Ratings Quality over the Business Cycle*

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Abstract

The reduced accuracy of credit ratings on structured finance products in the boom just preceding the financial crisis has prompted investigation into the business of Credit Rating Agencies (CRAs). While CRAs have long held that reputational concerns discipline their behavior, the value of reputation depends on economic fundamentals that vary over the business cycle. We analyze a dynamic model of ratings where reputation is endogenous and the market environment may vary over time. We find that ratings accuracy is countercyclical. Specifically, a CRA is more likely to issue less-accurate ratings when income from fees is high, competition in the labor market for analysts is tough, and default probabilities for the securities rated are low. Persistence in economic conditions can diminish our results, while mean reversion exacerbates them. The presence of naive investors reduces overall accuracy, but ratings accuracy remains countercyclical. Finally, we demonstrate that competition among CRAs yields similar qualitative results.

Keywords: Credit rating agencies, reputation, ratings accuracy

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1 Introduction

The current financial crisis has prompted an examination of the role of credit rating agencies (CRAs). With the rise of structured finance products, the agencies rapidly expanded their ratings business and earned dramatically higher profits (Moody's, for example, tripled its profits between 2002 and 2006). Yet ratings quality seems to have suffered, as the three main agencies increasingly gave top ratings to structured finance products shortly before the financial markets collapsed. This type of behavior has been brought to the public's (and regulators') attention many times, such as during the East Asian Financial Crisis (1997) and the failures of Enron (2001) and Worldcom (2002). Beyond the issue of why the CRAs were off-target, these repeated instances raise the question of *when* the CRAs are more likely to be off-target.

In this paper, we examine theoretically how the incentives of CRAs to provide quality ratings change in different economic environments, specifically in the booms and recessions of business cycles. Our analysis highlights that both the effective costs of providing high-quality ratings and the benefits to the CRA of doing so vary through the business cycle. Specifically, we show that reputational incentives lead naturally to countercyclical ratings quality.

Several economic fundamentals suggest that ratings quality is lower in booms and improves in recessions. First, consider that a CRA's primary expenditure is for skilled human capital. In boom periods, the outside options of current and prospective employees improve substantially, making it more difficult and expensive for a CRA to maintain the same quality of analyst resources.[1] Next, boom periods are likely to be associated with higher revenues for a CRA, both directly—through higher volume of issues and, perhaps, through higher fees—and indirectly—through advisory and other ancillary services. Lastly, if issues are relatively unlikely to default in boom periods, monitoring a CRA's activities is less effective, and the CRA's returns from investing in ratings quality are likely to be diminished. Default probabilities may actually be higher towards the end of a boom if lower-quality issuers seek to be rated. Our analysis allows us to understand the impact of each of these economic fundamentals individually.

[1] For example: "At the height of the mortgage boom, companies like Goldman offered million-dollar pay packages to workers like Mr. Yukawa who had been working at much lower pay at the rating agencies, according to several former workers at the agencies. Around the same time that Mr. Yukawa left Fitch, three other analysts in his unit also joined financial companies like Deutsche Bank." This is from "Prosecutors Ask if 8 Banks Duped Rating Agencies," by L. Story, *New York Times*, May 12, 2010.

We formalize these intuitions in a simple model of ratings reputation. We construct an infinite-period model where a CRA chooses in each period how much to spend on the accuracy of its ratings by hiring better analysts. The CRA continues to receive fees from issuers as long as it maintains its reputation with investors, who withdraw their business only after an investment with a good rating defaults.

In our baseline result, where future shocks to economic fundamentals are iid draws from a probability distribution, we find that ratings quality is countercyclical. We then examine the correlation between shocks in different periods. Our findings may be diminished when there is substantial persistence in shocks (positive correlation), but may actually be exacerbated when there is mean-reversion in shocks (negative correlation). When we introduce naive customers, that reduces accuracy in both booms and recessions, but does not change the comparison between the states. We also extend the model to allow for competition between CRAs and we demonstrate that similar results hold.

The idea that ratings quality may be countercyclical is consistent with recent empirical work on the market for structured finance products. As a relatively new market for hard-to-evaluate investments, the structured finance market opened up the possibility for accuracy and reputation management by CRAs. Ashcraft, Goldsmith-Pinkham, and Vickery (2010) show that the mortgage-backed security-issuance boom from 2005 to mid-2007 led to ratings quality declines. Griffin and Tang (2012) demonstrate that CRAs made mostly positive adjustments to their models' predictions of credit quality and that the amount adjusted increased substantially from 2003 to 2007. These adjustments were positively related to future downgrades.

Our results are relevant to the current policy debate regarding the role of CRAs. We show that if reputation losses are higher, there are greater incentives to provide accurate ratings. Recent SEC rules promoting full disclosure of ratings history can make it easier for investors to know when a CRA is performing poorly and to punish it. The Dodd-Frank financial reform bill makes CRAs more exposed to liability claims for poor performance.[2] This may give the investors a stick to make punishment credible.

White (2010) highlights the role that regulation has played in enhancing the importance and market power of the three major rating agencies (by granting them a special status and

[2] The higher standard of liability for CRAs required by the Dodd-Frank bill has not been enforced, as an initial attempt to do so caused CRAs to pull their ratings from asset-backed securities, freezing the market. The SEC decided, in response, to delay implementation for further study, and there is discussion about eliminating the requirement (see Jessica Holzer, "House Panel Votes to Free Raters From ABS Liability," *Wall Street Journal* Online, Jul 20, 2011).

having capital and investment requirements tied to ratings). Given the "protected" position of these agencies,[3] the reputational concerns that constrain CRAs' behavior should be understood somewhat more broadly than the reduced-form approach taken in our model.[4] The model views these reputational concerns as arising from investors' withdrawal of their business from products rated by the CRA. Although this might appear stark, it may apply well to innovative financial instruments, which have been the focus of public and policy concerns. Indeed, the structured finance market (and the need for ratings) dried up as the crisis hit, and stock market valuations for Moody's fell significantly. In addition, concerns regarding a regulatory environment that is relatively more or less sympathetic to the CRAs may also determine the CRAs' reputational incentives. Lastly, although something similar has not occurred in the recent crisis, the downfall of Arthur Andersen represents a severe punishment to a certification intermediary in a similar business line (auditing).

Our paper may also be cast more broadly in terms of incentives for certification intermediaries who are paid by those seeking certification. In this light, evidence on equity analysts appears consistent with our model. Michaely and Womack (1999) and Lin and McNichols (1998) find substantial evidence of biased recommendations when analysts' employers had underwriting relationships with the firms being analyzed. Jackson (2005) shows that both optimistic recommendations and better reputation (in the form of a higher ranking in an investor survey) generate higher trading volumes with the analyst's employer. Hong and Kubik (2003) show that accuracy becomes less important (and optimism somewhat more important) for equity analysts moving up the job hierarchy in boom times.[5]

In the following subsection, we review related theoretical work. In Section 2, we set up the model and analyze the case of a monopoly CRA. In Section 3, we study the duopoly case. In Section 4, we formulate the predictions of the model as hypotheses and examine support from recent empirical work on ratings. Section 5 concludes. Unless indicated that the proof is in the Online Appendix, all proofs are in the Appendix.

1.1 Related Theoretical Literature

Mathis, McAndrews, and Rochet (2009) is the closest paper to this one in examining how a CRA's concern for its reputation affects its ratings quality. They present a dynamic

[3] The Dodd-Frank bill and current rulemaking by the SEC will most likely diminish regulatory barriers to entry.

[4] And in related models of endogenous reputation, such as Mathis, McAndrews and Rochet (2009).

[5] Specifically, they compare accuracy in the 1996-2000 boom period with accuracy in the period 1986-1995.

model of reputation where a monopolist CRA may mix between lying and truthtelling to build up/exploit its reputation. The authors focus on whether an equilibrium in which the CRA tells the truth in every period exists and demonstrate that truthtelling incentives are weaker when the CRA has more business from rating complex products.[6] Strausz (2005) is similar in structure to Mathis et al. (2009), but examines information intermediaries in general. Our model considers a richer environment in which CRA incentives are linked to a broad set of economic fundamentals that fluctuate and may persist through time. Our paper also examines competition and develops a connection with labor-market conditions.

Our model also builds on and further develops the understanding of firm behavior in business cycles. Several papers analyze how firms maintain collusive behavior through the business cycle, while we analyze incentives to build up or milk reputation. Rotemberg and Saloner (1986) and Dal Bo (2007) consider future states to be iid draws from a known distribution. Haltiwanger and Harrington (1991) consider a deterministic business cycle. Bagwell and Staiger (1997) and Kandori (1991) add correlation between periods, as we do in our model.

In addition to Mathis et al. (2009), there are several other recent theoretical papers on CRAs. Faure-Grimaud, Peyrache and Quesada (2009) look at corporate governance ratings in a market with truthful CRAs and rational investors. They show that issuers may prefer to suppress their ratings if they are too noisy. They also find that competition between rating agencies can result in less information disclosure. Mariano (2012) considers how reputation disciplines a CRA's use of private information when public information is also available. Fulghieri, Strobl and Xia (2011) focus on the effect of unsolicited ratings on CRA and issuer incentives. Bolton, Freixas, and Shapiro (2012) demonstrate that competition among CRAs may reduce welfare due to shopping by issuers. Conflicts of interest for CRAs may be higher when exogenous reputation costs are lower and there are more naïve investors. Skreta and Veldkamp (2009) and Sangiorgi, Sokobin and Spatt (2009) assume that CRAs relay their information truthfully, and they demonstrate how noisier information creates more opportunity for issuers to take advantage of a naive clientele through shopping. Bouvard and Levy (2010) examine the two-sided nature of CRA reputation. In Pagano and Volpin (2010), CRAs also have no conflicts of interest, but can choose ratings to be more or less opaque depending on what the issuer asks for. They show that opacity can enhance

[6] Mathis et al. (2009) provide examples of reputation cycles where the CRA's reputational incentives fluctuate, depending on the current level of reputation. These are not related to economic fundamentals of the business cycle, as they are in our model.

liquidity in the primary market but may cause a market freeze in the secondary market. Winton and Yeramilli (2011) study a related market, that of originate-to-distribute lending, in a dynamic model where reputation may discipline monitoring incentives.

Our model probes the interaction between the business cycle and incentives. Bond and Glode (2011) analyze a problem where individuals may become bankers or regulators (of bankers). They find that in booms, banks grab the most-talented regulators, making the regulation of the system more fragile. In this paper, this type of cream-skimming by banks from CRAs forms the basis of our observations on the analyst labor market. Relatedly, Bar-Isaac and Shapiro (2011) examine incentives for analysts at CRAs and find that CRA accuracy is non-monotonic in the probability that analysts have outside offers from banks; it increases at first because of more effort from the analysts, but then may decrease due to lower CRA training incentives. Povel, Singh, and Winton (2007) study firms' incentives to commit fraud when investors may engage in costly monitoring, and find that fraud is more likely to occur in good times.

2 Monopoly

We begin by presenting a model with a single CRA and many issuers and investors who can interact over an infinite number of discrete periods.[7] Economic fundamentals change from period to period. We suppose that there are two states $s \in \{R, B\}$, R corresponding to a recessionary period and B to a boom. We specify the difference in the two states after defining the model.

Each period, an issuer has a new investment, which can be good (G) or bad (B). A good investment never defaults and pays out 1, while a bad investment defaults with probability p_s. If it defaults, its payout is zero; otherwise, its payout is 1. The probability that an investment is good is λ_s. The issuer has no private information about the investment. This implies that the CRA can have a welfare-increasing role of information production by identifying the quality of the investment. Both the issuer and the investors observe the ratings and performance of the investment.

The issuer approaches the CRA at the beginning of the period to evaluate its investment. If the CRA gives a good rating, the issuer pays the CRA an amount π_s. The CRA is not paid for bad ratings. This is a version of the shopping effect described in Bolton, Freixas, and Shapiro (2012) and Skreta and Veldkamp (2009). Mathis, McAndrews, and Rochet (2009) assume that no issue takes place if the rating is bad and that the CRA is

[7] Issuers and investors may be long- or short-lived in the model, whereas the CRA is long-lived.

not paid in this case, which is equivalent to our approach.

Our focus is on the CRA's ratings policy—i.e., how they invest in increasing the likelihood that their analyses are correct. We model this as a direct cost to the CRA for improving its accuracy. There is no direct conflict of interest, as in Bolton, Freixas, and Shapiro (2012), and we remain agnostic about whether CRAs intentionally produce worse quality ratings. In our model, increasing rating quality is costly, and the CRA maximizes profits given the reality of the business environment.

The cost that the CRA pays for accurate ratings could represent improving analytical models and computing power; performing due diligence on the underlying assets; the staffing resources allocated to ratings; or hiring and retaining better analysts. For the sake of concreteness, we will focus on the employment channel: Hiring better analysts is more costly to the CRA.[8] Investors cannot directly observe the CRA's policy, but must infer it from their equilibrium expectations and from their previous observations of defaults on rated investments.

We model the analyst labor market in a reduced-form manner. In a given period, a CRA pays a wage $w_s \in [0, \overline{w}]$ to get an analyst of ability $z(w_s, \gamma_s) \in [0, 1]$, where γ_s is a parameter that captures labor-market conditions.[9] When there is no confusion, we suppress the arguments and write ability as z_s. We suppose that it is harder to attract and retain higher-ability analysts and that it becomes even harder at the top end of the wage distribution, meaning that $\frac{\partial z}{\partial w_s} > 0$ and $\frac{\partial z^2}{\partial w_s^2} < 0$. We also assume that $\frac{\partial z}{\partial w_s} \to \infty$ as $w_s \to 0$, $z(0, \gamma_s) = 0$, and $\frac{\partial z}{\partial w_s}|_{\overline{w}} = 0$. With respect to the labor-market conditions, we suppose that when γ_s is larger, the labor market is tighter, and it is more difficult to get high-quality workers, so that $\frac{\partial z}{\partial \gamma_s} < 0$ and $\frac{\partial^2 z}{\partial \gamma_s \partial w_s} < 0$. This implies that a higher wage must be paid in order to maintain quality.

Ability is important for gathering information and figuring out whether the investment is good or bad. All analysts can identify a good investment perfectly ($p(G|G) = 1$). They

[8] While there is no empirical work on CRA staffing, internal emails uncovered by the Senate Permanent Subcomittee on Investigations (2010) shed light on the CRAs' staffing situation right before the recent crisis. For example, a Standard & Poor's employee wrote on 10/31/2006: "While I realize that our revenues and client service numbers don't indicate any ill [e]ffects from our severe understaffing situation, I am more concerned than ever that we are on a downward spiral of morale, analytical leadership/quality and client service."

[9] Note that, here, the wage w is the wage per issue rated and, so, lower quality might reflect either a less able analyst or an analyst (of equal quality) who spends less time on a rating. This latter interpretation is also present in Khanna, Noe, and Sonti (2008), who demonstrate in a static setting that IPO screening may suffer when the demand for financing increases due to a fixed supply of labor available to underwriters.

may, however, make an error about the bad investment with positive probability $1-z$, where $p(B|B) = z$. Therefore, the CRA, through its wage, is choosing its tolerance for mistakes based on both the costs of hiring *and* the incentives for accuracy that are embedded in the dynamics of the model.[10]

These incentives for accuracy arise since we assume that if investors suspect that the CRA is not investing sufficiently in ratings quality (say, $z < \overline{z}$), then they would not purchase the investment product; however, if investors believe that the CRA has hired sufficiently good analysts ($z \geq \overline{z}$), then the rating is of sufficient quality to lead investors to purchase. The cutoff $\overline{z}$ is exogenous here, but it represents the investor's decision to allocate money to this investment as opposed to other opportunities; that is, it could be derived from a participation-constraint or portfolio-allocation problem for the investor.[11] We suppose throughout, that while the CRA maintains its reputation, the constraint $z \geq \overline{z}$ does not bind. Trivially, if the constraint is ever violated, then investors would not purchase and so issuers would not seek ratings; in such circumstances, the CRA would not be active.

As in any infinitely repeated game, there are many equilibria. We focus on the equilibrium in which the CRA is most likely to report honestly or, equivalently, minimize mistakes: i.e., the equilibrium supported by grim-trigger strategies (see Abreu, 1986). Issuers and investors observe only three states: a good report where the investment returns 1; a bad report; and a good report where the investment defaults. A grim-trigger strategy here is that investors never purchase an investment rated by a CRA that had previously produced a good report for an investment that subsequently defaulted. This grim outcome is an equilibrium in the continuation game since, if investors do not purchase, then it is optimal for the CRA to set $w = 0$ so that $z < \overline{z}$. Moreover, this equilibrium of the infinitely repeated game has a natural interpretation corresponding to reputation. As in the seminal work of Klein and Leffler (1981), and developed in a wide-ranging literature discussed in Section 4 of Bar-Isaac and Tadelis (2008), the CRA sustains its reputation as long as it is not found to give a good rating to a bad investment, but loses its reputation if it is ever found to do so.

We distinguish between booms and recessions by assuming throughout the paper that booms involve higher fees ($\pi_B > \pi_R$), tighter labor-market competition ($\gamma_B > \gamma_R$), a

[10] While we have a very simplistic rating structure, we are able to capture the idea that ratings may be inflated—i.e., risky investments receive a stamp of being less risky.

[11] In order to endogenize $\overline{z}$, we would have to consider the investor's problem explicitly and introduce additional parameters to capture their preferences and opportunities. Note that although these may vary according to the business cycle, it is sufficient to consider $\overline{z}$ as the highest threshold in any state.

greater proportion of good projects ($\lambda_B > \lambda_R$), and lower probabilities of default ($p_B < p_R$).[12]

The changes in fundamentals may be correlated: positive correlation, which implies that a boom is more likely to be followed by a boom, or negative correlation, where a boom is likely to revert to a recession. Define τ_s as the probability that there is a transition from the current state s to the other state. Note that both τ_B and $1 - \tau_R$ represent the probabilities of moving to a recessionary state in the next period (when starting from the boom and recessionary states, respectively). When $\tau_B = 1 - \tau_R$, each period's state is an independent and identically distributed draw from the same distribution. When $\tau_B < 1 - \tau_R$, there is persistence or positive correlation among states: A boom state is more likely to follow a boom state than a recessionary state, and a recessionary state is more likely to follow a recessionary state. When $\tau_B > 1 - \tau_R$, there is reversion to the mean or negative correlation among states. These transition probabilities are related to the duration of a boom or recession: A higher value of τ_s implies a shorter duration for the state s and a rapid move towards the other state.

2.1 Analysis

The CRA chooses only the current wage, and takes the continuation values as given. We assume that investors anticipate that wages are high enough in each state such that they would purchase after observing a good rating; that is, $z(w_s, \gamma_s) > \overline{z}$ for $s = R$ or B. These conditions can be verified after characterizing the equilibrium wages w_R^* and w_B^*.

We now consider a value function for each state:

$$V_B = \max_{w_B} \pi_B(\lambda_B + (1-\lambda_B)(1-z_B)) - w_B + \delta(1-(1-\lambda_B)(1-z_B)p_B)((1-\tau_B)V_B + \tau_B V_R).$$
$$V_R = \max_{w_R} \pi_R(\lambda_R + (1-\lambda_R)(1-z_R)) - w_R + \delta(1-(1-\lambda_R)(1-z_R)p_R)((1-\tau_R)V_R + \tau_R V_B). \quad (1)$$

The CRA pays the wage w_s and earns the fee whenever it reports a good project, which occurs when the project is good (with probability λ_s) or when the project is bad, and it is misreported (that is, with probability $(1-\lambda_s)(1-z_s)$). The probability that the project is bad, the agency misreports, and the project defaults is $(1-\lambda_s)(1-z_s)p_s$; then, in the continuation, no issuer returns to the CRA (anticipating that the CRA would set

[12] In reality, λ_s may actually be decreasing, and p_s may be increasing at some point if booms attract lower-quality issuers or investments to get ratings. These cases can be analyzed easily in our model, given our comparative statics results.

$w_s = 0$), and the CRA's continuation value is 0. Otherwise, the CRA earns the expected continuation value $(1 - \tau_s)V_s + \tau_s V_{-s}$.

We denote equilibrium values with an asterisk (*). We begin by proving the existence and uniqueness of a solution.

Lemma 1 *There exists a unique solution* (V_B^*, V_R^*) *with associated* w_B^* *and* w_R^* *to the system of equations (1).*

We are interested in the difference between accuracy during booms and during recessions. We begin by writing the first-order conditions for the decision variables w_B and w_R, respectively:

$$\frac{\partial z}{\partial w}(w_B^*, \gamma_B) = \frac{1}{1-\lambda_B} \frac{1}{\delta p_B((1-\tau_B)V_B^* + \tau_B V_R^*) - \pi_B}. \tag{2}$$

$$\frac{\partial z}{\partial w}(w_R^*, \gamma_R) = \frac{1}{1-\lambda_R} \frac{1}{\delta p_R((1-\tau_R)V_R^* + \tau_R V_B^*) - \pi_R}. \tag{3}$$

We distinguish between booms and recessions by suggesting that booms involve higher fees, a greater proportion of good projects, lower probabilities of default, and tighter labor-market competition. While it seems natural that the first three effects suggest that it is more valuable to be in a boom than in a recession, so that $V_B^* > V_R^*$, the last force might act in the opposite direction (i.e., if it is sufficiently expensive to hire labor in the boom, then the CRA may actually prefer to be in a recession). Considering transition probabilities, if booms are worth more, then transitioning more often to booms from a boom increases the relative value of being in a boom, while transitioning more often from a recession to a recession decreases the relative value of being in a recession.

These intuitions are formalized in the Proposition below:

Proposition 1 *The difference between the value of being in a boom rather than in a recession* ($V_B^* - V_R^*$)*:*

(i) decreases in the probability of default in a boom (p_B) *and the competitiveness of labor-market conditions* (γ_B) *and increases in the proportion of good projects* (λ_B) *and the fee* (π_B)*;*

(ii) increases in the probability of default in a recession (p_R) *and the competitiveness of labor-market conditions* (γ_R) *and decreases in the proportion of good projects* (λ_R) *and the fee* (π_R)*;*

(iii) decreases in the probability of transitioning from a boom to a recession (τ_B) and increases in the probability of transitioning from a boom to a recession (τ_R) if and only if it is more valuable to be in the boom state ($V_B^* > V_R^*$).

In general, the comparison between V_B^* and V_R^* is ambiguous. However, as $V_B^* > V_R^*$ seems to be the interesting (and intuitive) case, we assume this to be true for presentation purposes throughout the remainder of the paper.[13] Although this is an assumption on endogenous values, it trivially holds where γ_B and γ_R are close enough and we order the other fundamentals according to the ordering assumed above.

Assumption A1: The value to a CRA of being in a boom is larger than the value of being in a recession ($V_B^* > V_R^*$)

We now examine how accuracy compares in booms and recessions. Define continuation values from the boom and recession states, respectively, as:

$$EV_B^* := (1 - \tau_B)V_B^* + \tau_B V_R^*, \text{ and} \tag{4}$$

$$EV_R^* := (1 - \tau_R)V_R^* + \tau_R V_B^*. \tag{5}$$

Given the first-order conditions (2) and (3), it follows that $w_B^* \leq w_R^*$ and there is more accuracy in recessions than in booms when:

$$(1 - \lambda_B)(\delta p_B EV_B^* - \pi_B) \leq (1 - \lambda_R)(\delta p_R EV_R^* - \pi_R). \tag{6}$$

As we stated earlier, when $\tau_B = 1 - \tau_R$, each period's state is an iid draw from the same distribution. This implies that the continuation values from a boom and from a recession are identical, $EV_B^* = EV_R^*$; i.e., the likelihood of transitioning to a boom (or a recession) is the same irrespective of whether there is a recession or boom today. In this case, we get a very strong result: Ratings quality is lower in boom states than in recessionary states.

Proposition 2 *If states are independent across time ($\tau_B = 1 - \tau_R$), then there is more investment in ratings quality in a recession than in a boom.*

As the CRA treats the future as fixed when shocks are iid, only the current incentive to milk reputation varies between the different states of the economy. Therefore, the higher

[13] Results on the opposite case ($V_B^* < V_R^*$) can be summarized easily, given the proofs in the Appendix.

fees that arise in a boom mean that the CRA wants to be less accurate to collect more fees. A similar logic holds with the proportion of good investments. Lower default probabilities imply a lower likelihood of getting caught for reduced accuracy, while a tighter labor market means that hiring good analysts is more costly. All of these point to lower accuracy in boom states.

If booms and recessions do not arise independently of history, then Proposition 2 cannot be applied directly. Condition (6) is not necessarily easy to verify since the continuation values EV_B^* and EV_R^* are endogenously determined. However, given Assumption A1, we can state the following:

Proposition 3 *If there is negative correlation between states, then there is more investment in ratings quality in a recession than in a boom.*

This is a direct result of Condition (6). Negative correlation or mean reversion implies that the future expected value when in a recession is larger than that in a boom because of the increased likelihood of transitioning to the boom. In the recession, the CRA builds up its reputation so as to reap the benefits of the approaching boom. In the boom, the incentive is to milk reputation since the recession is likely to come soon. This, then, implies that there are more-accurate ratings in a recession than in a boom.

In the case of positive correlation, we find that ratings may also be countercyclical. Formally, Condition (6) is slack when states are independent across time, suggesting that at least for “small” levels of positive correlation, the condition would not be violated and that ratings are countercyclical. Numerical simulations suggest that the range in which countercyclical ratings arise can be significant, as illustrated in Figure 1.

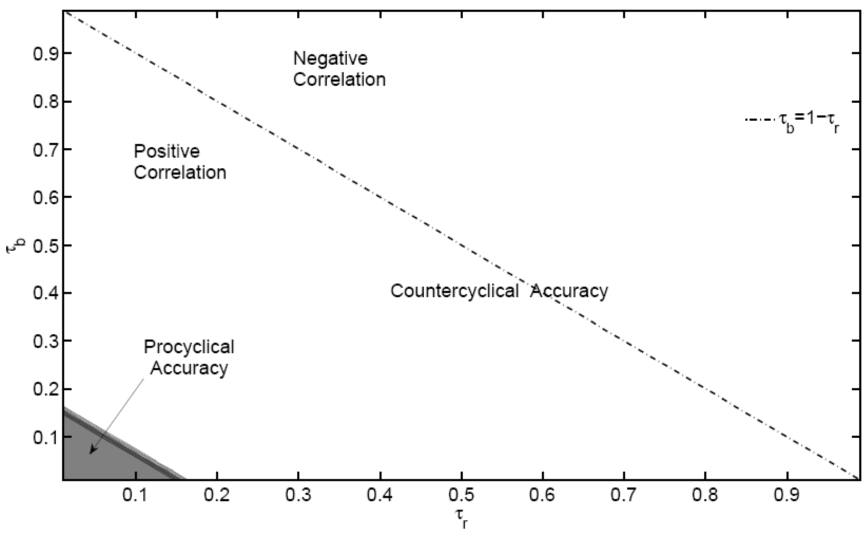

Figure 1: Countercyclical or Procyclical Accuracy as a Function of Correlation of Shocks over Time (parameters used: $\delta = .95$, $p_B = p_R = .5$, $\lambda_B = \lambda_R = .5$, $\gamma_B = \gamma_R = .5$, $\pi_B = .5$, $\pi_R = .25$; $z(w,\gamma) = \frac{\sqrt{w}}{\gamma}$).

To better understand the implications of correlation, we now study continuous changes in correlation between states (increasing/decreasing the amount of correlation):

Proposition 4 *(i) Decreasing the probability of transitioning from boom to recessionary states (reducing τ_B) increases investment in ratings quality in the boom state (w_B^*) and in the recessionary state (w_R^*)*

(ii) Decreasing the probability of transitioning from recessionary to boom states (reducing τ_R) decreases investment in ratings quality in the boom state (w_B^*) and in the recessionary state (w_R^*)

Decreasing the probability of transitioning from booms to recessions (or, equivalently, increasing the duration of booms) increases ratings quality in both states. In the boom, there is less likelihood that the good times will end soon, meaning that there is less desire to milk reputation. In the recession, the payoff of a transition to a boom increases, meaning that it is a good time to build up reputation. For analogous reasons, increasing the duration of recessions has the reverse effect.

Turning next to changing persistence or mean reversion in *both* states (specifically, changing the transition probabilities from both states equally), Proposition 4 suggests two contradictory effects. First, decreasing the persistence of a boom state (increasing τ_B) reduces ratings quality in both the boom and the recession, but decreasing the persistence of a recessionary state (increasing τ_R) increases ratings quality. As shown in Proposition 5, either effect can dominate. Intuitively, the effect through the change in τ_B is likely to dominate if the CRA is very often in the boom state (this is likely the case when τ_B is low and τ_R is high), and vice versa.

Proposition 5 *Decreasing the persistence of states (equivalently, increasing mean reversion) equally (increasing τ_B and τ_R by the same amount):*

(i) increases investment in ratings quality in the boom state if and only if $\tau_B - \tau_R > \frac{1}{\delta(1-(1-\lambda_R)(1-z_R)p_R)} - 1$; and

(ii) increases investment in ratings quality in the recessionary state if and only if $\tau_B - \tau_R > 1 - \frac{1}{\delta(1-(1-\lambda_R)(1-z_R)p_R)}$.

Note that $\frac{1}{\delta(1-(1-\lambda_R)(1-z_R)p_R)} - 1 > 0 > 1 - \frac{1}{\delta(1-(1-\lambda_R)(1-z_R)p_R)}$, and so it is never the case that decreasing persistence of states equally can lead to an increase of ratings quality in booms and a decrease of ratings quality in recessions; however, other combinations of outcomes can arise, depending on parameters.

In the special case that booms and recessions are of the same duration ($\tau_B = \tau_R$), or sufficiently close, we can obtain a more definitive result:

Corollary 1 *If booms and recessions are of the same duration ($\tau_B = \tau_R$), then decreasing the persistence of states (equivalently, increasing mean reversion) equally (decreasing τ_B and τ_R by the same amount) decreases investment in ratings quality in the boom state (w_B^* decreases) and increases investment in ratings quality in the recessionary state (w_R^* increases).*

Therefore, starting from a benchmark where booms and recessions are of similar duration, increasing persistence diminishes the result that ratings quality is lower in a boom than in a recession. On the other hand, decreasing persistence (or increasing mean reversion) exacerbates the result.

The results in this section and subsequent results below on the countercyclicality of ratings provide insight into the recent crisis and past events. In helping to explain what occurred, it might suggest limits to some of the proposed solutions—in particular, the claims of the CRAs that reputational incentives can be strong enough to preclude the need for any regulatory intervention. It is worth noting that we do not undertake a full welfare analysis in this paper; doing so would require fully modelling investor utility. However, it is interesting to observe that some of the forces highlighted in the analysis of CRA incentives might also suggest that from a welfare perspective, counter-cyclical ratings quality may be beneficial. For example, if analysts are much more expensive in a boom then from a welfare perspective it may be optimal to hire analysts of lower quality in a boom than in a recession.

For empirical work, it may be interesting to characterize default probabilities for rated products. Note that the probability that a product is rated is given by $\lambda_s + (1 - \lambda_s)(1 - z_s)$, and so the expected probability of default is given by $\frac{(1-\lambda_s)(1-z_s)p_s}{\lambda_s+(1-\lambda_s)(1-z_s)}$. Since this probability is monotonically decreasing in z_s, an increase in fees, π_s, or in the competitiveness of the labor market, γ_s, increases the probability of default for rated products. For the fraction of good projects, λ_s, and the likelihood that a bad project defaults, p_s, there are both direct effects on this default probability and indirect effects through the firm's hiring (w_s, which, in turn, affects z_s). These act in opposite directions, so their overall effect is ambiguous.

2.2 Naive Investors

Some of the potential investors in rated issues are sophisticated: They understand a CRA's incentives and rationally withdraw their business when evidence of poor rating quality persists. However, it is likely that a fraction of investors are naive, in that they are willing to buy investments with good ratings irrespective of the quality of the ratings. This may be due to poor incentives to do due diligence, as some have alleged is the case for pension fund managers whose compensation only marginally depends on the ex-post return of the assets they manage. Moreover, the complexity of some investments make it more costly to evaluate their worth. Regulation that forces managers to purchase only investments with good ratings could also provide incentives to be trusting.[14] In this section, we study the

[14]In a study of the CRA credit watch mechanism, Boot, Milbourn, and Schmeits (2006) model investors who take ratings at face value, calling them institutional investors. Similarly, Hirshleifer and Teoh (2003) model investors with "limited attention and processing power." Skreta and Veldkamp (2009) model naive investors as not realizing the selection bias that shopping for rating agencies induces. In Bolton, Freixas, and Shapiro (2012), naive investors also take ratings at face value, but punish rating agencies when evidence

effect of incorporating naive investors into our dynamic model.

We model naive investors as being willing to invest in products with good ratings irrespective of any evidence, such as defaults, on poor accuracy. This will generally impede the reputation mechanism, as a fraction of well-rated products have a guaranteed market. The proportion of fees that CRAs generate from good ratings given to issuers who sell to naive investors will be denoted by ω. We will assume that this proportion is constant across states for simplicity.[15] We define $\bar{V}_s$ as the continuation value for the CRA in state s when the CRA has lost its sophisticated customers but retains its naive customers. In this subgame, the CRA pays a wage $w_s^* = 0$ for $s \in \{B, R\}$, as it retains its naive investors irrespective of performance. We can write $\bar{V}_s$ for $s \in \{B, R\}$ as:

$$\bar{V}_s = (1-\tau_s)(\omega\pi_s + \delta\bar{V}_s) + \tau_s(\omega\pi_{-s} + \delta\bar{V}_{-s}).$$

Solving these equations for $\bar{V}_s$ gives us:

$$\bar{V}_s = \frac{1-\tau_s-\delta(1-\tau_s-\tau_{-s})}{(1-\delta)(1-\delta(1-\tau_s-\tau_{-s}))}\omega\pi_s + \frac{\tau_s}{(1-\delta)(1-\delta(1-\tau_s-\tau_{-s}))}\omega\pi_{-s}. \tag{7}$$

The value function for state s (where $s \in \{B, R\}$) when sophisticated investors are still willing to purchase rated investments is:

$$V_s = \max_{w_s} \pi_s(\lambda_s+(1-\lambda_s)(1-z_s)) - w_s + \delta(1-(1-\lambda_s)(1-z_s)p_s)((1-\tau_s)V_s+\tau_s V_{-s}) + \delta(1-\lambda_s)(1-z_s)p_s\bar{V}_s. \tag{8}$$

Existence and uniqueness of equilibrium can be shown using the approach of Lemma 1.

As one would expect, the effort that a CRA invests in accuracy while sophisticated investors are still purchasing rated investments decreases as the fraction of naive investors grows larger. The reduction of market discipline from investors reduces investment in standards.

Proposition 6 *Investment in ratings quality in both states $s \in \{B, R\}$ decreases with the fraction of naive investors.*

(default) proves malfeasance.

[15] Although this proportion is fixed across states, this leads to different levels of fees from selling to naive investors across states.

The proof is in the Online Appendix.

Now we examine the effect of naive investors on the countercyclicality of ratings accuracy.

We begin by writing the first-order conditions for the decision variables w_s, $s \in \{B, R\}$:

$$\frac{\partial z}{\partial w}(w_s^*, \gamma_s) = \frac{1}{1-\lambda_s} \frac{1}{\delta p_s((1-\tau_s)V_s^* + \tau_s V_{-s}^*) - \delta p_s \bar{V}_s - \pi_s}. \tag{9}$$

It follows that $w_B^* \leq w_R^*$, and there is more accuracy in recessions than in booms when:

$$(1-\lambda_B)(\delta p_B(EV_B^* - \bar{V}_B) - \pi_B) \leq (1-\lambda_R)(\delta p_R(EV_R^* - \bar{V}_R) - \pi_R). \tag{10}$$

By definition, $EV_s^* > \bar{V}_s$. We also know that $\pi_B > \pi_R$ implies that $\bar{V}_B > \bar{V}_R$. Given Assumption A1, this implies that $EV_B^* - \bar{V}_B < EV_R^* - \bar{V}_R$ when states are independent across time or when there is negative correlation between states. Therefore, our results of countercyclical ratings accuracy are robust to the presence of naive investors.

3 Duopoly

In the main model, we considered a monopoly CRA. Nevertheless, it is important to learn whether the main insights of that model hold when competition is taken into account. While S&P, Moody's, and Fitch certainly exercise some market power, they also compete for market share. In this section, we model competition between two rating agencies. In order to deal with the tractability issue of an infinite-period reputation model of competition, we model competition in a very simple fashion, by supposing that the fee that the CRA charges (and/or the volume of issues) depends not only on the state, but also on the extent of competition among CRAs. Specifically, we write $\pi_{D,s}$ to denote the fee charged by a duopolist in state s and $\pi_{M,s}$ to denote the fee charged by a monopolist in state s, where $\pi_{M,s} > \pi_{D,s}$ and $s \in \{B, R\}$.

We allow for correlation between the products that the agencies are rating. In practice, CRAs rate portfolios of issues with the fraction of issues that they rate in common varying by type of product.[16] We will maintain the assumption from the previous section that in

[16] While in the corporate bond market, almost all rated issues are rated by S&P and Moody's (with Fitch's share varying a large amount; see Becker and Milbourn (2011)), in the structured finance market, there is subtantially more variance in terms of which firms are rating. Benmelech and Dlugosz's (2010) sample of asset-backed securities shows that approximately 75% were rated by two or fewer CRAs (Table 10). Among those that were rated by two CRAs (about 60%), about 88% were rated by S&P and Moody's. White (2010) displays figures from an SEC filing in 2009 that shows that S&P, Moody's and Fitch rated

each period, a CRA rates one product, and we will incorporate correlation by defining ρ as the probability that CRA i and CRA j are rating the same product.

In analyzing the reputational equilibria and CRA incentives, we consider two possibilities for the way that investors react when they detect ratings inflation (by observing investments given a good rating that subsequently default). Specifically, we first consider a grim-trigger-strategy equilibrium in which investors who observe that an issue with a positive rating from CRA j defaults stop buying investments rated by CRA j. This is similar to our punishment strategy in the monopoly case.

The second possibility that we address links the punishment of the CRAs: If both CRAs give the same issue that defaults a positive rating, they are not punished. Any other default of an issue with a positive rating from CRA j has investors withdrawing their business from CRA j.[17] This might be thought of as investors being unsure whether the joint error was a problem with the CRAs' investment in accuracy or a one-time shock that was difficult to predict. Rating agencies certainly made this claim regarding their mistakes on mortgage-backed securities. As we discussed in the introduction, there did seem to be some punishment, but it was not as severe as being forced out of business.

3.1 The first punishment strategy: a CRA loses reputation if rated product defaults

When one CRA loses the confidence of investors, the market becomes a monopoly. When a CRA acts as a monopolist, the analysis of Section 2 applies. It is straightforward in this case to characterize optimal wages in each state, $w^*_{M,s}$, the continuation value associated with each state, $V^*_{M,s}$, and the expected continuation value, $EV^*_{M,s} = (1-\tau_s)V^*_{M,s} + \tau_s V^*_{M,-s}$ (where $-s$ represents the other state) . These have properties identical to those characterized in Section 2.

Using this characterization of the monopoly case, we can, in effect, work backwards to

198,200, 109,281, and 77,480 asset-backed securities respectively.

Some of the non-overlap in structured products is the result of ratings shopping by issuers. Modeling shopping is beyond the scope of this paper, but insight on shopping can be found in Bolton, Freixas, and Shapiro (2012) and Skreta and Veldkamp (2009).

[17] Stolper (2009) examines a similar type of joint reputation in a game in which a regulator is actively monitoring and punishing CRAs. There are additional punishment mechanisms that are of interest. For example, consumers may stop trusting all CRAs if one was found to have incorrectly rated an investment. In this case, the analysis would look similar to that of the monopoly case but with lower per-period payoffs. In addition, CRAs may collude; however, this may require the somewhat unreasonable assumption that they observe each other's wage policies.

consider duopoly behavior. In particular, we can write down the value for CRA i of being in a duopoly in state s and paying a wage $w_{i,s}$, given that its rival, CRA j, is expected to be paying a wage $w_{j,s}$:

$$\begin{aligned} V_{i,s} = \; & \pi_{D,s}(\lambda_s + (1-\lambda_s)(1-z_{i,s})) - w_{i,s} + \delta[\rho(1-(1-\lambda_s)(1-z_{i,s}z_{j,s})p_s) \\ & +(1-\rho)(1-(1-\lambda_s)(1-z_{i,s})p_s)(1-(1-\lambda_s)(1-z_{j,s})p_s)]EV^*_{D,s} \\ & +\delta[\rho((1-\lambda_s)(1-z_{j,s})z_{i,s}p_s) + (1-\rho)(1-(1-\lambda_s)(1-z_{i,s})p_s)(1-\lambda_s)(1-z_{j,s})p_s]EV^*_{M,s}, \end{aligned} \tag{11}$$

where $EV^*_{D,s} = (1-\tau_s)V^*_{i,s} + \tau_s V^*_{i,-s}$ and $s \in \{R, B\}$.

This expression is the duopoly analogue of Expression (1). Here, however, the future value for CRA i, if it succeeds in sustaining its reputation, incorporates both the possibility that its rival sustains its reputation, so that the CRA continues as a duopolist in the future, and the possibility that the rival firm is found to have assigned a good rating to a bad investment that defaulted, in which case the CRA becomes a monopolist. These probabilities depend on the likelihood ρ that both CRAs are rating the same investment in the current period.

In equilibrium, $w^*_{i,s}$ is optimally chosen and satisfies the following first-order condition:

$$\begin{aligned} 0 = -1 + \frac{\partial z_{i,s}}{\partial w}(1-\lambda_s)\{ & -\pi_{D,s} + \delta p_s[\rho z_{j,s} + (1-\rho)(1-(1-\lambda_s)(1-z_{j,s})p_s)]EV^*_{D,s} \\ & +\delta p_s[\rho(1-z_{j,s}) + (1-\rho)(1-\lambda_s)(1-z_{j,s})p_s]EV^*_{M,s}\}. \end{aligned} \tag{12}$$

In the following lemma, we demonstrate an important property of CRA choices.

Lemma 2 *The CRAs' wage choices are strategic substitutes.*

This lemma demonstrates that if CRA i raises its wages, CRA j would lower its wages in response, and vice-versa.[18] By raising its wages, CRA i increases the likelihood that it would be around in the subsequent periods. This, then, reduces the future payoffs for CRA j (and maintains its current payoffs), creating an incentive for CRA j to reduce its accuracy.

[18] Perotti and Suarez (2002) find that banks' decision to take on more risk in a dynamic framework are strategic substitutes for a reason similar to the accuracy decision in our model - if one bank is more risky, it may be more likely to stop operating, giving market power to the remaining bank and making that bank less likely to take on risk.

The lemma also ensures there is a unique symmetric equilibrium.[19] Imposing symmetry, we write the equilibrium wage for this duopoly case as $w^*_{D,s}$, drop the i and j subscripts on the $z_{i,s}$ and $z_{j,s}$ functions, and rewrite the CRA's first-order condition, Equation (12), as:

$$\begin{aligned} 0 = &-1 + \frac{\partial z_s}{\partial w}(1-\lambda_s)\{-\pi_{D,s} + \delta p_s[\rho z_s + (1-\rho)(1-(1-\lambda_s)(1-z_s)p_s)]EV^*_{D,s} \\ &+\delta p_s[\rho(1-z_s) + (1-\rho)(1-\lambda_s)(1-z_s)p_s]EV^*_{M,s}\}. \end{aligned} \quad (13)$$

We can use this to derive the following result on investment in ratings accuracy when states are iid draws.

Proposition 7 *When states are independent across time ($\tau_B = 1 - \tau_R$), there is lower investment in ratings quality in a boom than in a recession (that is, $w^*_{D,B} < w^*_{D,R}$) when booms and recessions differ in terms of duopoly fees, default rates, and/or the proportion of good investments. However, the effect of labor-market conditions is ambiguous.*

There are now two effects on the incentives of the CRA: the direct effect, as in the monopoly case, and a strategic effect. The direct effect clearly has the same effect on incentives to provide quality ratings as in our monopoly model and is examined in Proposition 2. A strategic effect arises since a change in the parameters affects the action of a rival CRA, which may then affect the probability of becoming a monopolist rather than a duopolist in the future, altering the CRA's tradeoff between current and future payoffs. Our analysis demonstrates that the direct effect outweighs the strategic effect for three of the four parameters: fees, default probability, and the proportion of good investments. This is not true for labor market tightness. Tighter labor-market conditions (an increase in γ_s), holding all else constant, reduce the quality of the rival's ratings and, hence, give a CRA an incentive to raise quality in opposition to the direct effect of a larger cost for accuracy. This leads to the ambiguous result on labor-market tightness.

For the duopoly model with correlation, as in the case of monopoly, we make the assumption that it is more valuable for a CRA to be in a boom state than in a recessionary state. Thus, we use assumption A1 here, which with the new notation, amounts to $V^*_{M,B} > V^*_{M,R}$. We also add an analogous assumption for the duopoly case:

Assumption A2: The value to a CRA in a duopoly of being in a boom is larger than that of being in a recession ($V^*_{D,B} > V^*_{D,R}$).

[19] This does not rule out the existence of asymmetric equilibria.

Proposition 8 *If there is negative correlation between states ($\tau_B > 1 - \tau_R$), and A1 and A2 hold, there is lower investment in ratings quality in a boom than a recession (that is, $w^*_{D,B} < w^*_{D,R}$) when booms and recessions differ in terms of duopoly fees, default rates, and/or the proportion of good investments. However, the effect of labor-market conditions is ambiguous.*

Therefore, countercyclical ratings quality also may be a feature of a competitive ratings market. While competition here changes the value of maintaining a CRA's reputation relative to a market dominated by a monopolist, the economic fundamentals shift incentives in a way mostly similar to that of the monopolist. The exception, of course, is that tighter labor markets in booms can bring about either procyclical or countercyclical accuracy in ratings. This arises from the strategic effect.

3.2 The second punishment strategy: a CRA does not lose its reputation when both CRAs rate products that default

We now consider a different punishment strategy: if both CRAs give the same issue that defaults a positive rating, they are not punished. Any other default of an issue with a positive rating has investors withdrawing their business from the culpable CRA. The value function for a duopolist in state s (where $s \in \{R, B\}$) is, therefore:

$$\begin{aligned} V_{i,s} = \; & \pi_{D,s}(\lambda_s + (1-\lambda_s)(1-z_{i,s})) - w_{i,s} + \delta[\rho(1-(1-\lambda_s)(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})p_s) \\ & +(1-\rho)(1-(1-\lambda_s)(1-z_{i,s})p_s)(1-(1-\lambda_s)(1-z_{j,s})p_s)]EV^*_{D,s} \\ & +\delta[\rho((1-\lambda_s)(1-z_{j,s})z_{i,s}p_s) + (1-\rho)(1-(1-\lambda_s)(1-z_{i,s})p_s)(1-\lambda_s)(1-z_{j,s})p_s]EV^*_{M,s}. \end{aligned} \quad (14)$$

The only difference between Expression (14) and Equation (11) is the part where both CRAs give good ratings and the issue subsequently defaults. Here, they still end up in duopoly. Previously, both were out of the ratings business.

In equilibrium, the wage $w^*_{i,s}$ for $s \in \{R, B\}$ is optimally chosen and so satisfies the first-order condition:

$$\begin{aligned} 0 = -1 + \frac{\partial z_{i,s}}{\partial w}(1-\lambda_s)\{-\pi_{D,s} + \delta p_s[\rho(-1+2z_{j,s}) + (1-\rho)(1-(1-\lambda_s)(1-z_{j,s})p_s)]EV^*_{D,s} \\ +\delta p_s[\rho(1-z_{j,s}) + (1-\rho)(1-\lambda_s)(1-z_{j,s})p_s]EV^*_{M,s}\}. \end{aligned} \quad (15)$$

In the following lemma, we demonstrate an important property of CRA wages.

Lemma 3 *1. If* $[\rho + (1-\rho)(1-\lambda_s)p_s]\left[EV^*_{D,s} - EV^*_{M,s}\right] + \rho EV^*_{D,s} < 0$, *the CRAs' wage choices are strategic substitutes.*

2. If $[\rho + (1-\rho)(1-\lambda_s)p_s]\left[EV^*_{D,s} - EV^*_{M,s}\right] + \rho EV^*_{D,s} > 0$, *the CRAs' wage choices are strategic complements.*

This lemma demonstrates that the strategic nature of wage choices has changed. Both the likelihood of being in a duopoly and the benefit of being incorrect are larger (due to no punishment if both CRAs get the rating wrong).

If CRA i raises its wage, it increases its probability of being around in subsequent periods but it decreases the future benefit of the situation in which both get caught lying and are not punished. So, while the current benefits for CRA j are fixed, the future benefits for CRA j can move in either direction. When they move down, the situation is similar to that in the previous section, and CRA j responds by lowering its wages. When they move up, CRA j will increase its wages to make it more likely to capture the future benefit. So, even though we have added a scenario where punishment is less likely, this may give the CRAs an incentive to increase investment in accuracy.

3.2.1 Strategic Substitutes

When the CRA's strategies are strategic substitutes, as characterized in part 1 of Lemma 3, there is a unique symmetric equilibrium.[20] Imposing symmetry, we write the equilibrium wage for this duopoly case as $w^*_{D,s}$, drop the i and j subscripts on the $z_{i,s}$ and $z_{j,s}$ functions, and rewrite the CRA's first-order condition (Equation 15) as:

$$\begin{aligned}0 = -1 + \frac{\partial z_s}{\partial w}(1-\lambda_s)\{-\pi_{D,s} + \delta p_s[\rho(-1+2z_s) + (1-\rho)(1-(1-\lambda_s)(1-z_s)p_s)]EV^*_{D,s} \\ +\delta p_s[\rho(1-z_s) + (1-\rho)(1-\lambda_s)(1-z_s)p_s]EV^*_{M,s}\}. \qquad (16)\end{aligned}$$

Given that the nature of the strategic interaction is similar to the case of the first punishment strategy, which we analyzed in Section 3.1, as one might expect, we find results analogous to Propositions 7 and 8.

Proposition 9 *When wages are strategic substitutes with the second punishment strategy, if states are independent across time ($\tau_B = 1 - \tau_R$), there is lower investment in ratings quality in a boom than in a recession (that is, $w^*_{D,B} < w^*_{D,R}$) when booms and recessions*

[20] This does not rule out the existence of asymmetric equilibria.

differ in terms of duopoly fees, default rates, and/or the proportion of good investments. However, the effect of labor-market conditions is ambiguous.

The proof is in the Online Appendix.

Proposition 10 *When wages are strategic substitutes with the second punishment strategy, if there is negative correlation between states ($\tau_B > 1-\tau_R$), and A1 and A2 hold, there is lower investment in ratings quality in a boom than in a recession (that is, $w^*_{D,B} < w^*_{D,R}$) when booms and recessions differ in terms of duopoly fees, default rates, and/or the proportion of good investments. However, the effect of labor-market conditions is ambiguous.*

The proof is in the Online Appendix.

3.2.2 Strategic Complements

In the case of strategic complements, characterized in part 2 of Lemma 3, it is possible to have multiple symmetric equilibria and/or a corner solution. We start by writing out conditions that guarantee existence and uniqueness of a symmetric equilibrium, all of which are consistent with the model. We then analyze the difference between CRA investments in accuracy in booms and recessions, and we find results similar to the cases of strategic substitutes.

The following conditions are sufficient to guarantee existence and uniqueness of an equilibrium. This equilibrium will be symmetric—i.e., both CRAs will make the wage choices in both states.

Condition 1 $EV^*_{M,s} > EV^*_{D,s}$ *for* $s \in \{B, R\}$.

Condition 2 $\pi_{D,s}$ *is small for* $s \in \{B, R\}$.

Condition 3 $\frac{\partial^3 z}{\partial w^3} \leq 0$.

Condition 1 states that the expected equilibrium value of a CRA is larger if the CRA is a monopolist than if it is a duopolist. Condition 2 is consistent with Condition 1. Both of these could represent a situation of Bertrand competition, in which case $\pi_{D,s} = 0$ for $s \in \{B, R\}$ and $\pi_{M,s} > 0$ for $s \in \{B, R\}$. In the proof of existence and uniqueness, we will be more specific about how small $\pi_{D,s}$ should be. The last condition is on the third derivative of $z(w)$. This is satisfied by a range of functions.

Proposition 11 *Given that Conditions 1-3 hold, there exists a unique equilibrium.*

We now analyze how the wages differ between booms and recessions given the unique equilibrium. The unique equilibrium is symmetric, so we will impose symmetry on the CRA's first-order condition, giving us Equation (16). This gives us the following results:

Proposition 12 *Given that conditions 1-3 hold, when wages are strategic complements with the second punishment strategy, if states are independent across time ($\tau_B = 1 - \tau_R$), there is lower investment in ratings quality in a boom than in a recession (that is, $w^*_{D,B} < w^*_{D,R}$) when booms and recessions differ in terms of duopoly fees, default rates, the proportion of good investments, and/or labor-market tightness.*

The proof is in the Online Appendix.

The results from the strategic substitute case for fees, probability of default, and the fraction of good issues remain the same. Moreover, we now get an unambiguous result with respect to labor-market tightness. When the labor market is tighter, investment in ratings quality strictly decreases, as it did in the monopoly case. The obvious reason is because the strategic effect has switched; an increase in labor-market tightness makes the competitor CRA lower its wage, which induces the CRA to lower its own wage (which it also wants to do because its costs have also gone up).

The result on negative correlation holds, as well.

Proposition 13 *When wages are strategic complements with the second punishment strategy, if there is negative correlation between states ($\tau_B > 1-\tau_R$), and A1 and A2 hold, there is lower investment in ratings quality in a boom than in a recession (that is, $w^*_{D,B} < w^*_{D,R}$) when booms and recessions differ in terms of duopoly fees, default rates, the proportion of good investments, and/or labor-market tightness.*

The proof is in the Online Appendix.

Once again, there is countercyclical ratings accuracy for all variables. Labor-market tightness is no longer ambiguous due to the strategic effect going in the same direction as the direct effect.

4 Empirical Implications

In this section, we examine evidence related to testable implications of the model. To examine our hypotheses, we use a set of very recent empirical papers focused on CRAs and ratings quality.

The model shows that ratings quality may be countercyclical. This effect is likely to be exacerbated when economic shocks are negatively correlated and diminished when economic shocks are positively correlated. While we are unable to find direct evidence relating the nature of business cycles to ratings quality, some recent papers document a decrease in ratings quality in the recent boom. Ashcraft, Goldsmith-Pinkham, and Vickery (2010) find that as the volume of mortgage-backed security issuance increased dramatically from 2005 to mid-2007, the quality of ratings declined. Specifically, when conditioning on the overall risk of the deal, subordination levels[21] for subprime and Alt-A MBS deals decreased over this time period. Furthermore, subsequent ratings downgrades for the 2005 to mid-2007 cohorts were dramatically greater than for previous cohorts. Griffin and Tang (2012) find that adjustments by CRAs to their models' predictions of credit quality in the CDO market were positively related to future downgrades. These adjustments were overwhelmingly positive, and the amount adjusted (the width of the AAA tranche) increased sharply from 2003 to 2007 (from six percent to 18.2 percent). The adjustments are not well explained by natural covariates (such as past deals by collateral manager, credit enhancements, and other modeling techniques). Furthermore, 98.6 percent of the AAA tranches of CDOs in their sample failed to meet the CRAs' reported AAA standard (for their sample from 1997 to 2007). They also find that adjustments increased CDO value by, on average, $12.58 million per CDO.

Larger current revenues should lead to lower ratings quality. He, Qian, and Strahan (2012) find that MBS tranches sold by larger issuers performed significantly worse (market prices decreased) than those sold by small issuers during the boom period of 2004-2006. They define larger by market share in terms of deals. As a robustness check, they also look at market share in terms of dollars and find similar results. Faltin-Traeger (2009) shows that when one CRA rates more deals for an issuer in a half-year period than does another CRA, the first CRA is less likely to be the first to downgrade that issuer's securities in the next half-year.

More-complex investments imply lower ratings quality. Increasing the complexity of investments has two implications for ratings quality. First, it implies more noise regarding the performance of the investment, making it harder to detect whether a CRA can be faulted for poor ratings quality. Second, it implies that CRAs may require more

[21] The subordination level that they use is the fraction of the deal that is junior to the AAA tranche. A smaller fraction means that the AAA tranche is less 'protected' from defaults and, therefore, less costly from the issuer's point of view.

expensive/specialized workers to maintain a given level of quality. Both of these channels decrease the return to investing in ratings quality. Structured finance products are certainly more complex (and the methodology for evaluating them less standardized) than corporate bonds, which provides casual evidence for the recent performance of structured finance ratings. Within the structured finance arena, Ashcraft, Goldsmith-Pinkham, and Vickery (2010) find that the MBS deals that were most likely to underperform were ones with more interest-only loans (because of limited performance history) and lower documentation—i.e., loans that were more opaque or difficult to evaluate.

We also offer two outcomes of the model that are testable but not yet examined, to the best of our knowledge.

1. Ratings-quality decisions between CRAs may be strategic substitutes or strategic complements. When one CRA chooses to produce better ratings, do other CRAs have incentives to worsen or improve their ratings? With the first punishment strategy, we show that choices are likely to be strategic substitutes. However, if the second punishment strategy is more likely, then it is possible that CRA investments will be strategic complements. Kliger and Sarig (2000) use a natural experiment that seems tailored for testing this question: Moody's' switch to a finer ratings scale. While their focus is on the informativeness of ratings, it would be interesting to study the strategic aspect of how this affects the quality of Standard and Poor's' ratings.

2. When forecasts of growth/economic conditions are better, ratings quality should be higher. This is because reputation-building is needed for milking in good times, and forecasts should be directly related to CRAs' future payoffs. This is also a prediction of the models of Mathis, McAndrews, and Rochet (2009) and Bolton, Freixas, and Shapiro (2012).

5 Conclusion

In this paper, we analyze how CRAs' incentives to provide high-quality ratings vary over the business cycle. We define booms as having tighter labor markets, larger revenue for CRAs, and lower average default probabilities than recessions have. When economic shocks are iid, booms have strictly lower quality ratings than do recessions, due to the incentive to milk reputation. These incentives are exacerbated when shocks are negatively correlated (mean reversion) and diminished when shocks are positively correlated. Adding naive investors does not change the qualitative results. We also put forth a model of competition that accounts for CRAs rating similar investments and different market reactions to evidence of CRA inaccuracy. This model demonstrates that countercyclical ratings quality also holds with in a competitive environment. Lastly, we find some empirical support for the model and make suggestions for future empirical work.

In order to make our model tractable, we have made several simplifications. In our model, we have simplified both investor and issuer behavior. Providing more structure on their decision making might provide additional insight and allow us to endogenize some of the parameters that we have taken to be exogenous. It could also prove useful to model the business cycle in a more realistic manner. Lastly, we have focused on initial ratings, but CRAs follow investments over time and develop reputations from the upgrading/downgrading process.

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A Proofs

Proof of Lemma 1

Proof. First, consider existence. Note that $\pi_B(\lambda_B + (1-\lambda_B)(1-z_B)) - w_B$ is bounded from above and that $\pi_R(\lambda_R + (1-\lambda_R)(1-z_R)) - w_R$ is bounded from above. Say that both are strictly less than A; then, trivially, $V_B < \frac{A}{1-\delta}$ and $V_R < \frac{A}{1-\delta}$. Define two functions from Equations (1), $V_B(V_R)$ and $V_R(V_B)$. Note that both are increasing and continuous functions, and that both $V_B(0) > 0$ and $V_R(0) > 0$ are positive. Since $V_B(\frac{A}{1-\delta}) < \frac{A}{1-\delta}$ and $V_R(\frac{A}{1-\delta}) < \frac{A}{1-\delta}$, it follows that there must be an odd number of solutions. This is easy to see graphically in the illustrative figure. However, we argue below that $V_B(.)$ and $V_R(.)$ are convex and, thereby, show that there cannot be more than two solutions. This will then prove that the solution is unique.

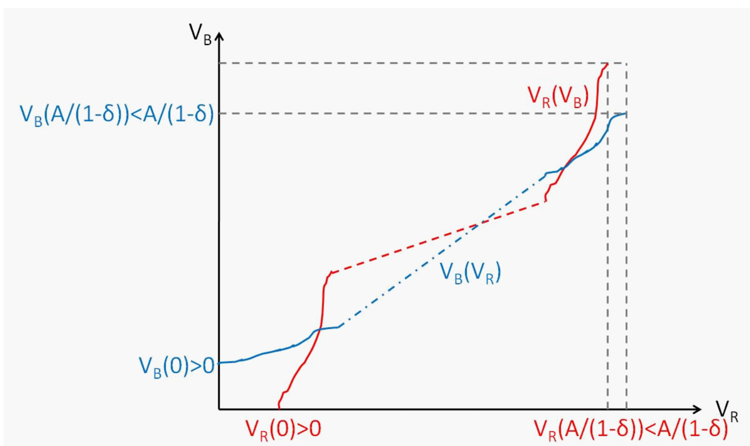

Figure 2: Odd number of solutions

It remains to demonstrate that $V_B(.)$ and $V_R(.)$ are convex. Note, first, that we can consider:

$$w_B^* = \arg\max_w \pi_B(\lambda_B+(1-\lambda_B)(1-z_B))-w+\delta(1-(1-\lambda_B)(1-z_B)p_B)((1-\tau_B)V_B+\tau_B V_R). \tag{17}$$

First, we claim that $\frac{dw_B^*}{dV_R} > 0$. We use the implicit function theorem to do so. Consider the first-order condition of the CRA's maximization problem:[22]

$$-\pi_B \frac{\partial z_B}{\partial w}|_{w_B^*} - \frac{1}{1-\lambda_B} + \delta p_B((1-\tau_B)V_B + \tau_B V_R)\frac{\partial z_B}{\partial w}|_{w_B^*} = 0. \tag{18}$$

[22] It can be shown that the second-order condition is satisfied when λ_B, λ_R and δ are close enough to 1.

Taking the derivative of the FOC with respect to V_R and rearranging yields:

$$\frac{dw_B^*}{dV_R} = \frac{\tau_B \delta p_B}{\pi_B - \delta p_B((1-\tau_B)V_B + \tau_B V_R)} \frac{\frac{\partial z_B}{\partial w}}{\frac{\partial^2 z_B}{\partial w^2}} \tag{19}$$

Note that the assumption that the CRA's second-order condition is negative implies that the denominator of the first fraction is negative, and so, since $\frac{\partial^2 z_B}{\partial w^2} < 0$ and $\frac{\partial z_B}{\partial w} > 0$, it follows that $\frac{dw_B^*}{dV_R} > 0$.

Now,

$$\frac{dV_B}{dV_R} = \frac{\partial V_B}{\partial w_B^*}\frac{dw_B^*}{dV_R} + \frac{\partial V_B}{\partial V_R} = \frac{\partial V_B}{\partial V_R} \tag{20}$$

since w_B^* is chosen to maximize V_B (the envelope condition), and so we can write

$$\begin{aligned}\frac{dV_B}{dV_R} &= (\tau_B + (1-\tau_B)\frac{dV_B}{dV_R})\delta(1-(1-\lambda_B)(1-z_B)p_B) \\ &= \frac{\tau_B \delta(1-(1-\lambda_B)(1-z_B)p_B)}{1-(1-\tau_B)\delta(1-(1-\lambda_B)(1-z_B)p_B)} > 0.\end{aligned} \tag{21}$$

Next, to prove convexity, note that

$$\begin{aligned}\frac{d^2V_B}{dV_R^2} &= \frac{d}{dz_B}(\frac{\tau_B \delta(1-(1-\lambda_B)(1-z_B)p_B)}{1-(1-\tau_B)\delta(1-(1-\lambda_B)(1-z_B)p_B)})\frac{\partial z_B}{\partial w}\frac{dw_B^*}{dV_R} \\ &= \frac{(1-\lambda_B)p_B\tau_B\delta}{(1-(1-\tau_B)\delta(1-(1-\lambda_B)(1-z_B)p_B))^2}\frac{\partial z_B}{\partial w}\frac{dw_B^*}{dV_R} > 0.\end{aligned} \tag{22}$$

Analogously, $\frac{d^2V_R}{dV_B^2} > 0$. ■

Proof of Proposition 1

Proof. We start by introducing some additional notation:

$$G_s(p_R, p_B, \gamma_R, \gamma_B, \pi_R, \pi_B, \lambda_R, \lambda_B, \delta) := \begin{array}{c} -V_s + \pi_s(\lambda_s + (1-\lambda_s)(1-z_s)) - w_s \\ +\delta(1-(1-\lambda_s)(1-z_s)p_s)((1-\tau_s)V_s + \tau_s V_{-s}) \end{array}. \tag{23}$$

We suppress the arguments for G_B and G_R and can then rewrite the Equations (1) as $G_B = G_R = 0$.

We apply the implicit function theorem, which here implies that

$$\frac{dV_R^*}{da} = -\frac{\det\begin{bmatrix}\frac{\partial G_B}{\partial a} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial a} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix}}{\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix}} \text{ and } \frac{dV_B^*}{da} = -\frac{\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial a}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial a}\end{bmatrix}}{\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix}}, \tag{24}$$

where a is an arbitrary parameter. We begin by analyzing the (common) denominator of both expressions.

As we show in the Lemma below, this determinant is negative. ■

Lemma 4 $\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*}$ *is negative.*

Proof. First, note:

$$\frac{\partial G_B}{\partial V_B^*} = \delta(1-(1-\lambda_B)(1-z_B)p_B)(1-\tau_B) - 1 < 0 \tag{25}$$

$$\frac{\partial G_B}{\partial V_R^*} = \delta(1-(1-\lambda_B)(1-z_B)p_B)\tau_B > 0 \tag{26}$$

$$\frac{\partial G_R}{\partial V_B^*} = \delta(1-(1-\lambda_R)(1-z_R)p_R)\tau_R > 0 \tag{27}$$

$$\frac{\partial G_R}{\partial V_R^*} = \delta(1-(1-\lambda_R)(1-z_R)p_R)(1-\tau_R) - 1 < 0, \tag{28}$$

where we have used the envelope theorem to simplify expressions. This, then, allows us to rewrite

$$\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*} = \delta^2\tau_B\tau_R\alpha_R\alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R)), \tag{29}$$

where $\alpha_s := (1-(1-\lambda_s)(1-z_s)p_s)$ and, thus, $\alpha_s \in (0,1)$.

Next, note that

$$\frac{\partial}{\partial \tau_B}\left(\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*}\right) = \delta\alpha_B(\delta\alpha_R - 1) < 0, \tag{30}$$

where the inequality follows since $1 > \alpha_s > 0$.

Finally, note that at $\tau_B = 0$,

$$\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*} = -(1-\delta\alpha_B)(1-\delta\alpha_R(1-\tau_R)) < 0. \tag{31}$$

■

Resumption of Proof of Proposition 1

Proof. Given Lemma 4, we can apply the implicit function theorem and note that $\frac{d}{da}(V_B^* - V_R^*)$ has the same sign as $\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial a}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial a}\end{bmatrix} - \det\begin{bmatrix}\frac{\partial G_B}{\partial a} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial a} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix}$.

We consider several parameters of interest; proofs for other parameters are similar, and so are omitted.

The effect of a change in the probability of default in a boom (p_B)

Consider, first, the comparative static with respect to p_B: $\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial p_B}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial p_B}\end{bmatrix} - \det\begin{bmatrix}\frac{\partial G_B}{\partial p_B} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial p_B} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix} = -\frac{\partial G_B}{\partial p_B}(\frac{\partial G_R}{\partial V_R^*} + \frac{\partial G_R}{\partial V_B^*})$, since $\frac{\partial G_R}{\partial p_B} = 0$. Now $\frac{\partial G_B}{\partial p_B} = -\delta(1-\lambda_B)(1-z_B)((1-\tau_B)V_B^* + \tau_B V_R^*) < 0$ and $\frac{\partial G_R}{\partial V_R^*} + \frac{\partial G_R}{\partial V_B^*} = -1 + \delta(1-(1-\lambda_R)(1-z_R)p_R) < 0$.

Consequently, $\frac{d(V_B^* - V_R^*)}{dp_B} < 0$.

The effect of a change in labor-market conditions in a recession (γ_R):

$\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \gamma_R}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \gamma_R}\end{bmatrix} - \det\begin{bmatrix}\frac{\partial G_B}{\partial \gamma_R} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial \gamma_R} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix} = \frac{\partial G_R}{\partial \gamma_R}(\frac{\partial G_B}{\partial V_R^*} + \frac{\partial G_B}{\partial V_B^*})$, since $\frac{\partial G_B}{\partial \gamma_R} = 0$.

Note that $\frac{\partial G_B}{\partial V_R^*} + \frac{\partial G_B}{\partial V_B^*} = \delta(1-(1-\lambda_B)(1-z_B)p_B) - 1 < 0$ and since $\frac{\partial G_R}{\partial \gamma_R} = (\delta(1-\lambda_R)p_R\,((1-\tau_R)V_R^* + \tau_R V_B^*) - \pi_R(1-\lambda_R))\frac{\partial z}{\partial \gamma_R} < 0$ by the second-order condition and since $\frac{\partial z}{\partial \gamma_R} < 0$. It follows that $\frac{d(V_B - V_R)}{d\gamma_R} > 0$.

The effect of a change in the transition probabilities

i) First, we examine the change with respect to a change in τ_B

$\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \tau_B}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \tau_B}\end{bmatrix} - \det\begin{bmatrix}\frac{\partial G_B}{\partial \tau_B} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial \tau_B} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix} = -\frac{\partial G_B}{\partial \tau_B}(\frac{\partial G_R}{\partial V_R^*} + \frac{\partial G_R}{\partial V_B^*})$ since $\frac{\partial G_R}{\partial \tau_B} = 0$.

As above, $\frac{\partial G_R}{\partial V_R^*} + \frac{\partial G_R}{\partial V_B^*} < 0$ and $\frac{\partial G_B}{\partial \tau_B} = -\delta(1-(1-\lambda_B)(1-z_B)p_B)(V_B^* - V_R^*)$. It follows that $sign(\frac{\partial G_B}{\partial \tau_B}) = -sign(V_B^* - V_R^*)$. Therefore, $sign\frac{d(V_B^* - V_R^*)}{d\tau_B} = -sign(V_B^* - V_R^*)$.

ii) Second, we examine the change with respect to a change in τ_R

$\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \tau_R}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \tau_R}\end{bmatrix} - \det\begin{bmatrix}\frac{\partial G_B}{\partial \tau_R} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial \tau_R} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix} = (\frac{\partial G_B}{\partial V_R^*} + \frac{\partial G_B}{\partial V_B^*})\frac{\partial G_R}{\partial \tau_R}$

As above, $\frac{\partial G_B}{\partial V_B^*} + \frac{\partial G_B}{\partial V_R^*} < 0$. Also, $\frac{\partial G_R}{\partial \tau_R} = \delta(1-(1-\lambda_R)(1-z_R)p_R)(V_B^* - V_R^*)$. It follows that $sign(\frac{\partial G_R}{\partial \tau_R}) = sign(V_B^* - V_R^*)$. Therefore, $sign\frac{d(V_B^*-V_R^*)}{d\tau_R} = -sign(V_B^* - V_R^*)$. ■

Proof of Proposition 4

Proof. First, consider the first-order condition that characterizes w_B^*:

$$-\pi_B(1-\lambda_B)\frac{\partial z_B}{\partial w} - 1 + \delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}((1-\tau_B)V_B^* + \tau_B V_R^*) = 0. \tag{32}$$

Taking the total derivative with respect to τ_B, we obtain

$$\begin{aligned} 0 = {} & \delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}(V_R^* - V_B^*) + (1-\lambda_B)\frac{\partial^2 z_B}{\partial w^2}\frac{dw_B^*}{d\tau_B}(\delta p_B((1-\tau_B)V_B^* + \tau_B V_R^*) - \pi_B) \\ & + \delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}((1-\tau_B)\frac{\partial V_B^*}{\partial \tau_B} + \tau_B\frac{\partial V_R^*}{\partial \tau_B}) \end{aligned} \tag{33}$$

First, note that (using the results from Lemma 4 and the definition of α_s $(s = B, R)$ from the same Lemma):

$$\begin{aligned} sign(\frac{\partial V_B^*}{\partial \tau_B}) &= sign(\det\begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \tau_B} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \tau_B} \end{bmatrix}) = -sign\frac{\partial G_B}{\partial \tau_B}\frac{\partial G_R}{\partial V_R^*} \\ &= -sign(\delta\alpha_B(1-\delta\alpha_R(1-\tau_R))(V_B^* - V_R^*) = -sign(V_B^* - V_R^*)) \end{aligned}$$

and

$$\begin{aligned} sign(\frac{\partial V_R^*}{\partial \tau_B}) &= sign(\det\begin{bmatrix} \frac{\partial G_B}{\partial \tau_B} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial \tau_B} & \frac{\partial G_R}{\partial V_B^*} \end{bmatrix}) = sign(\frac{\partial G_B}{\partial \tau_B}\frac{\partial G_R}{\partial V_B^*}) \\ &= sign(\delta^2\alpha_B\alpha_R\tau_R(V_R^* - V_B^*)) = -sign(V_B^* - V_R^*)). \end{aligned}$$

Now, consider (33): Since $\delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w} > 0$ and $\frac{\partial^2 z_B}{\partial w^2} < 0$ and $1-\lambda_B > 0$, it follows that $\frac{dw_B^*}{d\tau_B}$ has the same sign as $sign(V_B^* - V_R^*) * sign(\pi_B - \delta p_B((1-\tau)V_B^* + \tau V_R^*))$. Rearranging the FOC as $-\pi_B + \delta p_B((1-\tau)V_B^* + \tau V_R^*) = \frac{1}{(1-\lambda_B)\frac{\partial z_B}{\partial w}}$ and noting that the right-hand side is positive gives the result for w_B^*; that is, $sign(\frac{dw_B^*}{d\tau_B}) = -sign(V_B^* - V_R^*)$.

Analogously, $sign(\frac{dw_R^*}{d\tau_R}) = -sign(V_R^* - V_B^*) = sign(V_B^* - V_R^*)$.

Next, we turn to consider $\frac{dw_B^*}{d\tau_R}$.

Taking the derivative of (32) with respect to τ_R, we obtain:

$$0=(1-\lambda_B)\frac{\partial^2 z_B}{\partial w^2}\frac{dw_B^*}{d\tau_R}(\delta p_B((1-\tau_B)V_B^*+\tau_B V_R^*)-\pi_B)+\delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}((1-\tau_B)\frac{\partial V_B^*}{\partial \tau_R}+\tau_B\frac{\partial V_R^*}{\partial \tau_R}).$$

As above, $(\delta p_B((1-\tau_B)V_B^*+\tau_B V_R^*)-\pi_B)>0$ and $\frac{\partial^2 z_B}{\partial w^2}<0$ so that $sign(\frac{dw_B^*}{d\tau_R}) = sign(\delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}((1-\tau_B)\frac{\partial V_B^*}{\partial \tau_R}+\tau_B\frac{\partial V_R^*}{\partial \tau_R})) = sign((1-\tau_B)\frac{\partial V_B^*}{\partial \tau_R}+\tau_B\frac{\partial V_R^*}{\partial \tau_R}))$ where the second inequality follows since $\delta p_B(1-\lambda_B)\frac{\partial z_B}{\partial w}>0$.

Consider

$$\begin{aligned} sign(\frac{\partial V_B^*}{\partial \tau_R}) &= sign(\det\begin{bmatrix}\frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \tau_R}\\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \tau_R}\end{bmatrix}) = sign(\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial \tau_R}) \\ &= sign(\delta^2\alpha_B\tau_B\alpha_R(V_B^*-V_R^*)) = sign(V_B^*-V_R^*), \text{ and} \end{aligned} \tag{34}$$

$$\begin{aligned} sign(\frac{\partial V_R^*}{\partial \tau_R}) &= sign(\det\begin{bmatrix}\frac{\partial G_B}{\partial \tau_R} & \frac{\partial G_B}{\partial V_B^*}\\ \frac{\partial G_R}{\partial \tau_R} & \frac{\partial G_R}{\partial V_B^*}\end{bmatrix}) = -sign(\frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial \tau_R}) \\ &= sign((1-\delta\alpha_B(1-\tau_B))\delta\alpha_R(V_B^*-V_R^*)) = sign(V_B^*-V_R^*). \end{aligned}$$

This implies $sign(\frac{dw_B^*}{d\tau_R}) = sign(V_B^*-V_R^*)$. Analogously, $sign(\frac{dw_R^*}{d\tau_B}) = -sign(V_B^*-V_R^*)$.

■

Proof of Proposition 5

Proof. Define $\tau_B=\tilde{\tau}_B+\varepsilon$ and $\tau_R=\tilde{\tau}_R+\varepsilon$. We now examine the effect of a change in ε on wages. Taking the derivative of Equation (32) with respect to ε yields:

$$\frac{\partial^2 z_B}{\partial w^2}\frac{\partial w_B^*}{\partial \varepsilon}(-\pi_B+\delta p_B((1-\tau_B)V_B^*+\tau_B V_R^*))+\delta p_B\frac{\partial z_B}{\partial w}(V_R^*-V_B^*+(1-\tau_B)\frac{\partial V_B^*}{\partial \varepsilon}+\tau_B\frac{\partial V_R^*}{\partial \varepsilon})=0.$$

We know that $\frac{\partial^2 z_B}{\partial w^2}<0$ and $-\pi_B+\delta p_B((1-\tau_B)V_B^*+\tau_B V_R^*)>0$, so $sign\ (\frac{\partial w_B^*}{\partial \varepsilon}) = sign(V_R^*-V_B^*+(1-\tau_B)\frac{\partial V_B^*}{\partial \varepsilon}+\tau_B\frac{\partial V_R^*}{\partial \varepsilon})$.

We have

$$\frac{\partial V_R^*}{\partial \varepsilon} = -\frac{\det\begin{bmatrix} \frac{\partial G_B}{\partial \varepsilon} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial \varepsilon} & \frac{\partial G_R}{\partial V_B^*} \end{bmatrix}}{\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*}} = -\frac{\frac{\partial G_B}{\partial \varepsilon}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial \varepsilon}}{\delta^2\tau_B\tau_R\alpha_R\alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}$$
$$= -(V_B - V_R)\delta\alpha_R\frac{1-\delta\alpha_B(1-\tau_B+\tau_R)}{\delta^2\tau_B\tau_R\alpha_R\alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}$$

and

$$\frac{\partial V_B^*}{\partial \varepsilon} = -\frac{\det\begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \varepsilon} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \varepsilon} \end{bmatrix}}{\frac{\partial G_B}{\partial V_R^*}\frac{\partial G_R}{\partial V_B^*} - \frac{\partial G_B}{\partial V_B^*}\frac{\partial G_R}{\partial V_R^*}} = -\frac{\frac{\partial G_R}{\partial \varepsilon}\frac{\partial G_B}{\partial V_R^*} - \frac{\partial G_R}{\partial V_R^*}\frac{\partial G_B}{\partial \varepsilon}}{\delta^2\tau_B\tau_R\alpha_R\alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}$$
$$= -(V_B - V_R)\delta\alpha_B\frac{-1+\delta\alpha_R(1-\tau_R+\tau_B)}{\delta^2\tau_B\tau_R\alpha_R\alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}.$$

Therefore, we have

$$V_R^* - V_B^* + (1-\tau_B)\frac{\partial V_B^*}{\partial \varepsilon} + \tau_B\frac{\partial V_R^*}{\partial \varepsilon}$$
$$= (V_B^* - V_R^*)(-1 - (1-\tau_B)\delta\alpha_B\frac{-1+\delta\alpha_R(1-\tau_R+\tau_B)}{\delta^2\tau_B\tau_R\alpha_R\alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}$$
$$-\tau_B\delta\alpha_R\frac{1-\delta\alpha_B(1-\tau_B+\tau_R)}{\delta^2\tau_B\tau_R\alpha_R\alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}),$$

so that

$$sign(V_R^* - V_B^* + (1-\tau_B)\frac{\partial V_B^*}{\partial \varepsilon} + \tau_B\frac{\partial V_R^*}{\partial \varepsilon})$$
$$= sign(V_B^* - V_R^*) * sign(-1 - (1-\tau_B)\delta\alpha_B\frac{-1+\delta\alpha_R(1-\tau_R+\tau_B)}{\delta^2\tau_B\tau_R\alpha_R\alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}$$
$$-\tau_B\delta\alpha_R\frac{1-\delta\alpha_B(1-\tau_B+\tau_R)}{\delta^2\tau_B\tau_R\alpha_R\alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))})$$
$$= sign(V_B^* - V_R^*) * sign(\frac{\delta\alpha_R(1+\tau_B-\tau_R)-1}{(1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R)) - \delta^2\tau_B\tau_R\alpha_R\alpha_B}).$$

Given Assumption A1, and since by Lemma 4 $(1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R)) - \delta^2\tau_B\tau_R\alpha_R\alpha_B > 0$, it follows that $sign(\frac{\partial w_B^*}{\partial \varepsilon}) = sign(\delta\alpha_R(1+\tau_B-\tau_R)-1)$.

We now find $\frac{\partial w_R^*}{\partial \varepsilon}$. The FOC wrt w_R^* is:

$$-\pi_R \frac{\partial z_R}{\partial w} - 1 + \delta p_R \frac{\partial z_R}{\partial w}((1-\tau_R)V_R^* + \tau_R V_B^*) = 0.$$

Taking the derivative of the FOC wrt ε:

$$\frac{\partial^2 z_R}{\partial w^2}\frac{\partial w_R^*}{\partial \varepsilon}(-\pi_R + \delta p_R((1-\tau_R)V_R^* + \tau_R V_B^*)) + \delta p_R \frac{\partial z_R}{\partial w}(V_B^* - V_R^* + (1-\tau_R)\frac{\partial V_R^*}{\partial \varepsilon} + \tau_R \frac{\partial V_B^*}{\partial \varepsilon}) = 0,$$

so $sign \frac{\partial w_R^*}{\partial \varepsilon} = sign(V_B^* - V_R^* + (1-\tau_R)\frac{\partial V_R^*}{\partial \varepsilon} + \tau_R \frac{\partial V_B^*}{\partial \varepsilon})$, where $\frac{dV_R^*}{d\varepsilon} = -\frac{\det \begin{bmatrix} \frac{\partial G_B}{\partial \varepsilon} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial \varepsilon} & \frac{\partial G_R}{\partial V_B^*} \end{bmatrix}}{\det \begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial V_B^*} \end{bmatrix}}$ and $\frac{dV_B^*}{d\varepsilon} = -\frac{\det \begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial \varepsilon} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial \varepsilon} \end{bmatrix}}{\det \begin{bmatrix} \frac{\partial G_B}{\partial V_R^*} & \frac{\partial G_B}{\partial V_B^*} \\ \frac{\partial G_R}{\partial V_R^*} & \frac{\partial G_R}{\partial V_B^*} \end{bmatrix}}$, and so $\frac{\partial V_R^*}{\partial \varepsilon} = -\frac{\delta^2 \alpha_B \alpha_R \tau_R (V_R^* - V_B^*) - (\delta \alpha_B (1-\tau_B) - 1)\delta \alpha_R (V_B^* - V_R^*)}{\delta^2 \tau_B \tau_R \alpha_R \alpha_B - (1 - \delta \alpha_B (1-\tau_B))(1 - \delta \alpha_R (1-\tau_R))}$ and $\frac{\partial V_B^*}{\partial \varepsilon} = -\frac{\delta \alpha_B \tau_B \delta \alpha_R (V_B^* - V_R^*) - \delta \alpha_B (V_R^* - V_B^*)(\delta \alpha_R (1-\tau_R) - 1)}{\delta^2 \tau_B \tau_R \alpha_R \alpha_B - (1 - \delta \alpha_B (1-\tau_B))(1 - \delta \alpha_R (1-\tau_R))}$. Therefore, $V_B^* - V_R^* + (1-\tau_R)\frac{\partial V_R^*}{\partial \varepsilon} + \tau_R \frac{\partial V_B^*}{\partial \varepsilon} = (V_B^* - V_R^*)\frac{1 - \delta \alpha_B (1 - \tau_B + \tau_R)}{(1 - \delta \alpha_B (1-\tau_B))(1 - \delta \alpha_R (1-\tau_R)) - \delta^2 \tau_B \tau_R \alpha_R \alpha_B}$. Again, the denominator is positive by Lemma 4. ■

Proof of Lemma 2

Proof. For notational convenience, define

$$\begin{aligned} Y_s = {} & (1-\lambda_s)\{-\pi_{D,s} + \delta p_s[\rho z_{j,s} + (1-\rho)(1-(1-\lambda_s)(1-z_{j,s})p_s)]EV_{D,s}^* \\ & + \delta p_s[\rho(1-z_{j,s}) + (1-\rho)(1-\lambda_s)(1-z_{j,s})p_s]EV_{M,s}^*\}. \end{aligned}$$

Assuming that the first-order condition is satisfied, we know that $Y_s > 0$.

We begin by taking the derivative of CRA i's first-order condition with respect to $w_{j,s}$:

$$\frac{\partial^2 z_{i,s}}{\partial w^2}\frac{dw_{i,s}}{dw_{j,s}} Y_s + \frac{\partial z_{i,s}}{\partial w}\frac{\partial z_{j,s}}{\partial w}\delta[\rho(1-\lambda_s)p_s + (1-\rho)(1-\lambda_s)^2 p_s^2]\left[EV_{D,s}^* - EV_{M,s}^*\right] = 0,$$

Rearranging gives us:

$$\frac{dw_{i,s}}{dw_{j,s}} = -\frac{\frac{\partial z_{i,s}}{\partial w}}{\frac{\partial^2 z_{i,s}}{\partial w^2}} \frac{\frac{\partial z_{j,s}}{\partial w}\delta[\rho(1-\lambda_s)p_s + (1-\rho)(1-\lambda_s)^2 p_s^2][EV^*_{D,s} - EV^*_{M,s}]}{Y_s}. \tag{35}$$

Since $\frac{\frac{\partial z_{i,s}}{\partial w}\frac{\partial z_{j,s}}{\partial w}}{\frac{\partial^2 z_{i,s}}{\partial w^2}} < 0$, $Y_s > 0$, and, as shown in the Lemma below, $EV^*_{D,s} - EV^*_{M,s} < 0$, it follows that $\frac{dw_{i,s}}{dw_{j,s}} < 0$. ■

Lemma 5 $EV^*_{M,s} > EV^*_{D,s}$

Proof. We will prove by contradiction. Supppose that $EV^*_{M,B} < EV^*_{D,B}$. Then, it must be the case that either $V^*_{M,B} < V^*_{D,B}$ or $V^*_{M,R} < V^*_{D,R}$ or both; that is, there are three cases to consider: (i) $V^*_{M,B} < V^*_{D,B}$ and $V^*_{M,R} > V^*_{D,R}$; (ii) $V^*_{M,B} > V^*_{D,B}$ and $V^*_{M,R} < V^*_{D,R}$; and, (iii) $V^*_{M,B} < V^*_{D,B}$ and $V^*_{M,R} < V^*_{D,R}$. Note that in the text and in other proofs, we represent CRA i's payoffs in a duopoly in state s using $V^*_{i,s}$, $z_{i,s}$, and $w_{i,s}$. Here we denote the same expressions as $V^*_{D,s}$, $z_{D,s}$, and $w_{D,s}$ to clarify the difference between monopoly and duopoly. All expressions will be from the point of view of CRA i.

We consider each case in turn:

Case (i) $EV^*_{M,B} < EV^*_{D,B}$, $V^*_{M,B} < V^*_{D,B}$ and $V^*_{M,R} > V^*_{D,R}$

Substituting $EV^*_{D,B}$ for $EV^*_{M,B}$ in Expression (11) allows us to write

$$\begin{aligned} V^*_{D,B} &< \pi_{D,B}(\lambda_B + (1-\lambda_B)(1-z^*_{D,B})) - w^*_{D,B} + \delta(1-(1-\lambda_B)(1-z^*_{D,B})p_B)EV^*_{D,B} \\ &= \pi_{D,B}(\lambda_B + (1-\lambda_B)(1-z^*_{D,B})) - w^*_{D,B} + \delta(1-(1-\lambda_B)(1-z^*_{D,B})p_B)((1-\tau_B)V^*_{D,B} + \tau_B V^*_{D,R}), \end{aligned}$$

where $w^*_{D,B}$ is the optimal duopoly wage in the boom state and $z^*_{D,B}$ the associated accuracy.

Since $\pi_{M,B} > \pi_{D,B}$ and $V^*_{M,R} > V^*_{D,R}$, it follows that

$$V^*_{D,B} < \pi_{M,B}(\lambda_B+(1-\lambda_B)(1-z^*_{D,B}))-w^*_{D,B}+\delta(1-(1-\lambda_B)(1-z^*_{D,B})p_B)((1-\tau_B)V^*_{D,B}+\tau_B V^*_{M,R}). \tag{36}$$

Now, let

$$V_1 = \max_w \pi_{M,B}(\lambda_B+(1-\lambda_B)(1-z_B(w)))-w+\delta(1-(1-\lambda_B)(1-z_B(w))p_B)((1-\tau_B)V^*_{D,B}+\tau_B V^*_{M,R}).$$

Then, V_1 is greater than or equal to the right-hand side of Equation (36), and so $V_1 > V^*_{D,B}$.

Next, define

$$V_n = \max_w \pi_{M,B}(\lambda_B + (1-\lambda_B)(1-z_B(w))) - w + \delta(1-(1-\lambda_B)(1-z_B(w))p_B)((1-\tau_B)V_{n-1} + \tau_B V^*_{M,R}).$$

Then, following similar reasoning, $V_n \geq V_{n-1}$. It is clear every element of $\{V_n\}$ is bounded above by $\frac{\pi_{M,B}}{1-\delta}$, and since the sequence is non-decreasing and bounded, it must converge and the limit $V_n \to V^*_{M,B}$ since $V^*_{M,B}$ solves

$$V^*_{M,B} = \pi_{M,B}(\lambda_B + (1-\lambda_B)(1-z^*_B)) - w^*_{M,B} + \delta(1-(1-\lambda_B)(1-z^*_B)p_B)((1-\tau_B)V^*_{M,B} + \tau_B V^*_{M,R}). \tag{37}$$

It follows that $V^*_{M,B} > V^*_{D,B}$, which is a contradiction.

Case (ii) $EV^*_{M,B} < EV^*_{D,B}$, $V^*_{M,B} > V^*_{D,B}$, and $V^*_{M,R} < V^*_{D,R}$

Given that $EV^*_{M,B} < EV^*_{D,B}$, we can conclude

$$\begin{aligned}
V^*_{D,R} = {} & \pi_{D,R}(\lambda_B + (1-\lambda_R)(1-z^*_{D,R})) - w^*_{D,R} \\
& +\delta[\rho(1-(1-\lambda_R)(1-z^*_{D,R}z_{j,R})p_R) \\
& +(1-\rho)(1-(1-\lambda_R)(1-z^*_{D,R})p_R)(1-(1-\lambda_R)(1-z_{j,R})p_R]((1-\tau_R)V^*_{D,R} + \tau_R V^*_{D,B}) \\
& +\delta[\rho(1-\lambda_R)(1-z_{j,R})z^*_{D,R}p_R \\
& +(1-\rho)(1-(1-\lambda_R)(1-z^*_{D,R})p_R(1-\lambda_R)(1-z_{j,R})p_R]((1-\tau_R)V^*_{M,R} + \tau_R V^*_{M,B}) \\
< {} & \pi_{D,R}(\lambda_B + (1-\lambda_R)(1-z^*_{D,R})) - w^*_{D,R} \\
& +\delta[\rho(1-(1-\lambda_R)(1-z^*_{D,R}z_{j,R})p_R) \\
& +(1-\rho)(1-(1-\lambda_R)(1-z^*_{D,R})p_R)(1-(1-\lambda_R)(1-z_{j,R})p_R]((1-\tau_R)V^*_{D,R} + \tau_R V^*_{M,B}) \\
& +\delta[\rho(1-\lambda_R)(1-z_{j,R})z^*_{D,R}p_R \\
& +(1-\rho)(1-(1-\lambda_R)(1-z^*_{D,R})p_R(1-\lambda_R)(1-z_{j,R})p_R]((1-\tau_R)V^*_{D,R} + \tau_R V^*_{M,B}) \\
= {} & \pi_{D,R}(\lambda_B + (1-\lambda_R)(1-z^*_{D,R})) - w^*_{D,R} + \delta(1-(1-\lambda_B)(1-z^*_{D,R})p_R)((1-\tau_R)V^*_{D,R} + \tau_R V^*_{M,B}),
\end{aligned}$$

where the second expression follows from the first by noting that $V^*_{M,B} > V^*_{D,B}$ and $V^*_{M,R} < V^*_{D,R}$, and the third line follows by rearranging terms.

Working from the last expression and following the same reasoning as in Case (i) ensures that $V^*_{M,R} > V^*_{D,R}$ and provides a contradiction.

Case (iii) $EV^*_{M,B} < EV^*_{D,B}$, $V^*_{M,B} < V^*_{D,B}$, and $V^*_{M,R} < V^*_{D,R}$

For this case, it must also be true that $EV^*_{M,R} < EV^*_{D,R}$. Since $EV^*_{M,s} < EV^*_{D,s}$ for

$s \in \{B, R\}$, and substituting $EV^*_{D,s} = (1-\tau_s)V_{D,s} + \tau_s V_{D,-s}$,we can write:

$$V_{D,s} < \pi_{D,s}(\lambda_s + (1-\lambda_s)(1-z^*_{D.s})) - w^*_{D,s} + \delta(1-(1-\lambda_s)(1-z^*_{D,s})p_s)((1-\tau_s)V_{D,s} + \tau_s V_{D,-s}), \tag{38}$$

for $s \in \{B, R\}$.

We can then construct a $V^n_{D,s}$ and V^n_{D-s} as follows. First, define

$$V^1_{D,s} := \max_{w_s} \pi_{D,s}(\lambda_s + (1-\lambda_s)(1-z_s)) - w_s + \delta(1-(1-\lambda_s)(1-z_s)p_s)((1-\tau_s)V^1_{D,s} + \tau_s V_{D,-s}).$$

Then $V^1_{D,s}$ is larger than the right-hand side of (38), and so $V^1_{D,s} > V_{D,s}$. We can recursively define

$$V^n_{D,s} = \max_{w_s} \pi_{D,s}(\lambda_s + (1-\lambda_s)(1-z_s)) - w_s + \delta(1-(1-\lambda_s)(1-z_s)p_s)((1-\tau_s)V^n_{D,s} + \tau_s V^{n-1}_{D,-s}).$$

$V^n_{D,s}$ and $V^n_{D,-s}$ are non-decreasing in n, and since for any n both are bounded above $\frac{\pi_{D,B}}{1-\delta}$, the limits as $n \to \infty$ must be well defined; let $\tilde{V}_{D,s} := \lim_{n\to\infty} V^n_{D,s}$, then it is immediate that $(1-\tau_s)\tilde{V}_{D,s} + \tau_s \tilde{V}_{D,-s} \geq EV^*_{D,s}$.

Now compare $\tilde{V}_{D,s}$ with $V^*_{M,s}$:

$$\tilde{V}_{D,s} = \max_w \begin{array}{c} \pi_{D,s}(\lambda_s + (1-\lambda_s)(1-z_s(w))) - w \\ +\delta(1-(1-\lambda_s)(1-z_s(w))p_s)((1-\tau_s)\tilde{V}_{D,s} + \tau_s \tilde{V}_{D,-s}). \end{array} \tag{39}$$

$$V^*_{M,s} = \max_w \begin{array}{c} \pi_{M,s}(\lambda_s + (1-\lambda_s)(1-z_s(w))) - w \\ +\delta(1-(1-\lambda_s)(1-z_s(w))p_s)((1-\tau_s)V^*_{M,s} + \tau_s V^*_{M,-s}). \end{array} \tag{40}$$

Note that the only difference between $\tilde{V}_{i,s}$ and $V^*_{M,s}$ are the fees $\pi_{M,s}$ and $\pi_{D,s}$, which are exogenous. Define Equation (40) as two equations (one for each state) where the $V_{M,s}$ is brought over to the right-hand side and the left-hand side equals zero. Label these two equations G_B and G_R. We can now apply the implicit function theorem as in Proposition

1. We define α_s as in Lemma 4. This implies that:

$$\begin{aligned}\frac{dV^*_{M,R}}{d\pi_{M,R}} &= -\det\begin{bmatrix}\frac{\partial G_B}{\partial \pi_{M,R}} & \frac{\partial G_B}{\partial V^*_{M,B}}\\ \frac{\partial G_R}{\partial \pi_{M,R}} & \frac{\partial G_R}{\partial V^*_{M,B}}\end{bmatrix} / \det\begin{bmatrix}\frac{\partial G_B}{\partial V^*_{M,R}} & \frac{\partial G_B}{\partial V^*_{M,B}}\\ \frac{\partial G_R}{\partial V^*_{M,R}} & \frac{\partial G_R}{\partial V^*_{M,B}}\end{bmatrix}\\
&= -\frac{(\lambda_R + (1-\lambda_R)(1-z_{M,R}))(1-\delta\alpha_B(1-\tau_B))}{\delta^2\tau_B\tau_R\alpha_R\alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}; \text{ and,}\\
\frac{dV^*_{M,R}}{d\pi_{M,B}} &= -\det\begin{bmatrix}\frac{\partial G_B}{\partial \pi_{M,B}} & \frac{\partial G_B}{\partial V^*_{M,B}}\\ \frac{\partial G_R}{\partial \pi_{M,B}} & \frac{\partial G_R}{\partial V^*_{M,B}}\end{bmatrix} / \det\begin{bmatrix}\frac{\partial G_B}{\partial V^*_{M,R}} & \frac{\partial G_B}{\partial V^*_{M,B}}\\ \frac{\partial G_R}{\partial V^*_{M,R}} & \frac{\partial G_R}{\partial V^*_{M,B}}\end{bmatrix}\\
&= -\frac{(\lambda_B + (1-\lambda_B)(1-z_{M,B}))\delta\alpha_R\tau_R}{\delta^2\tau_B\tau_R\alpha_R\alpha_B - (1-\delta\alpha_B(1-\tau_B))(1-\delta\alpha_R(1-\tau_R))}.\end{aligned}$$

In Lemma 4, we demonstrated that the denominator of both expressions is negative. The overall expressions are, therefore, positive. It is easy to see that both $\frac{dV^*_{M,B}}{d\pi_{M,B}}$ and $\frac{dV^*_{M,B}}{d\pi_{M,R}}$ are also positive. Therefore, since $\pi_{M,s}$ is larger than $\pi_{D,s}$ for $s \in \{B, R\}$, $V^*_{M,s}$ is larger than $\tilde{V}_{D,s}$ for $s \in \{B, R\}$. This then implies that $EV^*_{M,s} > EV^*_{D,s}$ for $s \in \{B, R\}$, which is a contradiction. ∎

Proof of Proposition 7

Proof. Consider the first-order condition with symmetry imposed in Equation (13). Define:

$$\begin{aligned}A_s := -1 + \frac{\partial z_s}{\partial w}(1-\lambda_s)\{-\pi_{D,s} + \delta p_s[\rho z_s + (1-\rho)(1-(1-\lambda_s)(1-z_s)p_s)]EV^*_{D,s}\\ +\delta p_s[\rho(1-z_s) + (1-\rho)(1-\lambda_s)(1-z_s)p_s]EV^*_{M,s}\}\end{aligned} \tag{41}$$

and

$$\begin{aligned}\hat{Y}_s = (1-\lambda_s)\{-\pi_{D,s} + \delta p_s[\rho z_s + (1-\rho)(1-(1-\lambda_s)(1-z_s)p_s)]EV^*_{D,s}\\ +\delta p_s[\rho(1-z_s) + (1-\rho)(1-\lambda_s)(1-z_s)p_s]EV^*_{M,s}\}.\end{aligned}$$

Then

$$\frac{dA_s}{dw} = \frac{\partial^2 z_s}{\partial w^2}\hat{Y}_s - \delta(\frac{\partial z_s}{\partial w})^2(1-\lambda_s)p_s[\rho + (1-\rho)(1-\lambda_s)p_s](EV^*_{M,s} - EV^*_{D,s}) < 0,$$

where the inequality follows since $\hat{Y}_s > 0$ (assuming that there is an interior solution), $\frac{\partial^2 z_s}{\partial w^2} < 0$, $\frac{\partial z_s}{\partial w} > 0$ and $EV^*_{D,s} < EV^*_{M,s}$ by Lemma 5 above.

Note that we can represent the first-order condition as:

$$\frac{1}{\frac{\partial z_s}{\partial w}} = \hat{Y}_s. \tag{42}$$

The left-hand side is increasing in w. It also shifts up if γ is larger.

The right-hand side is decreasing in w. We now examine what happens to the right-hand side when the parameters change. In particular, how can we compare $\hat{Y}_B$ and $\hat{Y}_R$? This is a similar exercise to the one we performed for the monopoly case.

When states are independent across time, $\tau_B = 1 - \tau_R$, which implies that $EV^*_{D,B} = EV^*_{D,R}$ and $EV^*_{M,B} = EV^*_{M,R}$.

We consider each of our parameters in turn.

First, consider $\pi_{D,s}$:

$$\frac{d\hat{Y}_s}{d\pi_{D,s}} = -(1-\lambda_s) < 0. \tag{43}$$

Next,

$$\frac{d\hat{Y}_s}{dp_s} = (1-\lambda_s)\{\delta(2(1-\rho)p_s+\rho)EV^*_{D,s}+\delta(1-z_s)[\rho+2(1-\rho)(1-\lambda_s)p_s](EV^*_{M,s}-EV^*_{D,s})\} > 0. \tag{44}$$

Turning next to λ_s,

$$\frac{d\hat{Y}_s}{d\lambda_s} = -\frac{\hat{Y}_s}{1-\lambda_s} + (1-\lambda_s)\delta(1-\rho)(1-z_s)p_s^2(EV^*_{D,s} - EV^*_{M,s}) < 0. \tag{45}$$

Finally, we turn to consider γ_s,

$$\frac{d\hat{Y}_s}{d\gamma_s} = -\frac{\partial z_s}{\partial \gamma_s}\delta p_s(1-\lambda_s)[\rho + (1-\rho)(1-\lambda_s)p_s](EV^*_{M,s} - EV^*_{D,s}) > 0. \tag{46}$$

Using Equation (42) and the subsequent results, it is clear that when $\pi_{D,B} > \pi_{D,R}$, $p_B < p_R$, and $\lambda_B > \lambda_R$, it is true that $w^*_B < w^*_R$. This effect on wages is ambiguous for $\gamma_B > \gamma_R$, as higher γ_s pushes both the left-hand side and right-hand side of Equation (42) up. ■

Proof of Proposition 8

Proof. Given assumption A1, $EV^*_{M,B} < EV^*_{M,R}$. Assumption A2 implies that $EV^*_{D,B} < EV^*_{D,R}$. Using the proof of the previous proposition, this implies that $\hat{Y}_B < \hat{Y}_R$ when

$\pi_{D,B} > \pi_{D,R}$, $p_B < p_R$, and $\lambda_B > \lambda_R$ hold (and setting $\gamma_B = \gamma_R$), which means that $w^*_{D,B} < w^*_{D,R}$. As in the previous proposition, setting $\gamma_B > \gamma_R$ has an ambiguous effect. ■

Proof of Lemma 3

Proof. For notational convenience, define

$$\tilde{Y}_s = (1-\lambda_s)\{-\pi_{D,s} + \delta p_s[\rho(-1+2z_{j,s}) + (1-\rho)(1-(1-\lambda_s)(1-z_{j,s})p_s)]EV^*_{D,s} \\ +\delta p_s[\rho(1-z_{j,s}) + (1-\rho)(1-\lambda_s)(1-z_{j,s})p_s]EV^*_{M,s}\}. \quad (47)$$

Assuming that the first-order condition is satisfied, we know that $\tilde{Y}_s > 0$.

We begin by taking the derivative of CRA i's first-order condition with respect to $w_{j,s}$:

$$0 = \frac{\partial^2 z_{i,s}}{\partial w^2}\frac{dw_{i,s}}{dw_{j,s}}\tilde{Y}_s + \frac{\partial z_{i,s}}{\partial w}\frac{\partial z_{j,s}}{\partial w}\delta(1-\lambda_s)p_s[\rho + (1-\rho)(1-\lambda_s)p_s]\left[EV^*_{D,s} - EV^*_{M,s}\right] \\ +\frac{\partial z_{i,s}}{\partial w}\frac{\partial z_{j,s}}{\partial w}\delta(1-\lambda_s)p_s\rho EV^*_{D,s}, \quad (48)$$

and, so

$$\frac{dw_{i,s}}{dw_{j,s}} = -\frac{\frac{\partial z_{i,s}}{\partial w}}{\frac{\partial^2 z_{i,s}}{\partial w^2}}\frac{\frac{\partial z_{j,s}}{\partial w}\delta(1-\lambda_s)p_s\{[\rho + (1-\rho)(1-\lambda_s)p_s]\left[EV^*_{D,s} - EV^*_{M,s}\right] + \rho EV^*_{D,s}\}}{\tilde{Y}_s}. \quad (49)$$

Since $\frac{\frac{\partial z_{i,s}}{\partial w}\frac{\partial z_{j,s}}{\partial w}}{\frac{\partial^2 z_{i,s}}{\partial w^2}} < 0$, and $\tilde{Y}_s > 0$ the result follows. ■

Proof of Proposition 11

Proof. Define the reaction functions for state s as $w_{i,s}(w_{j.s})$ and $w_{j,s}(w_{i.s})$. From the definition of strategic complements, we know that $w'_{i,s}(w_{j.s})$ and $w'_{j,s}(w_{i.s})$ are greater than zero. Next, in order to ensure that the curves cross, we need two conditions. First, we need $w_{i,s}(0)$ and $w_{j,s}(0)$ to be greater than zero. The reaction functions are implicitly defined by the first-order conditions given by Equation (15). We will look at the case of CRA i. Taking $w_{j.s} = 0$, we know that $z_{j,s}(0) = 0$. The first-order condition simplifies to:

$$1 = \frac{\partial z_{i,s}}{\partial w}(1-\lambda_s)\{-\pi_{D,s} + \delta p_s[\rho + (1-\rho)(1-\lambda_s)p_s](EV^*_{M,s} - EV^*_{D,s}) + \delta p_s(1-\rho)EV^*_{D,s}\}.$$

In order for $w_{i,s}(0) > 0$ to be true, we need the expression in curly brackets to be positive. That would imply that $\frac{\partial z_{i,s}}{\partial w} > 0$. Conditions 1 and 2 guarantee this.

We also need the reaction functions to cross the 45-degree line. Consider $w_{j,s} = \bar{w}$, the maximum wage. Denote $z(\bar{w}) = \bar{z} \leq 1$. Then, the first-order condition in equation 15 is:

$$1 = \frac{\partial z_{i,s}}{\partial w}(1-\lambda_s)\{-\pi_{D,s}+\delta p_s(1-\bar{z})[\rho+(1-\rho)(1-\lambda_s)p_s](EV^*_{M,s}-EV^*_{D,s})+\delta p_s(1-\rho(1-\bar{z}))EV^*_{D,s}\}.$$

Once again, from Conditions 1 and 2, we know that the expression in curly brackets is positive. This implies that $\frac{\partial z_{i,s}}{\partial w}$ is positive and finite. Since $\frac{\partial z^2}{\partial w^2} < 0$ and $\frac{\partial z}{\partial w}|_{\overline{w}} = 0$, it must be that $w_{i,s}(\bar{w}) < \bar{w}$.

Lastly, for uniqueness, we will show that Condition 3 implies that the reaction functions are concave. To reduce notation slightly, label the strategic complements condition $[\rho + (1-\rho)(1-\lambda_s)p_s]\left[EV^*_{D,s} - EV^*_{M,s}\right] + \rho EV^*_{D,s}$ as Ψ. The slope of the reaction function $w_{i,s}(w_{j.s})$ is given by Equation (49), which we can rewrite as:

$$\frac{dw_{i,s}}{dw_{j,s}} = -\frac{\frac{\partial z_{i,s}}{\partial w}}{\frac{\partial^2 z_{i,s}}{\partial w^2}}\frac{\frac{\partial z_{j,s}}{\partial w}\delta(1-\lambda_s)p_s\Psi}{\tilde{Y}_s}.$$

The second-order condition for the reaction function is given by:

$$\frac{d^2 w_{i,s}}{dw^2_{j,s}} = -\delta(1-\lambda_s)p_s\Psi\frac{(\frac{\partial^2 z_{i,s}}{\partial w^2}\frac{dw_{i,s}}{dw_{j,s}}\frac{\partial z_{j,s}}{\partial w} + \frac{\partial z_{i,s}}{\partial w}\frac{\partial^2 z_{j,s}}{\partial w^2})\frac{\partial^2 z_{i,s}}{\partial w^2}\tilde{Y}_s - (\frac{\partial^3 z_{i,s}}{\partial w^3}\frac{dw_{i,s}}{dw_{j,s}}\tilde{Y}_s + \frac{\partial^2 z_{i,s}}{\partial w^2}\frac{\partial \tilde{Y}_s}{\partial w_{j,s}})\frac{\partial z_{i,s}}{\partial w}\frac{\partial z_{j,s}}{\partial w}}{(\frac{\partial^2 z_{i,s}}{\partial w^2}\tilde{Y}_s)^2}.$$

Note that from the property of strategic complements, $\Psi > 0$ and $\frac{dw_{i,s}}{dw_{j,s}} > 0$. As above, Conditions 1 and 2 give us that $\tilde{Y}_s > 0$. Condition 3 gives us that $\frac{\partial^3 z_{i,s}}{\partial w^3}$ is positive. The only remaining term left to sign is $\frac{\partial \tilde{Y}_s}{\partial w_{j,s}}$. Taking the derivative $\tilde{Y}_s$, as in Equation (47), gives us $\delta(1-\lambda_s)p_s\frac{\partial z_{j,s}}{\partial w}\Psi$, which is positive. This proves that $\frac{d^2 w_{i,s}}{dw^2_{j,s}}$ is negative. ■