\infty}^{\varepsilon^*} m^2(\varepsilon)[Ae^{\varepsilon}-A_w]g(\varepsilon)d\varepsilon=1\end{aligned} \tag{20}$$

and

$$-\int_{\varepsilon^*}^{+\infty} m^1(\varepsilon)g(\varepsilon)d\varepsilon+\int_{\varepsilon^*}^{+\infty} m^2(\varepsilon)g(\varepsilon)d\varepsilon+\frac{\partial\phi}{\partial B}k^{\alpha}\int_{-\infty}^{\varepsilon^*} m^2(\varepsilon)[Ae^{\varepsilon}-A_w]g(\varepsilon)d\varepsilon=0, \tag{21}$$

where ε^* is a shorthand for $\varepsilon^*(k, \phi, B)$.

The first two terms in both (20) and (21) are in common with the necessary conditions of the no-agency case. In fact, they constitute the immediate generalization of conditions (9) and (10). The last term on the left-hand-side of either equation is instead special to the environment with agency, reflecting the effects of k and B, respectively, on the end-of-period shareholders' choice of ϕ.

Consider the last term on the left-hand-side of (21). It can be established that, in equilibria where $B < A_w k^\alpha$, $\partial\phi/\partial B > 0$. Everything else equal, a lower debt induces end-of-period shareholders to select a lower risk loading ϕ. This is the defining feature of the asset substitution problem. Since the term that multiplies $\partial\phi/\partial B$ is negative, ceteris paribus initial shareholders will want to lower their debt choice to prevent end-of-period shareholders from taking inefficiently high risks.

In our incomplete-market environment, however, this consideration does not guarantee that agency leads to a lower equilibrium's corporate debt. The higher ϕ induced by agency also impacts the investors' consumption processes, differentially so for shareholders and bondholders. In turn, this modifies the firms' incentives to issue debt when catering to investors hedging needs.

We saw in Section 3 that, absent the choice of ϕ, it is optimal for firms to increase debt as long as the marginal gain from bondholders is greater than the marginal loss of shareholders. The first two terms in (21) highlight that such trade-off is still present when ϕ is a choice variable. Differentiating those terms with respect to ϕ informs us on how equilibrium changes in ϕ affect that trade-off:

$$\frac{\partial \varepsilon^*}{\partial \phi}\left[m^1(\varepsilon^*) - m^2(\varepsilon^*)\right] g(\varepsilon^*) + \int_{\varepsilon^*(\phi,k,B)}^{+\infty} \left[\frac{\partial m^2(\varepsilon)}{\partial \phi} - \frac{\partial m^1(\varepsilon)}{\partial \phi}\right] g(\varepsilon) d\varepsilon. \tag{22}$$

The first addendum in (22) reflects the effect resulting from the marginal increase in the probability of default. The second illustrates the differential effect of greater aggregate output risk on the state prices of agents 1 and 2, respectively, over the solvency region. Unfortunately, general results about their respective signs are not forthcoming. However, in our numerical exercises the first is negative, while the second is positive – and larger in absolute value terms.

In Figure 15 we display the state prices with (blue, dashed) and without agency (red, solid).[21] An increase in ϕ increases the likelihood of default, i.e. $\partial\varepsilon^*/\partial\phi > 0$. Since $m^1(\varepsilon^*) < m^2(\varepsilon^*)$, a higher ϕ reduces the net marginal gain from higher debt.

Consider now the second addendum. Since the bondholders' equilibrium consumption for $\varepsilon > \varepsilon^*$ does not depend on ϕ, $\partial m^2(\varepsilon)/\partial\phi = 0$ for all $\varepsilon \in (\varepsilon^*, +\infty)$. Turning to

[21] To help with the illustration, the equilibrium of which in the figure obtains for $\mu = -0.08125$, $\sigma = 0.45$, $\chi_2 = 4$, $c = 0.02$, and $A_w = 1.8 < A\mathbb{E}(e^\varepsilon) = 2.5e^{0.02}$.

equityholders, notice that $\partial m^1(\varepsilon)/\partial\phi < 0$ for $\varepsilon > \log(A_w/A)$ and $\partial m^1(\varepsilon)/\partial\phi > 0$ for $\varepsilon \in (\varepsilon^*, \log(A_w/A))$. While in general it is not possible to sign the term, the property $A_w < A$, by ensuring $\partial m^1(\varepsilon)/\partial\phi < 0$ for $\varepsilon > 0$, all but guarantees that it is positive.[22]

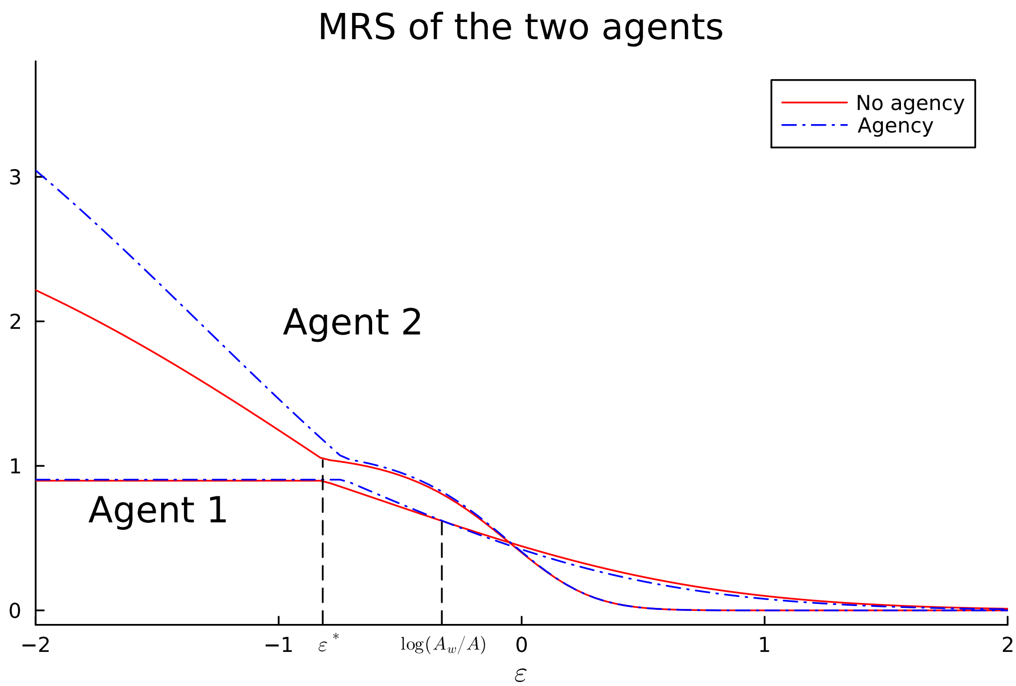

Figure 15: Equilibrium effects of higher ϕ on state prices

In summary, with incomplete markets agency affects optimal capital structure via two channels. The first is the textbook asset substitution mechanism that leads to lower debt. The second is the general equilibrium effect of the higher risk loading on investors' marginal rates of substitution. In general, the sign of the latter is ambiguous. In the case of our example, however, it is positive.

In Figure 16, we illustrate how equilibria with agency (blue, dash-dot lines) and without agency (red, solid lines) vary with χ_2. Consider the scenario without agency first. As it was the case in Section 3, firms cater to larger type-2 agents' hedging needs by issuing more debt. Since a larger debt level increases agent 1's state prices over the solvency region, however, it is now optimal for firms to accompany a higher debt with higher levels of both ϕ and k.

With agency, firms always opt for a higher ϕ. The rationale can be found in the asset substitution effect. Interestingly, the difference between the loadings that obtain in the two scenarios is increasing in χ_2. When debt is essentially riskless, there is no asset

[22] If $\varepsilon^* \geq \log(A_w/A)$, then $\partial m^1(\varepsilon)/\partial\phi < 0$ for $\varepsilon > \varepsilon^*$.

substitution to speak of. Since shareholders also suffer the negative consequences of higher ϕ, i.e. lower firm cash-flows for low realizations of ε, they have no interest in increasing risk. Higher values of χ_2, commanding higher and riskier debt, generate the incentives for shareholders to select higher ϕ. Agency costs, as substantiated in excess risk, are higher when hedging demand is higher.

In our example, agency leads to a small decrease in debt issuance. The asset substitution effect prevails over the hedging demand effect: Initial shareholders reduce debt to restrain end-of-period shareholders from introducing too much risk. To detect the hedging effect, we report in Figure 16 (black dashed lines) the firms' optimal choices with agency, when investors' state prices are set equal to those arising in the equilibrium without agency. When we suppress the general equilibrium effect of higher ϕ on consumption, the debt chosen by firms is lower. The difference is due to the hedging effect.

Additional evidence on the interaction between agency and hedging demand is provided by the comparative statics of the interest rate, illustrated in the bottom-right panel of Figure 16. The risk-free rate is lower with agency, because the larger risk loading ϕ leads the consumption of type-2 investors to decrease in bad states of nature and hence their discount factor to increase.

For the same reason, the price of debt is also higher with agency, and so is market leverage. This is unlike the standard partial-equilibrium treatment of asset substitution where, asset prices being exogenous, market leverage is invariably lower with agency.

5 Conclusions

In a general equilibrium model with production and incomplete markets, the capital structure is pinned down by investors' hedging needs. When the latter increase, firms issue more debt. A portion of the greater proceeds accrue to shareholders in the form of equity payout, while the remainder finances physical capital accumulation, attenuating the impact on default risk.

This result carries through when investors can rely on alternative hedging means such as derivatives on corporate securities or government-issued risk-free assets in finite supply. Indeed, the model rationalizes the post-financial crisis increase in corporate leverage as firms' equilibrium response to a rise in hedging demand not met by a proportionate increase in safe asset.

When capital structure is jointly shaped by hedging demand and supply considerations – the latter, in the form of an asset–substitution problem – we find that, as in partial

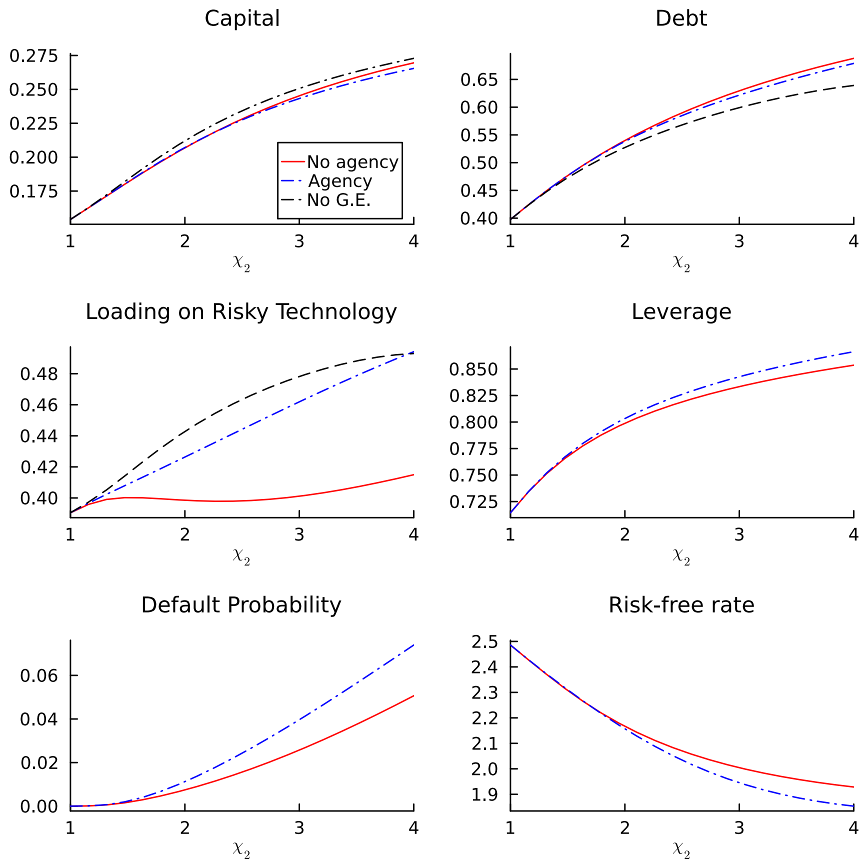

Figure 16: Firms' choices with agency

equilibrium, asymmetric information leads to greater risk. The impact on debt, however, is ambiguous. The partial-equilibrium incentive effect is still present, as initial shareholders choose a lower debt to reduce incentives to exploit bondholders ex-post. However, in general equilibrium the greater risk in production ends up affecting state prices, potentially calling for higher debt.

The definition of a firms' objective function under incomplete markers is not a trivial task. In our framework, firms are postulated to operate under rational conjectures, a construct introduced by Louis Makowski. Building on his work, we can show that equilibria exist, feature unanimity, and display appealing welfare properties.

The next step, which we leave for future work, consists in adapting the equilibrium concept and extending the analysis to the dynamic economies typically considered in macroeconomics and finance.

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A Appendix

A.1 Proofs of unanimity and constrained optimality of equilibria

We prove here the unanimity and constrained-optimality properties of competitive equilibria, stated in Section 2. We establish them for a specification of our model that is more general than above. The production function is $y = e^{\varepsilon} f(k)$, with f a strictly increasing and strictly concave function. There are I different types of households, each of unit mass. A type-i household's endowment consists of $w_0^i \geq 0$ units of commodity at $t = 0$ and $w_1^i(\varepsilon)$ at $t = 1$. Her initial equity stake in the firm is $\theta_0^i \geq 0$. Households' preferences are described by a common strictly increasing, strictly quasi-concave, Von Neumann-Morgerstern utility function over random consumption sequences $\left\{c_0^i, c_1^i(\varepsilon)\right\}$:

$$U(c_0^i, c_1^i) \equiv u(c_0^i) + \beta \mathbb{E}\left[u(c_1^i)\right], \quad \beta > 0.$$

The optimization problem of a type i households is:

$$\max_{c_0^i, \theta^i, b^i, c_1^i(\varepsilon)} u(c_0^i) + \beta \mathbb{E}\left[u(c_1^i)\right] \tag{23}$$

$$\text{s.t.} \quad c_0^i = w_0^i + \theta_0^i V - q\theta^i - pb^i, \tag{24}$$

$$c_1^i(\varepsilon) = w_1^i(\varepsilon) + \theta^i d^e(\varepsilon) + b^i d^b(\varepsilon) \;\; \forall\, \varepsilon,$$

$$\theta^i \geq 0, \;\; b^i \geq 0.$$

For simplicity, we restrict notation to symmetric equilibria and allocations, where all firms make the same choices.

Definition 1 ***Competitive Equilibrium.*** *A competitive equilibrium consists of firms' choices k, B, cum-dividend value V, asset prices q, p, price conjectures $p(k, B)$, $q(k, B)$ for all possible k, B, as well as consumption choices $\left(c_0^i, c_1^i(\varepsilon)\right)$ and portfolio choices $\left(\theta^i, b^i\right)$ for each agent $i = 1, \ldots, I$, such that (i) k, B attain the maximum in problem (3), (ii) V is the value of problem (3), (iii) $\left(c_0^i, c_1^i(\varepsilon)\right)$ and $\left(\theta^i, b^i\right)$ solve problem (23) for each agent $i = 1, \ldots, I$, (iv) $p(k, B)$ and $q(k, B)$ are rational, i.e. satisfy (4)-(5), (v) price conjectures and asset payoffs at the equilibrium choices k, B are consistent, i.e.*

$$q = q(k, B), \;\; p = p(k, B), \;\; d^e = d^e(k, B), \;\; d^b = d^b(k, B),$$

and (vi) markets clear:

$$\sum_i b^i \leq B, \;\; \sum_i \theta^i \leq 1.$$

Definition 2 ***Unanimity.*** *At a competitive equilibrium, initial shareholders unanimously support the firm's decision to adopt plan (k, B), when the firm's market value under this plan is greater or equal than the marginal utility of each initial shareholder for any possible plan (k', B'), keeping constant the equity initially held and the pro-rata debt issued.*

This definition relies on the fact that, with a continuum of consumers and a continuum of ex-ante identical firms, we restrict our attention to the case where each investor is endowed with at most a negligible amount of equity of any individual firm. Hence, the effects on the shareholder's utility of alternative choices of a single firm can be evaluated using her intertemporal marginal rate of substitution at equilibrium m^i, for $i = 1, \ldots, I$.

Proposition 1 *Let* (k, B) *be the firms' choice and* $\left(c_0^i, c_1^i\right)_{i=1}^I$ *the consumption allocation at a competitive equilibrium. Then initial shareholders unanimously support the choice of* (k, B).

Proof of Proposition 1. Consider an initial shareholder of type i. The maximum value of her marginal utility, across all possible plans of the firm k', B', when he holds the equity he is initially endowed and (pro rata) the debt issued by the firm, is:

$$V^i = \max_{k', B'} \left\{ -k' + \mathbb{E}\left[m^i d^e(k', B')\right] + \mathbb{E}\left[m^i d^b(k', B')\right] B' \right\}$$

On the other hand, the value of the firm at equilibrium V, that is the amount agent i receives for the firm in the market and enters his budget constraint (24), is greater or equal than V^i for all $i \in I$. This follows from the fact that, when firms operate on the basis of rational price conjectures (4-5) and choose the plan which maximizes their market value, as in (3), we have:

$$V = \max_{k', B'} \left\{ -k' + \max_i \mathbb{E}\left[m^i d^e(k', B')\right] + \max_i \mathbb{E}\left[m^i d^b(k', B')\right] B' \right\}$$

That is every initial shareholders weakly prefers selling the firm, which proves the statement. ■

Definition 3 Constrained optimality. *An equilibrium allocation* $\left(c_0^i, c_1^i(\varepsilon)\right)_{i=1}^I$ *is constrained-Pareto optimal if there does not exist an* attainable *allocation which Pareto-dominates it. We say that a consumption allocation* $\left(c_0^i, c_1^i(\varepsilon)\right)_{i=1}^I$ *is attainable if:*
(i) it satisfies resource feasibility, i.e. there exists a production plan k *such that*

$$\sum_i c_0^i + k \leq \sum_i w_0^i$$

$$\sum_i c_1^i(\varepsilon) \leq \sum_i w_1^i + e^{\varepsilon} f(k);$$

(ii) the consumption profile of each consumer type i *is attainable with the existing*

asset structure, i.e. there exists $B \geq 0$ and a pair $\theta^i, b^i \geq 0$ such that

$$c_1^i(\varepsilon) = w_1^i(\varepsilon) + \theta^i d^e(k, B, \varepsilon) + b^i d^b(k, B, \varepsilon).$$

Proposition 2 ***Constrained efficiency*** *Competitive equilibria are constrained-Pareto optimal.*

The proof of Proposition 2 relies on an important implication of the rationality of firms' conjectures: a competitive equilibrium is equivalent to one where markets for all 'types' of equity and bonds, associated to all possible choices of k, B, are open for trade to consumers at the prices $p(k, B)$, $q(k, B)$ satisfying (4)-(5). At those prices, consumers do not wish to trade any of the additional securities and firms do not wish to issue them.

Lemma 1 *Assume that the set of assets available for trade by each consumer $i \in I$ in competitive equilibrium is expanded to include the equity and bond associated to any other possible choice k', B' of a firm, at the prices $p(k', B')$, $q(k', B')$ satisfying the* rationality of conjectures (4)-(5)*. Then, the consumers' choices and the equilibrium allocation are unchanged. The markets for all the additional securities clear with zero trade.*

Proof of Lemma 1 Consider the first-order conditions of any consumer i for all additional securities, evaluated at zero trade. For all (k', b') different from the firm's choice,

$$q(k', B') \leq \mathbb{E}\left[m^i d^e(k', B')\right],$$
$$p(k', B') \leq \mathbb{E}\left[m^i d^e(k', B')\right].$$

When price conjectures satisfy the rationality condition, the above conditions are always satisfied, establishing that at those prices no consumer wants to trade a positive amount of those equity and bonds. Similarly, firms at those prices prefer to choose k, B, which solve (3), and hence do not wish to issue any positive amountof debt and equity associated to k', B'. ■

Proof of Proposition 2. Consider the competitive equilibrium allocation $\left(c_0^i, c_1^i(\varepsilon), \theta^i, b^i\right)_{i=1}^I$ and equilibrium firms choices (k, B). Proceeding by contradiction, let $\left(\tilde{c}_0^i, \tilde{c}_1^i(\varepsilon)\right)_{i=1}^I$ be an admissible consumption allocation that is Pareto improving. By the definition of admissibility, there exists a firm's production plan $\tilde{k}$, debt choice $\tilde{B}$ and portfolios $\left(\tilde{\theta}^i, \tilde{b}^i\right)_{i=1}^I$ such that $\left(\tilde{c}_0^i, \tilde{c}_1^i(\varepsilon)\right)_{i=1}^I$ is feasible and attainable with those trades of debt and equity associated to $\tilde{k}, \tilde{B}$.

At the competitive equilibrium $\left(c_0^i, c_1^i(\varepsilon), \theta^i, b^i\right)$ must be a solution of the consumers' optimization problem (3) at the prices q, p and payoffs of debt and equity associated to firms' choice k, B. By Lemma 1, the consumers' problem is equivalent to one where all types of debt and equity, associated to all values of k', B', thus $\tilde{k}, \tilde{B}$ included, are available for trade. Hence, it must be that

$$\tilde{c}_0^i + q(\tilde{k}, \tilde{B})\tilde{\theta}^i + p(\tilde{k}, \tilde{B})\tilde{b}^i \geq c_0^i + q\theta^i + pb^i.$$

Equivalently,

$$\tilde{c}_0^i + q(\tilde{k}, \tilde{B})\tilde{\theta}^i + p(\tilde{k}, \tilde{B})\tilde{b}^i \geq \omega_0^i + (-k + pB + q)\theta_0^i.$$

Since this inequality must hold for all i, strictly for at least one consumer type j, summing over i yields:

$$\sum_{i \in I} \tilde{c}_0^i + q(\tilde{k}, \tilde{B}) + q(\tilde{k}, \tilde{B})\tilde{B} \geq \sum_{i \in I} \omega_0^i - k + pB + q. \tag{25}$$

The fact that k solves the firms' optimization problem at equilibrium in turn implies that

$$-k + q + pB \geq -\tilde{k} + q(\tilde{k}, \tilde{B}) + p(\tilde{k}, \tilde{B})\tilde{B},$$

or

$$-k \geq -q - pB - \tilde{k} + q(\tilde{k}, \tilde{B}) + p(\tilde{k}, \tilde{B})\tilde{B}$$

Using the above inequality to substitute for $-k$ in the term on the right-hand side of (25), we obtain

$$\sum_i \tilde{c}_0^i + \tilde{k} > \sum_i \omega_0^i,$$

The improving allocation violates feasibility at date 0, which is a contradiction. ■

This result does not extend to the economy with agency frictions studied in Section 4. With agency frictions, constrained efficiency may fail as the incentive constraint (17) generates a pecuniary externality. According to (17), the value of the firm's risk loading ϕ does not only depend on the firm's choices of k and B, but also on the equilibrium stochastic discount factors m^i. The consumers' marginal rate of substitution contributes to determine the conjectured market value of equity for all possible values of ϕ. While the marginal rates of substitution are taken as given by the firm, they depend on the consumption allocation. An admissible change in this allocation, by affecting the constraint, may allow to achieve a welfare improvement.

The pecuniary externality we just highlighted is the only source of inefficiency in our economy. Since unanimity always holds, welfare losses cannot be imputed to conflicts among shareholders or to a misallocation of equity ownership.

A.2 Equilibrium capital structure

With complete segmentation, the second-order effect of a marginal change in B at a candidate equilibrium where (10) holds is

$$\left[m^1(\varepsilon^*)g(\varepsilon^*) - m^2(\varepsilon^*)g(\varepsilon^*)\right]\frac{1}{B}dB^2.$$

It follows that the sufficient condition for firm value maximization is $m^1(\varepsilon^*) < m^2(\varepsilon^*)$. We have verified that it always holds in the numerical example considered above.

Next, consider an equilibrium with partial segmentation. Since the Euler equation for equity holds with equality for both agents, firm value (8) is no longer guaranteed to be differentiable at the equilibrium. It turns out it is.

Lemma 2 *With incomplete segmentation, at the firm's optimal choice* $(\bar{k}, \bar{B})$*,* $\int_{\varepsilon^*}^{+\infty} m^1(\varepsilon)g(\varepsilon)d\varepsilon = \int_{\varepsilon^*}^{+\infty} m^2(\varepsilon)g(\varepsilon)d\varepsilon$*.*

Proof (Step 1). By contradiction, suppose that $\int_{\varepsilon^*}^{+\infty} m^1(\varepsilon)g(\varepsilon)d\varepsilon > \int_{\varepsilon^*}^{+\infty} m^2(\varepsilon)g(\varepsilon)d\varepsilon$. Consider an infinitesimal decrease in the debt choice B, i.e. $dB < 0$. The variation in equity valuation is

$$\max_{i=1,2}\left\{-\left(\int_{\varepsilon^*}^{+\infty} m^i(\varepsilon)g(\varepsilon)d\varepsilon\right)dB\right\} = \left[\min_{i=1,2}\left\{-\int_{\varepsilon^*}^{+\infty} m^i(\varepsilon)g(\varepsilon)d\varepsilon\right\}\right]dB = -\left[\int_{\varepsilon^*}^{+\infty} m^1(\varepsilon)g(\varepsilon)d\varepsilon\right]dB.$$

Thus, the value of the firm changes by

$$-\left[\int_{\varepsilon^*}^{+\infty} m^1(\varepsilon)g(\varepsilon)d\varepsilon\right]dB + \left[\int_{\varepsilon^*}^{+\infty} m^2(\varepsilon)g(\varepsilon)d\varepsilon\right]dB > 0,$$

which contradicts the optimality of $(\bar{k}, \bar{B})$.

(Step 2). Proceeding again by constradiction, suppose that at $(\bar{k}, \bar{B})$ we have $\int_{\varepsilon^*}^{+\infty} m^1(\varepsilon)g(\varepsilon)d\varepsilon < \int_{\varepsilon^*}^{+\infty} m^2(\varepsilon)g(\varepsilon)d\varepsilon$. Consider an infinitesimal increase in the debt choice B, i.e. $dB > 0$. The variation in equity valuation is

$$\max_{i=1,2}\left\{-\left(\int_{\varepsilon^*}^{+\infty} m^i(\varepsilon)g(\varepsilon)d\varepsilon\right)dB\right\} = -\min_{i=1,2}\left\{\left(\int_{\varepsilon^*}^{+\infty} m^i(\varepsilon)g(\varepsilon)d\varepsilon\right)dB\right\} = -\left(\int_{\varepsilon^*}^{+\infty} m^1(\varepsilon)g(\varepsilon)d\varepsilon\right)dB.$$

Thus, the value of the firm changes by

$$-\left[\int_{\varepsilon^*}^{+\infty} m^1(\varepsilon)g(\varepsilon)d\varepsilon\right]dB + \left[\int_{\varepsilon^*}^{+\infty} m^2(\varepsilon)g(\varepsilon)d\varepsilon\right]dB > 0,$$

again contradicting the optimality of $(\bar{k}, \bar{B})$. ■

Lemma 2 immediately implies that the optimum $(\bar{k}, \bar{B})$ must satisfy the first-order conditions (9)-(10). Let's now turn to the sufficient conditions. Recall that type-2 agents value debt strictly more than type-1's at $(\bar{k}, \bar{B})$. Therefore, local deviations in the value of debt are evaluated by means of m^2. If local deviations in the value of equity are also evaluated by means of m^2, firm value is invariant to local perturbations around $\bar{B}$. It follows that $(\bar{k}, \bar{B})$ is a maximum if type-2s value equity strictly more than type-1s around $(\bar{k}, \bar{B})$, while it is a local minimum otherwise.

Lemma 3 implies that $(\bar{k}, \bar{B})$ attains value maximization if and only if $m^2(\varepsilon^*) > m^1(\varepsilon^*)$.

Lemma 3 *With incomplete segmentation, in an open set around $(\bar{k}, \bar{B})$, equity is priced by type-2 consumers, i.e.*

$$\int_{\varepsilon^*}^{+\infty} (Ak^{\alpha}e^{\varepsilon} - B)m^2(\varepsilon)g(\varepsilon)d\varepsilon > \int_{\varepsilon^*}^{+\infty} (Ak^{\alpha}e^{\varepsilon} - B)m^1(\varepsilon)g(\varepsilon)d\varepsilon,$$

if and only if $m^2(\varepsilon^*) > m^1(\varepsilon^*)$.

Proof The difference between type-2's and type-1's valuation of equity is

$$\int_{\varepsilon^*}^{+\infty} (Ak^{\alpha}e^{\varepsilon} - B)m^2(\varepsilon)g(\varepsilon)d\varepsilon - \int_{\varepsilon^*}^{+\infty} (Ak^{\alpha}e^{\varepsilon} - B)m^1(\varepsilon)g(\varepsilon)d\varepsilon.$$

Differentiating with respect to B yields

$$-\left[\int_{\varepsilon^*}^{+\infty} m^2(\varepsilon)g(\varepsilon)d\varepsilon - \int_{\varepsilon^*}^{+\infty} m^1(\varepsilon)g(\varepsilon)d\varepsilon\right] dB.$$

By Lemma 2, this expression equals zero at $(\bar{k}, \bar{B})$. The second order effect of a marginal change in B is

$$\left[m^2(\varepsilon^*) - m^1(\varepsilon^*)\right] g(\varepsilon^*)\frac{1}{B}dB^2,$$

which is strictly positive if and only if $m^2(\varepsilon^*) > m^1(\varepsilon^*)$. ■

A.3 Firms' objective function

The literature on incomplete markets with production has emphasized the problems concerning the specification of the firms' objective function. These problems do not arise for the equilibrium notion we propose, where both unanimity and constrained efficiency hold. The key difference lies in the specification of the firms' price conjectures. It is useful then to compare the (*Makowski criterion* for) *rational conjectures* we consider to the two main alternative specifications in the literature, the *Dreze* and the *Grossman-Hart criteria*.

Applied to our environment, the criterion proposed by Dreze (1974) for equity price conjectures is as follows:

$$q(k,B) = \mathbb{E}\left[\sum_i \theta^i m^i d^e(k,B)\right],\ \forall\, k,B. \qquad \text{(D criterion)}$$

It requires the conjectured price of equity for any plan k, B to equal – pro rata – the marginal valuation of the agents who in equilibrium are shareholders of the firm (that is, the agents who value the most the plan chosen by the firm in equilibrium and hence choose to buy equity). It does not however require that the firm's shareholders are those who value any other possible plan of the firm the most. Intuitively, the choice of a plan which maximizes the firm's value with $q(k, B)$ as in the *D criterion* corresponds to a situation in which the firm's shareholders choose the plan which is optimal for them without contemplating the possibility of selling the firm in the market, to allow the buyers of equity to operate the plan they instead prefer.

Equivalently, the value of equity for out of equilibrium production and financial plans is determined using the – possibly incorrect – conjecture that the agents who in equilibrium own the equity of a firm remain the firm's shareholders also for any alternative production and financial plan.[23]

Grossman and Hart (1979) propose an alternative criterion for price conjectures which, when applied to the price of equity in our environment, requires

$$q(k,B) = \mathbb{E}\left[\sum_i \theta_0^i m^i d^e(k,B)\right],\ \forall k,B.$$

We can interpret this specification as describing a situation where the firm's plan is chosen by the initial shareholders (i.e., those with some predetermined endowment of equity at the beginning of date 0) so as to maximize their welfare, again without contemplating the possibility of selling the equity to other consumers who value it more. According to this criterion, the value of equity for all production and financial plans is derived on the basis of the conjecture that the firm's initial shareholders stay in control of the firm whatever is the plan.

In contrast, according to the Makowski criterion for rational conjectures each firm evaluates different production and financial plans using possibly different marginal valua-

[23] It is easy to see that any allocation constituting an equilibrium with rational conjectures is also an equilibrium under the *D criterion*: all shareholders of a firm have in fact the same valuation for the firm's production and financial plan and their marginal utility for any other possible plan is lower, hence a fortiori the chosen plan maximizes the weighted average of the shareholders' valuations. But the reverse implication is not true, i.e., an equilibrium under the *D criterion* is not always an equilibrium under rational conjectures.

tions (that is, possibly different pricing kernels, but all still consistent with the consumers' marginal rate of substitution at the equilibrium allocation). This is essential to ensure the unanimity of shareholders' decisions and is a key difference with respect to Dreze (1974) and Grossman and Hart (1979), both of whom rely on the use of a single pricing kernel.

Turning then to asymmetric information and agency frictions, most of the competitive equilibrium concepts which have been proposed for production economies build on the one proposed by Prescott and Townsend (1984) for exchange economies, therefore exhibiting no traded equity.[24]

Prescott and Townsend's approach, rooted in mechanism design, is rather different from ours, which instead relies on the extension of *rational conjectures* to economies with asymmetric information. Yet, our equilibrium notion is equivalent to the one of Prescott and Townsend, once it is extended to economies with incomplete markets where firms rather than consumers face agency frictions. Indeed, consider the equilibrium concept adopted by Prescott and Townsend (1984) for exchange economies with asymmetric information. In this concept prices depend both on observable and unobservable choices (or states) and this is sustained, drawing a parallel with mechanism design formulations of related problems relying on the Revelation Principle, by restricting admissible choices to those which are incentive compatible. This is analogous to what we do in the firm's problem (16)-(17), where price conjectures also depend on the choice of the risk loading ϕ, though this choice is not observable by outside investors, but the values of ϕ are restricted by the agency constraint (17). Via this constraint, the level of ϕ is determined by the observable choices of the firm, k, B. Hence, price conjectures reflect the correct anticipation of the firm's unobservable choices.

Nonetheless, interesting and important conceptual differences emerge between the properties of equilibria in the environments studied by Prescott and Townsend and in ours. While competitive equilibria are always constrained efficient in the exchange economies with moral hazard considered by Prescott and Townsend, this is not the case in production economies, where agency frictions enter the firms' choice problem. The nature of the equilibrium concept considered plays no role in this, given the equivalence recalled above. Rather, it is the fact that the incentive constraint in the firm's choice problem features a pecuniary externality.[25]

[24] See, e.g., Magill and Quinzii (2002), Prescott and Townsend (2006), and Zame (2007).

[25] Prescott and Townsend also assume that markets are complete, while we do not. But whether markets are complete or not, and hence whether marginal rates of substitution are equalized or not across consumers, it is not crucial for the welfare result. What is key is that the marginal rates of substitution enter the incentive constraint.

A.4 Short sales

We provide here a more detailed analysis of the model with intermediated short-sales discussed in Section 3.4. We allow intermediaries to issue derivatives not only on corporate debt but also on equity. In both cases the origination of a derivative entails a cost, due to the assumption that consumers taking a short position partly default, inducing a deadweight cost.

For simplicity, we assume that only an exogenously given fraction $(1-\delta)$ of the amount due by consumers on their short positions reaches the intermediaries.[26]

To ensure its ability to meet its future obligations in the presence of default risk, an intermediary who issues long and short positions in a derivative in bonds (or equity), respectively, will hold some amount of corporate debt (resp. equity) as reserves. The intermediary's problem consists then in the choice of the amount H^b, H^e issued of long and short positions in the derivatives on debt and equity and the amounts γ^b, γ^e of debt and equity held as reserve, to maximize its total revenue at $t = 0$ subject to the solvency constraints:

$$\begin{aligned} &\max_{H,\gamma \in R^2_+} \left[(p^+ - p^-)H^b - p\gamma^b + (q^+ - q^-)H^e - q\gamma^e\right], \\ \text{subject to }\; &H^b \leq H^b(1-\delta) + \gamma^b, \\ &H^e \leq H^e(1-\delta) + \gamma^e. \end{aligned}$$

The latter ensure the reserves held suffice to allow the intermediary to cover all shortfalls in future revenue due to consumers' default and hence to meet all its future obligations. The presence of a bid ask spread on the long and short positions issued allows the intermediary to cover the cost of the debt and equity held as reserve.

A solution to the intermediary's problem exists, provided that

$$p \geq \frac{p^+ - p^-}{\delta}, \; q \geq \frac{q^+ - q^-}{\delta} \tag{26}$$

and is characterized by $H^j > 0$ and $\gamma^j = \delta H^j$, $j = b, e$, only if the inequalities in (26) hold as equalities.

Let $h^{i,j}_+ \in \Re_+$ denote consumer i's holdings of long positions in the derivative $j = b, e$ issued by intermediaries, and $h^{i,j}_- \in \Re_+$ her holdings of short positions. The consumer's choice problem consists in maximizing her expected utility subject to the budget con-

[26] Our assumption could be justified by setting $1 - \delta$ as the cutoff above which the intermediary would gain from enforcing a court ruling again the defaulting agent. Notice further that the analysis and results extend to situations where default rates are endogeneously chosen by consumers.

straints[27]

$$c_0^i = w_0^i \ + \theta_0^i V - q\theta^i - q^+ h_+^{i,e} + q^- h_-^{i,e} - pb^i - p^+ h_+^{i,b} + p^- h_-^{i,b} \tag{27}$$

$$c_1^i(\varepsilon) = w_1^i(s) + R^b(\varepsilon)(b^i + h_+^{i,b} - h_-^{i,b}) + R^e(\varepsilon)(\theta^i + h_+^{i,e} - h_-^{i,e}) \tag{28}$$

and $\left(\theta^i, b^i, h_+^{i,b}, h_-^{i,b}, h_+^{i,e}, h_-^{i,e}\right) \geq 0$.

The asset market clearing conditions for debt and equity become

$$\gamma^b + \sum_{i \in I} b^i \leq B,$$
$$\gamma^e + \sum_{i \in I} \theta^i \leq 1.$$

For the derivative securities, we have

$$\sum_{i \in I} h_+^{i,j} = \sum_{i \in I} h_-^{i,j} = H^j, \ \ j = b, e.$$

The firm's choice problem is the same as in Section 2. The most significant change concerns the conditions specifying the *rationality* of the price conjectures for debt and equity, which need to reflect that intermediaries demand debt and equity in the market:

$$p(k,B) \ = \ \max \left\{ \begin{array}{c} \max_i \ \mathbb{E}\left[m^i d^b(k,,B)\right], \\ \frac{\max_i \ \mathbb{E}\left[m^i d^b(k,B)\right] - \min_i \mathbb{E}\left[m^i d^b(k,B)\right]}{\delta} \end{array} \right\}, \tag{29}$$

$$q(k,B) \ = \ \max \left\{ \begin{array}{c} \max_i \ \mathbb{E}\left[m^i d^e(k,B)\right], \\ \frac{\max_i \ \mathbb{E}\left[m^i d^e(k,B)\right] - \min_i \mathbb{E}\left[m^i d^e(k,B)\right]}{\delta} \end{array} \right\} \tag{30}$$

for all k, B. The above expressions state that the conjecture of a firm over the prices of its debt and equity when it chooses the plan k, B equals the maximal marginal valuation of the corresponding payoffs, *among both intermediaries and consumers.* The second term on the right hand–side of the above expressions is in fact the intermediaries' marginal valuation for debt and equity and can be interpreted as the *value of intermediation.*

Since an appropriate amount of debt and equity are needed, as reserves, to ensure the intermediary can operate and fulfil its obligations, the intermediary's willingness to pay for debt and equity is determined by the consumers' marginal valuation for the corresponding derivative claims which can be issued.[28] Hence, the above specification of the price

[27] In the investors' $t = 1$ budget constraint, we consider the case where default is actually negligible, hence the loss δ incurred by intermediaries on these position consists almost exclusively of the deadweight costs of default. When default is a not negligible fraction $\xi < \delta$, the loss δ is due partly to actual default ξ and partly to deadweight costs $\delta - \xi$. In this case, $h_-^{i,b}$ and $h_-^{i,e}$ in (28) should be both multiplied by $1 - \xi$ and the price conjectures below should be modified accordingly. All the rest remains unchanged.

[28] More precisely, the first term on the numerator of the term in the second line of (29) equals the

conjectures allows firms to take into account the effects on the value of intermediation of the willingness to pay of consumers for the derivatives issued.

This model captures the relationship between the financial claims issued by firms and the intermediation process. The key feature is that the derivatives issues by intermediaries are backed by the claims issued by firms in two ways. First, the yields of these derivatives are pegged to the yield of the claims issued by firms; second, the intermediaries must hold some amount of these claims to back the derivatives issued. Hence, part of the demand for the firms' claims now also comes from intermediaries (as such claims enter as a sort of input in the intermediation technology).

A competitive equilibrium of the economy with intermediated short sales is defined along similar lines to Definition 1 in Section 2. The properties of shareholders' unanimity and constrained efficiency can then be established by similar arguments as the ones for Propositions 1 and 2. It is interesting to compare this optimality result with Theorem 5 in Allen and Gale (1991), where it is shown that the competitive equilibria of an economy where consumers face a finite, exogenous bound K on short sales are constrained inefficient. In their set-up, long and short positions trade at the same price, i.e., the bid ask spread is zero. The inefficiency result in Allen and Gale (1991) then follows from the fact that firms maximize their market value as determined by price conjectures which ignore the effect of their decisions on the value of intermediation. In other words, a firm does not take into account the possible gains arising from the demand for short positions in the firm's equity or debt. In contrast, in our economy, when a firm makes its production and financial decisions the firm considers the value of its equity and debt not only for the consumers but also for the intermediaries who use these assets as an input in the intermediation process. The gains from trade due to intermediation are so taken into account by firms.

Furthermore, it is straightforward to show that, at an equilibrium, either (i) $p = (p^+ - p^-)/\delta > p^+$ and intermediation for debt derivatives is full (the whole amount of outstanding debt is purchased by intermediaries) or (ii) $p = p^+$ and intermediation is partial (some if not all the amount of outstanding debt is held by consumers). Similarly, for equity derivatives. In other words, at an equilibrium where debt intermediation is full, debt sells at a premium over the long positions on the derivative claim issued by the intermediary, due to its additional value as input in the intermediation technology. Intermediaries in turn recoup the higher cost of debt through a sufficiently high spread $p^+ - p^-$

consumers' marginal valuation for long positions in the derivative on debt, the second one their valuation for short positions; dividing by δ yields the revenue from intermediation, per unit of debt purchased. Similarly, for equity, in (30).

between the price of long and short positions on the derivative. When intermediation is partial, debt and long positions in the derivative trade at the same price, intermediaries may not be active in equilibrium and the bid ask spread $p^+ - p^-$ is low (in particular, less or equal than δp).