

Effects of Information Disclosure Under First- and Second-Price Auctions in a Supply Chain Setting

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We study a supply chain where an upstream supplier auctions his inventory or capacity as a bundle. The importance of this setting is twofold: From a practical point of view, there are several examples, both in manufacturing (e.g., auctioning the capacity of a wafer fabrication facility) and service industries (e.g., auctioning the sponsorship of a website), where a supplier's capacity is sold as a single piece; from a theoretical side, it highlights the information asymmetry introduced on the downstream supply chain parties when the auction result is disclosed.

We formulate the problem as a two-stage supply chain comprising a single supplier and two resellers. Each reseller receives a signal of the consumer demand and bids for the capacity of the supplier. The supplier announces the winner as well as the auction price. Both resellers can get additional units in a procurement market and then engage in Cournot competition in the consumer market.

We analyze the impact of the information elicited by the supplier in the early stage of the game. We characterize sufficient conditions for the existence of equilibrium behavior, derive the equilibrium bidding functions under both first- and second-price auctions, and show that the bidding functions are lower than the corresponding ones for a single-shot auction without resale. Our computational experiments indicate that both the supplier and resellers are better off running a second-price auction and that consumers benefit if the resellers have very different signals on the total demand. Overall, our results suggest that traditional auctions may have a profound impact in the context of a supply chain because of the information disclosure in the upstream stages.

Key words: supply chain; auctions; information asymmetry; demand effort; interdependent values

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1. Introduction

Auctions and the advances in information technology constitute two of the most significant innovations incorporated in supply chains since the middle 1990s. The advent of the Internet, the consequent reduction in transaction costs, and the easier access to more participants have contributed to the explosive growth that online auctions have exhibited during the last several years (e.g., see the online auction overview by Pinker et al. 2003). Nowadays, the wide spread of auctions in both virtual and traditional markets creates rich opportunities for firms to reinvent their procurement and selling practices, and in general to redesign the interface through which they interact

with the market, from the acquisition of raw material or input services to the delivery of finished goods or value-added services. Auctions take place among suppliers themselves, including both manufacturers and service providers, between suppliers and resellers, and between resellers and final consumers.

Formally speaking, an auction is simply a set of rules defining a *mechanism* for how products are awarded to buyers, how information is revealed between sellers and buyers, and what payments are made based on the revealed information. Two of the standard (sealed-bid) auction mechanisms are the first-price and second-price (Vickrey) auctions. The object is awarded to the highest bidder in both

auctions, but the auction price differs: it is the winning bid in the first-price auction, but is the highest losing bid in the second-price auction.

In this paper, we study a supply chain where an upstream supplier auctions his inventory or capacity as a bundle. The importance of this setting is two-fold: From a practical point of view, there are several relevant examples spanning both manufacturing and service industries where a supplier's capacity is sold as a single piece; from a theoretical side, our supply chain setting highlights the information asymmetry introduced on the downstream parties when the auction result is disclosed.

Regarding the practical side, consider a small winery that seeks to sell its product through retailers. For a small production lot of a high-quality wine, the winery is often interested in dealing with one exclusive retailer, as opposed to arranging deals with several. Another reason for selling manufacturing capacity as a bundle is the economies of scale. One example is silicon wafer production in the commoditized semiconductor industry, the "front-end" operation in the manufacturing of microelectronic products (e.g., see Karabuk and Wu 2002). Wafer fabrication is characterized by long manufacturing cycle times for short life-cycle products, large setup costs, and long lead times for capacity expansion. Therefore, capacity allocation becomes a critical task, and capacity bundling seems a natural course of action to simplify the problem.

There are also service industry cases where capacity is sold as a bundle. Online retailers such as Amazon or Buy.com can auction a space in their webpage to advertise a credit card, but they would not split the sponsored space between two competing credit cards. In this case, the capacity is given by the potential number of visitors to the website. Other online companies like Yahoo are considering auctions as a way to clear excess inventory of advertising space.¹ In particular, they sell all the "nonpremium inventory" as a bundle (e.g., see Holahan 2006). Advertising placement companies bid for this space and resell it to final clients. In a similar spirit, media such as TV channels bid for the right of carrying special events. Of course,

media firms cannot purchase a partial right of syndicating a show or a sport event. As an example, ITV and BBC bid for the right to carry Football Association games in England (e.g., see Gibson 2006). The capacity is defined by the potential number of viewers. The winner then sells advertising time slots to sports-related companies.

Regarding the theoretical relevance of our setting, we emphasize that the realization of auctions in a supply chain enforces information revelation among players that may affect the performance of the overall system. In particular, the disclosure of bids at different stages of the chain reveals ex post signals of the demand received by different players. This observation may clearly affect ex ante the strategic behavior of the bidders. For example, in the supplier/reseller relationship in which the supplier uses the auction to sell the capacity to the resellers, these resellers may obtain some information advantage (or disadvantage) with respect to their competitors in the consumer market and consequently adopt different purchasing/selling strategies.

The information revealed from an auction critically depends on how the bids are announced. In practice, a variety of auction-related announcement policies are followed, one being the revelation of the closing price. This, however, induces a clear asymmetry between resellers under both first- and second-price auctions: In the first-price auction the winning bid is disclosed, whereas the highest losing bid is revealed in the second-price auction. This difference is irrelevant in the traditional single-shot auction without resale. In the context of supply chains, since the bidders are typically the resellers that compete among themselves after the auction, this information disclosure becomes crucial. The threat of disclosing resellers' bids raises a number of important research questions: How does the information disclosure after the auction affect the resellers' purchasing/selling strategies? How should resellers behave in a procurement auction when running the risk of having their bids disclosed? How much information content should a reseller embed in her bid, knowing that it could be learned ex post by a direct competitor? Which auction mechanism mitigates or emphasizes more the asymmetry introduced by the disclosure of information? Is there any mechanism under which both suppliers and resellers are better off?

¹ Typically, these online companies sell most of the available capacity through long-term contracts.

This paper makes progress in addressing these questions. We consider a single supplier who auctions his capacity to two resellers. He conducts either a first- or second-price auction and announces the winner and the payment in each case. Both resellers can get additional (or substitutable) units in a procurement market before engaging in Cournot competition in the consumer market. How much the resellers value the auctioned capacity depends on the expected revenue to be collected from the end consumers. However, the resellers are aware of the information revelation threat: One reseller will have her bid revealed, and the other one will keep hers hidden after the auction.² We investigate the impact of information revelation on the downstream resellers' bidding behavior, on the supplier's and resellers' expected payoffs, and on the volume of transactions in the consumer market.

1.1. Literature Review

A vast body of work on auction theory has been published since Vickrey (1961). See, for example, the influential paper of Milgrom and Weber (1982) and the recent book by Krishna (2002). Gupta and Lebrun (1999), Krishna (2002, §4.4), and Haile (2003) consider first-price auctions with resale, but the resale occurs among the same set of bidders, and their assumptions on the announcement policies are quite different from ours: After the auction, all bidders' values are announced in Gupta and Lebrun (1999), whereas in Krishna (2002) all bids are made public. In Haile (2003), bidders receive new signals after the auction, so all bidders still hold private information. Benoit and Dubra (2006) consider the information-revelation issue in auctions as well, but their concentration is on whether bidders are willing to disclose their private information before submitting their bids. Hafalir and Krishna (2008) investigate the impact of asymmetry between bidders when the winner makes an offer to the loser after the auction. They assume that the winning bid is announced in both the first- and second-price auctions. Zheng (2002) assumes that each bidder is only aware of whether she wins the auction and

how much she pays. He focuses on the design of the optimal auction mechanism with resale. Moresi (2000) develops a model of information acquisition prior to an open auction, in which the common value of the item has two distinct components. Each of the two bidders conducts some research and specializes in one component independently, and then decides how much to bid. This model provides a rationale for bidders to differentiate themselves by conducting different lines of research.

Several papers have addressed the impact of a supplier's announcement policy and the derived information sharing in more traditional supply chain contexts (e.g., see the surveys by Cachon 2003, Chen 2003, and references therein). Our paper is closely related to the information sharing with horizontal Cournot competition. Along this research stream, Li (1985) focuses on the incentives for oligopolists to share information about either the demand function or the individual cost functions. The model includes an outside agency that controls the information leakage process. Li (2002) considers a supply chain in which a supplier sells the products to multiple retailers. He allows the retailers to decide, before receiving their private signals, whether to share their information with the supplier. In the Cournot competition setting, he shows that the possible information leakage from the supplier to other retailers eliminates each retailer's incentive to share the demand information. In a one-supplier-two-manufacturer setting, Anand and Goyal (2008) investigate whether the information leakage reduces the leading (incumbent) manufacturer's incentive to acquire demand information. Tunca (2008) studies an industrial market in which both the manufacturers (buyers) and suppliers (sellers) possess private information and the information disclosure is carried out through the market-clearing price. He shows that because of the strategic interaction between the market participants, inefficiency can arise as an emerging feature even if the number of participants approaches infinity.

Unlike these papers, we explicitly model and illustrate how the information leakage takes place under a particular revelation mechanism: an auction. Specifically, in our setting the downstream participants can only disclose their information through their bidding behavior *after* receiving their signals. We argue that

² When the supplier is allowed to split his capacity, then retailers' bids may be revealed if both retailers receive a portion of it, and the asymmetry is mitigated.

the use of auctions can serve as a credible mechanism for both the downstream resellers and the upstream supplier to disclose or withhold valuable information.

Different streams of research that relate supply chain management to auction theory include the design of procurement contracts (e.g., see Dasgupta and Spulber 1989, Chen 2007, and the survey by Elmaghraby 2000) and the structure of joint inventory policies and auction design decisions (e.g., van Ryzin and Vulcano 2004, Huh and Janakiraman 2008). However, in all these papers, there is no integration of procurement, auctions, and competition among resellers.

In our setting, resellers compete horizontally in the consumer market. We consider a linear demand model based on resellers' promotional efforts. In a competitive retailing environment, the demand-enhancing effort of one reseller may increase the demand of other resellers. These spillovers may lead to *free riding*, where one reseller enjoys a higher demand because of the efforts of others without exerting his own effort (see, e.g., Lal 1990). Hula (1993) develops two profit-maximization models of firm behavior that incorporate industry-demand externality effects of firm price changes, advertising, and research and development expenditures. On the operational side, Krishnan et al. (2004) study mechanisms to achieve coordination in a simple supply chain setting with one manufacturer and one reseller, accounting for the reseller's promotional effort.

Some research on treasury bill auctions is also related to our work (e.g., see Bikhchandani and Huang 1989, Chatterjea and Jarrow 1998, and references therein). These papers model the primary market of stocks/bonds as an auction, where traders operate with private information about future values of the financial assets. The purpose of trading in the primary market is to resell the securities afterwards to other uninformed traders or market makers. In particular, Bikhchandani and Huang (1989) consider a multi-unit auction setting with a resale market, but the resale price is determined by the uninformed traders' expectation of securities' true values. They discuss the impact of announcing the winning bid and the highest losing bid on the market equilibrium and traders' behavior. However, no oligopolistic competition is considered in the resale market.

Finally, we complement this review with the computer science literature on trading agent design, a prominent application area in artificial intelligence. The main focus of this stream of research is on the automated decision-making processes of a supply chain agent in terms of procurement, selling, and production/inventory management (e.g., see Buffett and Scott 2004, Wellman et al. 2003).

1.2. Overview of Main Results

We characterize the resellers' equilibrium bidding functions under the first- and second-price auctions. We provide sufficient conditions for the existence of an incentive-compatible equilibrium (i.e., an equilibrium where each reseller bids according to the demand signal that she got from the market). We show that the threat of revealing private information discourages the resellers from bidding as high as they would bid in the conventional counterpart (i.e., auctions without resale). Moreover, under the first-price auction, the condition for the equilibrium bidding function being monotonic in the reseller's own signal is more restrictive than under the second-price auction.

We also show that the consumers are better off (worse off) if the resellers receive very different (similar) demand signals. In the extreme case where the resellers receive very different signals, the first-price auction raises the loser's expectation of the consumer demand because the winner's bid is announced. However, in the second-price auction, the loser has no access to the winner's signal. Consequently, she underestimates the consumer demand and chooses an abnormally low quantity.

We verify numerically that the supplier's expected revenue is lower in the first-price auction than in the second-price auction. This result is in contrast with the revenue linkage principle (e.g., see Krishna 2002, chap. 7): in a conventional auction, if the signals are independent, the supplier should receive the same revenue from both auction formats. In our setting, because the winning bid is announced in the first-price auction, the winner collects on average less profit and hence is unwilling to pay as much as in the second-price auction. In fact, the announcement of the winning bid translates into some information disadvantage; in this sense, winning brings "bad news."

This can be regarded as another sort of the so-called *winner's curse*—the possibility that the winner pays more than the “real value” of the object (see Krishna 2002, chap. 6).

We also find that not only the supplier but also the resellers are better off when the second-price auction is used. An explanation for this is that the combined first-mover and information advantage for the winning reseller mitigates the horizontal competition between the resellers, and therefore the profit of the entire supply chain is driven up. This demonstrates that the choice between traditional auction mechanisms upstream in the supply chain has a great impact on performance if the subsequent competition is taken into consideration.

The rest of this paper is organized as follows. In §2 we describe our model setup, then discuss the model assumptions. We derive the equilibrium analysis of the two-stage game and relevant payoffs in §3. We provide numerical results in §4 and present our conclusions in §5. All proofs and supplementary technical details are in the online supplement.

2. Model Description

A single supplier runs an auction to sell his capacity C to two competitive resellers who do not possess any capacity endowment. The resellers can get additional units in a procurement market right after the auction closes. Once the resellers obtain their capacities, they engage in Cournot (quantity) competition in the consumer market. The inverse demand function is common knowledge, and is described by

$$P(Q) = \theta - Q, \quad (1)$$

where the intercept θ is a random variable and $Q = q_1 + q_2$ is the total aggregated supply—i.e. the sum of the continuous quantities q_i provided by resellers $i = 1, 2$. This linear demand model is commonly adopted in the literature of economics, marketing, and operations (e.g., see Tirole 1995). Even though the resellers face a stochastic demand, we neglect the nonnegativity constraint over the price for computational convenience. This assumption is plausible when the quantities chosen in equilibrium drive down the likelihood of a negative price to a negligible level (e.g., see Li 2002).

2.1. Sequence of Events

For ease of presentation, we divide the sequence of events into four periods:

Period 1: Reseller i receives a private signal s_i regarding the intercept θ . We keep the following assumption throughout:

ASSUMPTION 1. *The maximum possible demand is $\theta = \theta_0 + s_1 + s_2$, where θ_0 is a common knowledge constant, and s_1, s_2 are independently and $\text{Unif}[0, 1]$ distributed random variables.*

Neither the other reseller nor the supplier knows the true value of reseller i 's signal s_i , but they do know that it is a draw from a $\text{Unif}[0, 1]$ distribution. The contribution s_i to the total demand is a proxy for reseller i 's demand effort (e.g., it could be related to advertising levels).

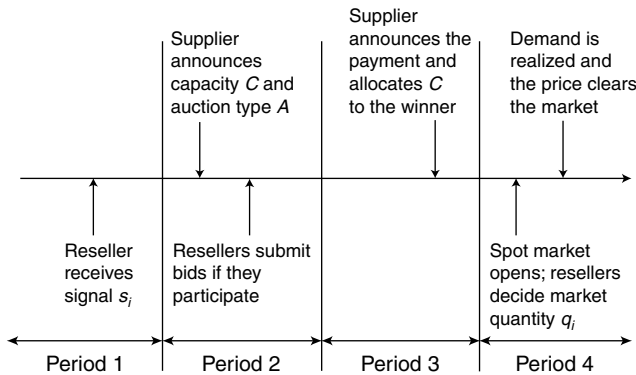
Period 2: At the beginning of this period, the supplier announces his capacity C and sells it as a bundle through an auction. Therefore, although auctioning multiple units, the supplier conducts in fact a single-unit auction. He announces the auction type A , which can be either the first- or second-price auction (i.e., $A = I$ or $A = II$, respectively). Then the auction takes place. Both resellers participate in it, each one submitting a bid b_i , $i = 1, 2$. We assume that the individual rationality constraint of the resellers is satisfied (i.e., we are implicitly assuming that by not participating, a reseller can guarantee herself a null payoff). In §3, we verify that this is indeed the case.

Period 3: The auction closes and the supplier announces who the winner is and how much she should pay for the capacity C . Therefore, in the first-price auction, the winner's bid is made available to the public,³ and in the second-price auction, the losing bid is revealed.

Period 4: At the beginning of this period, resellers can pay a constant marginal cost c for additional units from a procurement market with unlimited capacity. Finally, reseller i puts q_i units on the market. The demand is then revealed and the market clears. The

³ Although revealing the price in a first-price auction is not the most usual practice, by 2007 there are some websites that implement it. Examples are Timeshare Resales International (<http://www.tri-timeshare.com>) and The Chicago Wine Company (<http://www.tcwc.com>).

Figure 1 Time Flow Diagram for Sequence of Events



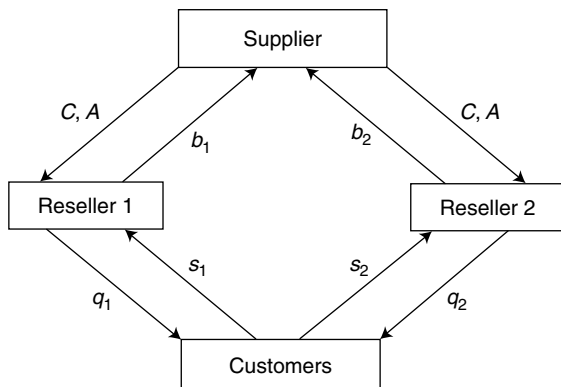
price is determined, and the resellers’ payoffs are realized.

The timing of this model is summarized in Figure 1. The information flow is presented in Figure 2: the supplier announces the auction type A and the capacity C to the resellers, and then resellers submit bids b_1, b_2 to the supplier. The resellers acquire demand information s_1 and s_2 from the final consumers before the auction and provide quantities q_1 and q_2 to the consumer market after the procurement process takes place.

2.2. Discussion of Model Assumptions

Our framework is an *interdependent-value* model with symmetric bidders. In an auction with N bidders and interdependent values, a bidder’s valuation may depend on other bidders’ signals. Let s_1, \dots, s_N denote the signals of bidders whose prior symmetric distribution $F(s_1, \dots, s_N)$ is common knowledge. If we let V_i denote bidder i ’s valuation of the auction object,

Figure 2 Information Flow Diagram of Relevant Events



then V_i can be expressed as $V_i = v_i(s_1, \dots, s_N)$. Therefore, until bidder i receives the signal s_i , she does not know exactly her own valuation, and knowing what other bidders also receive affects the expectation of her own value.⁴

A well-established theorem for revenue comparison of conventional auctions under the interdependent-value setting is the revenue linkage principle (see Milgrom and Weber 1982; Krishna 2002, chap. 7). An application of this principle is that if the signals s_1, \dots, s_N are independent (even though the valuations V_1, \dots, V_N are interdependent, as in our case with $N = 2$), then the seller’s expected revenue is the same under the first- and second-price conventional auctions. In §4 we compare our result to this well-known principle.

We assume that signals are independently distributed (i.e., they are *nonaffiliated*) and additive to the demand. This is a plausible assumption in situations regarding information acquisition (e.g., see Froot et al. 1992, Moresi 2000), and its tractability allows us to prove our results analytically. This assumption corresponds to the case in which the resellers put independent efforts to increase the market demand through strategies such as advertising and promotions and to the fact that the realization of a reseller effort could be observed only by the reseller herself. Thus, the signals capture the resellers’ private information following their private actions.

For ease of presentation and to emphasize the different situations that a winner and a loser of the auction will face in the consumer market, we consider only two resellers. If there were more than two participants in the auction, a similar approach would apply since only one bid would be disclosed. We further assume that these resellers do not possess any capacity endowment before participating in the auction and that their signals have the same precision. Thus, both resellers are *ex ante* symmetric. If their

⁴ More specifically, if $V_i = v_i(s_i)$, i.e., bidder i ’s valuation depends only on her own signal, it degenerates to a *private-value* model. If $V_i = v(s_1, \dots, s_N)$, we call it a *common-value* model. In this case, if bidders know all signals in the end, their valuations will become the same. An example for the common value model fit is the allocation of U.S. mineral rights. In our case, though, a reseller’s expected payoff will be affected by the other reseller’s signal concerning demand.

inventory endowments were different, their bids may depend not only on their signals but also on their inventory levels. It is known that in auctions with interdependent values, the asymmetry among bidders may destroy the revenue linkage principle (see Krishna 2002, chap. 8). If the precision of their signals concerning the realized demand varies across bidders, there may not exist a pure-strategy equilibrium. By considering ex ante symmetric resellers, we isolate the effect of information revelation that arises when they bid for the supplier's capacity.

In our setting, the supplier sells his capacity as a bundle. As we have mentioned in §1, this is appropriate in several relevant practical settings. From a modeling perspective, it stresses the information asymmetry introduced by different announcement policies: if the seller were allowed to split his capacity, then both resellers' signals would be revealed if both received some units. The paper by Anton and Yao (1989) also touches on the issue of divisible versus indivisible awards, for a complete information procurement auction. They show that under a split-award procurement, the two bidders implicitly collude. Therefore, the auctioneer strictly prefers a single-source award auction.

The assumption that the procurement market has a given price c is merely made for tractability. It is reasonable if these two resellers are relatively small with respect to the aggregated market for this product. There are several papers in the literature along these lines. In Peleg et al. (2002), a firm is allowed to replenish its inventory via two channels: a long-term contract or an online search. Under the long-term contract, the price is fixed ex ante, and in the online search the price is random, reflecting different market situations. These two procurement alternatives, also labelled as *two-source factor purchasing* in Elmaghraby (2000), are similar to our procurement market and the auction, respectively. In fact, many suppliers provide auctions as a complementary sales channel besides their long-term relationship with resellers (e.g., see Grey et al. 2005).

Finally, we would like to emphasize that the case where the winner of the auction cannot purchase additional units in the procurement market can be conveniently incorporated into our model by assuming that $c = \infty$ for the winner (although the loser may

still be eligible for purchasing from the procurement market). We have investigated this alternative scenario and found that all the insights are robust against such a modification.

3. Equilibrium Analysis of Game

Because of the sequential nature of our four-period game under incomplete information, we will characterize a perfect Bayesian equilibrium (e.g., see Osborne and Rubinstein 1994 for more details):

DEFINITION 1. In a perfect Bayesian equilibrium, players' strategies and beliefs satisfy the following conditions:

1. *Bayesian updating:* Players have correct initial beliefs. Moreover, after observing players' actions at a stage, they use Bayes' rule to update only the corresponding beliefs.

2. *Sequential rationality:* Given their beliefs, the players' actions must be the best responses.

In the sequel, we use backward induction to analyze the players' behavior: first, we derive the fourth-period equilibrium in the consumer market, assuming that a strictly increasing equilibrium has been established in the second period (i.e., the auction game). Note that in the second period, each reseller's strategy is a bidding function that maps her signal to her bid; i.e., $\beta: [0, 1] \rightarrow R_+$.

DEFINITION 2. An equilibrium is said to be strictly increasing if the equilibrium bidding function is strictly increasing in a bidder's own signal.

When the bidding function is strictly increasing, as players observe the bid, they can invert the bidding function to obtain a competitor's signal. Therefore, it is also fully separating.

In our setting, the supplier discloses the auction price. In the first-price auction, the winning bid is announced. Thus, the winning reseller's private information becomes public. In contrast, if a second-price auction is adopted, the supplier announces the loser's bid.

Table 1 summarizes the information that a bidder possesses at the end of *Period 3*, where s_w and s_l are the signals received by the winner and the loser, respectively. Note that while competing in the consumer market, the loser has information advantage in the first-price auction; the winner is the more informed reseller if the second-price auction

Table 1 Information Obtained by Winner and Loser When Auction Closes

	First-price auction	Second-price auction
Winner	$\{s_w\}, \{s_l < s_w\}$	$\{s_w\}, \{s_l\}$
Loser	$\{s_l\}, \{s_w\}$	$\{s_l\}, \{s_w > s_l\}$

is adopted. In what follows, we use superscript *I* to denote terms associated with the first-price auction, and superscript *II* for terms associated with the second-price auction. The subscripts *w* and *l* refer to the variables for the winner and the loser, respectively.

3.1. Consumer Market Game

We now derive the equilibrium in the consumer market given the two auction formats chosen by the supplier. Recall that at this stage of the game (*Period 4*), the resellers decide how many units, q_w and q_l , they provide to the consumer market.

3.1.1. Second-Price Auction Procurement Case.

We first consider the case when the supplier runs a second-price auction and assume that the resellers bid truthfully in the auction. The winning reseller's goal is to maximize her expected payoff, given that she knows both signals and that she has to pay c per unit in the procurement market (if her optimal quantity supplied to the consumer market exceeds the auctioned capacity C). Hence, the winner's objective is as follows:

$$\max_{q_w \geq 0} \{(\theta - q_w - q_l)q_w - c(q_w - C)^+\},$$

where $a^+ \equiv \max\{a, 0\}$ is the positive part of the real number a . Similarly, the loser's objective is

$$\max_{q_l \geq 0} E_{s_w} [(\theta - q_w - q_l - c)q_l \mid s_w > s_l, s_l].$$

Let $q_w^I(s_w, s_l)$ and $q_l^I(s_l)$ denote the optimal quantities chosen by the winner and the loser, respectively, where the arguments depict their corresponding information knowledge at the moment of decision. Proposition A1 in the appendix summarizes the equilibrium quantities provided by the two resellers in the consumer market.

3.1.2. First-Price Auction Procurement Case.

Similarly, we can obtain the equilibrium quantities after the resellers bid truthfully in the first-price

auction. In this case, because the winner's bid is announced, the loser knows both signals, but the winner only knows her own signal. The resellers' objective functions are, respectively,

$$\text{Winner: } \max_{q_w \geq 0} E_{s_l} [(\theta - q_w - q_l)q_w - c(q_w - C)^+ \mid s_w, s_l < s_w]$$

$$\text{Loser: } \max_{q_l \geq 0} \{(\theta - q_w - q_l - c)q_l\}.$$

After differentiating the objective functions and applying the same argument as in Proposition A1, we obtain the resellers' best responses. These quantities are described in Proposition A3 in the appendix.

3.2. Auction Game

We now proceed to the previous stage of the game (*Period 2*). At this point, the resellers' decision is about how to bid in the auction. We restrict ourselves to the symmetric (and strictly increasing) equilibrium throughout this section. An equilibrium in the auction mechanism A is said to be symmetric if there exists a bidding function $\beta^A(s)$ such that a reseller who receives a signal s submits the bid $\beta^A(s)$, independent of the reseller's identity. We establish in the sequel the uniqueness of such an equilibrium under both the first- and second-price auctions.

Suppose a reseller receives a signal s . If she participates in auction A and pretends her signal is z , we denote by $\Pi^A(z \mid s)$ her ex ante expected payoff. A reseller's objective is to choose a strategy that maximizes it, and therefore a truth-telling equilibrium requires

$$s \in \arg \max_z \{\Pi^A(z \mid s) : 0 \leq z \leq 1\}.$$

To verify this fact for both auction mechanisms, we introduce the quantities $\pi_w^A(\cdot)$ and $\pi_l^A(\cdot)$: the partial payoffs under auction mechanism A in case of winning and losing, respectively. By *partial payoff*, we refer to the expected revenue in the consumer market minus the procurement cost in the procurement market. Thus, we do not account for the procurement cost through the auction.

3.2.1. Second-Price Auction. Even though *bid your own value* is a dominant strategy in auctions where bidders possess private values, in an

interdependent-value model bidders do consider the bidding function their opponent uses. This is because based on this the bidder may be able to estimate her opponent’s signal and hence the “right value” of the auctioned object. Therefore, strategic interaction scatters the hope of finding a dominant strategy equilibrium.

If the supplier sells the capacity through a second-price auction, the expected payoff of a type s reseller who behaves as a type z reseller, assuming that her opponent adopts the bidding function $\beta^{II}(y)$, is

$$\begin{aligned} \Pi^{II}(z | s) = & \int_0^z (\pi_w^{II}(s, y) - \beta^{II}(y)) dy \\ & + \int_z^1 \pi_i^{II}(z, s, y) dy, \end{aligned} \quad (2)$$

where z and s belong to the inspected reseller, and y refers to the opponent’s type. The first term represents the event that she wins the auction, in which case she gets the object and pays the opponent’s bid $\beta^{II}(y)$. Because of the monotonicity of $\beta^{II}(y)$, this event happens when the opponent’s signal is less than z . Her partial payoff $\pi_w^{II}(s, y)$ is independent of her own bid $\beta^{II}(z)$ because in the second-price auction, the supplier does not announce the winning bid. The second term corresponds to the event that she loses in the auction while pretending to be type z : $\pi_i^{II}(z, s, y)$ is the partial payoff of a loser whose signal is s and plays as a type z , given that her opponent—i.e., the winner—receives signal y . Note that if she loses while playing as a type z , her bid $\beta^{II}(z)$ will be revealed so the opponent will infer a wrong type z and will choose the best response accordingly.

In the following we explicitly write down the winner’s and loser’s partial payoffs. The winner’s partial payoff $\pi_w^{II}(s, y)$ is simply the product of the realized price and the quantity, minus the purchasing cost in the procurement market. Thus, we can express it as follows:

$$\begin{aligned} \pi_w^{II}(s, y) = & (\theta_0 + s + y - q_w^{II}(s, y) - q_i^{II}(y)) \\ & \cdot q_w^{II}(s, y) - c(q_w^{II}(s, y) - C)^+, \end{aligned}$$

where $q_w^{II}(s, y)$ and $q_i^{II}(y)$ are given by Equation (5) in the appendix. Similarly, we can write down the loser’s partial payoff as follows:

$$\pi_i^{II}(z, s, y) = q_i^{II}(z, s)(\theta_0 + y + s - q_w^{II}(y, z) - q_i^{II}(z, s) - c),$$

where

$$\begin{aligned} q_i^{II}(z, s) = & \frac{1}{2} \left(\mathbb{E}_y[\theta | y > z, s] \right. \\ & \left. - \int_z^1 q_w^{II}(y, z) \frac{1}{1-z} dy - c \right) \end{aligned} \quad (3)$$

follows from Proposition A1.

Similarly to Milgrom and Weber (1982), we focus on the cases where the expected (total) payoff is unimodal in the reported type. The unimodal pattern of the expected payoff can be rationalized by the following argument: As the bidder increases her bid, her payoff will first increase because of the higher probability of winning. However, the bidder cannot increase her bid unboundedly because at some point she will pay too much. The following assumption is needed to guarantee the monotonicity of the reseller bidding function:

$$\text{ASSUMPTION 2. } q_w^{II}(s, s) \geq \frac{8}{5} q_i^{II}(s) + \frac{3}{16}, \quad \forall s.$$

This condition simply requires that a truth-telling type s reseller provide a significantly higher quantity when she gets the capacity than when she loses. Note that we could express this condition in terms of the primitive values C , c , and θ_0 by using Propositions A1 and A2, but it would become harder to read and less intuitive to understand.

Now we are ready to characterize the equilibrium bidding function of the second-price auction. Assuming that the objective function is differentiable and there exists an interior solution to the first-order condition of $\Pi^{II}(z | s)$ in Equation (2), we have the following result:

THEOREM 1. *Suppose Assumption 2 holds. The unique symmetric, strictly increasing equilibrium bidding function $\beta^{II}(s)$ in the second-price auction is*

$$\beta^{II}(s) = \pi_w^{II}(s, s) - \pi_i^{II}(s, s, s) + \int_s^1 \frac{\partial}{\partial v} \pi_i^{II}(v, s, y) \Big|_{v=s} dy.$$

Moreover, $\beta^{II}(s) \leq \pi_w^{II}(s, s) - \pi_i^{II}(s, s, s)$.

Note that $\pi_w^{II}(s, s) - \pi_i^{II}(s, s, s)$ can be regarded as the equilibrium bidding function in the second-price auction with no resale: a bidder with signal s is asked to bid an amount such that if she were to win the auction with that bid, she would just “break even” (e.g., see Krishna 2002, chap. 6). Theorem 1 tells us

that the bidding function here is lower than that in the conventional auction. This is because the bid of a type s reseller will be announced with some probability (i.e., when she loses); hence the threat of revealing her private information lowers her bid.

3.2.2. First-Price Auction. If the supplier uses a first-price auction in *Period 2*, the expected payoff of a type s reseller who behaves as a type z reseller, assuming that her opponent adopts the same equilibrium bidding function, is

$$\Pi^I(z|s) = \int_0^z (\pi_w^I(z, s, y) - \beta^I(z)) dy + \int_z^1 \pi_l^I(s, y) dy. \quad (4)$$

The first term accounts for the case where the reseller wins the auction. In this case, her payment is her bid $\beta^I(z)$. She wins because the opponent's signal y is less than z , and the opponent believes that she receives the signal z . After the auction, the Cournot game takes place, and $\pi_w^I(z, s, y)$ depends on the bidder's reported z rather than her true signal (which is not disclosed). The second term corresponds to the case in which the reseller loses in the auction. In this case, the partial payoff $\pi_l^I(s, y)$ is independent of the losing bid because it is not disclosed.

As demonstrated for the second-price auction, we can also explicitly express the winner's and loser's partial payoffs, $\pi_w^I(z, s, y)$ and $\pi_l^I(s, y)$, as functions of the equilibrium quantities $q_w^I(z, s)$ and $q_l^I(y, z)$, where the arguments indicate the information available when the reseller makes the quantity decisions. The closed-form expressions of $q_w^I(z, s)$ and $q_l^I(y, z)$ are given in §A.2 in the appendix. In deriving the equilibrium bidding function for the first-price auction, we would like to focus on the case where the expected (total) payoff is unimodal and the bidding function increasing, so the first-order condition can be applied. We first characterize a sufficient condition similar to Assumption 2:

$$\text{ASSUMPTION 3. } q_w^I(s, s) \geq 4q_l^I(s, s), \forall s.$$

Note that the condition is stronger than Assumption 2. In Milgrom and Weber (1982), the sufficient equilibrium condition for the first-price auction is also stronger than the one for the second-price auction. In the second-price auction, Milgrom and Weber only require the valuation $v(s, y)$ be strictly increasing in both s and y , whereas in the first-price auction, they

need the signal affiliation to obtain the unimodality of bidders' payoffs.

The next theorem characterizes the equilibrium bidding function.

THEOREM 2. *Suppose Assumption 3 holds. In the first-price auction, the unique symmetric strictly increasing equilibrium bidding function is*

$$\beta^I(s) = \frac{1}{s} \int_0^s (\pi_w^I(y, y, y) - \pi_l^I(y, y) + \int_0^y \frac{\partial}{\partial u} \pi_w^I(u, y, v) \Big|_{u=y} dv) dy.$$

Moreover, $\beta^I(s) \leq (1/s) \int_0^s (\pi_w^I(y, y, y) - \pi_l^I(y, y)) dy$.

The term $(1/s) \int_0^s (\pi_w^I(y, y, y) - \pi_l^I(y, y)) dy$ can be regarded as the equilibrium bidding function in the conventional first-price auction by a slight modification of Milgrom and Weber (1982) to account for the type-dependent participation constraint. Put in terms of their notation, $v_i(s, y) \neq 0$, where s is the bidder's own signal, y is her opponent's signal, and v_i is the utility when the bidder does not win the object. More specifically, when $v_i(s, y) = 0, \forall s, \forall y$, Milgrom and Weber (1982) show that $\beta^{II}(s) = v_w(s, s)$, where $v_w(s, s)$ is the expected value of the object if both bidders receive signal s . With a Unif[0, 1] signal distribution, the equilibrium bidding function under the first-price auction is $\beta^I(s) = \int_0^s v_w(y, y)(1/s) dy$. Allowing $v_i(s, y)$ to depend on the signals as well, it can be verified that $\beta^{II}(s) = v_w(s, s) - v_l(s, s)$ and $\beta^I(s) = \int_0^s (v_w(y, y) - v_l(y, y))(1/s) dy$.

Thus, Theorem 2 shows that the winner's expected payment here is lower than her expected payment in the conventional counterpart. This is because the bid of a type s reseller will be announced with some probability, and hence the threat of revealing her private information lowers her bid.

The monotonicity conditions of the bidding functions are also worth noting. $\pi_w^I(s, s, s) - \pi_l^I(s, s)$ and $\pi_w^{II}(s, s) - \pi_l^{II}(s, s, s)$ are the differences of equilibrium payoffs of a type s reseller between the cases where she wins and where she loses in the auctions, provided that her opponent also receives the same signal. Because our bidding functions $\beta^{II}(s)$ and $\beta^I(s)$ have an extra integral term, namely $\int_s^1 (\partial/\partial v) \pi_l^{II}(v, s, y) \Big|_{v=s} dy$ and $\int_0^y (\partial/\partial u) \pi_w^I(u, y, v) \Big|_{u=y} dv$, respectively, it is not enough to ensure that the difference of payoffs is

increasing. This is different from Milgrom and Weber (1982). In both auctions, a sufficient condition for the monotonicity of the bidding function is that $v_w(s, s) - v_l(s, s)$ is increasing in s .

In the first-price auction, we observe in the proof of Theorem 2 (see Appendix A.2.2) that the third term in the integrand is either $-\frac{1}{2} \int_0^y q_w^l(u, u) du$ or $-\frac{1}{4} \int_0^y q_w^l(u, u) du$; in either case, it is decreasing in y . This implies that we need a stronger condition to guarantee the monotonicity of $\beta^l(s)$; i.e., $\pi_w^l(s, s, s) - \pi_l^l(s, s)$ being strictly increasing does not suffice. More specifically, we require the integrand $\pi_w^l(y, y, y) - \pi_l^l(y, y) + \int_0^y (\partial/\partial u) \pi_w^l(u, y, v)|_{u=y} dv$ be strictly increasing in y .

On the contrary, in the second-price auction, we can find situations where the integral

$$\int_s^1 \frac{\partial}{\partial v} \pi_l^{II}(v, s, y)|_{v=s} dy$$

is nonmonotonic (for instance, in Region II in Figure A1.2 in the online supplement). Thus, in some cases the extra integral term actually helps to sustain the monotonicity of $\beta^{II}(s)$. We conclude that it is more difficult to sustain a fully revealing bidding equilibrium in the first-price auction than in the second-price auction.

Finally, note that the resellers' participation constraint is implicitly taken into consideration under both auction mechanisms: if a reseller does not want to participate, she can always join and submit a type 0 bid (i.e., $\beta^{II}(0)$ or $\beta^l(0)$). In this way, she will lose, and her true signal will not be revealed.

3.3. Outcome of Game

3.3.1. Supplier's Expected Revenue. Under the second-price auction, the supplier's expected revenue is $E[R^{II}] = \int_0^1 \beta^{II}(s) f_{(2)}(s) ds$, where $f_{(2)}(s)$ is the probability density function of $\min\{s_1, s_2\}$, because the payment is the highest losing bid. Given the independently uniformly distributed s_1 and s_2 , $f_{(2)}(s) = 2(1-s)$.

Under the first-price auction, when the payment equals the winning bid, the supplier's expected revenue is $E[R^I] = \int_0^1 \beta^l(s) f_{(1)}(s) ds$, where $f_{(1)}(s)$ is the density function of $\max\{s_1, s_2\}$, and in the context of our model, $f_{(1)}(s) = 2s$.

3.3.2. Quantities in the Consumer Market. Given the inverse linear demand function (1), the total aggregated consumers' surplus given auction format A is $\Pi_C^A(s_1, s_2) = \frac{1}{2}(Q^A(s_1, s_2))^2$, where $Q^A(s_1, s_2)$ is the total quantity that resellers provide to the consumer market.⁵ Note that for a given pair of signals (s_1, s_2) , $\Pi_C^A(s_1, s_2)$ is strictly increasing in $Q^A(s_1, s_2)$. Therefore, to compare consumers' surplus in both auctions, it suffices to consider the equilibrium quantities.

According to the auction announcement policy, the equilibrium quantities can be defined as $Q^I(s_1, s_2) = q_w^I(s_w) + q_l^I(s_l, s_w)$ and $Q^{II}(s_1, s_2) = q_w^{II}(s_w, s_l) + q_l^{II}(s_l)$, where $s_w = \max\{s_1, s_2\}$ and $s_l = \min\{s_1, s_2\}$. We obtain the following:

THEOREM 3. *If the difference between resellers' signals is large, consumers are better off in the first-price auction. But if the difference between resellers' signals is small, consumers are better off in the second-price auction.*

In the extreme case in which the resellers receive very different signals (s_1, s_2) , the real demand is actually close to its mean. According to Theorem 3, in the first-price auction, the disclosure of the winner's bid raises the loser's expectation of the demand. Therefore, the loser sets the quantity at the normal level. However, in the second-price auction, the loser has no access to the winner's signal. Therefore, she underestimates the demand and this results in an abnormally low quantity. In this case, consumers are better off in the first-price auction because of the correction of the loser's belief.

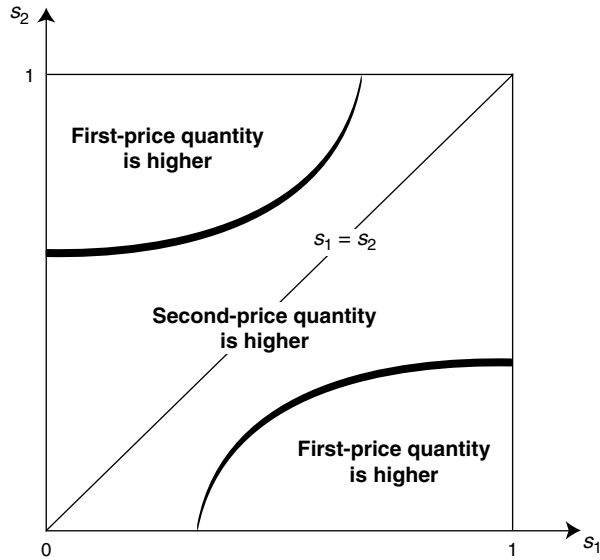
Figure 3 illustrates the quantity difference between the first-price and second-price auctions. The graph is symmetric because these two resellers are symmetric, and the boundary between the two regions in the lower triangle is nondecreasing in s_1 . Moreover, the closer a point is to the lower-right corner (equivalently, the upper-left corner), the larger the difference between $Q^I(s_1, s_2)$ and $Q^{II}(s_1, s_2)$.

4. Numerical Results

In this section, we provide several numerical examples to illustrate the previous analytical results and

⁵ Note that the for the inverse linear demand function $P(Q) = \theta - Q$, the total aggregated consumers' surplus is $\int_0^Q P(q) dq - P(Q)Q = \frac{1}{2}Q^2$.

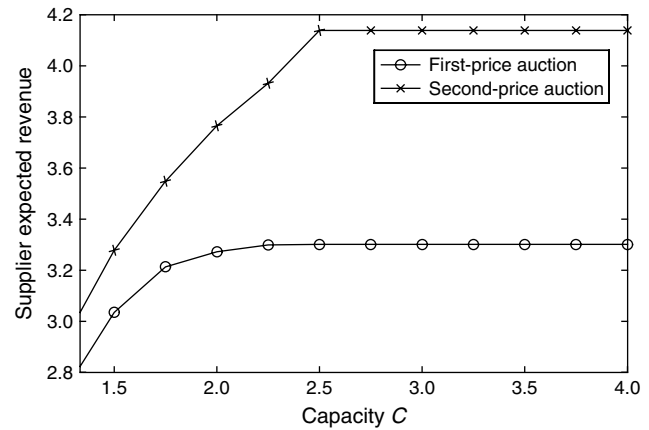
Figure 3 Comparison Between Aggregated Quantities Provided to the Consumer Market Under First-Price and Second-Price Auctions



get some insights. The parameters θ_0 , c , and C are carefully chosen such that the nonnegativity of realized prices and quantities is satisfied in all subgames. In particular, unless explicitly mentioned, we fix $\theta_0 = 3$, $C = 1.5$, and $c = 2.5$ in all the following examples. In fact, given $\theta_0 = 3$, we observed similar qualitative results with different values of c . Note that in all cases, we need Assumptions 2–4 to hold in order to guarantee the unimodality of the resellers’ payoffs and the monotonicity of the bidding functions.

Figure 4 presents the supplier’s expected revenue under both mechanisms. The supplier’s expected revenue, derived in §3.3.1, equals the expected payment from the resellers while taking expectation with respect to their signals. This amount turns out to be higher in the second-price auction than in the first-price auction. Note that this result is in contrast with the revenue linkage principle: in a conventional auction, if the signals are independent, as in our case, the revenue linkage principle claims that the supplier should receive the same revenue from both the first- and second-price mechanisms. Here, the foreseeable information disadvantage in the consumer market constitutes a sort of winner’s curse in that the winner in the first-price auction collects on average less profit from winning the auction. Therefore, she is unwilling to pay as much as she would pay under the

Figure 4 Comparison of Supplier’s Revenues $E[R^I]$ and $E[R^{II}]$



second-price auction. This result directly lowers the supplier’s expected revenue in the first-price auction.

From Figure 4, we also observe that as the supplier increases the capacity level C , his expected revenue gets saturated after certain thresholds. This is related to the maximum quantity that the market demand may request (recall the downward sloping demand function in Equation (1)), and the resellers are unwilling to pay more for excess capacity because it does not bring more profit. Suppose that the auction mechanism is given. The supplier can decide his optimal capacity, taking into account his own cost of building capacity. Our results further suggest, for example, that providing capacity higher than $C = 2.5$ is a weakly dominated strategy, independent of supplier’s cost structure (as long as capacity cost is increasing). The fact that the saturation comes later in the second-price auction (at $C = 2.5$ in the figure) is also worth noting: it implies that the resellers are still willing to pay for marginal increase of capacity C because of the combined first-mover and information advantage.

Next, we investigate the reseller’s willingness to pay in both auctions as opposed to the given spot price c , where willingness to pay is defined as the expected payment, given that the reseller wins. In other words, in the first-price auction, the willingness to pay is the bidding function, whereas in the second-price auction, it becomes the expected losing bid. Figure 5 shows that the willingness to pay under both auctions increases linearly in c and runs always below c (the 45° line). The participation of the resellers

Figure 5 Reseller's Bidding Function Under a Second-Price Auction vs. Spot Price c , When $s_i = 0.5$

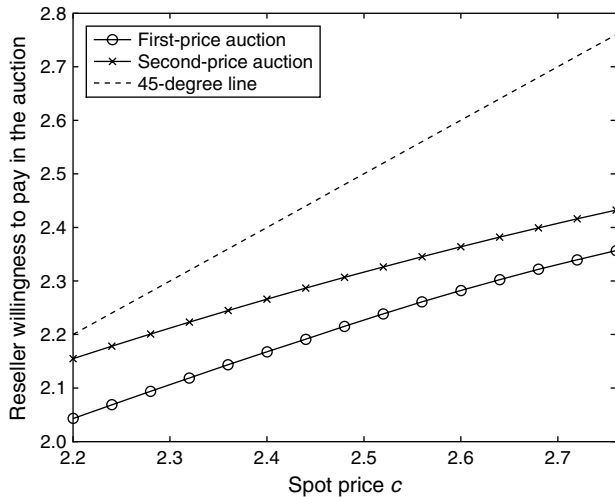
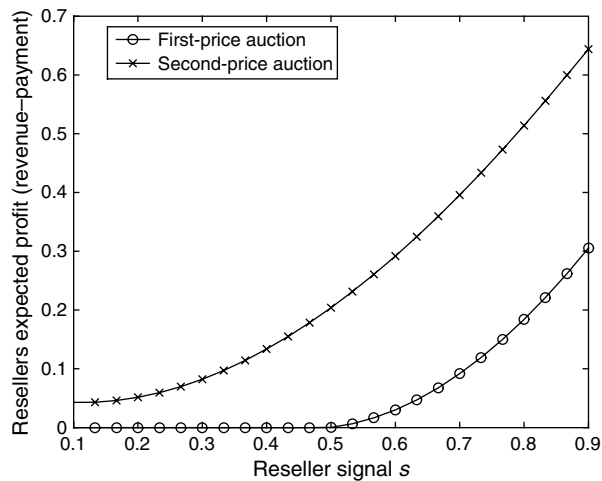


Figure 6 Comparison of Reseller Expected Profit Under Both Auction Mechanisms



in the auction can be attributed to two factors: gathering information about the opponent's private signal and lowering the expected procurement cost. Thus, a supplier with limited capacity would sell it at a lower price than the unlimited procurement market. This also justifies why dual sourcing is possible: neither source is dominated by the other.

Figure 6 exhibits the resellers' expected profit under both auction mechanisms. We draw two curves that come from Equations (2) and (4), respectively, when replacing z by the true type s . The figure shows that a reseller is better off in the second-price auction, and

the gap is expanded as the signal becomes higher. This observation jointly with Figure 4 implies that the second-price auction in fact yields a higher combined payoff for the entire supply chain, because the supplier and the resellers are all better off. Allowing the winner to get both the first-mover and the information advantages reduces the competition tension between downstream resellers, and hence ex ante the entire supply chain benefits. Note also that in the first-price auction, the resellers with low signals receive just their reservation utilities eventually, so bidders with low signals do not get information rent.

We can also examine the monotonicity of the bidding function. Let us take the parameters ($\theta_0 = 3$, $C = 1.5$, $c = 2.5$) as the reference point and change them one at a time. Fix $\theta_0 = 3$ and $C = 1.5$; the monotonicity holds for both auctions when $c \in [2.1, \infty)$. Fix $\theta_0 = 3$ and $c = 2.5$; it holds when $C \in [0.83, \infty)$. And if $C = 1.5$ and $c = 2.5$, the bidding functions are monotonic when $\theta_0 \leq 3.7$. The monotonicity fails as either the gain from the auction becomes small or the value of information advantage in the consumer market becomes large. Naturally, the resellers are unwilling to reveal their signals, as the procurement market price is small, the capacity is small, and the potential demand is large. Moreover, in all our numerical experiments, as long as the bidding function under the first-price auction is monotonic, then the monotonicity for the bids under the second-price auction holds. This suggests that the separating equilibrium is relatively harder to sustain in the first-price auction, as predicted by our analysis. The numerical experiments support our interpretation of Theorems 1 and 2.

Finally, in the online supplement we exhibit figures about the quantities provided to the consumer market and about the bidding functions (comparing them with the conventional counterparts) under both auction mechanisms.

5. Conclusions

Motivated by supply chain settings where a supplier's capacity is sold as a bundle, occurring both in manufacturing (e.g., commoditized capacity subject to large setup costs) and service industries (e.g., sponsorship of a website or an event), we analyze a two-stage supply chain model, where two symmetric resellers

bid for the capacity of a supplier and then compete in the consumer market. After the auction closes, both resellers can get additional units in a procurement market. We study this model under first- and second-price auctions and analyze the impact of the auction price information elicited in the first stage of the game.

We find that the threat of revealing private information induces lower bids than the ones that would be submitted under conventional auctions (i.e., auctions with no resale). Comparing the supply chain performance under both auction formats, the second-price auction leads to a higher payoff for all players because it aligns the first-mover and the information advantages for the winner, reducing the competition tension between downstream resellers.

Regarding the consumers, the first-price auction makes them better off if the resellers receive very different demand signals, while under a second-price auction, consumers are better off if the resellers receive similar signals.

Our model shows that traditional auctions have a significant impact when put to work in the context of a supply chain because of the information asymmetry that may be introduced when announcing the results. It constitutes a contribution to understanding the underlying economics in supply chain management, using auctions as a means to facilitate transactions.

A possible extension of the model would be to include budget constraints on the resellers. Note that the equilibrium of a Cournot game like ours is independent of the resellers' cash endowments. The impact of budget constraint on bidders' strategies has been studied in Che and Gale (1998). They focus on the conventional auction and show that bids are distorted because of the budget constraint. In our model, a more intriguing trade-off prevails. If a reseller spends too much acquiring the capacity C from the auction, she may not have enough budget to trade in the procurement market after observing the information revealed in the auction. Another extension would be to allow the supplier to split the capacity sold through the auction. When both resellers get part of it, because the supplier announces all the transaction prices, both signals might be revealed.

This might mitigate the information asymmetry when the downstream resellers compete afterwards.

We point out here that there is an ongoing discussion in the auction theory and experimental economics communities about whether the game theoretic models like the one adopted in this paper can be used as valid decision support for the bidders, dating back to the paper by Cox et al. (1982) and including Rothkopf and Harstad (1994) and Lucking-Reiley (1999). It would be interesting to conduct experiments to test how resellers bid in practice and to validate our predictions.

Finally, in our model the supplier is committed to announcing the auction price regardless of the auction mechanism, which leads to significant information disadvantage for the winner in the first-price auction. A natural question to ask is whether under the first-price auction, the supplier should adopt a different announcement policy or even withhold all the information. More generally, should the supplier strategically disclose the information in favor of one reseller *after* receiving the bids? Exploring the impact of such partial information disclosure on the supply chain performance is intriguing and should provide explanations to some recent trends in industrial procurement of moving from full disclosure feedback—including the closing price—to a less informative disclosure such as rank feedback (Elmaghraby 2007). The resellers' sensitivity to disclosed information is one of the arguments that sustain this shift.

Electronic Companion

An electronic companion to this paper is available on the *Manufacturing & Service Operations Management* website (<http://msom.pubs.informs.org/ecompanion.html>).

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Appendix

In this appendix we sketch the proofs of the technical results of the consumer market game and the auction game. The detailed proofs, including those for the auxiliary lemmas and propositions, are in the online supplement.

A.1. Consumer Market Game

A.1.1. Second-Price Auction Procurement Case

The following proposition summarizes the equilibrium quantities provided by these two resellers in the consumer market.

PROPOSITION A1. *Suppose that the second-price auction is used. In the consumer market, there exists a unique Nash equilibrium in which*

$$q_w^{II}(s_w, s_l) = \begin{cases} \frac{1}{2}(\theta_0 + s_w + s_l - q_l^{II}(s_l)), & \text{if } s_w < S_1^{II}(s_l) \\ C, & \text{if } S_1^{II}(s_l) \leq s_w \leq S_2^{II}(s_l) \\ \frac{1}{2}(\theta_0 + s_w + s_l - q_l^{II}(s_l) - c), & \text{if } s_w > S_2^{II}(s_l), \end{cases} \quad (5)$$

$$q_l^{II}(s_l) = \frac{1}{2}(E_{s_w}[\theta \mid s_w > s_l, s_l] - E_{s_w}[q_w(s_w, s_l) \mid s_w > s_l] - c),$$

where the two thresholds $S_1^{II}(s_l)$ and $S_2^{II}(s_l)$ are

$$S_1^{II}(s_l) = \min\{\max\{s_l, 2C - \theta_0 - s_l + q_l^{II}(s_l)\}, 1\} \quad \text{and}$$

$$S_2^{II}(s_l) = \min\{\max\{s_l, 2C - \theta_0 - s_l + q_l^{II}(s_l) + c\}, 1\}.$$

PROOF. Let $\Pi_w^{II} \equiv (\theta - q_w - q_l)q_w - c(q_w - C)^+$ denote the winner's payoff function. We can then differentiate Π_w^{II} with respect to q_w , and the optimal quantities follow from the comparison between marginal revenue and marginal cost. \square

Now we characterize the structural property when the capacity constraint is binding.

PROPOSITION A2. *The threshold function $S_1^{II}(s_l)$ can be divided into three regions by two values s_1^* and s_1^{**} , where $s_1^* \leq s_1^{**}$.*

If $0 \leq s_l \leq s_1^$, then $S_1^{II}(s_l) = 1$; i.e., the capacity constraint is not binding for all s_w . If $s_1^* \leq s_l \leq s_1^{**}$, then $S_1^{II}(s_l)$ decreases from 1 until $S_1^{II}(s_l) = s_l$, and $S_1^{II}(s_l) - S_1^{II}(s_2) < s_2 - s_1$, $\forall s_1 < s_2$. Finally, when $s_1^{**} \leq s_l \leq 1$, then $S_1^{II}(s_l) = s_l$; i.e. $q_w^{II}(s_w, s_l) \geq C$ for all s_w .*

Likewise, there exist two corresponding thresholds s_2^ and s_2^{**} for $S_2^{II}(s_l)$ such that*

$$s_2^* \geq s_1^*, \quad s_2^{**} \geq s_1^{**}, \quad s_2^{**} \geq s_2^*, \quad S_2^{II}(s_1) - S_2^{II}(s_2) < s_2 - s_1, \quad \forall s_1 < s_2.$$

PROOF. We first state a technical lemma required to show this proposition:

LEMMA A1. *In the equilibrium quantities, $0 < q_l^{II}(s_2) - q_l^{II}(s_1) < s_2 - s_1$, for signals $s_1 < s_2$.*

We will focus on $S_1^{II}(s)$; the proof for $S_2^{II}(s)$ goes along the same argument. We first observe that $S_2^{II}(s) \geq S_1^{II}(s)$, $\forall s$. Let s_1 and s_2 denote two distinct signals, and let $s_1 < s_2$. Consider the case $S_1^{II}(s_2) = 1$. In this case, the

capacity constraint is never binding when $s_l = s_1$, so $S_1^{II}(s_1) = 1$. Next, let us consider the case $s_1 < S_1^{II}(s_1) < 1$ and $s_2 < S_1^{II}(s_2) < 1$. Here the capacity constraint is not binding for some s_w and is binding for others, so we obtain that $S_1^{II}(s_2) < S_1^{II}(s_1)$ from Lemma A1. Finally, when $S_1^{II}(s_1) = s_1$, we have $S_1^{II}(s_2) \neq 1$ and thus $S_1^{II}(s_2) = s_2$ from Lemma A1. \square

A.1.2. First-Price Auction Procurement Case

After differentiating the corresponding objective functions and applying the same argument as in Proposition A1, we obtain the resellers' best responses. The proof involves routine algebra and hence is omitted.

PROPOSITION A3. *Suppose the first-price auction is used. Define thresholds S_1^I and S_2^I as*

$$S_1^I = \min\{1, (2C - \frac{2}{3}c - \frac{2}{3}\theta_0)^+\} \quad \text{and}$$

$$S_2^I = \min\{1, (2C + \frac{2}{3}c - \frac{2}{3}\theta_0)^+\}.$$

In the consumer market, there exists a unique Nash equilibrium that satisfies the following: If $s_w < S_1^I$, then $q_w^I(s_w) = \frac{1}{3}\theta_0 + \frac{1}{2}s_w + \frac{1}{3}c$, and $q_l^I(s_l, s_w) = \frac{1}{3}\theta_0 + \frac{1}{4}s_w + \frac{1}{2}s_l - \frac{2}{3}c$, $\forall s_l \leq s_w$. If $S_1^I \leq s_w \leq S_2^I$, then $q_w^I(s_w) = C$, and $q_l^I(s_l, s_w) = \frac{1}{2}(\theta_0 + s_w + s_l - C - c)$, $\forall s_l \leq s_w$. Finally, if $s_w > S_2^I$, then $q_w^I(s_w) = \frac{1}{3}\theta_0 + \frac{1}{2}s_w - \frac{1}{3}c$, and $q_l^I(s_l, s_w) = \frac{1}{3}\theta_0 + \frac{1}{4}s_w + \frac{1}{2}s_l - \frac{1}{3}c$, $\forall s_l \leq s_w$.

A.2. Equilibrium Analysis of Auction Game

A.2.1. Second-Price Auction

PROOF OF THEOREM 1.

To prove this theorem, we need several technical lemmas, whose economic intuition is given in the online supplement:

LEMMA A2. $0 \leq q_w^{II}(y, z_2) - q_w^{II}(y, z_1) < \frac{1}{2}(z_2 - z_1)$, $\forall z_1 < z_2$.

LEMMA A3. $\frac{1}{2}(z_2 - z_1) \leq q_l^{II}(z_2) - q_l^{II}(z_1) < \frac{3}{4}(z_2 - z_1)$, $\forall z_1 < z_2$.

LEMMA A4. $\int_s^1 (\partial/\partial z) \pi_l^{II}(z, s, y)|_{z=s} dy \leq 0$.

We will first derive a necessary condition that an equilibrium bidding function must satisfy. The first-order condition with respect to z is as follows:

$$\frac{\partial \Pi^{II}(z \mid s)}{\partial z} = [\pi_w^{II}(s, z) - \beta^{II}(z) - \pi_l^{II}(z, s, z)] + \int_z^1 \frac{\partial}{\partial v} \pi_l^{II}(v, s, y) \Big|_{v=z} dy.$$

The truth-telling equilibrium requires that

$$\begin{aligned} \beta^{II}(s) &= \pi_w^{II}(s, s) - \pi_l^{II}(s, s, s) + \int_s^1 \frac{\partial}{\partial z} \pi_l^{II}(z, s, y) \Big|_{z=s} dy \\ &\leq \pi_w^{II}(s, s) - \pi_l^{II}(s, s, s), \end{aligned}$$

where the last inequality is given by Lemma A4. The equilibrium bidding function is uniquely determined by the above equation.

Let us now verify the monotonicity of $\beta^{II}(s)$. For the second-price auction, our goal is to show that $\beta^{II}(s)$ is

strictly increasing in s ; i.e., $\lim_{z \rightarrow s} (\beta^{II}(z) - \beta^{II}(s)) / (z - s) > 0$, $\forall s \in [0, 1]$. Except the case in which the capacity constraint is binding for $q_w^{II}(y, y)$, we obtain

$$\begin{aligned} & \pi_w^{II}(z, z) - \pi^{II}(z, z, z) - [\pi_w^{II}(s, s) - \pi^{II}(s, s, s)] \\ &= (2 - 2\rho_l)q_w^{II}(s, s) - (1 - \frac{1}{2}\rho_l)q_l^{II}(s, s) + \rho_l c \mathbb{1}\{s \in V\}, \end{aligned}$$

where we have ignored the second-order terms and ρ_l is such that $q_l^{II}(z, z) = q_l^{II}(s, s) + \rho_l(z - s)$. We can then divide the analysis into different cases depending on the thresholds $S_1^{II}(s)$ and $S_2^{II}(s)$ and verify that the bidding function $\beta^{II}(s)$ is strictly increasing.

Finally, we verify the incentive compatibility. Following Milgrom and Weber (1982), we should verify that a type s bidder's payoff is unimodal in her reported type, and it achieves the maximum at the truth-telling value s . Suppose the other player adopts that bidding function. Differentiating the expected payoff with respect to z , we obtain

$$\frac{\partial \Pi^{II}(z | s)}{\partial z} = (\pi_w^{II}(s, z) - \pi_l^{II}(z, s, z)) - (\pi_w^{II}(z, z) - \pi_l^{II}(z, z, z)).$$

By dividing the analysis into cases depending on the thresholds $S_1^{II}(s)$ and $S_2^{II}(s)$, we can show that $\partial \Pi^{II}(z | s) / \partial z < 0$ if $z < s$, and $\partial \Pi^{II}(z | s) / \partial z > 0$ if $z > s$. Thus, the payoff is unimodal. \square

A.2.2. First-Price Auction

PROOF OF THEOREM 2.

We first present the equilibrium quantities. Note that $q_l^I(y, z)$ are obtained from Proposition A3, and following them we calculate $q_w^I(z, s)$ as the corresponding best responses.

PROPOSITION A4. *Suppose that the first-price auction is used. If a type s bidder that behaves as a type z wins the auction, then: If $z \leq S_1^I$, then $q_l^I(y, z) = \frac{1}{3}\theta_0 + \frac{1}{4}z + \frac{1}{2}y - \frac{2}{3}c$, and*

$$q_w^I(z, s) = \begin{cases} \frac{1}{3}\theta_0 + \frac{1}{2}s + \frac{1}{3}c, & s \leq 2(C - \frac{1}{3}\theta_0 - \frac{1}{3}c), \\ C, & 2(C - \frac{1}{3}\theta_0 - \frac{1}{3}c) \leq s \leq 2(C - \frac{1}{3}\theta_0 + \frac{1}{6}c), \\ \frac{1}{3}\theta_0 + \frac{1}{2}s - \frac{1}{6}c, & s \geq 2(C - \frac{1}{3}\theta_0 + \frac{1}{6}c). \end{cases}$$

If $S_1^I \leq z \leq S_2^I$, then $q_l^I(y, z) = \frac{1}{2}(\theta_0 + z + y - C - c)$, and

$$q_w^I(z, s) = \begin{cases} \frac{1}{4}\theta_0 + \frac{1}{2}s - \frac{1}{8}z + \frac{1}{4}C + \frac{1}{4}c, & s \leq \frac{3}{2}C - \frac{1}{2}\theta_0 + \frac{1}{4}z - \frac{1}{2}c, \\ C, & \frac{3}{2}C - \frac{1}{2}\theta_0 + \frac{1}{4}z - \frac{1}{2}c \leq s \leq \frac{3}{2}C - \frac{1}{2}\theta_0 + \frac{1}{4}z + \frac{1}{2}c, \\ \frac{1}{4}\theta_0 + \frac{1}{2}s - \frac{1}{8}z + \frac{1}{4}C - \frac{1}{4}c, & s \geq \frac{3}{2}C - \frac{1}{2}\theta_0 + \frac{1}{4}z + \frac{1}{2}c. \end{cases}$$

Finally, if $z \geq S_2^I$, then $q_l^I(y, z) = \frac{1}{3}\theta_0 + \frac{1}{4}z + \frac{1}{2}y - \frac{1}{3}c$, and

$$q_w^I(z, s) = \begin{cases} \frac{1}{3}\theta_0 + \frac{1}{2}s + \frac{1}{6}c, & s \leq 2C - \frac{2}{3}\theta_0 - \frac{1}{3}c, \\ C, & 2C - \frac{2}{3}\theta_0 - \frac{1}{3}c \leq s \leq 2C - \frac{2}{3}\theta_0 + \frac{2}{3}c, \\ \frac{1}{3}\theta_0 + \frac{1}{2}s - \frac{1}{3}c, & s \geq 2C - \frac{2}{3}\theta_0 + \frac{2}{3}c. \end{cases}$$

The payoffs $\pi_w^I(z, s, y)$ and $\pi_l^I(s, y)$ in both cases can be expressed as follows:

$$\begin{aligned} \pi_w^I(z, s, y) &= q_w^I(z, s)(\theta_0 + s + y - q_w^I(z, s) - q_l^I(y, z)) \\ &\quad - c(q_w^I(z, s) - C)^+, \\ \pi_l^I(s, y) &= q_l^I(s, y)(\theta_0 + y + s - q_w^I(y, y) - q_l^I(s, y) - c). \end{aligned}$$

Now let us prove the theorem. We first derive the necessary condition for the equilibrium bidding function. The first-order condition of $\Pi^I(z | s)$ is

$$\begin{aligned} & \pi_w^I(z, s, z) - \beta^I(z) - \pi_l^I(s, z) + \int_0^z \pi_{w,1}^I(z, s, y) dy \\ & \quad - \int_0^z (\beta^I)'(z) dy = 0, \end{aligned}$$

which leads to $(\beta^I)'(z) = (1/z)(\pi_w^I(z, s, z) - \pi_l^I(s, z) + \int_0^z \pi_{w,1}^I(z, s, y) dy - \beta^I(z))$. The closed-form solution can be obtained from this differential equation by the same procedure in Krishna (2002, Proposition 6.3):

$$\beta^I(s) = \int_0^s (\pi_w^I(y, y, y) - \pi_l^I(y, y) + \int_0^y \pi_{w,1}^I(y, y, v) dv) dL(y | s),$$

where $L(y | s) = \exp(-\int_y^s (1/v) dv) = y/s$. Plugging $L(y | s)$ in the above equation, we obtain the (unique) bidding function $\beta^I(s)$.

Next, we will show that $\pi_w^I(z, s, y)$ is decreasing in z , for all pair (s, y) . This leads to the conclusion $\beta^I(s) < (1/s) \int_0^s (\pi_w^I(y, y, y) - \pi_l^I(y, y)) dy$. The differentiation can be expressed as follows:

$$\begin{aligned} \frac{\partial}{\partial z} \pi_w^I(z, s, y) &= \frac{\partial}{\partial z} q_w^I(z, s)(\theta_0 + s + y - q_w^I(z, s) - q_l^I(y, z)) \\ &\quad - c \frac{\partial}{\partial z} (q_w^I(z, s) - C)^+ \\ &\quad + q_w^I(z, s) \left(-\frac{\partial}{\partial z} q_w^I(z, s) - \frac{\partial}{\partial z} q_l^I(y, z) \right). \end{aligned}$$

After some algebra, we can show that $(\partial/\partial z)\pi_w^I(z, s, y)$ is negative according to Proposition A4. We now verify the monotonicity of $\beta^I(s)$. Similar to Milgrom and Weber (1982), a sufficient condition for the monotonicity of bidding function in the first-price auction is that the integrand of $\beta^I(s)$ is increasing. Recall that $(\partial/\partial u)\pi_w^I(u, y, v)|_{u=y} = -q_w^I(y, y)(\partial/\partial u)q_l^I(v, u)|_{u=y}$, the integral is $-\frac{1}{2} \int_0^y q_w^I(u, u) du$ if $S_1^I \leq y \leq S_2^I$ and is $-\frac{1}{4} \int_0^y q_w^I(u, u) du$ otherwise. Rewriting the payoff difference as

$$\begin{aligned} & \pi_w^I(y, y, y) - \pi_l^I(y, y) \\ &= q_w^I(y, y)[\theta_0 + y + y - q_w^I(y, y) - q_l^I(y, y) - c(q_w^I(y, y) - C)^+] \\ & \quad - q_l^I(y, y)[\theta_0 + y + y - q_w^I(y, y) - q_l^I(y, y) - c], \end{aligned}$$

we can obtain the desired monotonicity of $\beta^I(s)$ by dividing the analysis into cases.

Finally, we verify the incentive compatibility. Plugging the bidding function $\beta^I(z)$ into the expected payoff,

we obtain

$$\begin{aligned} \frac{\partial}{\partial z} \Pi^I(z|s) &= \pi_w^I(z, s, z) - \beta^I(z) - \pi_1^I(s, z) \\ &= (\pi_w^I(z, s, z) - \pi_1^I(s, z)) - (\pi_w^I(z, z, z) - \pi_1^I(z, z)). \end{aligned}$$

We can then divide the problem into cases to prove that the differentiation is positive when $z < s$ and negative if $z > s$; i.e., the expected payoff of a type s bidder is unimodal in the reported type z with maximum achieved at $z = s$. \square

PROOF OF THEOREM 3. To prove Theorem 3, it suffices to show that when $q_w^I(s_w) + q_1^I(s_w, s_1) \geq q_w^{II}(s_w, s_1) + q_1^{II}(s_1)$, for all $y \geq s_w$, $q_w^I(y) + q_1^I(y, s_1) \geq q_w^{II}(y, s_1) + q_1^{II}(s_1)$, $\forall y \geq s_w$, and $q_w^I(s_w) + q_1^I(s_w, z) \geq q_w^{II}(s_w, z) + q_1^{II}(z)$, $\forall z \leq s_1$. We let ρ_w^I, ρ_w^{II} denote the coefficients of the winner's signals in the first-price and second-price auctions, respectively. From Lemma A3 and the fact that $q_1^{II}(s_1)$ is independent of the winner's signal, we obtain that $\frac{1}{2} \leq \rho_w^I \leq \frac{3}{4}$, and $0 \leq \rho_w^{II} \leq \frac{1}{2}$. Therefore, $\rho_w^I \geq \rho_w^{II}$. Now consider the case $z \leq s_1$. We can again compare the coefficients of loser's signals between these two auctions, labelled as ρ_1^I, ρ_1^{II} . Combining Proposition A3, Proposition A1, and Lemma A3, we obtain that $\rho_1^I = \frac{1}{2}$ and $\frac{1}{2} \leq \rho_1^{II} \leq \frac{5}{4}$. Hence, $\rho_1^I \leq \rho_1^{II}$. \square

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