Effects of Information Disclosure under First- and Second-Price Auctions in a Supply Chain Setting

ONLINE SUPPLEMENT

Ying-Ju Chen[†] Gustavo Vulcano[‡]

In this online supplement we provide supplementary materials of the paper, including the detailed materials of the technical results and supplementary numerical examples. For ease of presentation, we include the statements of auxiliary propositions and lemmas in this online supplement.

A1. The consumer market game

A1.1 The second-price auction procurement case

Recalling from Assumption 1 that $\theta = \theta_0 + s_1 + s_2$, the following proposition summarizes the equilibrium quantities provided by these two resellers in the consumer market:

Proposition A1. *Suppose that the second-price auction is used. In the consumer market, there exists a unique Nash equilibrium in which*

$$\begin{aligned} q_w^{II}(s_w, s_l) &= \begin{cases} \frac{1}{2}\left(\theta_0 + s_w + s_l - q_l^{II}(s_l)\right), & \text{if } \; s_w < S_1^{II}(s_l) \\ C, & \text{if } \; S_1^{II}(s_l) \le s_w \le S_2^{II}(s_l) \\ \frac{1}{2}\left(\theta_0 + s_w + s_l - q_l^{II}(s_l) - c\right), & \text{if } \; s_w > S_2^{II}(s_l), \end{cases} \\ q_l^{II}(s_l) &= \frac{1}{2}\left(\mathrm{E}_{s_w}[\theta | s_w > s_l, s_l] - \mathrm{E}_{s_w}[q_w(s_w, s_l) | s_w > s_l] - c\right), \end{aligned} \tag{A1.1}$$

where the two thresholds $S_1^{II}(s_l)$ and $S_2^{II}(s_l)$ are

$$\begin{aligned} S_1^{II}(s_l) &= \min\left\{\max\{s_l, 2C - \theta_0 - s_l + q_l^{II}(s_l)\}, 1\right\}, \\ S_2^{II}(s_l) &= \min\left\{\max\{s_l, 2C - \theta_0 - s_l + q_l^{II}(s_l) + c\}, 1\right\}. \end{aligned} \tag{A1.2}$$

[†] University of California, 4121 Etcheverry Hall, Berkeley, CA 94720, chen@ieor.berkeley.edu.

[‡] Stern School of Business, New York University, 44 West 4th Street, Suite 8-76, New York, NY 10012, gvulcano@stern.nyu.edu.

Proof. We first disregard the capacity constraint of the winner, and derive her first-order conditions given s_l.

Let $\Pi_w^{II} \equiv (\theta - q_w - q_l)q_w - c(q_w - C)^+$ denote the winner's payoff function. By differentiating Π_w^{II} with respect to q_w where $q_w \leq C$, we obtain the first-order condition

$$q_w^*(s_w, s_l) = \frac{1}{2}\left(\theta - q_l(s_l) - c\mathbb{1}\{q_w^*(s_w, s_l) \geq C\}\right),$$

where $\mathbb{1}\{\cdot\}$ is the indicator function. Similarly, we derive the first-order condition for the loser:

$$q_l^*(s_l) = \frac{1}{2}\left(\mathrm{E}_{s_w}[\theta | s_w > s_l] - \mathrm{E}_{s_w}[q_w(s_l) | s_w > s_l] - c\right).$$

Observing that the differentiation $\theta - q_l^* - 2q_w - c\mathbb{1}\{q_w \geq C\})$ is decreasing in q_w, the marginal change of the winner payoff will look like Figure A1.1. Now the optimal quantity $q_w^{II}(s_w, s_l)$ follows immediately from the comparison between marginal revenue and marginal cost. Note that the discontinuity occurs when capacity C is hit. In Figure A1.1, s_0 falls into the first case of equation (A1.1) since the marginal change becomes negative before hitting C. s_1 there equals $S_1^{II}(s_l)$ because the marginal change at C from the right just turns negative. Likewise, $s_2 = S_2^{II}(s_l)$ because s_2 is the largest point whose marginal change is positive for all $q_w \leq C$. By continuity of $q_w^{II}(s_w, s_l)$, the values of $S_1^{II}(s_l)$ and $S_2^{II}(s_l)$ can be obtained by equating the capacity C and $q_w^{II}(s_w, s_l)$ at the boundary points. ☐

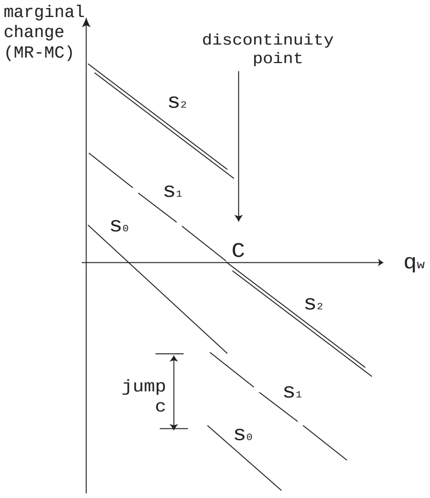


Figure A1.1: Marginal change of the winner's payoff vs quantity.

Note that these two thresholds are greater than s_l, and when $S_2^{II}(s_l)$ does not hit the boundary 1, $S_2^{II}(s_l) = S_1^{II}(s_l) + c$, i.e. the second threshold is higher. Now we characterize the structural property of the points at which the capacity constraint is binding. Figure A1.2 illustrates the general shape of $S_1^{II}(s_l)$ and $S_2^{II}(s_l)$ as functions of s_l.

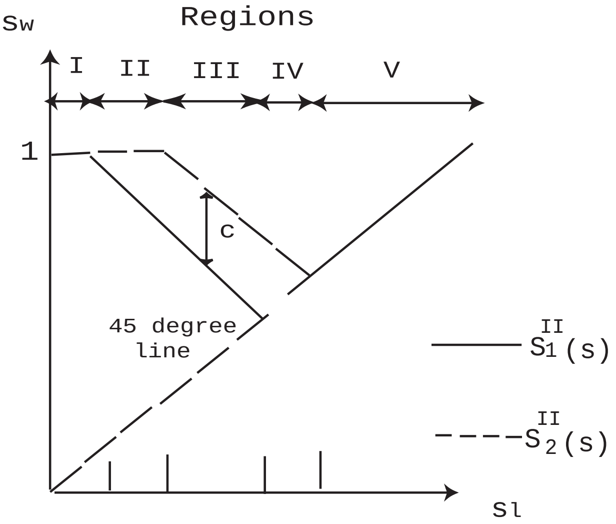


Figure A1.2: An example to show the shape of $S_1^{II}(s)$ and $S_2^{II}(s)$ under the second-price auction procurement case

Proposition A2. *The threshold function $S_1^{II}(s_l)$ can be divided into three regions by two values s_1^* and s_1^{**}, where $s_1^* \leq s_1^{**}$.*

If $0 \leq s_l \leq s_1^*$, $S_1^{II}(s_l) = 1$, i.e., the capacity constraint is not binding for all s_w.

If $s_1^* \leq s_l \leq s_1^{**}$, $S_1^{II}(s_l)$ decreases from 1 until it hits the 45-degree line, and

$$S_1^{II}(s_1) - S_1^{II}(s_2) < s_2 - s_1, \ \forall s_1 < s_2.$$

Finally, when $s_1^{**} \leq s_l \leq 1$, $S_1^{II}(s_l) = s_l$, i.e. $q_w^{II}(s_w, s_l) \geq C$ for all s_w.

Likewise, there exist two corresponding thresholds s_2^* and s_2^{**} for $S_2^{II}(s_l)$ such that

$$\begin{gathered} s_2^* \geq s_1^*, \ s_2^{**} \geq s_1^{**}, \ s_2^{**} \geq s_2^*, \\ S_2^{II}(s_1) - S_2^{II}(s_2) < s_2 - s_1, \ \forall s_1 < s_2. \end{gathered} \tag{A1.3}$$

Note that according to the values of some parameters, a region could be indistinguishable in the graph.

Proof. The following lemma is needed to show this proposition:

Lemma A1. *In the equilibrium quantities described in equation (A1.1),*

$$0 < q_l^{II}(s_2) - q_l^{II}(s_1) < s_2 - s_1, \ \textit{for signals } s_1 < s_2.$$

Proof of Lemma A1: The proof is by contradiction. Suppose that there exists s_l^1 such that $(q_l^{II})'(s_l^1) = \rho_l^1 > 1$. We can rewrite equation (A1.1) as

$$q_l^{II}(s_l) = \frac{1}{2}[\theta_0 + \frac{3}{2}s_l - \mathrm{E}_{s_w}[q_w^{II}(s_w, s_l)|s_w > s_l] + \frac{1}{2} - c].$$

The coefficient s_w term in $q_w^{II}(s_w, s_l)$ is lower bounded by 0 according to equation (A1.1), and hence $(q_l^{II})'(s_l^1) = \rho_l^1 > 1$ implies that there exists a signal s_w^1 such that $\frac{\partial}{\partial s_l}(q_w^{II})'(s_w, s_l)|_{s_l=s_l^1} < \frac{3}{2} - 2\rho_l^1 < 0$. Since q_w^{II} will not change if the capacity is binding, this can happen only in either the first or the third case. But then this implies that $\frac{1}{2}(1-\rho_l^1) \leq \frac{3}{2} - 2\rho_l^1$, which leads to $\rho_l^1 \leq \frac{2}{3}$, a contradiction.

Similarly, suppose that there exists s_l^2 such that $(q_l^{II})'(s_l^2) = \rho_l^2 < 0$. Since from equation (A1.1) the coefficient s_w term in q_w^{II} is upper bounded by $\frac{1}{2}$, the term associated to s_w in $\mathrm{E}_{s_w}[q_w^{II}(s_w, s_l)|s_w > s_l]$ contributes at most $\frac{1}{2}\frac{1+s_l}{2}$. We can show that for some s_w^2 whose $q_w^{II}(s_w^2, s_l^2)$ implies $\frac{1}{2}(1-\rho_l^2) \geq 1 - 2\rho_l^2 \ \Rightarrow \rho_l^2 \geq \frac{1}{3}$. Hence we conclude that $0 \leq q_l^{II}(s_2) - q_l^{II}(s_1) \leq s_2 - s_1, \ \forall \ s_1 \leq s_2$. □

Now we prove Proposition A2. We will focus on $S_1^{II}(s)$; the proof for $S_2^{II}(s)$ goes along the same argument. The first part of equation (A1.3) follows from the fact that $S_2^{II}(s) \geq S_1^{II}(s)$, $\forall s$. In the sequel, s_1 and s_2 are two distinct signals and $s_1 < s_2$.

Case a): $S_1^{II}(s_2) = 1$

We would like to prove that $S_1^{II}(s_1) = 1$ as well, i.e., the capacity constraint is never binding when $s_l = s_1$.

Note that $S_1^{II}(s_2) = 1$ means that

$$\frac{1}{3}\theta_0 + \frac{1}{2} + \frac{1}{4}s_2 + \frac{1}{3}c - \frac{1}{12} \leq C,$$

and therefore

$$\frac{1}{3}\theta_0 + \frac{1}{2} + \frac{1}{4}s_1 + \frac{1}{3}c - \frac{1}{12} < \frac{1}{3}\theta_0 + \frac{1}{2} + \frac{1}{4}s_2 + \frac{1}{3}c - \frac{1}{12} \leq C,$$

which implies $S_1^{II}(s_1) = 1$.

Case b): $s_1 < S_1^{II}(s_1) < 1$ and $s_2 < S_1^{II}(s_2) < 1$

Suppose $s_1 < S_1^{II}(s_1) < 1$ and $s_2 < S_1^{II}(s_2) < 1$. Our goal here is to prove that $S_1^{II}(s_2) < S_1^{II}(s_1)$. Since in both cases the capacity constraint is not binding for some s_w and binding for others,

$$\begin{aligned} S_1^{II}(s_2) &= 2C - \theta_0 - s_2 + q_l^{II}(s_2) \\ &= S_1^{II}(s_1) - (s_2 - s_1) + (q_l^{II}(s_2) - q_l^{II}(s_1)), \end{aligned}$$

and therefore by Lemma A1,

$$0 < S_1^{II}(s_1) - S_1^{II}(s_2) < s_2 - s_1.$$

Case c): $S_1^{II}(s_1) = s_1$.
In this case, we would like to prove that $S_1^{II}(s_2) = s_2$ as well. The proof is by contradiction. First we claim that $S_1^{II}(s_2) \neq 1$. If this is not the case, the capacity constraint is never binding when $s_l = s_2$. By the argument in Case a), $S_1^{II}(s_1) = 1$ as well since $s_1 < s_2$, which contradicts the fact $S_1^{II}(s_1) = s_1$.
Therefore, the only possibility is that $s_2 < S_1^{II}(s_2) < 1$. In this case, $S_1^{II}(s_2) = 2C - \theta_0 - s_2 + q_l^{II}(s_2)$, and $S_1^{II}(s_1) = s_1$ implies $\frac{1}{2}\left(\theta_0 + s_1 + s_1 - q_l^{II}(s_1)\right) \geq C$. Then we can establish the following inequality:

$$S_1^{II}(s_2) \leq 2s_1 + \theta_0 - q_l^{II}(s_1) - \theta_0 - s_2 + q_l^{II}(s_2)$$

Thus,

$$\begin{aligned} s_2 < S_1^{II}(s_2) &\leq 2s_1 + \theta_0 - q_l^{II}(s_1) - \theta_0 - s_2 + q_l^{II}(s_2), \\ &\Rightarrow 2(s_2 - s_1) < q_l^{II}(s_2) - q_l^{II}(s_1), \\ &\Rightarrow q_l^{II}(s_2) - q_l^{II}(s_1) > 2(s_2 - s_1), \end{aligned}$$

which contradicts Lemma A1. Hence we conclude that $S_1^{II}(s_2) = s_2$. □
In Figure A1.2, as s_l becomes larger, the threshold function $S_1^{II}(s_l)$ first stays at the upper bound, and then decreases until it hits the 45-degree line. After that, $S_1^{II}(s_l)$ coincides with s_l. $S_2^{II}(s_l)$ has a similar graph, except that the thresholds s_2^* and s_2^{**} occur at higher values. Note also that within region III, $S_2^{II}(s_l)$ could also stay at 1, depending on the ordering of s_2^* and s_1^{**}.

A1.2 The first-price auction procurement case

After differentiating the corresponding objective functions and applying the same argument as in Proposition A1, we obtain the resellers' best responses. The proof involves routine algebra and hence is omitted.

Proposition A3. *Suppose the first-price auction is used. Define thresholds S_1^I and S_2^I as*

$$S_1^I = \min\left\{1, \left(2C - \frac{2}{3}c - \frac{2}{3}\theta_0\right)^+\right\},$$
$$S_2^I = \min\left\{1, \left(2C + \frac{2}{3}c - \frac{2}{3}\theta_0\right)^+\right\}.$$

In the consumer market, there exists a unique Nash equilibrium that satisfies the following: If $s_w < S_1^I$*,*

$$\begin{aligned} q_w^I(s_w) &= \frac{1}{3}\theta_0 + \frac{1}{2}s_w + \frac{1}{3}c, \\ q_l^I(s_l, s_w) &= \frac{1}{3}\theta_0 + \frac{1}{4}s_w + \frac{1}{2}s_l - \frac{2}{3}c, \forall s_l \leq s_w. \end{aligned}$$

If $S_1^I \leq s_w \leq S_2^I$*, then*

$$\begin{aligned} q_w^I(s_w) &= C, \\ q_l^I(s_l, s_w) &= \frac{1}{2}(\theta_0 + s_w + s_l - C - c), \forall s_l \leq s_w. \end{aligned}$$

Finally, if $s_w > S_2^I$*,*

$$\begin{aligned} q_w^I(s_w) &= \frac{1}{3}\theta_0 + \frac{1}{2}s_w - \frac{1}{3}c, \\ q_l^I(s_l, s_w) &= \frac{1}{3}\theta_0 + \frac{1}{4}s_w + \frac{1}{2}s_l - \frac{1}{3}c, \forall s_l \leq s_w. \end{aligned}$$

Note that S_1^I and S_2^I are regulated by 0 and 1 because the signals have finite support $[0, 1]$. In the first case, the winner's optimum is feasible before hitting C, and the loser chooses the corresponding best response. In the second case, the global optimum of the winner without considering marginal cost c exceeds her capacity C, and the marginal revenue is less than c if we increase q_w above C, therefore the equilibrium turns out to be a corner solution. In the third case, the optimal order quantity exceeds C.

Observe that in the first case, the loser's quantity $q_l^I(s_l, s_w)$ can be expressed as

$$q_l^I(s_l, s_w) = \frac{1}{3}(\theta_0 + s_w + s_l) + \frac{1}{6}\left\{(\theta_0 + s_w + s_l) - \mathrm{E}_{s_l}[\theta | s_w, s_l < s_w]\right\} - \frac{2}{3}c,$$

where $\frac{1}{6}\left\{(\theta_0 + s_w + s_l) - \mathrm{E}_{s_l}[\theta | s_w, s_l < s_w]\right\}$ is the adjusted term resulting from the winner's bias on θ expectation. In other words, although the loser knows both signals and henceforth has the best prediction of θ, her best response still contains the winner's bias $\mathrm{E}_{s_l}[\theta | s_w, s_l < s_w]$. Likewise, we observe the same phenomenon in the last case.

A2. Equilibrium analysis of the auction game

A2.1 Second-price auction: Proof of Theorem 1

Preliminaries

We start discussing several technical lemmas that lead to Theorem 1 and provide their economic intuition.

If the losing bid $\beta^{II}(z)$ is higher, the winner should expect the demand to be higher, and therefore the quantity she puts in the consumer market $q_w^{II}(y,z)$ should also be larger, and the magnitude by which the winning quantity increases should be reasonably bounded. Hence, we have

Lemma A2.

$$0 \le q_w^{II}(y,z_2) - q_w^{II}(y,z_1) < \frac{1}{2}(z_2 - z_1), \ \forall z_1 < z_2. \tag{A2.1}$$

Proof: We show the monotonicity by dividing the shape of $S_1^{II}(z)$ into cases. We write $z_i \in I$ if z_i belongs to region I; $z_i \in II$ and $z_i \in III$ are defined analogously.

Case 1: $y \le S_1^{II}(z_1), y \le S_1^{II}(z_2)$

In this case, the capacity constraint is not binding, and therefore

$$q_w^{II}(y,z) = \frac{1}{3}\theta_0 + \frac{1}{2}y + \frac{1}{4}z + \frac{1}{3}c - \frac{1}{12},$$

which satisfies equation (A2.1).

Case 2: $(z_1, z_2) \in (II, II)$

Since when $z < S_1^{II}(z) < 1$, $S_1^{II}(z)$ is decreasing in z, the critical point where $q_w^{II}(y,z)$ hits the capacity C comes earlier. It suffices to show that while $q_w^{II}(y,z)$ is not hitting the capacity, it is increasing in z.

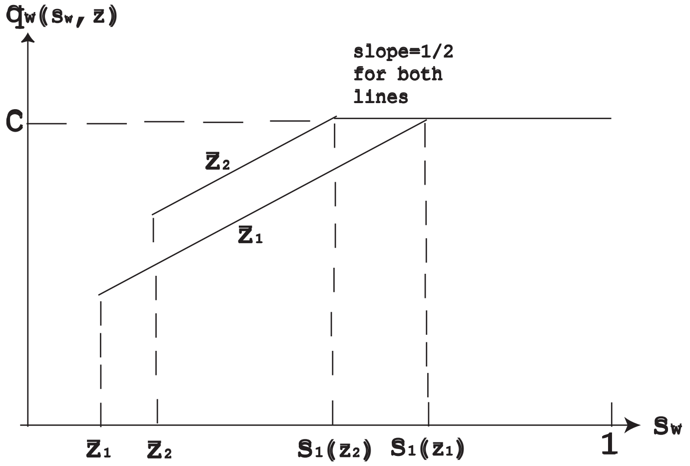
Following Proposition A1, we first observe that given z, $\frac{\partial}{\partial y} q_w^{II}(y,z) = \frac{1}{2}$, which means $q_w^{II}(y,z)$ is increasing in y with a fixed rate. Therefore, for all y such that $z_2 \le y \le S_1^{II}(z_2)$,

$$\begin{aligned} q_w^{II}(y,z_2) &= q_w^{II}(S_1^{II}(z_2), z_2) - \frac{1}{2}[S_1^{II}(z_2) - y] \\ &= C - \frac{1}{2}[S_1^{II}(z_2) - y] \\ &\ge C - \frac{1}{2}[S_1^{II}(z_1) - y], \quad \text{since } S_1^{II}(z_1) > S_1^{II}(z_2) \\ &= q_w^{II}(S_1^{II}(z_1), z_1) - \frac{1}{2}[S_1^{II}(z_1) - y], \quad \text{by definition of } S_1^{II}(z_1) \\ &= q_w^{II}(y, z_1). \end{aligned}$$

Moreover,

$$\begin{aligned} q_w^{II}(y,z_2) - q_w^{II}(y,z_1) &= [C - \frac{1}{2}(S_1^{II}(z_2) - y)] - [C - \frac{1}{2}(S_1^{II}(z_1) - y)] \\ &= \frac{1}{2}(S_1^{II}(z_1) - S_1^{II}(z_2)) \le \frac{1}{2}(z_2 - z_1), \end{aligned}$$

Figure A2.1: An example to show the relative positions of $z_1, z_2, S_1^{II}(z_2)$, and $S_1^{II}(z_1)$.



where the last inequality comes from Lemma A1 and equation (A1.2).

Remark: Figure A2.1 shows the relationship between $z_1, z_2, S_1^{II}(z_2)$, and $S_1^{II}(z_1)$. Referring to Figure A1.2, we fix the loser's signal s_l inside region II. Now we increase s_w along the vertical line that passes $(s_l, 0)$, and draw the winner's quantity q_w as a function of s_w in Figure A2.1.

Case 3: $S_1^{II}(z_1) \leq y \leq S_2^{II}(z_1), S_1^{II}(z_2) \leq y \leq S_2^{II}(z_2)$

In this case, the capacity constraint of the winner is binding for all z, i.e., $q_w^{II}(y, z) = C, \forall z$. Thus, the result holds.

Other Cases: The cases $y \geq S_2^{II}(z_1), S_1^{II}(z_2) \leq y \leq S_2^{II}(z_2)$ and $y \geq S_2^{II}(z_1), y \geq S_2^{II}(z_2)$ are very similar to Cases 2 and 3 respectively, and the results follow from analogous derivations. In other cases where z_1 and z_2 belong to different regions in Figure A1.2, we can use the triangle inequalities by inserting the boundaries of two regions s_l^* and s_l^{**} respectively. $\square$

Lemma A2 confirms the correctness of our intuition. When the losing bid increases, the winner's expectation of the demand in the consumer market also increases, and therefore she puts more equilibrium quantity. This can be interpreted as the winner's overestimation of the demand if the loser submits a bid higher than $\beta^{II}(s)$. However, the increment of $q_w^{II}(y, z)$ is bounded above.

Furthermore, if the loser's signal increases, her optimal quantity should also increase at a reasonable rate:

Lemma A3.

$$\frac{1}{2}(z_2 - z_1) \le q_l^{II}(z_2) - q_l^{II}(z_1) < \frac{3}{4}(z_2 - z_1), \ \forall z_1 < z_2. \tag{A2.2}$$

Proof: Recall from equation (A1.1) that

$$q_l^{II}(z) = \frac{1}{2}\theta_0 + \frac{3}{4}z - \frac{1}{2}c - \frac{1}{2}\mathrm{E}_y[q_w^{II}(y,z)|y \ge z].$$

If we increase z by Δz, the term $\frac{3}{4}z$ will increase by $\frac{3}{4}\Delta z$. The term $\mathrm{E}_y[q_w^{II}(y,z)|y \ge z]$ is bounded by 0 and $\frac{1}{2}$ according to Lemma A2. Therefore, equation (A2.2) is valid. □

Note that this lemma provides tighter upper and lower bounds for the first-order difference of $q_l^{II}(z)$ than Lemma A1. The next lemma says that if a type-s reseller loses in the auction, her expected partial payoff will be decreasing in her reported type.

Lemma A4. $\int_s^1 \frac{\partial}{\partial z}\pi_l^{II}(z,s,y)|_{z=s}dy \le 0$.

Proof: Define $g(s) = \int_s^1 \frac{\partial}{\partial z}\pi_l^{II}(z,s,y)|_{z=s}\mathrm{d}y$.

$$\begin{aligned} g(s) &= \int_s^1 \frac{\partial}{\partial z}\left(q_l^{II}(z,s)[\theta_0 + y + s - q_w^{II}(y,z) - q_l^{II}(z,s) - c]\right)|_{z=s}\mathrm{d}y \\ &= \int_s^1 \left(\frac{\partial q_l^{II}(z,s)}{\partial z}[\theta_0 + y + s - q_w^{II}(y,z) - q_l^{II}(z,s) - c]\right. \\ &\qquad \left. + q_l^{II}(z,s)[-\frac{\partial q_w^{II}(y,z)}{\partial z} - \frac{\partial q_l^{II}(z,s)}{\partial z}]\right)|_{z=s}\mathrm{d}y. \end{aligned} \tag{A2.3}$$

Recalling that $q_l^{II}(z,s) = q_l^{II}(z) + \frac{1}{2}(s-z)$ and Proposition A1, we have

$$\begin{aligned} \frac{\partial q_l^{II}(z,s)}{\partial z}|_{z=s} &= (q_l^{II})'(s) - \frac{1}{2}, \\ \frac{\partial q_w^{II}(y,z)}{\partial z}|_{z=s} &= \begin{cases} 0, & S_1^{II}(z) \le y \le S_2^{II}(z), \\ \frac{1}{2}[1 - (q_l^{II})'(s)], & \text{otherwise.} \end{cases} \end{aligned}$$

Thus, equation (A2.3) can be rewritten as

$$\begin{aligned} g(s) =& \left((q_l^{II})'(s) - \frac{1}{2}\right)\left(\mathrm{E}_y[\theta|s, y \ge s] - \mathrm{E}_y[q_w^{II}(y,s)|s, y \ge s] - q_l^{II}(s) - c\right) \\ &+ q_l^{II}(s)\left(-\frac{1}{2}(q_l^{II})'(s)\right)\left(1 - (S_2^{II}(s) - S_1^{II}(s))\right) - q_l^{II}(s)\left((q_l^{II})'(s) - \frac{1}{2}\right)(S_2^{II}(s) - S_1^{II}(s)) \\ =& \frac{1}{2}\left(1 - (S_2^{II}(s) - S_1^{II}(s))\right)q_l^{II}(s)\left((q_l^{II})'(s) - 1\right), \end{aligned} \tag{A2.4}$$

where the last equality follows from the fact $q_l^{II}(s) = \frac{1}{2}\left(\mathrm{E}_y[\theta|s, y \ge s] - \mathrm{E}_y[q_w^{II}(y,s)|s, y \ge s] - c\right)$. Since $1-(S_2^{II}(s)-S_1^{II}(s))$ and $q_l^{II}(s)$ are positive, and $\left((q_l^{II})'(s) - 1\right)$ is negative by Lemma A3, we conclude that $g(s) < 0$, which completes the proof. □

The average marginal change of a type-s loser's partial payoff is obtained by integrating over her opponent's signal y, with $y \geq s$. For notational convenience, we assume that $\pi_l^{II}(z, s, y)$ is differentiable with respect to z at boundary points. If it is not differentiable, then we should look for the subgradients rather than the gradient from the first-order condition. Now, we complete the proof of Theorem 1.

Main body of the proof

We will first derive a necessary condition that an equilibrium bidding function must satisfy, and then provide the verification of the proposed bidding function.

1. Necessary Condition

The first-order condition with respect to z is as follows:

$$\frac{\partial \Pi^{II}(z|s)}{\partial z} = [\pi_w^{II}(s, z) - \beta^{II}(z) - \pi_l^{II}(z, s, z)] + \int_z^1 \frac{\partial}{\partial v} \pi_l^{II}(v, s, y)|_{v=z} \mathrm{d}y. \tag{A2.5}$$

The truth-telling equilibrium requires that the bidder's optimal strategy is to reveal her own type. Thus, we conjecture the equilibrium bidding function $\beta^{II}(s)$ as follows:

$$\begin{aligned} \beta^{II}(s) &= \pi_w^{II}(s, s) - \pi_l^{II}(s, s, s) + \int_s^1 \frac{\partial}{\partial z} \pi_l^{II}(z, s, y)|_{z=s} \mathrm{d}y \\ &\leq \pi_w^{II}(s, s) - \pi_l^{II}(s, s, s), \end{aligned} \tag{A2.6}$$

where the last inequality is given by Lemma A4. Note that by Proposition A1, the equilibrium quantities $q_w^{II}(z, s)$ and $q_l^{II}(y)$ are all uniquely determined for any values z, s, y, and hence the terms $\pi_w^{II}(s, s)$, $\pi_l^{II}(s, s, s)$, and $\frac{\partial}{\partial z} \pi_l^{II}(z, s, y)|_{z=s}$ are all unique since these are generated from $q_w^{II}(z, s)$ and $q_l^{II}(y)$. Thus, within the class of symmetric equilibria, if there exists an equilibrium bidding function that is strictly increasing in the signal, then it must be uniquely determined by equation (A2.6).

2. Verification: Monotonicity of $\beta^{II}(s)$

For the second-price auction, our goal is to show that $\beta^{II}(s)$ is strictly increasing in s, i.e., $\lim_{z \to s} \frac{\beta^{II}(z) - \beta^{II}(s)}{z - s} > 0$, $\forall s \in [0, 1]$. Except region IV where the capacity constraint is binding for $q_w^{II}(y, y)$, we obtain

$$\pi_w^{II}(z, z) - \pi^{II}(z, z, z) - [\pi_w^{II}(s, s) - \pi^{II}(s, s, s)] = (2 - 2\rho_l) q_w^{II}(s, s) - (1 - \frac{1}{2}\rho_l) q_l^{II}(s, s) + \rho_l c \mathbb{1}\{s \in V\},$$

where we have ignored the second-order terms and ρ_l is such that $q_l^{II}(z, z) = q_l^{II}(s, s) + \rho_l(z - s)$. We will divide the analysis according to Figure A1.2. Note that $q_w^{II}(s, s)$ takes values along the 45-degree line.

Case 1: $s \in I$

In this case, $\rho_l = \frac{1}{2}$, $S_1^{II}(s) = S_2^{II}(s) = 1$. ¿From equation (A2.4),

$$g(s) = \int_s^1 \frac{\partial}{\partial z}\pi_l^{II}(z,s,y)|_{z=s}dy = \frac{1}{2}\left(1-(S_2^{II}(s)-S_1^{II}(s))\right)q_l^{II}(s)\left((q_l^{II})'(s)-1\right),$$

and hence $\beta^{II}(z) - \beta^{II}(s) = (z-s)(q_w^{II}(s,s) - \frac{3}{4}q_l^{II}(s,s) + \frac{1}{2}c - \frac{1}{8}) + o(z-s)$. In other words, $(\beta^{II})'(z) = q_w^{II}(s,s) - \frac{3}{4}q_l^{II}(s,s) + \frac{1}{2}c - \frac{1}{8} > 0$ by Assumption 2.

Case 2: $s \in II$

When $s \in II$, $S_2^{II}(s) = 1$ and $S_1^{II}(s) = 2C - \theta_0 - s + q_l^{II}(s)$. Therefore,

$$\beta^{II}(z) - \beta^{II}(s) = (2-2\rho_l)q_w^{II}(s,s) - \frac{1}{2}(\rho_l^2 - 3\rho_l + 3)q_l^{II}(s,s) - \frac{1}{2}\rho_l(1-\rho_l)S_1^{II}(s) + o(z-s).$$

Note that from Lemma A3, $\frac{1}{2} \le \rho_l \le \frac{3}{4}$, and $S_1^{II}(s) \le 1$. A sufficient condition for monotonicity under these parameters is that $q_w^{II}(s,s) > \frac{13}{16}q_l^{II}(s,s) + \frac{3}{16}$.

Case 3: $s \in III$

Here we have $S_2^{II}(s) - S_1^{II}(s) = 1 - c$, and therefore

$$\beta^{II}(z) - \beta^{II}(s) = (2-2\rho_l)q_w^{II}(s,s) - (1-\frac{1}{2}\rho_l)q_l^{II}(s,s) - \frac{1}{2}\rho_l(1-\rho_l) + c(1-\frac{1}{2}(1-\rho_l)) + o(z-s).$$

Note that when $\frac{1}{2} \le \rho_l \le \frac{3}{4}$, $c(1-\frac{1}{2}(1-\rho_l))$ is always positive. A sufficient condition for $\beta^{II}(s)$ being increasing is that $q_w^{II}(s,s) > \frac{5}{4}q_l^{II}(s,s) + \frac{3}{16}$.

Case 4: $s \in IV$

In this case, the capacity constraint is binding and $g(s) = -\frac{1}{2}(1-\rho_l)(1-s)q_l^{II}(s,s)$.

$$\beta^{II}(z) - \beta^{II}(s) = (2-2\rho_l)q_w^{II}(s,s) - 2q_l^{II}(s,s) - \frac{1}{2}\rho_l(1-\rho_l) + o(z-s).$$

$\frac{1}{2}\rho_l(1-\rho_l)$ achieves its maximum at $\rho_l = 2-\sqrt{2}$, and hence if $q_w^{II}(s,s) \ge \frac{8}{5}q_l^{II}(s,s) + \frac{(\sqrt{2}-1)(2-\sqrt{2})}{2\sqrt{2}}$, the monotonicity holds. The constant term is roughly 0.086.

Case 5: $s \in V$

In region V, $\rho = \frac{1}{2}$, $S_1^{II} = S_2^{II} = s$, and hence $g(s) = -\frac{1}{4}q_l^{II}(s,s)$. It can be verified that $\beta^{II}(z) - \beta^{II}(s) = (z-s)(q_w^{II}(s,s) - \frac{3}{4}q_l^{II}(s,s) - \frac{1}{8}) + o(z-s)$, and therefore monotonicity holds.

We conclude that the bidding function $\beta^{II}(s)$ is strictly increasing since Assumption 2 is sufficient for all cases.

3. Verification: Incentive compatibility

We will follow Milgrom and Weber (1982) to verify that the proposed bidding function is indeed an equilibrium. The idea is to show that a type-s bidder's payoff is unimodal in her reported type, and it achieves the maximum at the truth-telling value s.

Suppose the other player adopts that bidding function. Differentiating the expected payoff with respect to z, we obtain

$$\begin{aligned}\frac{\partial \Pi^{II}(z|s)}{\partial z} &= \left(\pi_w^{II}(s,z) - \beta^{II}(z) - \pi_l^{II}(z,s,z)\right) + \int_z^1 \frac{\partial}{\partial v}\pi_l^{II}(v,s,y)|_{v=z}\mathrm{d}y \\ &= \left(\pi_w^{II}(s,z) - \pi_l^{II}(z,s,z)\right) - \left(\pi_w^{II}(z,z) - \pi_l^{II}(z,z,z)\right).\end{aligned} \tag{A2.7}$$

Our goal is to show that if $z < s$, then

$$\frac{\partial \Pi^{II}(z|s)}{\partial z} < 0,$$

and if $z > s$, then

$$\frac{\partial \Pi^{II}(z|s)}{\partial z} > 0.$$

We divide the proof into four cases in the sequel.

Case 1: $z, s \leq S_1^{II}(z)$.

Recall that

$$\begin{aligned}\pi_w^{II}(s,z) - \pi_l^{II}(z,s,z) \;&=\; q_w^{II}(s,z)\left(\theta_0 + s + z - q_w^{II}(s,z) - q_l^{II}(z)\right) \\ &\quad -q_l^{II}(z,s)\left(\theta_0 + z + s - q_w^{II}(z,z) - q_l^{II}(z,s) - c\right),\end{aligned}$$

and $q_l^{II}(z,s) = q_l^{II}(z,z) + \frac{1}{2}(s-z)$. In this case, the capacity constraint is never binding, and thus $q_w^{II}(s,z) = q_w^{II}(z,z) + \frac{1}{2}(s-z)$. After simple manipulations, we can rewrite equation (A2.7) as follows

$$\begin{aligned}\frac{\partial \Pi^{II}(z|s)}{\partial z} \;&=\; \frac{1}{2}(s-z)[q_w^{II}(z,z) - q_l^{II}(z,z) \\ &\quad +(\theta_0 + s + z - q_w^{II}(s,z) - q_l^{II}(z)) - (\theta_0 + s + z - q_w^{II}(z,z) - q_l^{II}(z,s) - c)] \\ &=\; \frac{1}{2}(s-z)[q_w^{II}(z,z) - q_l^{II}(z,z) + c].\end{aligned}$$

The multiplicative term $q_w^{II}(z,z) - q_l^{II}(z,z) + c$ is always positive by Assumption 2. Hence $\partial \Pi^{II}(z|s)/\partial z$ is positive if $s - z > 0$ and negative if $s - z < 0$.

Case 2: $S_1^{II}(z) \leq z, s \leq S_2^{II}(z)$.

In this case, $q_w^{II}(s,z) = q_w^{II}(z,z) = C$.

$$\begin{aligned}\frac{\partial \Pi^{II}(z|s)}{\partial z} &= \frac{1}{2}(s-z)\left(C - \frac{1}{2}q_w^{II}(z,z) - \frac{1}{2}[\theta_0 + z + s - C - q_l^{II}(z,s) - c]\right) \\ &= \frac{1}{2}(s-z)\left(C - \frac{1}{2}[\theta_0 + z + \frac{s+z}{2} - c - C]\right).\end{aligned}$$

Note that $C = q_w^{II}(z,z)$ here and $\frac{1}{2}[\theta_0 + z + \frac{s+z}{2} - c - C] \leq \frac{1}{2}[\theta_0 + z + \frac{s+z}{2} - c - C] = q_l^{II}(z)$ since $s \leq 1$. Thus, by Assumption 2, the multiplicative term is positive and the payoff is unimodal.

Case 3: $z, s \geq S_2^{II}(z)$

Mimicking the treatment of Case 1, we get

$$\frac{\partial \Pi^{II}(z|s)}{\partial z} = \frac{1}{2}(s-z)\left(q_w^{II}(z,z) - q_l^{II}(z,z)\right),$$

where the term in brackets is positive by Assumption 2.

Case 4: General case.

Now we consider the case where the capacity constraint is binding only for one of $q_w^{II}(s,z)$ and $q_w^{II}(z,z)$. This includes both $z \leq S_1^{II}(z) \leq s \leq S_2^{II}(z)$ and $S_1^{II}(z) \leq z \leq S_2^{II}(z) \leq s$. In this case, $q_w^{II}(s,z) - q_w^{II}(z,z) = \frac{1}{2}\rho(s-z)$, where $0 \leq \rho \leq 1$. We can rewrite $\pi_w^{II}(s,z) - \pi_l^{II}(z,s,z)$ as follows:

$$\begin{aligned}&\pi_w^{II}(s,z) - \pi_l^{II}(z,s,z) \\ &= \left(q_w^{II}(z,z) + \frac{1}{2}(S_1^{II}(z) - z)\right)\left(q_w^{II}(z,z) + (s-z) - \frac{1}{2}(S_1^{II}(z) - z)\right) - \left(q_l^{II}(z,z) + \frac{1}{2}(s-z)\right)^2 \\ &= \pi_w^{II}(z,z) - \pi_l^{II}(z,z,z) + (s-z)\left(q_w^{II}(z,z) - q_l^{II}(z) + \frac{1}{2}\rho\left(1 - \frac{1}{2}\rho\right)(s-z) - \frac{1}{4}(s-z)\right),\end{aligned}$$

and hence

$$\frac{\partial \Pi^{II}(z|s)}{\partial z} = (s-z)\left(q_w^{II}(z,z) - q_l^{II}(z) + \frac{1}{2}\rho(1 - \frac{1}{2}\rho)(s-z) - \frac{1}{4}(s-z)\right).$$

Note that Cases 1 and 2 can be regarded as two special cases of this general case, with $\rho = 1$ and $\rho = 0$ respectively, which bounds $\frac{1}{2}\rho\left(1 - \frac{1}{2}\rho\right)(s-z)$ from below and above. Thus the multiplicative term is positive according to Cases 1 and 2, and the payoff is unimodal in this case as well. $\square$

Remark: It can be verified that the assumption needed to guarantee the monotonicity of bidding function $\beta^{II}(s)$ is stronger than that for the unimodality of resellers' expected payoffs (incentive compatibility). This phenomenon does not occur in the conventional auctions (e.g., Milgrom and Weber (1982)). Similar results apply to the first-price auction as well. See the proof of Theorem 2.

A2.2 First-price auction: Proof of Theorem 2

Preliminaries

We first present the equilibrium quantities. In all cases, the formulas for $q_l^I(y,z)$ are obtained from Proposition A3, and following them we calculate $q_w^I(z,s)$ as the corresponding best responses.

Proposition A4. *Suppose that the first-price auction is used. If a type-s bidder that behaves as a type-z wins the auction, then:*
If $z \le S_1^I$,

$$\begin{aligned} q_l^I(y,z) &= \frac{1}{3}\theta_0 + \frac{1}{4}z + \frac{1}{2}y - \frac{2}{3}c, \\ q_w^I(z,s) &= \begin{cases} \frac{1}{3}\theta_0 + \frac{1}{2}s + \frac{1}{3}c, & s \le 2\left(C - \frac{1}{3}\theta_0 - \frac{1}{3}c\right), \\ C, & 2\left(C - \frac{1}{3}\theta_0 - \frac{1}{3}c\right) \le s \le 2\left(C - \frac{1}{3}\theta_0 + \frac{1}{6}c\right), \\ \frac{1}{3}\theta_0 + \frac{1}{2}s - \frac{1}{6}c, & s \ge 2\left(C - \frac{1}{3}\theta_0 + \frac{1}{6}c\right). \end{cases} \end{aligned}$$

If $S_1^I \le z \le S_2^I$, *then*

$$\begin{aligned} q_l^I(y,z) &= \frac{1}{2}(\theta_0 + z + y - C - c), \\ q_w^I(z,s) &= \begin{cases} \frac{1}{4}\theta_0 + \frac{1}{2}s - \frac{1}{8}z + \frac{1}{4}C + \frac{1}{4}c, & s \le \frac{3}{2}C - \frac{1}{2}\theta_0 + \frac{1}{4}z - \frac{1}{2}c, \\ C, & \frac{3}{2}C - \frac{1}{2}\theta_0 + \frac{1}{4}z - \frac{1}{2}c \le s \le \frac{3}{2}C - \frac{1}{2}\theta_0 + \frac{1}{4}z + \frac{1}{2}c, \\ \frac{1}{4}\theta_0 + \frac{1}{2}s - \frac{1}{8}z + \frac{1}{4}C - \frac{1}{4}c, & s \ge \frac{3}{2}C - \frac{1}{2}\theta_0 + \frac{1}{4}z + \frac{1}{2}c. \end{cases} \end{aligned}$$

Finally, if $z \ge S_2^I$,

$$\begin{aligned} q_l^I(y,z) &= \frac{1}{3}\theta_0 + \frac{1}{4}z + \frac{1}{2}y - \frac{1}{3}c, \\ q_w^I(z,s) &= \begin{cases} \frac{1}{3}\theta_0 + \frac{1}{2}s + \frac{1}{6}c, & s \le 2C - \frac{2}{3}\theta_0 - \frac{1}{3}c, \\ C, & 2C - \frac{2}{3}\theta_0 - \frac{1}{3}c \le s \le 2C - \frac{2}{3}\theta_0 + \frac{2}{3}c, \\ \frac{1}{3}\theta_0 + \frac{1}{2}s - \frac{1}{3}c, & s \ge 2C - \frac{2}{3}\theta_0 + \frac{2}{3}c. \end{cases} \end{aligned}$$

The payoffs $\pi_w^I(z,s,y)$ *and* $\pi_l^I(s,y)$ *in both cases can be expressed as follows:*

$$\begin{aligned} \pi_w^I(z,s,y) &= q_w^I(z,s)\left(\theta_0 + s + y - q_w^I(z,s) - q_l^I(y,z)\right) - c(q_w^I(z,s) - C)^+, \\ \pi_l^I(s,y) &= q_l^I(s,y)\left(\theta_0 + y + s - q_w^I(y,y) - q_l^I(s,y) - c\right). \end{aligned} \tag{A2.8}$$

Main body of the proof

1. Necessary condition

The first-order condition of $\Pi^I(z|s)$ is

$$\pi_w^I(z,s,z) - \beta^I(z) - \pi_l^I(s,z) + \int_0^z \pi_{w,1}^I(z,s,y)\mathrm{d}y - \int_0^z (\beta^I)'(z)\mathrm{d}y = 0.$$

Rewriting the above equation, we find an expression of $(\beta^I)'(z)$ as follows:

$$(\beta^I)'(z) = \frac{1}{z}\left(\pi_w^I(z,s,z) - \pi_l^I(s,z) + \int_0^z \pi_{w,1}^I(z,s,y)\mathrm{d}y - \beta^I(z)\right),$$

The closed-form solution of $\beta^I(z)$ can be obtained from this differential equation by the same procedure in Krishna (2002, Proposition 6.3). Note that the proof of Proposition 6.3 in Krishna requires the signal affiliation to guarantee the unimodality for the last steps in his derivation. In our case signals are non-affiliated, but unimodality is guaranteed from Assumption 3.

Together with the incentive compatibility condition, the bidding function is

$$\beta^I(s) = \int_0^s \left(\pi_w^I(y,y,y) - \pi_l^I(y,y) + \int_0^y \pi_{w,1}^I(y,y,v)\mathrm{d}v\right) dL(y|s), \tag{A2.9}$$

where

$$L(y|s) = \exp\left(-\int_y^s \frac{1}{v}\mathrm{d}v\right) = \frac{y}{s}.$$

Plugging $L(y|s)$ in equation (A2.9), we obtain the bidding function $\beta^I(s)$. Since Proposition A4 guarantees that $q_w^I(z,s)$ and $q_l^I(y,z)$ are unique, all terms in equation (A2.9) are known, i.e., there is only one bidding function that satisfies the first-order condition. Thus, if there exists a symmetric, strictly increasing equilibrium, it must be uniquely determined by equation (A2.9).

Next, we will show that $\pi_w^I(z,s,y)$ is decreasing in z, for all pair (s,y). This leads to the conclusion $\beta^I(s) < \frac{1}{s}\int_0^s \left(\pi_w^I(y,y,y) - \pi_l^I(y,y)\right)\mathrm{d}y$. Observing equation (A2.8), the differentiation can be expressed as follows:

$$\begin{aligned}\frac{\partial}{\partial z}\pi_w^I(z,s,y) = {} & \frac{\partial}{\partial z}q_w^I(z,s)\left(\theta_0 + s + y - q_w^I(z,s) - q_l^I(y,z)\right) - c\frac{\partial}{\partial z}(q_w^I(z,s) - C)^+ \\ & + q_w^I(z,s)\left(-\frac{\partial}{\partial z}q_w^I(z,s) - \frac{\partial}{\partial z}q_l^I(y,z)\right)\end{aligned} \tag{A2.10}$$

Note that by Proposition A4, $-\frac{1}{8} \le \frac{\partial}{\partial z}q_w^I(z,s) \le 0$ and $\frac{1}{4} \le \frac{\partial}{\partial z}q_l^I(y,z) \le \frac{1}{2}$. Thus, the last term of equation (A2.10) is negative. Observe that the first two terms can be combined into $q_w^I(z,s)\frac{\partial}{\partial z}q_w^I(z,s)$ regardless of whether $q_w^I(z,s) > C$ or not. Hence it is negative as well because $\frac{\partial}{\partial z}q_w^I(z,s) \le 0$, and we conclude that $\pi_w^I(z,s,y)$ is decreasing in z.

2. Verification: Monotonicity of $\beta^I(s)$

Similar to Milgrom and Weber (1982), a sufficient condition for the monotonicity of bidding function in the first-price auction is that the integrand of $\beta^I(s)$ is increasing, i.e.,

$$\pi_w^I(y,y,y) - \pi_l^I(y,y) + \int_0^y \frac{\partial}{\partial u}\pi_w^I(u,y,v)|_{u=y}\mathrm{d}v$$

is increasing in y. Recall that $\frac{\partial}{\partial u}\pi_w^I(u,y,v)|_{u=y} = -q_w^I(y,y)\frac{\partial}{\partial u}q_l^I(v,u)|_{u=y}$ by equation (A2.10), the integral is $-\frac{1}{2}\int_0^y q_w^I(u,u)\mathrm{d}u$ if $S_1^I \leq y \leq S_2^I$, and is $-\frac{1}{4}\int_0^y q_w^I(u,u)\mathrm{d}u$ otherwise. Note also that

$$\begin{aligned}\pi_w^I(y,y,y) - \pi_l^I(y,y) &= q_w^I(y,y)[\theta_0 + y + y - q_w^I(y,y) - q_l^I(y,y) - c(q_w^I(y,y) - C)^+] \\ &\quad -q_l^I(y,y)[\theta_0 + y + y - q_w^I(y,y) - q_l^I(y,y) - c].\end{aligned}$$

Now we will divide our analysis into three cases, depending on the regions to which y belongs.

Case 1: $y \leq S_1^I$

We first consider the case $y \in S_1^I$. Let

$$\begin{aligned}\beta^I(z) - \beta^I(y) &= \pi_w^I(z,z,z) - \pi_l^I(z,z) + \textstyle\int_0^z \frac{\partial}{\partial u}\pi_w^I(u,z,v)|_{u=z}\mathrm{d}v] \\ &\quad -[\pi_w^I(y,y,y) - \pi_l^I(y,y) + \textstyle\int_0^y \frac{\partial}{\partial u}\pi_w^I(u,y,v)|_{u=y}\mathrm{d}v].\end{aligned}$$

Our goal is to show that $\lim_{z\to y}\frac{\beta^I(z)-\beta^I(y)}{z-y} > 0$, $\forall y$. Note that in this case $q_w^I(z,z) = q_w^I(y,y) + \frac{1}{2}(z-y)$ and $q_l^I(z,z) = q_l^I(y,y) + \frac{3}{4}(z-y)$. Therefore after some algebra, $\beta^I(z) - \beta^I(y)$ can be rewritten as $(z-y)[\frac{2}{4}q_w^I(y,y) - q_l^I(y,y) + \frac{1}{2}c] + o(z-y)$, where $\lim_{x\to 0} o(x)/x = 0$. Hence $\lim_{z\to y}\frac{\beta^I(z)-\beta^I(y)}{z-y} > 0$ by Assumption 3 in this case.

Case 2: $y > S_2^I$

Similar to Case 1 except that $\beta^I(z) - \beta^I(y) = (z-y)[\frac{2}{4}q_w^I(y,y) - q_l^I(y,y)] + o(z-y)$, and hence the result holds.

Case 3: $S_1^I \leq y \leq S_2^I$

In this case the capacity constraint is binding, i.e., $q_w^I(z,z) = q_w^I(y,y) = C$, and $q_l^I(z,z) = q_l^I(y,y) + (z-y)$. It can be verified that $\beta^I(z) - \beta^I(y) = (z-y)(\frac{1}{2}q_w^I(y,y) - 2q_l^I(y,y)) + o(z-y)$, which is strictly positive under Assumption 3. The proof of monotonicity is now complete.

3. Verification: Incentive compatibility

We will show that the expected payoff of a type-s bidder is unimodal in the reported type z with maximum achieved at $z = s$. Plugging the bidding function $\beta^I(z)$ into the expected payoff, we obtain

$$\begin{aligned}\frac{\partial}{\partial z}\Pi^I(z|s) &= \pi_w^I(z,s,z)-\beta^I(z)-\pi_l^I(s,z)\\ &\quad+\int_0^z \frac{\partial}{\partial v}\pi_w^I(v,s,y)|_{v=z}\mathrm{d}y-\int_0^z (\beta^I)'(z)\mathrm{d}y\\ &= \left(\pi_w^I(z,s,z)-\pi_l^I(s,z)\right)-\left(\pi_w^I(z,z,z)-\pi_l^I(z,z)\right).\end{aligned}$$

Recall that

$$\begin{aligned}\pi_w^I(z,s,z)-\pi_l^I(s,z) &= q_w^I(z,s)[\theta_0+s+z-q_w^I(z,s)-q_l^I(z,z)-c(q_w^I(z,s)-C)^+]\\ &\quad -q_l^I(s,z)[\theta_0+z+s-q_w^I(z,z)-q_l^I(s,z)-c], \qquad \text{(A2.11)}\end{aligned}$$

and

$$q_l^I(s,z)=q_l^I(z,z)+\frac{1}{2}(s-z).$$

In the following, we will divide the problem into cases to prove that the differentiation is positive when $z<s$ and negative if $z>s$.

Case 1: $q_w^I(z,s)<C$ and $q_w^I(z,z)<C$

In this case, the capacity constraint is not binding. Thus

$$q_w^I(z,s)=q_w^I(z,z)+\frac{1}{2}(s-z).$$

Plugging it into equation (A2.11), we obtain

$$\frac{\partial}{\partial z}\Pi^I(z|s)=\frac{1}{2}(s-z)\left(q_w^I(z,z)-q_l^I(z,z)+c\right),$$

where the multiplicative term $q_w^I(z,z)-q_l^I(z,z)+c$ is positive by Assumption 3.

Case 2: $q_w^I(z,s)=q_w^I(z,z)=C$

In this case, there is no difference between $q_w^I(z,s)$ and $q_w^I(z,z)$. After some algebra, the differentiation becomes

$$\frac{\partial}{\partial z}\Pi^I(z|s)=\frac{1}{2}(s-z)\left(C-\frac{1}{2}(q_l^I(z,z)+q_l^I(s,z))\right).$$

Now the multiplicative term is

$$2[q_w^I(z,z)-q_l^I(z,z)-\frac{1}{4}(s-z)]\geq 2\left(q_w^I(z,z)-q_l^I(z,z)-\frac{1}{4}(1-z)\right),$$

and hence the differentiation is positive if and only if $z<s$.

Case 3: General case.

We first rewrite the expression for $\pi_w^I(z,s,z) - \pi_l^I(s,z)$. After some algebra, we obtain that if $S_1^I \leq z \leq S_2^I$, $\pi_w^I(z,s,z) = q_w^I(z,s)\left(q_w^I(z,s) + \frac{1}{4}z\right)$; otherwise, $\pi_w^I(z,s,z) = q_w^I(z,s)\left(q_w^I(z,s) + \frac{3}{4}z\right)$. Thus

$$\frac{\partial}{\partial z}\Pi^I(z|s) = \begin{cases} (s-z)\big(q_w^I(z,z) - q_l^I(z,z) + (\frac{1}{2}\rho(1-\frac{1}{2}\rho) - \frac{1}{4})(s-z) + \frac{3}{8}\rho z\big) & \text{if } z \leq S_1^I, \\ (s-z)\big(q_w^I(z,z) - q_l^I(z,z) + (\frac{1}{2}\rho(1-\frac{1}{2}\rho) - \frac{1}{4})(s-z) + \frac{1}{8}\rho z\big) & \text{otherwise.} \end{cases}$$

Let us consider the term $(\frac{1}{2}\rho(1-\frac{1}{2}\rho)-\frac{1}{4})(s-z)+\frac{1}{8}\rho z$. When $s-z<0$, $(\frac{\rho}{2}(1-\frac{\rho}{2})-\frac{1}{4})(s-z)>0$ since $\frac{\rho}{2}(1-\frac{\rho}{2}) \leq \frac{1}{4}$ for $\rho \in [0,1]$. With the last term being positive, we obtain by Assumption 3 that $\frac{\partial}{\partial z}\Pi^I(z|s)$ is negative for $z > s$.

It remains to show that this holds for $z < s$ as well. When $s-z>0$, both $(\frac{1}{2}\rho(1-\frac{1}{2}\rho)-\frac{1}{4})(s-z)$ and $\frac{1}{8}\rho z$ are minimized at $\rho = 0$, which corresponds to Case 2. Thus, the multiplicative term is bounded below by Case 2 and hence is positive for all cases. □

Remark: From the proofs of Theorems 1 and 2, the monotonicity of bidding functions is relatively hard to guarantee when the capacity constraint is binding in both auctions, i.e., Case III for the first-price auction and region IV for the second-price auction. In these regions, as the signal s increases, $\pi_w^I(s,s)$ and $\pi_w^{II}(s,s,s)$ are restricted by the capacity constraint, while $\pi_l^I(s,s,s)$ and $\pi^{II}(s,s)$ keep growing without any constraint. This phenomenon makes it harder to ensure that bidding functions are monotonic.

Proof of Theorem 3

To prove Theorem 3, it suffices to show the following monotonicity result:

Suppose that $q_w^I(s_w) + q_l^I(s_w,s_l) \geq q_w^{II}(s_w,s_l) + q_l^{II}(s_l)$. Then for all $y \geq s_w$,

$$q_w^I(y) + q_l^I(y,s_l) \geq q_w^{II}(y,s_l) + q_l^{II}(s_l), \ \forall y \geq s_w, \tag{A2.12}$$

and

$$q_w^I(s_w) + q_l^I(s_w,z) \geq q_w^{II}(s_w,z) + q_l^{II}(z), \ \forall z \leq s_l. \tag{A2.13}$$

Given that s_l is fixed and $y \geq s_w$ in inequality (A2.12), let ρ_w^I, ρ_w^{II} denote the coefficients of winner's signals in the first-price and second-price auctions respectively. ρ_w^I, ρ_w^{II} capture the marginal change of the total quantities resulting from one unit change in s_w. The monotonicity is equivalent to the fact $\rho_w^I \geq \rho_w^{II}$, and hence in the sequel we will verify it. According to Lemma A3, we know that $\frac{1}{2} \leq \rho_w^I \leq \frac{3}{4}$. Since $q_l^{II}(s_l)$ is independent of the winner's signal, the bounds of ρ_w^{II} are $0 \leq \rho_w^{II} \leq \frac{1}{2}$. Therefore, $\rho_w^I \geq \rho_w^{II}$ in all cases of inequality (A2.12).

Now we focus on inequality (A2.13). Given that s_w is fixed and $s_l \geq z$, we compare the coefficients of loser's signals between these two auctions (similarly to the treatment for inequality (A2.12)). Let ρ_l^I, ρ_l^{II} denote these coefficients. It suffices to show that $\rho_l^I \leq \rho_l^{II}$ in all cases. First we observe from Proposition A3 that $\rho_l^I = \frac{1}{2}$, $\forall s_w \in [0,1]$. Combining Proposition A1 and Lemma A3, we obtain

$$0 + \frac{1}{2} \leq \rho_l^{II} \leq \frac{1}{2} + \frac{3}{4},$$

which implies $\frac{1}{2} \leq \rho_l^{II} \leq \frac{5}{4}$. Hence, $\rho_l^I \leq \rho_l^{II}$. In fact it can be shown that ρ_l^{II} has tighter bounds, i.e., $\frac{1}{2} \leq \rho_l^{II} \leq \frac{3}{4}$. $\square$

A3. Supplement to the numerical experiments

Figure A3.1 compares the total quantities $Q^I(s_1, s_2)$ and $Q^{II}(s_1, s_2)$ provided to the consumer market. Since the resellers are symmetric, the graph is symmetric; and $Q^I(s_1, s_2)$ turns out to be higher in the upper-left and lower-right corners. This is consistent with Theorem 3 and Figure 3 since larger quantities in the consumer market corresponds to lower prices for the consumers.

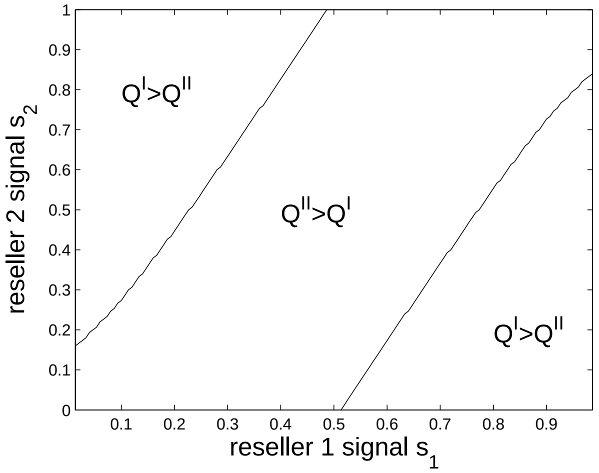


Figure A3.1: Total aggregated quantity provided to the consumer market under both auction mechanisms.

In Figure A3.2, we plot the bidding function of a type-s reseller. Note that the bidding function in a first-price auction is lower than that in a second-price auction for all signals. Since in the first-price auction the bid is the payment when the reseller wins, she decreases her bid to maintain her rent (similar results are reported in Krishna (2002, Chapter 6)). Moreover, the bidding functions inferred here are lower than the counterparts under the conventional auction (as mentioned in Section 3.2), and the difference becomes larger as the signal is higher.

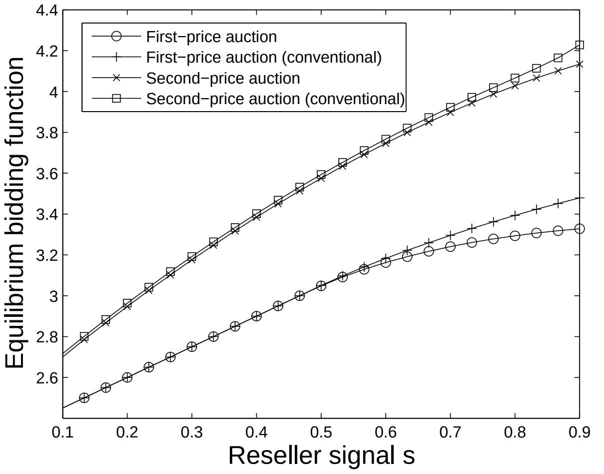


Figure A3.2: Comparison of reseller bidding functions under both auction mechanisms.