Financial Constraints and Investment-Cash Flow Sensitivities: New Research Directions*

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Abstract

A key assumption in the existing theoretical work on firm financial constraints is that these constraints translate entirely into higher *costs* of funds. This approach poses two types of difficulties for research on corporate investment. First, it inadvertently narrows our understanding about the impact of financial constraints on investment since, in practice, firms often face credit *quantity* constraints (rationing). Second, it is a matter of debate whether such an approach can deliver unambiguous implications for applied work. The current paper develops a theory explaining the relationship between investment demand and cash flows when firms face credit rationing. We show that when firms' investments and use of external finance are endogenously related, investment-cash flow sensitivities increase as credit constraints are relaxed. From an empirical perspective, our analysis suggests a consistent way of identifying the impact of financial constraints on firm investment. Our predictions, however, are markedly different from those examined in most empirical studies in this area.

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1 Introduction

A large and growing literature, starting with Fazzari, Hubbard, and Petersen [1988] (hereinafter FHP[1988]), examines the influence of financial constraints on corporate investment by looking at the empirical sensitivity of investment to cash flow.[1] The basic working assumption used in those studies is that the sensitivity of investment to cash flow should be uniformly higher for firms that face a larger wedge between the internal and external costs of funds (*monotonicity hypothesis*). For a large enough cost differential, investment spending should vary with the availability of internal funds, rather than just with the availability of positive net present value projects. Accordingly, one should be able to measure the degree of financial constraints faced by different firms by looking at cross-sectional differences in firm investment-cash flow sensitivities. Hoshi, Kashyap, and Scharfstein [1991] is a good example of such an approach. They estimate the investment-cash flow sensitivities of Japanese companies and find that firms which are associated with *keiretsu* groups have significantly lower sensitivities. The authors interpret this result as evidence that *keiretsu* firms are less financially constrained than other ordinary Japanese firms.

In recent years, the use of investment-cash flow sensitivities has become standard in the corporate finance literature. These sensitivities are one of the most important metrics used for drawing inferences about a variety of issues, such as efficiency in internal capital markets (Shin and Stulz [1998]), the effect of agency on firm investment (Hadlock [1998]), and the influence of managerial characteristics on corporate policies (Malmendier and Tate [2001] and Bertrand and Schoar [2002]), among others. Despite the interest investment-cash flow sensitivities have received, there is still an unresolved debate about the theoretical relation between those sensitivities and firms' external financing constraints. Kaplan and Zingales [1997] (KZ[1997]) have questioned the usefulness of investment-cash flow sensitivities, arguing that the monotonicity hypothesis is not a necessary implication of optimal investment under financial constraints. Their criticism is supported by empirical evidence suggesting that more (less) 'constrained' firms' investments are less (more) sensitive to changes in cash flows (see KZ[1997] and Cleary [1999]). Such findings have spurred a debate in the literature, as seen in Fazzari, Hubbard, and Petersen [2000] (FHP[2000]) and Kaplan and Zingales [2000] (KZ[2000]). FHP[2000] respond

[1] A partial list of papers in this literature includes Devereux and Schiantarelli [1990], Hoshi, Kashyap, and Scharfstein [1991], Fazzari and Petersen [1993], Calomiris and Hubbard [1995], Gilchrist and Himmelberg [1996], and Lamont [1997]. See Hubbard [1998] for a comprehensive survey.

to the KZ[1997] critique by providing conditions under which investment-cash flow sensitivities should be monotonic in financial constraints. However, KZ[2000] argue that those conditions are still insufficient to characterize a precise relationship between investment-cash flow sensitivities and financial constraints.

This paper contributes to the debate on corporate investment under financial constraints with a theory that delivers *precise* implications for the relationship between financial constraints and investment-cash flow sensitivities. The paper's ultimate goal is to move the research agenda forward, proposing new alternative avenues for studying the influence of those constraints on firm investment. Our starting observation is that the theoretical models in the existing literature (e.g., KZ [1997, 2000] and FHP [1988, 2000]) assume that financial constraints translate entirely into higher *costs* of external funds.[2] Although convenient for model-building, such an approach is less than satisfactory for most practical purposes. Real-world firms often face credit rationing, and one can argue that *quantity* financing constraints are at least as relevant as *cost* constraints. This very argument is found in a number of earlier papers on capital markets imperfections. Greenwald, Stiglitz, and Weiss [1984, p.194], for example, stress that "it is the availability of credit, not the price which they [firms] have to pay, which limits their investment."[3] This insight motivates our analysis. We develop a model in which financial constraints translate primarily into a *quantity* constraint on the amount of external funds that firms can raise. These credit constraints are endogenized in our model by conditioning a firm's ability to raise external finance on its investment spending. The endogeneity of financial constraints further sets our paper apart from the existing work in this area. Although we restrict the analysis to a detailed study of two situations in which borrowing constraints manifest themselves as a function of investment, we describe the necessary conditions for our intuition to hold more generally.[4]

Our main result can be summarized as follows. Consider examining the effect of a given cash flow shock on the investment of a cross-section of firms. Since constrained firms invest all of their internal funds, the *direct* impact of that shock on investment is similar (one-for-one) for all such firms. However, there is also an *indirect* effect associated with the income shock.

[2] Implicitly, firms can raise any amount of funds they wish so long as they pay the price.

[3] See also Stiglitz and Weiss [1981]. Consistent with this view, Petersen and Rajan's [1994, 1995] evidence on small firm financing suggests that the value of the relationship with a financial intermediary stems primarily from greater access to funds, rather than from cheaper costs of funds.

[4] Povel and Raith [2001] also revisit the theoretical foundations of investment-cash flow sensitivities. Similarly to the extant literature, however, their model focuses on the costs of external funds.

This latter effect stems from an endogenous change in borrowing capacity. For a given change in investment, the change in borrowing capacity will be greater for firms that can borrow against a higher proportion of the value of their investments. This indirect *amplification effect* drives the differences in investment-cash flow sensitivities across constrained firms in our model. Because less (more) constrained firms borrow more (less), their capacity for external finance — and thus their investment spending — is more (less) sensitive to a given cash flow innovation.[5] For sufficiently high levels of borrowing capacity and internal funds, on the other hand, the optimal (unconstrained) investment is achieved regardless of the current cash flow.

From an empirical perspective, our analysis suggests a theoretically consistent way of identifying the influence of financial constraints on corporate investment. However, the implication is *not* that investment-cash flow sensitivities should increase with the degree of financial constraints. Rather, our model predicts that sensitivities will *decrease* with financial constraints, so long as firms are not entirely unconstrained. If firms are unconstrained, then investment-cash flow sensitivities drop to zero. Our analysis thus identifies a precise relationship between sensitivities and financial constraints. Although it is always a difficult task to map the predictions of a theoretical model to observable data, we discuss a number of empirical implications of our model, and explore ideas about how they can be implemented.

While our theory supports the case for the use of investment-cash flow sensitivities as a measure of financial constraints, our predictions are inconsistent with the monotonicity hypothesis. We explain in detail how the differences between our analysis and that of FHP/KZ yield very distinct implications for investment-cash flow sensitivities. The main ideas are as follows. FHP/KZ assume that firms can raise any amount of external funds, so long as they pay the right price. A constrained firm thus essentially trades off the output from higher investment and the effect of higher investment on the deadweight costs of external funds. As a result, the comparative statics of their model depend on how financial constraints affect *both* the slope of the marginal (deadweight) cost of external funds and the slope of the marginal productivity of investment. Unfortunately, however, unless these cost and productivity functions have some specific properties, investment-cash flow sensitivities cannot be unambiguously interpreted. Our analysis, in contrast, yields an equilibrium in which the slope of the marginal cost of external

[5]The amplification effect we study is similar to the credit multiplier introduced by Kiyotaki and Moore [1997]. Our theory adds to the literature proposing that endogenous developments in the capital markets amplify and propagate relatively small shocks throughout the economy (see Bernanke, Gertler, and Gilchrist [1999]).

funds is similar for all constrained firms, and in which the slope of the capital demand curve does not matter, since it is not equal on the margin to the slope of the marginal cost of funds. Consequently, our results do not depend on sophisticated assumptions about the marginal productivity of investment and deadweight costs of finance. Finally, because in FHP/KZ new investment has no direct effect on the firm's ability to raise funds (for a given level of external finance), their analysis cannot capture the "multiplier" effect we describe in our model.[6]

We use the contracting framework of Hart and Moore [1994] to develop our basic argument about investment-cash flow sensitivities. As will become clear, the notion of inalienability of human capital naturally delivers the type of financial constraint we want to study. A specific feature of the Hart and Moore framework is that the optimal financial contract is most easily interpreted as collateralized debt. We emphasize, however, that our conclusions do not hinge on any particular element of the Hart and Moore framework. We argue that a larger class of models allowing for endogenous constraints of the type we consider will yield implications for investment-cash flow sensitivities that are identical to ours. To substantiate this claim, we consider a simple moral hazard framework (based on Holmstrom and Tirole [1997]), where the optimal financial contract can be interpreted as either debt or outside equity. We show that conducting our analysis in this framework produces results that are observationally equivalent to those of our basic model.

Our paper is related to a strand of recent literature addressing the relationship between financial constraints, investment, and cash flow. Erickson and Whited [2000], Gomes [2001], and Alti [2002] criticize the idea of running regressions of investment on cash flow and Tobin's Q. They argue that financial constraints are not necessary to generate a strong empirical relationship between cash flow and investment; essentially because Q does a poor job of controlling for investment opportunities.[7] In addition, Chirinko [1993] and Gomes [2001] argue that financial constraints might not be sufficient to generate a relationship between cash flow and investment after controlling for Q because stock prices should already reflect firm financial constraints. We note that these criticisms are primarily of an applied/empirical nature, and do not supersede the theoretical debate about whether more constrained firms should have higher sensitivities

[6] Froot, Scharfstein, and Stein [1993] show that the FHP/KZ model can be seen as a reduced-form costly state verification model in which the capacity for external finance depends on existing assets, but not on new investment. The FHP/KZ analysis therefore does not allow for any type of investment feedback effect.

[7] This point was originally made by Poterba [1988].

to cash flow shocks which are uncorrelated with investment opportunities. Our paper focuses precisely on this latter theoretical question. Moreover, as we argue below, running regressions of investment on cash flow and Q is not the only way of testing the impact of financial constraints on the cash flow sensitivity of investment.

The remainder of the paper is organized as follows. In the next section, we introduce a model in which firms face endogenous credit quantity constraints because the human capital necessary for production is inalienable. We derive predictions for investment-cash flow sensitivities, and contrast the results of our model with those of previous theoretical work. In section 3, we show that our conclusions are not particular to the inalienability set up. In section 4, we discuss some of the empirical implications of our main results. Section 5 concludes the paper.

2 The basic model

This section introduces our basic model of investment under credit rationing.[8] For now, we borrow our contracting framework from Hart and Moore [1994]. Using their framework allows us to show our main result in a simple, intuitive way. In particular, as we show next, the concept of inalienability of human capital naturally yields an endogenous credit quantity constraint as the contractual outcome. We argue in section 3, however, that our main results do not depend on the inalienability assumption.

2.1 Analysis

The model is structured as follows. Assume the firm has a production technology $f(I)$ that generates output from a physical investment I. $f(I)$ satisfies standard functional assumptions, but production only occurs if the entrepreneur inputs her human capital. By this we mean that if the entrepreneur abandons the project, only the physical investment I is left in the firm. We suppose that some amount of external financing, B, may be needed to initiate the project. Since human capital is inalienable, the entrepreneur cannot credibly commit her input to the production process. It is thus common knowledge that she may renege on any contract she

[8] The term "credit rationing" has two different definitions in the contracting literature (see Williamson [1986]). The first definition is what Stiglitz and Weiss [1981] call "pure" credit rationing: among a group of identical borrowers, some receive loans while others do not. The second, which is closest to the one we use, is studied by Tirole [2001]: a borrower might receive a smaller loan than he would like given the quoted interest rate because the borrower must retain a fraction of the cash flows generated by the investment project.

signs, forcing renegotiation at a future date. As shown in Hart and Moore [1994], the contractual outcome in this framework is such that creditors will only lend up to the expected value of the firm in liquidation.[9] This amount of credit can be sustained by a promised payment equal to the value of physical investment goods for creditors, and a covenant establishing a transfer of ownership to creditors in case the entrepreneur does not make the payment.

Let the physical goods invested by the firm have a current price equal to 1, and a price next period (the period when output is produced) equal to q. We introduce firm heterogeneity in our model by assuming that liquidation of a firm's assets by creditors entails firm-specific transaction costs that are proportional to the value of the assets. More precisely, if a firm's physical assets are seized by its creditors in the next period, a fraction $\tau \in (0, 1)$ of the proceeds qI is lost. τ is a simple way of measuring the degree of capital market imperfections in this environment. It can be seen as a function of factors such as the tangibility of the firm's physical assets and the legal environment that dictates the relations between borrowers and creditors.[10] Firms with low τ are able to borrow more because they invest in assets that are worth more to outside creditors.

The liquidation value of the assets to creditors, $(1-\tau)qI$, establishes the borrowing constraint faced by the firm:[11]

$$B \leq (1 - \tau)qI \tag{1}$$

Equation 1 exemplifies the type of financial constraint we want to study. Notice that this constraint is endogenous in nature: a firm's ability to raise investment funds from outside financiers is conditioned by the value of the new investment.

The entrepreneur chooses investment and debt in order to maximize the value of her equity in the firm, e_t, where $t = \{1, 2\}$ is the time index. Assuming that the firm has W (> 0) currently available, and that the discount rate is equal to zero, the entrepreneur solves the program:

$$\max_{I,B} \ (e_0 + e_1) \ s.t. \tag{2}$$

$$\begin{aligned} e_0 &= W - I + B \geq 0 \\ e_1 &= f(I) + qI - B \end{aligned}$$

[9] We are assuming for simplicity that the entrepreneur has all the bargaining power in the game that follows her withdrawal from the project.

[10] Myers and Rajan [1998] parametrize the liquidity of a firm's assets in a similar way.

[11] This particular borrowing constraint is discussed in Kiyotaki and Moore [1997].

$$B \leq (1-\tau)qI$$

The firm's optimal investment depends on whether the borrowing constraint is binding or not. If the borrowing constraint is not binding, the first best level of investment will obtain:

$$F'(I^{FB}) = 1 \tag{3}$$

where $F(I) \equiv f(I) + qI$. First best investment will be feasible so long as:

$$W + (1-\tau)qI^{FB} \geq I^{FB} \tag{4}$$

This expression suggests that for sufficiently high internal funds and borrowing capacity the (unconstrained) efficient level of investment is achieved. However, for a low enough W, equation 4 establishes the minimum value of τ which will lead to a constrained solution, and vice-versa. Investment will be constrained when:

$$\tau > \tau_{\min}(W) = 1 - \frac{I^{FB} - W}{qI^{FB}} \tag{5}$$

In this case, the optimal investment $I(W,\tau)$ is directly determined from the firm's budget (or credit) constraint:[12]

$$I(W,\tau) - W = (1-\tau)qI(W,\tau)$$

The model's general expression for the optimal level of investment is as follows:

$$\begin{aligned} I(W,\tau) &= \frac{W}{(1-q+\tau q)}, \text{ if } \tau > \tau_{\min}(W) \\ &= I^{FB}, \text{ otherwise} \end{aligned} \tag{6}$$

Figure 1 depicts how the level of investment is determined in our model. The figure compares the investment levels of two constrained firms that differ according to τ. Everything works as if firms had an infinitely elastic supply of funds until the equilibrium investment level $I(W,\tau)$, but a completely inelastic supply of funds after that level. As the figure illustrates, the firm with high τ (τ_H) invests less than a similar firm with lower τ (τ_L).

[12]For simplicity, we assume the price of the investment good next period is fixed. In particular, even though $\tau_{\min}(W)$ and $I(W,\tau)$ might also depend on q, we drop this dependence from our notation. We note that if the future price of the investment good is endogenized the multiplier effects we emphasize in the paper will be even larger (see Kiyotaki and Moore [1997]).

[- insert Figure 1 here -]

The investment-cash flow sensitivity of this model is given by:[13]

$$\frac{\partial I}{\partial W}(W,\tau) = \frac{1}{(1-q+\tau q)}, \text{ if } \tau > \tau_{\min}(W) \qquad (7)$$
$$= 0, \text{ if } \tau < \tau_{\min}(W)$$

2.2 Discussion

KZ[1997] discuss two ways of assessing the relative degree of financial constraints faced by firms. First, conditional on examining a cohort of constrained firms, one firm may be seen as relatively more constrained if it has fewer internal resources (W). Empirically, W should be positively (negatively) related to variables such as cash holdings (financial leverage). FHP[2000], however, argue against focusing on firm heterogeneity as measured by W. In their view, this proxy can be misleading because financial policy is endogenous to the firm.[14]

The second (less controversial) possibility is related to firms' intrinsic characteristics, which in our context are captured by the parameter τ. In this case, unlike KZ[1997], our model has precise implications for the relationship between financial constraints and investment-cash flow sensitivities. However, the relationship we obtain is *not* the one usually investigated in the investment-cash flow literature. From equation 7, one can see that investment-cash flow sensitivity will in fact be *higher* for the less constrained (low τ) firms, so long as such firms are not entirely unconstrained (i.e., so long as $\tau > \tau_{\min}(W)$).

We depict the investment-cash flow sensitivities predicted by our basic model in Figure 2. As illustrated, these sensitivities increase as τ decreases, dropping to zero when firms become unconstrained. Notice that this result agrees with the non-monotonicity argument of KZ[1997]. But in contrast to their critique, our model proposes a precise, empirically useful relationship

[13]This derivative does not exist at $\tau = \tau_{\min}(W)$. Note that since $\tau_{\min}(W)$ is continuos in W the constraint sensitivity is given exactly by the derivative of the constrained investment with respect to W. In particular, we do need to take into account the effect of W on $\tau_{\min}(W)$.

[14]For instance, a firm may have unusually high cash holdings precisely because it faces strong capital market imperfections, and thus it rationally accumulates cash as precautionary savings to avoid future costs of financial constraints (see, e.g., Calomiris, Himmelberg, and Wachtel [1995]).

between financial constraints and investment-cash flow sensitivities. This relationship, nevertheless, is markedly different from that of FHP[1988].

[- insert Figure 2 here -]

The effect we describe resembles the *credit multiplier* of Kiyotaki and Moore [1997], in which credit limits are responsible for amplifying and propagating transitory shocks. To see this, consider Figure 3, which depicts the impact of a positive cash flow innovation for two firms facing different degrees of capital market imperfections (measured by τ). The change in the availability of internal funds, ΔW, has a *direct* effect on constrained investment, which is the same for both firms (equal to ΔW). However, there is also an *indirect* effect that stems from the endogenous change in borrowing capacity (i.e., a relaxation in the credit constraint). This latter effect, which equals $(1-\tau)q\Delta I$, implies that the increase in borrowing capacity will be greater for the low τ firm. Consequently, an exogenous change in the availability of internal funds has a greater impact on the investment of the less constrained firm.

[- insert Figure 3 here -]

2.3 Why is our result different?

A key assumption of the FHP/KZ theory is that financial constraints translate entirely into higher *costs* of external funds. In contrast, in the model we study, financial constraints translate into a *quantity* constraint on the amount of funds that can be raised at a given cost. Furthermore, as we explain below, the FHP/KZ model does not allow for the endogenous change in debt capacity we analyzed in the last subsection. In this subsection, we explain how these two aspects lead to very distinct conclusions about investment-cash flow sensitivities.

The essence of FHP/KZ can be summarized as follows. Firms have an endowment of internal funds, equal to W, and a production technology $F(I)$ that satisfies ordinary assumptions. Any amount of external funds raised entails deadweight costs equal to $C(I-W,k)$. The parameter k is interpreted as a measure of the firm's wedge between internal and external costs of funds — the higher k is, the higher the deadweight costs. Firms can raise as much external finance as they wish by paying the appropriate deadweight costs $C(E,k)$. Investment is determined by solving the following problem:

$$\max_{I,E} \; F(I) - I - C(E,k) \; s.t.$$
$$I \leq W + E$$

One can show that this formulation is equivalent to a costly state verification (CSV) model in which, for a given level of external finance E $(= I - W)$, investment has no independent effect on the firm's *capacity* to raise external finance.[15] The only effect of an increase in investment (other than raising output) is to increase the deadweight *costs* of external funds. Effectively, the firm trades off the output from higher investment and the effect of higher investment on the deadweight costs of external funds. The equilibrium of FHP/KZ gives:

$$F^{'}(I^{*}) = 1 + C_k(I^{*} - W, k) \tag{8}$$

And the comparative statics are driven by changes in the marginal productivity of investment and marginal deadweight costs. As discussed in Froot, Scharfstein and Stein [1993], this result hinges on the assumption that contractual inefficiencies are such that the capacity for external finance is determined not by the cash flows of the new investment, but (entirely) by the cash flows from existing assets. This assumption essentially eliminates the investment feedback effect we describe.

In contrast to FHP/KZ, in our analysis, investment has a positive and independent effect on the capacity for external finance. This happens because capacity for external finance is endogenously determined by the level of the new investment and the degree of capital market imperfections. There are no ex-post inefficiencies in our model, and consequently no deadweight costs of external finance. The firm raises external finance by paying the opportunity cost of funds until the capacity for external finance is exhausted. The financial constraint expresses itself as a *quantity* constraint on the amount of finance the firm can raise:

$$I^{*} = W + (1-\tau)qI^{*}$$

Investment-cash flow sensitivities are driven by the fact that borrowing capacity, $(1-\tau)qI$,

[15] See Froot, Scharfstein, and Stein [1993] and Stein [2001]. In the CSV framework, the parameter k would be interpreted as a function of auditing costs; i.e., costs incurred by creditors to verify the truthfulness of the entrepreneur's report.

depends positively on the level of investment (amplification effect), and this effect is higher for less constrained firms.

Investment-Cash Flow Sensitivities

FHP/KZ measure the sensitivity of investment to cash flow across firms facing different degrees of financial constraints using the derivative $\frac{\partial^2 I}{\partial W \partial k}(W, k)$, which comes form equation 8. Figure 4 (the counterpart of Figure 3) illustrates their results by comparing the impact of similar changes in W on the investment of two firms with different k's. Investment-cash flow sensitivities depend on how financial constraints affect *both* the slope of the marginal (deadweight) costs of external funds and the slope of the marginal productivity of investment. To see how their model works, assume (as in FHP[2000]) that the marginal cost curve is steeper if firms face stronger capital market imperfections (that is, $C_{EEk} > 0$). A shock to cash flow will then have a larger impact on more constrained firms, since it is more expensive for them to adjust the amount of external funds they raise. This effect is depicted in Figure 4, where it is assumed that marginal costs and marginal productivity are linear in investment.

[- insert Figure 4 here -]

One major difficulty with this analysis concerns the robustness of its main result. To see this, simply suppose that the marginal cost curve is convex in the amount of external funds. In this case, less constrained firms are pushed into a range where the marginal costs of external finance are more sensitive to further changes in external funds. That is, shocks to cash flow will then have a *larger* impact on the investment of *less* constrained firms. Similarly, if the capital demand curve is convex in investment, then less constrained firms will also be in a range where changes in the supply of funds will have a larger effect on equilibrium investment. Since one cannot expect both the marginal cost C_E and the marginal productivity of investment $F^{'}(I)$ to be necessarily linear (or concave), the function relating investment-cash flow sensitivities to financial constraints can bear almost any shape. Investment-cash flow sensitivities cannot be unambiguously interpreted in FHP/KZ.

Remarkably, none of the effects that drive sensitivities in FHP/KZ are relevant in our analysis. This is because of our focus on quantity constraints, which sidesteps the difficulties just described. To see this, notice that the slope of the capital supply curve is the same across all firms in our model (see Figure 3). That is, all constrained firms are at the point where the supply of capital becomes inelastic, and therefore effects related to changes in the slope of the supply of capital are irrelevant. Moreover, as in any equilibrium with quantity constraints, the slope of the capital demand curve does not matter since it is not equal on the margin to the slope of the supply curve. These features of our solution do away with the need for (often strong) auxiliary assumptions in determining investment-cash flow sensitivities.[16]

3 Robustness

The particular framework which we use above revolves around the issue of inalienability of human capital. Notice, however, that the crucial features that drive our results are:

i) Firms face credit quantity constraints, and thus a firm's optimal investment is determined directly from its credit constraint:

$$I(W,\tau) = W + D[I(W,\tau),\tau]$$

where $D[I(W,\tau),\tau]$ represents the firm's capacity for external finance.

ii) $D[I(W,\tau),\tau]$ depends positively on the level of investment (amplification effect), and this effect is stronger for the less constrained firms.

Noteworthy, a large number of models in which constraints are linear on internal funds — consistent with *i*) and *ii*) — yield implications for investment-cash flow sensitivities that are identical to those we describe.[17] In other words, our results do not depend crucially on the

[16]We point out that figures 3 and 4 would suggest that one could approximate a quantity constraint using a particular cost function which becomes completely inelastic after a certain threshold. While it is difficult to provide a rationale for such (competing) cost schedule, our quantity constraint characterization gives an economically meaningful scenario for the influence of capital market imperfections on investment.

[17]Clearly, if debt capacity is given by a non-linear function of investment then investment-cash flow sensitivities will vary according to the level of investment. This will only pose a problem to our story when the constrained firm's marginal return to investment is decreasing. In this case, we are still able to show that as long as there are quantity constraints, there exists a $\tau^* < 1$, such that we have $\frac{\partial I}{\partial W}_{\tau=\tau^*} > \frac{\partial I}{\partial W}_{\tau=1}$. This means that cash flow sensitivities are always decreasing with financial constraints for some range of the parameter measuring the degree of the financial constraint. However, strict monotonicity conditions will depend on assumptions about $f^{''}(I)$.

inalienability set up. They also do not depend on the inability of the firm to raise outside equity in the Hart and Moore [1994] framework. To help substantiate this claim, we show that conducting our analysis in a simple moral hazard framework where the optimal contract can be interpreted as either debt or outside equity produces results that are observationally equivalent to those of our basic model.

3.1 The moral hazard model

To demonstrate that our results can be derived from a larger class of models we consider Holmstrom and Tirole's [1997] model of moral hazard in project choice.[18] In this set up, after an investment $I \in (0, \infty)$ is sunk, the entrepreneur chooses the probability of success of a project that is partially financed with external funds. If the project succeeds, it generates a payoff equal to RI. In case of failure, the payoff is zero. The probability of success can be either p_H or p_L (with $p_H > p_L$). If the manager chooses the probability p_L, she enjoys private benefits that are proportional to the level of investment ($= \phi I$). As in Holmstrom and Tirole, we consider the case where the project has a positive NPV only if the manager chooses the high probability of success. This is captured by the condition: $p_H R \geq 1 > p_L R + \phi$. Given constant returns to scale, the scale of the project varies freely and is subject only to financial constraints. We assume that private benefits are large enough in order to ensure that there is a finite scale with financial constraints: $1 + \frac{p_H \phi}{\Delta p} > p_H R$, where $\Delta p \equiv p_H - p_L$. We further suppose that there is an optimal scale (I^{FB}) that obtains in the absence of financial constraints.[19]

Investment is worth funding only if the contract induces the entrepreneur to maximize the project's probability of success. The optimal contract must therefore leave enough monetary benefits for the manager so that she foregoes private benefits and chooses the action associated with p_H. Since this poses a limit to the total monetary benefit that can be pledged to outside investors, credit might be rationed. To see how this happens, denote by R_e the part of the monetary income which stays with the entrepreneur, and by R_l ($= RI - R_e$) the amount which can be pledged to outside investors. For every investment level I, we must guarantee that the

[18] A similar framework is used in Holmstrom and Tirole [1998] and in Tirole [2001]. Here, we study a version of their model which assumes away monitoring by financial institutions.

[19] Clearly, one cannot differentiate constrained from unconstrained firms in this simplified version of the Holmstrom and Tirole [1997] model unless one introduces an exogenous optimal scale.

manager does not misbehave:

$$p_H R_e \geq p_L R_e + \phi I$$

The amount which can be pledged to outside investors is consequently capped:

$$R_l \leq RI - \frac{\phi I}{\Delta p}, \text{ for all } I.$$

Because of the pledgeability cap, the firm is credit rationed, and not all of its positive NPV projects are financed. Notice that external financing is positively related to the value of investment (since $R > \frac{\phi}{\Delta p}$).

In this model, the optimal contract can be interpreted either as debt or as outside equity. Under the outside equity interpretation, the firm sells some fraction x of the cash flow ($R_l = xRI$). Outside equity is feasible because RI is observable and verifiable. Under the debt interpretation, the firm raises debt of face value R_l, backed by future cash flows RI. In the discussion, we refer to outside investors as "creditors" and to the financial constraint as "credit constraint," but this does not mean that the financial contract must be interpreted as a debt contract.

Denote by W (> 0) the amount of internal funds that the firm has. In order to achieve an investment level equal to I, the entrepreneur must borrow an amount equal to $I - W$. Under limited liability, creditors will be willing to lend so long as:

$$I - W \leq p_H R_l$$

Thus, the firm can invest at the first best level I^{FB} if:

$$I^{FB} \leq W + p_H[RI^{FB} - \frac{\phi I^{FB}}{\Delta p}]$$

If this expression does not hold, the firm is constrained and will underinvest.[20] In this case, the equilibrium investment level will be determined from the credit constraint:

$$I^* = W + p_H[RI^* - \frac{\phi I^*}{\Delta p}] \tag{9}$$

Equation 9 can be manipulated to generate an expression for the investment-cash flow sensitivity in the moral hazard set up:

[20] This equation determines a minimum level of private benefits such that the firm is constrained, $\phi_{\mathsf{min}}(W)$. This value is analogous to $\tau_{\mathsf{min}}(W)$ in our basic model. Naturally, ϕ_{min} is also a function of other parameters, such as R and p_H, but we drop this dependence for simplicity of notation.

$$
\begin{aligned}
\frac{\partial I}{\partial W} &= \frac{1}{1+\frac{p_H\phi}{\Delta p}-p_H R}, \text{ if } \phi > \phi_{\min}(W) \\
&= 0, \text{if } \phi < \phi_{\min}(W)
\end{aligned}
\tag{10}
$$

This expression has properties that are identical to those of the investment-cash flow sensitivity formula obtained from the inalienability model. In particular, consider what happens when one compares two (otherwise identical) firms: one with a high ϕ, the other with a low ϕ (both higher than $\phi_{\min}$). From equation 10, one can verify that an increase in internal funds will generally increase I, but private benefits will dampen this effect relatively more in the high ϕ firm. That is, the lower the firm's ϕ (and thus the greater the firm's capacity for external finance), the higher the impact of the change in internal funds on its investment. This is identical to the effect that drives results in the inalienability model. The upshot is that we can straightforwardly derive our main results for cross-sectional differences in investment-cash flow sensitivities starting from another simple framework (in this case a moral hazard model) that features the two basic conditions specified above (in page 12). We can now discuss some of the empirical implications of our theory.

4 Empirical implications

The main implication of our model is as follows. If capacity for external finance is positively related to the value of the new investment (amplification effect) and firms are credit constrained, then investment-cash flow sensitivities increase as credit constraints are progressively relaxed because this relaxation increases the amplification effect. If firms are unconstrained, then investment is insensitive to cash flow. This establishes a precise (non-monotonic) specification for the expected relationship between sensitivities and financial constraints.

From an empirical perspective, our study suggests a theoretically consistent way of identifying the impact of financial constraints on corporate investment, at least under certain conditions. The fundamental condition is that financial market imperfections manifest themselves primarily in the form of credit quantity constraints. Given this condition, the amplification effects that we describe arise naturally because credit capacity is likely to depend on the value of investment. An additional empirical property of investment-cash flow sensitivities according to our model is that these sensitivities do not depend directly on the *level* of internal funds available. Conditional on using a sample of financially constrained firms, one can test the prediction that sensitivities are increasing in the degree of financial constraints without the need to control for variables such as cash stocks.

As we have argued, our conclusions will hold for a broad class of financing problems where investment and contractibility are utterly linked. While most of the extant empirical work on financial constraints has used some dimension of financial policy (such as dividend payout ratios) to sort constrained and unconstrained firms, our analysis suggests looking at the underlying conditions governing investment and contractibility. One example is asset specificity. The capital expenditures of otherwise similar firms may respond differently to cash flow innovations according to the ease with which those productive assets can be redeployed for alternative uses (i.e., on their expected value in liquidation). Another natural way of testing our theory is to look at data from businesses in which entrepreneurial effort has a significant influence on the productivity (or success) of investment, such is the case of the software development and biotech firms. Else the same, firms with stronger such characteristics should present lower sensitivity of investment to cash flow changes. Other avenues for empirical research along the lines of our analysis include exploring cross-sectional variations in corporate organization and charter form, as well as variations in the legal environment governing contractual rights.

A few papers have recently compared patterns in investment-cash flow sensitivities across different countries with an emphasis on the influence of country-specific institutional arrangements on corporate investment.[21] Although restricted to a small number of developed economies, those studies show strong evidence that sensitivities are higher (lower) in countries with strong (weak) legal investor protection according to the criteria of La Porta, Lopez-de-Silanes, Shleifer, and Vishny [1998]. These results agree with our intuition that investment will display greater sensitivity to changes in cash flows for firms operating in environments in which it is relatively easier to contract external finance.

Among the firm-level criteria previously used to characterize financially constrained firms, one could conjecture that firm size could capture — albeit imperfectly — the types of constraints

[21] See Bond, Elston, Mairesse, and Mulkay [1997], Hall, Mairesse, Bransteter, and Crepon [1999], and Mulkay, Hall, and Mairesse [2000].

we emphasize in our model. To our knowledge, only four published studies focus on differences in sensitivities across groups of firms in different size categories.[22] In all but one of those papers the evidence shows that smaller firms (which we presume are more credit constrained) have *significantly* lower investment-cash flow sensitivities than larger firms.[23] This evidence is consistently reported for a number of firm size measures, and across different countries and time periods.

Of course, most of the available firm-level evidence must be interpreted with caution given the results in Erickson and Whited [2000], Gomes [2001], Alti [2002], and Moyen [2002]. Since investment opportunities are very hard to measure, the common approach of regressing investment in Q and cash flow is problematic. However, as discussed by Stein [2001], running regressions of investment on cash flow and Q is not the only way to test implications of financial constraints (and other firm-specific characteristics) for cash flow sensitivities of investment and other variables. For example, one can also try to isolate shocks to the firm's internal funds which are uncorrelated to their investment opportunities. This is the approach used in papers such as Blanchard, Lopez-de-Silanes, and Shleifer [1994] and Lamont [1997].

Our results also have implications for the growing literature examining the influence of internal capital markets on corporate investment. A number of papers in this literature have used variants of traditional investment equations to determine whether or not internal capital markets are efficient (see, e.g., Shin and Stulz [1998]). One commonly tested hypothesis is that the sensitivity of diversified firms segments' investments to their own (segment-level) cash flows should be lower than that of stand-alone industry counterparts. Moreover, should internal capital markets allocate funds efficiently, segments in more "promising" industries should display even lower sensitivities. Shin and Stulz's results are consistent with the first hypothesis, but not with the second. While those authors find that segments with more profitable opportunities display higher *levels* of investment, they conclude that internal capital markets are inefficient based on investment *sensitivities*. According to our theory, however, it does not necessarily follow that higher investment-cash flow sensitivities for stand-alone firms means that they are more constrained. Rather, that the differences in sensitivities across stand-alones and subsidiary

[22] Devereux and Schiantarelli [1990] use data from UK firms, Athey and Laumas [1994] look at data from Indian firms, Gilchrist and Himmelberg [1996] use US data, and Kadapakkam, Kumar, and Riddick [1998] use data from six OECD countries, including the US.

[23] The exception is Gilchrist and Himmelberg, who, excluding very small firms form their sample, find mostly statistically insignificant differences in sensitivities across size categories.

firms might reflect the fact that the two groups of firms borrow from capital markets — external and internal, respectively — in which credit constraints manifest differently. Moreover, that the "best" subsidiaries of a constrained firm may display possibly higher investment-cash flow sensitivities can be attributed to investment feedback effects being more relevant for the divisions who invest more in the first place, rather than to allocation inefficiencies. In other words, the observed cross-segment differences in investment-cash flow sensitivities could be generated not by the inefficiency of internal capital markets, but by greater availability of credit (higher internal "borrowing" capacity) for the most promising segments.

Finally, our analysis also has implications for risk management at corporations. In our set up, the variability of cash flows leads to amplified changes in investment as financial constraints are relaxed. As shown by Froot, Scharfstein, and Stein [1993], an increase in the variability of investment will be undesirable if there are diminishing marginal returns to investment, which in turn creates a rationale for active corporate risk management. Our results suggest that risk management will be particularly important for firms with higher capacity for external finance, since variability in cash flows will be more costly when firms are less financially constrained. The empirical evidence provided in Nance, Smith, and Smithson [1993] is consistent with such prediction.

5 Conclusions

This paper develops a theory explaining the relationship between corporate investment and cash flow when firms face credit quantity constraints. Our model describes a robust, precise relationship between investment-cash flow sensitivities and financial constraints. However, in contrast to the established view, the derived implication is *not* that investment-cash flow sensitivities increase with the degree of financial constraints. Rather, we predict that sensitivities will *decrease* with financial constraints, so long as firms are not entirely unconstrained. This result is driven by our focus on credit quantity constraints, and by the endogenous amplification effects we allow for in our model. Both of these considerations are absent from previous theoretical work on investment-cash flow sensitivities.

From an empirical perspective, our results suggest a theoretically consistent way of identifying the impact of financial constraints on investment behavior, at least under certain conditions.

We show that if financial constraints primarily affect credit quantity constraints on firms, then investment-cash flow sensitivities are indeed useful measures of financial constraints. Overall, we believe that the main contribution of our paper is to suggest new alternative directions for studying corporate investment in imperfect capital markets.

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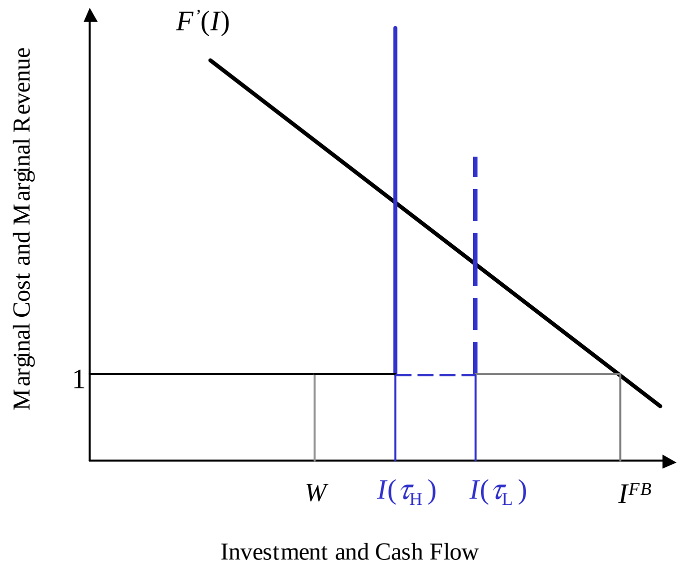

Figure 1 - Equilibrium investment

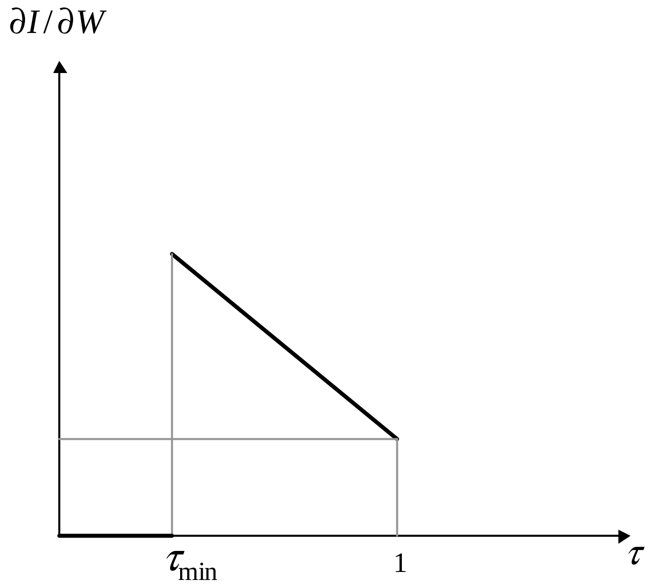

Figure 2 - Investment-cash flow sensitivity and financial constraints

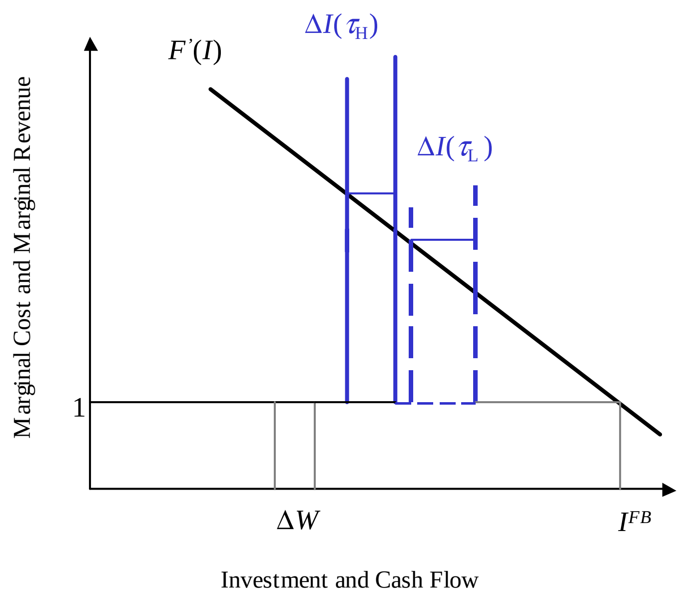

Figure 3 - A cash flow shock in the baseline model

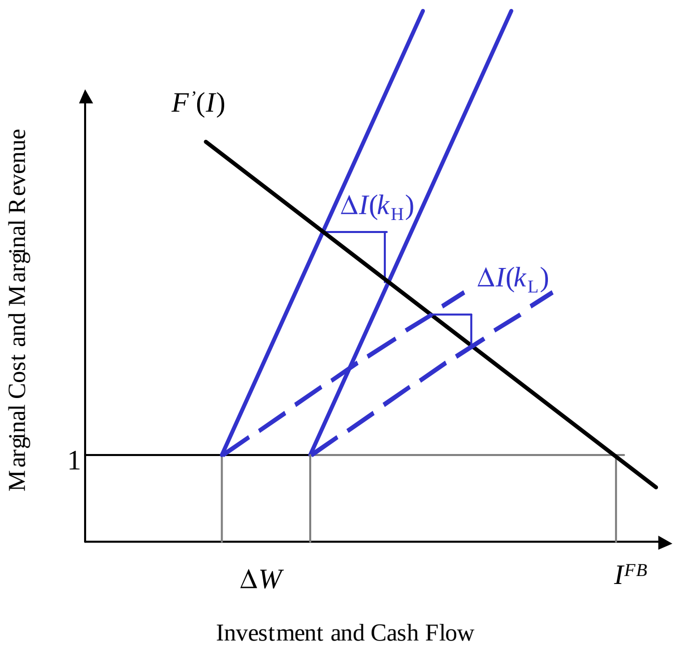

Figure 4 - A cash flow shock in the FHP/KZ model