

# Imperfect Competition and Reputational Commitment

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## Abstract

Competition can both aid and hinder reputational commitments for quality. These are self-sustaining depending on future profits after maintaining or deviating from the commitment, and on current costs of sustaining it. Competition can affect these three elements at different rates.

Key words: quality, competition, self-enforcing commitment

JEL code: L13 (Oligopoly and Other Imperfect Markets ), L14 (Transactional Relationships; Contracts and Reputation; Networks ), L15 (Information and Product Quality; Standardization and Compatibility)

## 1 Introduction

This note presents an example highlighting that competition can have a non-monotonic effect on the ability of a firm or an industry to sustain an equilibrium in which it produces high quality products. Moreover, inasmuch as we provide parameter examples in which high quality can be sustained in markets where the degree of competition is either very low or very high, but not where it is intermediate, the intuitions we provide might be of use in considering the difficulty of the transition to competitive markets

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\*I thank Adam Brandenburger, Luis Cabral, Johannes Hörner, Niko Matouschek, Felix Muenich, Roy Radner and participants at the First Interdisciplinary Symposium on Online Reputation Mechanisms for comments and advice. An earlier version of this note appeared as a chapter of my PhD thesis, I thank my advisers Margaret Bray and Leonardo Felli for advice and encouragement throughout. I also gratefully acknowledge the Center for Mathematical Studies in Economics and Managerial Sciences which hosted me through much of the overlong gestation of this note.

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in developing markets, or for example, the coexistence between the Anglo-Saxon competitive banking model and German/Japanese model of relationship-banking.

In general, whether reputational considerations will motivate a firm to perform some costly action, say producing high quality, depends on the trade-off between the short-term gain or saving in not performing the action and the long-term effects of beginning the following period with a relatively low reputation (say that the expectation will be that from then onwards the firm will produce only low quality). Suppose that producing high quality maintains a high reputation and failing to ensure a low reputation (as is typically assumed to be the case in the literature on relational contacts or equilibria supported by trigger strategies and as we will assume below). Then this trade-off can be summarized by the following familiar inequality—the key to our analysis—which ensures that the firm produces high quality:

$$\begin{aligned} & \text{short-term cost of producing high rather than low quality} \\ & \leq \text{discounted value of high reputation} - \text{discounted value of low reputation} \end{aligned} \tag{1}$$

This note essentially aims to point out that increased competition can affect all of these terms and at different rates, and so the overall effect of competition on such reputation incentives is ambiguous and may be non-monotonic.

Before introducing our example and analyzing it, it is perhaps worth providing some intuition, concerning the "punishment" for not exerting effort—the difference in the discounted values of continuing with a high or low reputation, though note that competition might also affect the left hand side of Inequality (1). On the one hand, if competition drives prices down then, since the firm can always exit and earn zero profits, it seems clear that competition reduces the discounted value of high reputation with no effect on the discounted value of low reputation (which stays at zero). In this case, competition reduces the punishment for deviating and so makes it more difficult to satisfy Inequality (1). On the other hand, a firm in a more competitive environment faces the prospect of a more severe fall in market share as well as a price drop on losing its reputation in the case where the discounted value

of low reputation is non-zero and so the punishment for not exerting effort increases with competition, making it easier to satisfy Inequality (1).

## 1.1 Related Literature

A literature beginning with Klein and Leffler (1981) and including, more recently, Hörner (2003) aims at examining the extent to which relational contracts or reputation concerns can ensure high quality provision. As discussed at great length below, our principal point of departure from this tradition is to assume that profits are driven to zero when there are many identical firms since we do not allow quantities to play a signalling role. Kranton (2003), perhaps the paper closest to this note in spirit inasmuch as it makes a similar assumption, makes the additional assumption that once a firm produces low quality, its continuation value is zero and so does not consider or allow for any positive affects of competition on the ability to sustain high quality equilibria—whereas the focus of this note is to highlight that competition has ambiguous effects on such equilibria.

To some extent providing high quality can be thought of as making an investment in reputation and so these results of ambiguous effects of competition on this kind of investment parallel a wide literature on Schumpeterian innovation and discussions on the ambiguous effects of market structure on advertising intensity (see, for example, Sutton (1991), Cabral (2000) and Martin (1993)).

With respect to our conclusion on the ambiguous role of competition on quality provision and efficiency, there is a considerable body of literature which, though focusing on different mechanisms, suggests that economic research is not always in agreement with the conventional wisdom on the benefits and effect of competition (for example Schmidt (1997), Spence (1975), and Stiglitz (1987)). Finally, to the extent, that this note suggests that firms might choose to operate in more competitive environments as a means to commit to “good” behavior both in the present and in the future, it is related to the literature on second-sourcing which considers a firm’s choice to allow for more competition in the future as a means to commit to “good” behavior both in the future (see Farrell and Gallini (1988) and Shepard (1987) and Dudey (1990) and Wernerfelt (1994) in a slightly different context).

## 2 Model

Suppose that there are  $n + 1$  identical firms. In every period, every firm simultaneously chooses quantity and quality. When producing low quality, the cost of production is 0, and when producing high quality, the per-unit cost of production is  $c > 0$ . Customers cannot observe a product's quality and we do not allow for warranties or other explicit quality-contingent contracts,<sup>1</sup> and so Firm  $i$ 's demand depends on customers' anticipation of the quality of its product and on those of its rivals. Specifically, we suppose that demand is given by the following inverse demand function:<sup>2</sup>

$$p_{it} = 1 - \frac{2x_{it}}{u_{it}^2} - \frac{2\sigma}{u_{it}} \sum_{j \neq i} \frac{x_{jt}}{u_{jt}}, \quad (2)$$

where  $x_{it}$  denotes the quantity of the good produced by Firm  $i$  in period  $t$  and  $u_{it} \in \{l, h\}$  denotes its *anticipated* quality, which may be either low or high and  $\sigma$  denotes the degree of substitution between different firms' outputs). In particular, this inverse demand function implies that goods are imperfect substitutes when  $\sigma < 1$  and that customers are willing to pay more when they anticipate high quality goods.

We assume that customers' and rivals' expectations of Firm  $i$ 's quality and future behavior are not affected by its quantity decision. This implies both that  $x_{it}$  plays no signalling role and precludes the possibility of collusion between firms.<sup>3</sup> Firms maximize future profits with a per-period discount factor  $\delta$ . Finally, suppose that  $h(1 - c) > l$ , which is sufficient to ensure that high quality provision is efficient and that firms would choose to produce high quality if it were observable.

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<sup>1</sup>In numerous applications, such as banking or many professional services it is hard to envisage that enforceable and complete outcome contingent contracts could be written and it is certainly the case that they are not.

<sup>2</sup>See Appendix 2.2 of Sutton (1998) for further details concerning this linear demand model with quality indices.

<sup>3</sup>Klein and Leffler (1981), for example, assume that customers make inferences from prices and so even with many identical firms, firms can enjoy a premium above costs and so high quality provision can be sustained.

We suppose that the assumption that quantity choices are observable, for example in a richer model with demand shocks, may be unrealistic and believe that relaxing this assumption for the purposes of this example is valuable. Kranton (2003) also makes a similar assumption to this note, though in her model while customers cannot use past prices to make inferences about current quality they can use current price and past quality realization to form their expectations of current quality.

### 3 Analysis

We aim to explore how the possibility that an industry can sustain a high quality equilibrium, that is one where each firm produces high quality, varies with the degree of competition (as measured either by  $\sigma$  or by  $n$ ).

Notice that since we assume that expectations of future quality depend only on past quality and not on past or current quantity decisions, if a high quality equilibrium is sustained then it will be one where each firm sets the quantity that would maximize profits in the case that it was maximizing static profits where it and all other firms were producing at high quality. We denote this profit-maximizing quantity by  $x_h$  and the associated per-period profits  $\pi_h$  and price  $p_h$ .

Suppose that customers and firms anticipated that Firm  $i$  was going to produce high quality and that all other firms produced high quality and produced  $x_h$  then Firm  $i$ 's most profitable deviation would be to produce low quality output and to change its quantity to  $x_d$ , where:

$$x_d = \arg \max_x \left(1 - \frac{2x}{h^2} - \frac{2\sigma}{h^2} nx_h\right)x, \quad (3)$$

and the associated single-period profit associated with this behavior is  $\pi_d$ .

Following Abreu (1988), to consider the viability of the equilibrium in which all firms produce high quality, one should consider a most severe feasible punishment continuation equilibrium for a firm that deviates, suppose that the per-period profit that the deviating firm earns in this most-severe-feasible-punishment equilibrium is  $\pi_{msfp}$  then the condition that ensures the existence of an equilibrium in which all firms produce high quality, analogous to Inequality (1), is given by:

$$\frac{1}{1-\delta}\pi_h \geq \pi_d + \frac{\delta}{1-\delta}\pi_{msfp}. \quad (4)$$

We return to analyze  $\pi_h$ ,  $\pi_d$ ,  $\pi_{msfp}$  and how  $\Delta = \frac{1}{1-\delta}\pi_h - (\pi_d + \frac{\delta}{1-\delta}\pi_{msfp})$  varies with our measures of competition. First however, it will be useful to consider the following.

### 3.1 Static benchmark: $m$ firms low, $n + 1 - m$ high

In our simple environment, where no other agents' decisions depend on Firm  $i$ 's quantity decision, it is enough to consider continuation equilibria where firms maximize static profits taken as given equilibrium quality decisions.

Consider, therefore the situation where  $m$  firms are producing low quality and  $n + 1 - m$  firms are producing high quality. Then let  $x_{lm}$  denote a low quality producer's output in this case and  $x_{hm}$  a high quality producer's output (and note that  $x_h = x_{h0}$ ). Then  $x_{lm}$  is given by:

$$x_{lm} = \arg \max_x \left( 1 - \frac{2x}{l^2} - \frac{2\sigma}{hl}(n + 1 - m)x_{hm} - \frac{2\sigma}{l^2}(m - 1)x_{lm} \right) x, \quad (5)$$

A similar expression will define  $x_{hm}$ . These will be useful for determining the profits for a low quality producer  $\pi_{lm}$  and for a high quality produce  $\pi_{hm}$  when  $m$  out of  $n + 1$  firms are producing low output. First order conditions and some algebraic manipulation will reveal that

$$\pi_{lm} = \chi_m \frac{l}{2} \left( \frac{2l + \sigma l(n - m) - \sigma h(1 - c)(n + 1 - m)}{4 + 2\sigma(n - 1) - \sigma^2 n} \right)^2, \quad (6)$$

and

$$\pi_{hm} = \chi_m \frac{1}{2} \left( \frac{2h(1 - c) + h\sigma(m - 1)(1 - c) - l\sigma m}{(2 + \sigma n)(2 - \sigma)} \right)^2 + (1 - \chi_m) \frac{1}{2} \left( \frac{h(1 - c)}{2 + \sigma(n - m)} \right)^2, \quad (7)$$

where

$$\chi_m = \begin{cases} 1 & \text{if } 2l + \sigma l(n - m) - \sigma h(1 - c)(n + 1 - m) > 0 \\ 0 & \text{otherwise} \end{cases}. \quad (8)$$

It can be shown that both  $\pi_{lm}$  and  $\pi_{hm}$  are continuous and increasing in  $m$  and that  $\pi_{hm} > \pi_{lm}$ .

### 3.2 Necessary and sufficient conditions for high quality equilibrium

Returning to our central question, the last sentence of the previous subsection implies that the most severe punishment will be the one where in the continuation equilibrium the deviating firm is supposed to produce at low quality and as many firms as possible produce high quality. Note, however, that a continuation equilibrium where one firm produces low quality and the remaining firms produce high quality may not be feasible. Nevertheless, it is clear that it is always the case that there is a continuation equilibrium where all firms produce low quality, and associated with this continuation equilibrium we can define:

$$\Delta_{suff} = \frac{1}{1-\delta}\pi_h - \pi_d - \frac{\delta}{1-\delta}\pi_{l(n+1)}. \quad (9)$$

In addition, following our earlier remarks, associated with the most severe equilibrium (which may not be feasible) we can define:

$$\Delta_{nec} = \frac{1}{1-\delta}\pi_h - \pi_d - \frac{\delta}{1-\delta}\pi_{l1}. \quad (10)$$

Finally, it follows that  $\Delta_{nec} \geq \Delta \geq \Delta_{suff}$  and so the figures below, plotted for specific parameter values, as described in the legends, are sufficient to substantiate our earlier claim that competition (as measured either by  $\sigma$  or by  $n$ ) can have ambiguous and, in particular, non-monotonic effects on the possibility of an equilibrium in which high quality output is produced, as summarized by  $\Delta$  which represents the trade-off between the short-term gain of deviating from such an hypothetical equilibrium and the long term costs of sticking to it.

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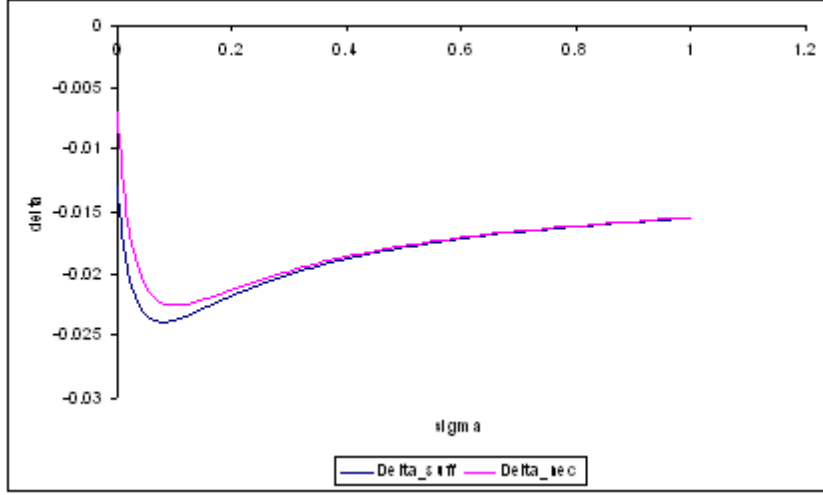


Figure 1:  $\Delta$  against  $\sigma$  with  $h=0.9$ ,  $l=0.05$ ,  $c=0.35$ ,  $n=25$  and  $\delta=0.55$

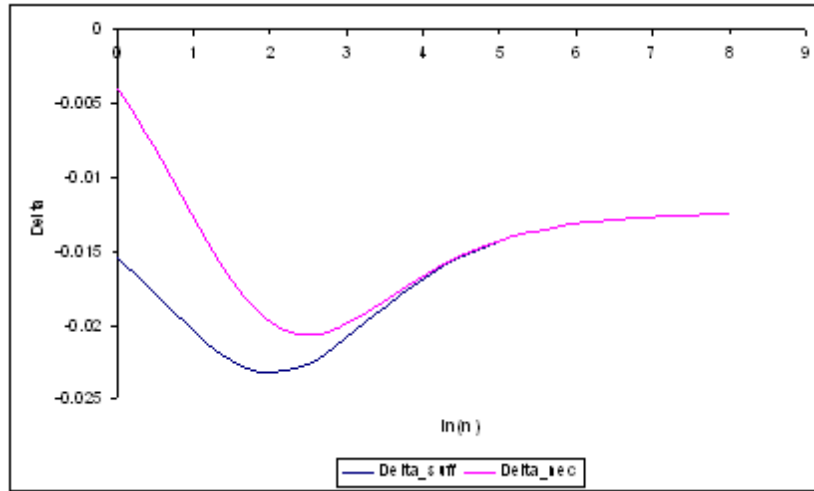


Figure 2:  $\Delta$  against  $\ln(n)$  with  $h=0.9$ ,  $l=0.05$ ,  $c=0.35$ ,  $\sigma=1$  and  $\delta=0.65$



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## Appendix

In this Appendix, we present the detailed calculations for  $x_{lm}$ ,  $x_{hm}$ ,  $p_{lm}$ ,  $p_{hm}$ ,  $\pi_{lm}$ , and  $\pi_{hm}$  and verify that the latter two are increasing in  $m$ . Next, using these expressions, we write down  $x_h = x_{h0}$  and so calculate  $x_d$ ,  $p_d$  and  $\pi_d$  and finally using the expressions for  $\pi_{lm}$  and  $\pi_{hm}$ , we write down  $\pi_{l(n+1)}$  and  $\pi_{l1}$  and so  $\Delta_{suff}$  and  $\Delta_{nec}$ , which should allow the reviewer to easily recreate the figures above.

Well, from Equation (5), so long as  $x_{lm} > 0$  then

$$0 = 1 - \frac{4x_{lm}}{l^2} - \frac{2\sigma}{hl}(n+1-m)x_{hm} - \frac{2\sigma}{l^2}(m-1)x_{lm}, \quad (11)$$

or equivalently

$$x_{lm} = \max\left\{0, \frac{l^2h - 2\sigma l(n+1-m)x_{hm}}{4h + 2\sigma h(m-1)}\right\} \quad (12)$$

and since  $x_{hm}$  satisfies

$$x_{hm} = \arg \max_x (p_{hm}(x) - c)x = \arg \max_x \left(1 - \frac{2x}{h^2} - \frac{2\sigma}{h^2}(n-m)x_{hm} - \frac{2\sigma}{hl}mx_{lm} - c\right)x \quad (13)$$

so

$$1 - \frac{4x_{hm}}{h^2} - \frac{2\sigma}{h^2}(n-m)x_{hm} - \frac{2\sigma}{hl}mx_{lm} - c = 0 \quad (14)$$

or equivalently

$$x_{hm} = \frac{(1-c)h^2l - 2\sigma hmx_{lm}}{(4 + 2\sigma(n-m))l} \quad (15)$$

Solving Equations (12) and (15) simultaneously yields

$$x_{lm} = \begin{cases} 0 & \text{if } l(2 + \sigma(n-m)) - \sigma h(1-c)(n+1-m) < 0 \\ \frac{l}{2} \frac{l(2+\sigma(n-m)) - \sigma h(1-c)(n+1-m)}{4 + 2\sigma(n-1) - \sigma^2 n} & \text{otherwise} \end{cases} \quad (16)$$

and

$$x_{hm} = \begin{cases} \frac{(1-c)h^2 l}{(4+2\sigma(n-m))l} & \text{if } l(2 + \sigma(n-m)) - \sigma h(1-c)(n+1-m) < 0 \\ \frac{h}{2} \frac{2h(1-c) + h\sigma(m-1)(1-c) - l\sigma m}{(2+\sigma(n+1)-\sigma)(2-\sigma)} & \text{otherwise} \end{cases}. \quad (17)$$

Then using these two equations and Equation (2)—the expression for the inverse demand function—and noting that the profit for a low quality producing firm is  $\pi_{lm} = p_{lm}x_{lm}$  and for a high quality producing firm that  $\pi_{hm} = (p_{hm} - c)x_{hm}$  then Equations (6) and (7) result.

Note that  $2l + \sigma l(n-m) - \sigma h(1-c)(n+1-m) > 0$  is increasing in  $m$  since  $h(1-c) > l$  and so  $\pi_{lm}$  and  $\pi_{hm}$  are increasing in  $m$ .

Now when all firms are producing high quality then  $m = 0$  and

$$x_h = x_{h0} = \frac{(1-c)h^2}{4 + 2\sigma n}. \quad (18)$$

From Equation (3), it follows that:

$$1 - \frac{4x_d}{h^2} - \frac{2\sigma}{h^2}nx_h = 0, \quad (19)$$

and so

$$x_d = \frac{h^2}{4} \left( 1 - \frac{2\sigma n(1-c)}{4 + 2\sigma n} \right), \quad (20)$$

and

$$\pi_d = \frac{h^2}{8} \left( \frac{2 + \sigma cn}{2 + \sigma n} \right)^2. \quad (21)$$

Finally, using this last expression and Equation (7) at  $m = 0$  and Equation (6) at  $m = 1$  and  $m = n + 1$  and noting that  $\chi_{n+1} = 1$ , we can write down  $\Delta_{suff}$  and  $\Delta_{nec}$  in terms of the parameters of the model as follows:

$$\Delta_{suff} = \frac{1}{1-\delta} \frac{1}{2} \left( \frac{h(1-c)}{2 + \sigma n} \right)^2 - \frac{h^2}{8} \left( \frac{2 + \sigma cn}{2 + \sigma n} \right)^2 - \frac{\delta}{1-\delta} \frac{l}{2} \left( \frac{(2-\sigma)l}{4 + 2\sigma(n-1) - \sigma^2 n} \right)^2 \quad (22)$$

and

$$\Delta_{nec} = \frac{1}{1-\delta} \frac{1}{2} \left( \frac{h(1-c)}{2+\sigma n} \right)^2 - \frac{h^2}{8} \left( \frac{2+\sigma cn}{2+\sigma n} \right)^2 - \frac{\delta}{1-\delta} \chi_1 \frac{l}{2} \left( \frac{2l+\sigma l(n-1)-\sigma h(1-c)n}{4+2\sigma(n-1)-\sigma^2 n} \right)^2. \quad (23)$$