Informed Lending and Security Design

ROMAN INDERST and HOLGER M. MUELLER

ABSTRACT

We examine the role of security design when lenders make inefficient accept or reject decisions after screening projects. Lenders may be either “too conservative,” in which case they reject positive-NPV projects, or “too aggressive,” in which case they accept negative-NPV projects. In the first case, the uniquely optimal security is debt. In the second case, it is levered equity. In equilibrium, profitable projects that are relatively likely to break even are financed with debt, while less profitable projects are financed with equity. Highly profitable projects are financed by uninformed arm’s-length lenders.

Professional lenders have specialized expertise in screening projects. Unlike the typical entrepreneur to whom they lend, they have been exposed to a wide variety of projects in the past. Professional lenders have experienced loan officers and analysts who know precisely what information is relevant, how to treat “soft” information such as management quality, and how to weigh the various pieces of information in making a forecast of the project’s cash flows and default risk.

This paper examines how securities should be optimally designed when lenders make inefficient accept or reject decisions after screening projects. In our model, screening generates an informative albeit private signal about the project’s profitability. High signals indicate a positive net present value (NPV) and low signals indicate a negative NPV, where the project’s NPV is increasing

Inderst is from the London School of Economics and CEPR; Mueller is from the NYU Stern School of Business and CEPR. We are especially grateful to an anonymous referee whose helpful comments substantially improved the paper. We also thank Rick Green (the editor), Andres Almazan, Jacques Cremer, Darrel Duffie, Anthony Lynch, Colin Mayer, Patrick Rey, Jean-Charles Rochet, Tony Saunders, Raghu Sundaram, Elu von Thadden (our WFA discussant), Jean Tirole, Tony Saunders, Raghu Sundaram, Elu von Thadden (our WFA discussant), and seminar participants at NYU, Yale, Kellogg, Rochester, UNC Chapel Hill, INSEAD, Wisconsin-Madison, LBS, LSE, Toulouse, Amsterdam, Frankfurt, Humboldt, the WFA Meetings in Los Cabos, the Oxford Finance Summer Symposium, and the European Summer Symposium in Financial Markets in Gerzensee for helpful comments. An earlier version of this paper circulated under the title “Credit Risk Analysis and Security Design.”

The signal may be interpreted as the lender’s internal rating of the borrower (see Treacy and Carey (2000)). As it is based on the lender’s personal and subjective evaluation of the borrower—often involving the use of qualitative “soft” information as in Stein (2002)—it will naturally be private information. (The lender could always argue that her impression of the borrower’s management skills is bad while in reality it is good.) Indeed, Brunner, Krahnen, and Weber (2000, p. 4) argue: “Internal ratings should therefore be seen as private information. Typically, banks do not inform their customers of the internal ratings or the implied POD (probability of default), nor do they publicize the criteria and methods used in deriving them.”

2137
in the signal. When the lender decides whether to accept a project, her cutoff signal above which she accepts may deviate from the first-best. In particular, the cutoff signal may be too high, in which case she rejects all negative- but also some positive-NPV projects, or it may be too low, in which case she accepts all positive- but also some negative-NPV projects. In the first case, the lender is “too conservative.” In the second case, she is “too aggressive.” As the decision to accept or reject the project at a given signal depends on the security in place, the objective of security design is to minimize distortions in the lender’s accept or reject decision.

Both banks and venture capitalists screen projects, yet they hold different securities. With few exceptions banks hold debt, while venture capitalists hold more equity-like securities (Kaplan and Strömbärg (2003)). In our model, debt and levered equity are both optimal—indeed, uniquely optimal—securities when the lender’s accept or reject decision deviates from the first-best: Debt is optimal when the lender is too conservative, and equity is optimal when she is too aggressive. Whether the lender is too conservative or too aggressive is endogenous and depends on ex ante observable project characteristics. Ultimately, debt is optimal for relatively safer projects that are quite likely to break even based on public information alone. In contrast, levered equity is optimal for projects that are less likely to break even based on public information. This accords with observed patterns of small business financing whereby transparent projects such as investments in tangibles and plant, property, and equipment are predominantly financed with debt. In contrast, opaque investments that are less likely to break even based on publicly available information are primarily financed with equity (e.g., Berger and Udell (1998), Brewer et al. (1996)).

Our result that debt is optimal when the lender is too conservative while levered equity is optimal when she is too aggressive suggests that the two securities play opposite roles. Debt maximizes the lender’s payoff in low cash flow states, thus maximizing her expected payoff at low signals at which the project’s expected NPV is low. This, in turn, shifts the cutoff signal above which the lender is willing to accept the project as far down as possible, thereby minimizing her conservatism. In contrast, levered equity minimizes the lender’s payoff in low cash flow states, thus minimizing her expected payoff at low signals. This, in turn, shifts the lender’s cutoff signal as far up as possible, thereby minimizing her aggressiveness.

While the precise type of optimal security depends on the nature of the problem (too conservative or too aggressive), the general objective of security design is to minimize the efficiency loss that arises when lenders make their accept decisions under private information. A fairly robust insight from our analysis is that the optimal security design will be deliberately sparse so as to limit the lender’s available options after observing her private signal. That is, in our specific setting allowing the lender to choose from a menu of contracts after observing the signal is not optimal. Intuitively, a menu creates an ex post “self-dealing problem” as the lender would always choose the security that is ex post optimal for her. For instance, at high signals she would not choose debt but
rather a more “equity-like” security that offers her a higher expected payoff at these signals. But this would undermine the basic principle that the expected payoff of lenders who are too conservative should be shifted away from high signals towards low signals. Similarly, at low signals the lender would not choose levered equity but rather a more “debt-like” security that offers her a higher expected payoff at these signals. Again, this would undermine the basic principle that the expected payoff of lenders who are too aggressive should be shifted away from low signals toward high signals.

Given that the lender optimally restricts herself to a single contract to avoid ex post self-dealing, her accept or reject decision takes a simple form: Accept the project under the terms specified in the initial contract, or reject it. In other words, there is no adjustment of the loan terms after the screening. This is consistent with the fact that in many lending markets loans are either granted at standardized terms or not at all: “Loan decisions made for many types of retail loans are reject or accept decisions. All borrowers who are accepted are often charged the same rate of interest and by implication the same risk premium” (Saunders and Thomas (2001, p. 323)).

Finally, we embed our screening model in a competitive lending market environment. This endogenizes the borrower’s reservation utility and determines, based on ex ante observable project characteristics, under what circumstances the lender is too conservative and too aggressive, respectively, and thus when she holds debt and when she holds equity. Our analysis also shows that for certain projects, informed lending may not be viable at all. In particular, when projects have a very high NPV based on public information alone, informed lenders may be unable to compete with uninformed arm’s-length lenders. Not only does screening add little value in this case, but informed lenders may be so conservative that they generate less surplus than uninformed lenders. The resulting prediction that highly profitable and financially secure firms are more apt to borrow at arm’s length is consistent with the empirical evidence (e.g., Petersen and Rajan (2002), Denis and Mihov (2003)).

Our paper adds to the small but growing literature on privately informed investors. In a related paper, DeMarzo, Kremer, and Skrzypacz (2004) study security-bid auctions in which investors with ex ante private information compete for a project by bidding with securities. The authors consider the role of security design holding fixed the auction format, the role of the auction format holding fixed the security design, and the optimal combination of auction format and security design. Other models of security-bid auctions with privately informed investors include Garmaise (2001) and Axelson (2003). Two recent papers that also have privately informed lenders are Bernhardt and Krasa (2004) and Manove, Padilla, and Pagano (2001). Both of these papers show that informed lenders may screen too little. In the first paper, this result is obtained because informed lenders try to protect their informational rents; in the second, it arises because lenders are protected by collateral.

2 See also Boot and Thakor (1993) and Fulghieri and Lukin (2001), who study the role of security design in providing incentives to acquire information.
Our paper provides a novel argument for debt and levered equity in settings in which lenders make their accept or reject decisions based on private signals. Most, albeit not all, of the existing security design literature focuses on debt contracts. In particular, it has been shown that debt mitigates moral hazard (Innes (1990)), verification costs (Townsend (1979), Gale and Hellwig (1985)), cash flow stealing (Bolton and Scharfstein (1990), DeMarzo and Fishman (2003)), and the lemons problem that arises when firms have ex ante private information (Nachman and Noe (1994)). DeMarzo and Duffie (1999) and Biais and Mariotti (2005), on the other hand, consider the case in which firms become privately informed after the security design.

The rest of this paper is organized as follows. Section I presents the model. Section II characterizes the lender’s optimal accept or reject decision and illustrates why the lender may be either too conservative or too aggressive. Section III considers the optimal security design in these two cases. It shows that debt is uniquely optimal in the first case, while levered equity is uniquely optimal in the second case. It also shows that a menu of securities is suboptimal after the project screening. Section IV closes the model by embedding it into a competitive lending market environment. It characterizes under what circumstances the lender is too conservative and too aggressive, respectively, and thus when she holds debt and when she holds equity. It also discusses some empirical implications of the model. Section V concludes. All proofs are in the Appendix.

I. The Model

A. Project Technology and Screening

A penniless entrepreneur (“the borrower”) has a project that requires a fixed investment $k$. The project’s cash flow $x$ is stochastic with support $X := [0, \bar{x}]$, where $\bar{x}$ may be finite or infinite. In our base case model, we assume that there is a single lender. In Section IV, we embed our model in a competitive lending market environment.

The lender can screen the project prior to deciding whether to finance it. Screening generates a private signal $s \in [0, 1]$, which represents the lender’s personal and subjective assessment of the project (see footnote 1 in the Introduction). The signal is drawn from the distribution function $F(s)$, which is common knowledge. We assume that $F(s)$ is absolutely continuous with $F(0) = 0$ and positive density $f(s)$ for all $s \in (0, 1)$. Each signal is associated with a distribution function over project cash flows $G_s(x)$, which is absolutely continuous in $x$ with $G_s(\bar{x}) = 0$. The density $g_s(x)$ is positive and continuous in $s$ for all $x$. The expected project cash flow given the signal is $\mu_s := \int_x x g_s(x) dx$.

We assume that the signal is informative about $x$ in the sense of the Monotone Likelihood Ratio Property (MLRP): For any pair $(s, s') \in [0, 1]$ with $s' > s$, the ratio $g_{s'}(x)/g_s(x)$ is strictly increasing in $x$ for all $x \in X$. Accordingly, high signals are good news for the borrower in the sense that they put more probability
mass on high cash flows than low signals. MLRP is satisfied by many standard distributions (Milgrom (1981)).

Our assumption that the signal is informative has a natural interpretation: The lender has specialized expertise, which allows her to make a more accurate prediction of the project’s expected future cash flows, that is, while the borrower only knows the “coarse” expected cash flow estimate \( \mu := \int_{0}^{1} \mu_{s} f(s) ds \), the lender observes the finer estimate \( \mu_{s} \).

We can think of at least two inefficiencies associated with project screening: (i) the lender accepts bad projects and rejects good ones, and (ii) she devotes too little time and effort to screening. The second inefficiency has been studied previously (e.g., Manove, Padilla, and Pagano (2001), Fulghieri and Lukin (2001)). To focus on the first inefficiency, we take the informativeness of the lender’s signal as given.

B. Lending Process and Financial Contracts

The timing is as follows. In \( \tau = 0 \), the lender offers a contract specifying an investment \( k \) and a repayment \( t(x) \) out of the project’s cash flow. The offer is conditional on the project being accepted. In \( \tau = 1 \), the lender screens the project. Depending on her private signal, she either accepts the project at the terms specified in the initial offer or she rejects it.

Several comments are in order. The first concerns the assumption of a single contract. While the lender cannot offer a contract that directly conditions on her private signal, she could, in principle, offer a menu of contracts from which she chooses after the screening. We consider this possibility in Section III.C.

Second, how reasonable is the assumption that the lender offers a contract before the screening? Put differently, in practice do lenders really tell loan applicants “these are the loan terms that you will get if your loan is approved”? At least with respect to small business loans, this appears to be true. At Chase Manhattan, for instance, applicants for small business loans are shown a pricing chart that explains what interest rate they will get if their loan is subsequently approved. The interest rate is a function of verifiable loan characteristics and is defined as a spread over the prime rate. Moreover, while we assume that the lender makes her offer prior to screening the project, we allow the lender to offer a menu of contracts, which implies the loan terms may—at least in principle—depend on the screening outcome (see the above paragraph). We also allow the lender and borrower to renegotiate the initial offer after the screening. We consider renegotiations in Section III.D.

3 “As a result, banks are likely to be more knowledgeable about some aspects of project quality than many of the entrepreneurs they lend to … This is why banks are, and ought to be, in the project-evaluation business” (Manove et al. (2001, p. 727–728)). Our assumption that professional lenders are better at estimating the success likelihood of projects is consistent with the observation that bank-financed firms have higher survival rates than do firms financed by family members (Reid (1991)).

4 A copy of the pricing chart is available from the authors. The fact that lenders post their rates upfront allows borrowers to shop for the lowest rate without first having to go through a tedious loan application process.
Third, the fact that the lender makes the offer does not mean she can extract all the surplus. Below we introduce the borrower’s ex ante participation constraint. By varying the borrower’s reservation utility, we can vary the amount of surplus the lender can possibly extract.\(^5\)

If the project is undertaken, we assume the borrower forgoes a wage \(w \geq 0\) from alternative employment. We might expect \(w\) to be large if the borrower is a start-up entrepreneur leaving a highly paid job to pursue a business idea. On the other hand, if the project is merely an expansion of the borrower’s existing business, \(w\) might be small or zero. To allow surplus sharing, we introduce an ex ante participation constraint that requires that the borrower’s expected utility in \(\tau = 0\) be at least \(\bar{V}\). Given that the borrower can always pursue his employment opportunity, it must hold that \(\bar{V} \geq w\). We also make the standard assumption that both \(t(x)\) and \(x - t(x)\) are nondecreasing (e.g., Innes (1990), DeMarzo and Duffie (1999)).

Finally, we rule out any transfer payments to the borrower except when the project is undertaken and the cash flow is positive. In particular, this rules out the possibility that the lender “buys” the project before screening it. It also rules out transfer payments to the borrower if the project is rejected or if it has been accepted but the cash flow is zero. We motivate this assumption by appealing to a version of Rajan’s (1992) “fly-by-night operator” argument. Suppose there is a potentially infinitely large pool of crooks, or fly-by-night operators, who have zero reservation utility and “fake” projects that generate \(x = 0\) for sure. Any prospect of receiving a positive transfer payment would attract these operators and cause the lender to make a sure loss.\(^6\)

\section*{II. The Lender’s Problem}

\subsection*{A. Optimal Lending Decision}

Working backward, we start with the lender’s optimal accept or reject decision in \(\tau = 1\). To obtain a benchmark, we first derive the socially optimal (or first-best) lending decision. The first-best decision is to accept the project if and only if its NPV \(\mu_s - k - w\) is positive. To rule out trivial cases in which the lender’s signal is irrelevant, we assume that \(\mu_0 < k + w\) and \(\mu_1 > k + w\). Accordingly, the project’s NPV is positive (negative) at high (low) signals. Moreover, \(\mu_s\) is continuous and strictly increasing in \(s\) by MLRP and continuity of \(g_s(x)\). The first-best decision rule is thus a simple cutoff rule: Accept the project if and only if \(s > s_{FB}\), where the first-best cutoff signal \(s_{FB} \in (0, 1)\) is uniquely defined by \(\mu_{s_{FB}} = k + w\).\(^7\)

\(^5\) Instead of having the lender make a contract offer, we could alternatively have the lender and borrower bargain over an offer ex ante. The qualitative results are the same (see Inderst and Mueller (2004)).

\(^6\) The “fly-by-night operator” argument rules out the possibility that \(t(x) < 0\) when \(x = 0\). In conjunction with the assumption that \(t(x)\) is nondecreasing, this immediately implies that \(t(x) \geq 0\) for all \(x\).

\(^7\) If the NPV is exactly zero, we specify that the project be rejected. Given that \(s = s_{FB}\) is a zero-probability event, this is without loss of generality.
The lender’s privately optimal accept or reject decision depends on her costs and benefits. We assume that when she is indifferent, the lender rejects the project. Given that this is a zero-probability event (see below), the assumption is without loss of generality. Let \( u_s(t) := \int_X t(x) g_s(x) dx \) denote the lender’s expected gross payoff from accepting the project. The lender consequently accepts if and only if \( u_s(t) > k \). We can safely rule out the case in which \( t(x) = 0 \) for all \( x \). Given that \( k > 0 \), this would imply that the lender never accepts. If \( t(x) > 0 \) for some \( x \), however, it follows immediately from MLRP and the fact that \( t(x) \) is nondecreasing that \( u_s(t) \) must be strictly increasing in \( s \). Accordingly, the lender’s privately optimal decision rule is (also) a simple cutoff rule: Accept the project if and only if \( s > s^* \), where the lender’s privately optimal cutoff signal \( s^* = s^*(t) \) depends on the contract in place. If \( u_0(t) > k \), we have that \( s^* = 0 \). In all other cases, there exists a unique interior cutoff signal \( s^* \in (0, 1) \) given by \( u_{s^*}(t) = k \).

B. The Lender’s Maximization Problem

The fact that the lender uses a cutoff rule in \( \tau = 1 \) simplifies the security design problem in \( \tau = 0 \). Accordingly, the lender chooses a contract \( t(x) \) to maximize her expected payoff

\[
U(t) = \int_{s^*}^1 [u_s(t) - k] f(s) ds,
\]

subject to the borrower’s ex ante participation constraint

\[
V(t) = \int_{s^*}^1 v_s(t) f(s) ds + F(s^*) w \geq \bar{V},
\]

and the constraint that \( \bar{V} \geq w \) (see Section I.B), where \( v_s := \int_X [x - t(x)] g_s(x) dx = \mu_s - u_s(t) \) denotes the borrower’s expected utility conditional on \( s \), and where the cutoff signal \( s^* \) depends on \( t(x) \) as described above.\(^8\)

The following lemma simplifies the lender’s problem further.

**Lemma 1**: The borrower’s participation constraint (2) must hold with equality.

**Proof**: See Appendix.

Inserting (2) into (1), we can rewrite the lender’s objective function more succinctly as

\[
\int_{s^*}^1 v_s(t) \frac{f(s)}{1 - F(s^*)} ds,
\]

is never less than his opportunity cost from undertaking the project, \( w \). Indeed, if \( \bar{V} > w \), the expected payoff of an accepted borrower is strictly greater than his opportunity cost.
\[ U(t) = \int_{s^*}^{1} [\mu_s - k - w] f(s) ds - (\bar{V} - w). \] (3)

By inspection, the lender is the residual claimant to any expected surplus in excess of \( \bar{V} - w \). Since \( \mu_s - k - w > 0 \) if and only if \( s > s_{FB} \), this expected surplus is smaller the greater is the difference between \( s^* \) and \( s_{FB} \). The more \( s^* \) lies above \( s_{FB} \), the more likely is it that positive-NPV projects are rejected. Conversely, the more \( s^* \) lies below \( s_{FB} \), the more likely is it that negative-NPV projects are accepted. This discussion lucidly illustrates the objective of the lender’s security design problem: Find a contract \( t(x) \) that minimizes the gap between \( s^* \) and \( s_{FB} \).

When \( \bar{V} \) is too large, a solution to the lender’s problem may not exist. In what follows, we assume that \( \bar{V} \) is sufficiently small in that sense. In Section IV we show that if \( \bar{V} \) is “too large,” the lender will be unable to attract the borrower.

C. Too Aggressive or Too Conservative?

We show below that the optimal contract is different depending on whether \( s^* < s_{FB} \) or \( s^* > s_{FB} \). In the first case the lender is “too aggressive,” that is, she accepts all positive- but also some negative-NPV projects. In the second case she is “too conservative,” that is, she rejects all negative- but also some positive-NPV projects. As \( s^* \) depends on \( t(x) \), it is useful to define “aggressive” and “conservative” in a way that is independent of the contract choice. We shall say that the lender is too aggressive if, for any feasible contract satisfying the borrower’s participation constraint (2) with equality, it holds that \( s^* < s_{FB} \). Similarly, we shall say that the lender is too conservative if, for any feasible contract satisfying (2) with equality, it holds that \( s^* > s_{FB} \).

What makes the lender too aggressive or too conservative? There are two countervailing effects. The first is that when deciding whether to accept the project, the lender does not internalize the borrower’s opportunity cost \( w \). When \( w \) is large, the lender may therefore accept projects that have a negative NPV. The second effect is surplus sharing. When the borrower’s reservation utility \( \bar{V} \) exceeds his private opportunity cost \( w \), the lender must share surplus with the borrower. As \( \bar{V} \) increases, the lender’s own share of the project cash flow will at some point become too small to cover her investment cost, even though the project itself may have a positive NPV. The lender will then only accept projects whose NPV is sufficiently positive. As a consequence, when \( w \) is sufficiently large relative to \( \bar{V} - w \), we expect that the first effect outweighs the second, implying the lender is too aggressive; conversely, when \( w \) is sufficiently small relative to \( \bar{V} - w \), we expect that the lender is too conservative.

Let us give two examples. In the first example, \( w \) is “sufficiently small” relative to \( \bar{V} - w \). Suppose that \( \bar{V} > w = 0 \). Since \( w = 0 \), the lender internalizes the full cost of undertaking the project. If she obtained the full project cash
flow, she would therefore make the first-best decision. But since $\bar{V} > 0$, the lender must share surplus with the borrower, implying that $t(x) < x$ for some $x$. Hence, regardless of the contract in place, the lender’s expected payoff $u_s(t)$ is less than the expected project cash flow $\mu_s$ for all $s$, which immediately implies that $s^* > s_{FB}$, that is, the lender is too conservative.\(^9\)

In the second example, $w$ is “sufficiently large” relative to $\bar{V} - w$. Suppose that $\bar{V} = w > 0$. Since $\bar{V} = w$, there is no surplus sharing. Accordingly, the lender only needs to make sure that the borrower’s expected utility conditional on being accepted equals $w$. The crux, however, is that she cannot pay the borrower a fixed “wage” of $x - t(x) = w$.\(^{10}\) Consequently, while accepted borrowers must obtain $w$ on average, the borrower will obtain more than $w$ at high signals and less than $w$ at low signals. In particular, at $s = s^*$ it holds that $v_{s^*} < w$, implying that $\mu_{s^*} := u_{s^*}(t) + v_{s^*}(t) < k + w$ and $s^* < s_{FB}$, that is, the lender is too aggressive.\(^{11}\)

### III. Optimal Security Design

In the following two subsections we derive the optimal contract when the lender is too conservative and too aggressive, respectively.

#### A. Optimal Security Design When the Lender Is Too Conservative

When the lender is too conservative, her private cutoff signal $s^*$ exceeds the first-best cutoff $s_{FB}$ for any feasible contract satisfying (2) with equality. The optimal contract consequently minimizes $s^*$, thereby minimizing the gap between $s^*$ and $s_{FB}$. We have the following result.

**Proposition 1**: The uniquely optimal contract when the lender is too conservative is standard debt, that is, $t^*(x) = \min\{x, R^*\}$, where $R^* > k$.

**Proof**: See Appendix.

We argued in Section II.C that the lender is too conservative because the borrower’s ex ante participation constraint (2) requires that the lender shares surplus. However, this constraint does not impose any requirement on how the surplus ought to be shared. All it requires is that in expectation the borrower’s

---

\(^9\) Recall that both $\mu_s$ and $u_s(t)$ are strictly increasing in $s$. As $u_s(t) < \mu_s$ for all $s$, this implies that $s^* > s_{FB}$.

\(^{10}\) This is because the “fly-by-night-operator” argument in conjunction with the monotonicity of $t(x)$ implies that $t(x) \geq 0$ (see Section I.B).

\(^{11}\) Formally, from $w > 0$, $t(0) \geq 0$, the fact that $x - t(x)$ is nondecreasing, and MLRP, it follows that $v_s(t)$ is strictly increasing in $s$. Since

$$\int_{c}^{1} v_s(t) \frac{f(s)}{1 - F(s')} ds = w$$

by Lemma 1 and $\bar{V} = w > 0$, this implies that $v_s(t) < w$ at low signals and $v_s(t) > w$ at high signals in $(s^*, 1]$. 

payoff must equal $\bar{V}$. Under a debt contract, the borrower receives as much as possible of his surplus share in good states in which the project’s cash flow is high. In turn, the lender receives as much as possible (subject to the constraint that $t(x)$ be nondecreasing) of her surplus share in bad states in which the project’s cash flow is low. Given that low cash flows are relatively more likely after low signals (by MLRP), a debt contract therefore maximizes—subject to leaving the borrower $\bar{V}$ in expectation—the lender’s expected payoff $u_s(t) - k$ at low signals. This pushes the cutoff signal $s^*$ at which the lender breaks even as far down as possible and hence closer toward the first-best cutoff $s_{FB}$. Consequently, standard debt minimizes the lender’s conservatism.

The above discussion lucidly illustrates the role of security design in our model. While the ex ante participation constraint (2) merely requires that the borrower receive a given payoff in expectation, the role of security design is to allocate the borrower’s payoff across cash flow states, or signals, so that the underlying efficiency is minimized. When the lender is too conservative, the optimal allocation requires shifting as much as possible of the borrower’s payoff into high cash flow states. As we will see below, the fundamental role of security design when the lender is too aggressive is similar, except that the optimal allocation requires shifting as much as possible of the borrower’s payoff into low cash flow states.

Recall that above we mentioned the constraint that $t(x)$ be nondecreasing. As the optimal solution requires shifting as much as possible of the borrower’s payoff into high cash flow states, a nonmonotonic discontinuous “live-or-die” (LD) contract $t(x) = x$ if $x \leq \hat{x}$ and $t(x) = 0$ if $x > \hat{x}$ might otherwise—that is, in the absence of a monotonicity constraint—be optimal. Whether or not LD is optimal in this case is not obvious, however; while it can implement a lower cutoff signal than debt, it also leaves the lender with zero payoff in high cash flow states. The lender’s expected payoff $u_s(t) - k$ may consequently become negative at high signals, implying that she rejects precisely those projects with the highest NPV. Whether or not LD is superior to debt ultimately depends on the distributional assumptions of the model.

**B. Optimal Security Design When the Lender Is Too Aggressive**

When the lender is too aggressive, her cutoff signal lies below the first-best cutoff for any feasible contract satisfying (2) with equality. The optimal contract consequently maximizes $s^*$, thereby minimizing the difference between $s^*$ and $s_{FB}$. We have the following result.

**Proposition 2:** The uniquely optimal contract when the lender is too aggressive is levered equity, that is, $t^*(x) = \max\{0, x - R^*\}$, where $R^* > 0$.

**Proof:** See Appendix.

The intuition is similar to that for debt, except in the opposite direction. Levered equity shifts as much as possible of the borrower’s payoff into low cash
flow states. This minimizes the lender’s payoff in low cash flow states and thus minimizes her expected payoff at low signals. This in turn pushes the lender’s cutoff signal as far up as possible, and hence closer toward \( s_{FB} \). Consequently, levered equity minimizes the lender’s aggressiveness.

As in the case of the debt result, there is a binding monotonicity constraint.\(^{12}\) In this case, it is the constraint that \( x - t(x) \) be nondecreasing. As the optimal solution requires shifting as much as possible of the borrower’s payoff into low cash flow states, a discontinuous contract \( t(x) = 0 \) if \( x < \hat{x} \) and \( t(x) = x \) if \( x > \hat{x} \) would otherwise—that is, in the absence of a monotonicity constraint—be optimal.

C. Menu of Contracts

Given that the lender makes her accept or reject decision in \( \tau = 1 \) under private information, a natural question to ask is whether efficiency can be improved by letting the lender choose from a menu of contracts after the project screening. We have the following result.

**Proposition 3:** A menu of contracts is strictly suboptimal if the lender is either too conservative or too aggressive.

**Proof:** See Appendix.

Not only is our restriction to single contracts without loss of generality, a menu of contracts is generally suboptimal; the only exception is when the first-best can already be attained with a single contract. A menu of contracts is suboptimal for the same reason that debt and levered equity are optimal single contracts. Suppose the lender is too conservative. In this case, the optimal solution requires maximizing the lender’s expected payoff at low signals. But this implies that the lender must be able to commit to leave the borrower a sufficiently high expected payoff at high signals, or else the participation constraint (2) cannot be satisfied. If the lender offers a single contract, this commitment ability is naturally given. If she offers a menu of contracts it is not given, as a menu creates an ex post “self-dealing problem” whereby the lender always chooses the contract that is ex post optimal for her given the observed signal. In particular, at high signals she would choose a “more equity-like” contract that offers her a higher expected payoff than the optimal debt contract. To satisfy the borrower’s participation constraint, the lender must then adjust the menu in a way that leaves the borrower relatively more at low signals. But this implies that the menu provides the lender a lower expected payoff at low signals—and thus implements a higher cutoff signal—than the optimal single contract from Proposition 1.

---

\(^{12}\) Also important for the levered equity result is the assumption that the lender cannot make transfer payments (see Section 1.B). If \( \bar{V} - w > 0 \), for instance, the first-best could otherwise be attained with a fixed-wage contract \( t(x) = x - w \) for all \( x \).
The intuition when the lender is too aggressive is analogous. The optimal solution now requires minimizing the lender's expected payoff at low signals. Under a menu, however, the lender would choose a contract at low signals that provides her with a higher expected payoff than levered equity, that is, she would choose a "more debt-like" contract. Consequently, a menu implements a strictly lower cutoff signal than the optimal single contract from Proposition 2.

Proposition 3 suggests that it is not optimal to fine-tune the loan terms after the project screening. Instead, it is optimal to offer a single contract ex ante and accept or reject the borrower on the basis of this contract. While this implies that loan terms will be insensitive with respect to interim information, that is, the lender's signal, it does not imply that they are insensitive with respect to project risk as such. Naturally, borrowers with different ex-ante signal distributions \( F(s) \), and thus different risk profiles, will obtain different contracts. Similarly, the optimal contract varies with the investment size and other ex ante characteristics. Our result is consistent with the observation that in many lending markets borrowers are either accepted at prespecified loan terms or not at all. (See the quote by Saunders and Thomas (2001) in the Introduction.) It is also consistent with Petersen and Rajan's (1994) finding that the interest rate charged on small business loans is insensitive with respect to interim information about borrowers. The availability of credit, in contrast, is sensitive with respect to this kind of information.

The main message of this section is that informed lenders may face an ex post self-dealing problem that can make it optimal to restrict their ex post choices. The optimality of a single contract is an extreme version of such a restriction, depending, among other things, on our assumption that the investment size is fixed. If the first-best optimal investment size varied with the lender's signal, a single contract would generally not be optimal. Yet, the lender would still want to trade off the benefits of a richer menu, namely, that it can implement different investment sizes at different signals, against the potential costs of a richer menu, namely, that it creates opportunities for ex post self-dealing.

**D. Renegotiations**

When the lender is too conservative or too aggressive, she inefficiently rejects or accepts the project at some signals. This potentially provides scope for mutually beneficial renegotiations: After the lender observes the signal, or equivalently, after she makes her decision, either she or the borrower might want to propose a new contract to improve efficiency.

When the lender is too conservative, she inefficiently rejects the borrower at signals \( s \in (s_{FB}, s^*) \). To make it more attractive for the lender to accept him, the borrower might then want to propose a new contract that is more favorable from the lender's perspective (e.g., a new debt contract with repayment \( R > R^* \)). This is precisely what would happen if the signal were jointly observable. Given that only the lender observes the signal, however, the borrower does not know whether \( s \in (s_{FB}, s^*) \), in which case he would indeed be better off under a new contract, or if \( s > s^* \), in which case replacing the existing optimal debt contract with a new contract would merely constitute a wealth transfer to the lender.
A necessary condition for the borrower to accept a new contract \( t \) that is more favorable for the lender is therefore that \( t \) implements a lower cutoff signal than the existing optimal debt contract \( t^* \). As can be easily shown, however, the lender would prefer this new contract not only if \( s \in (s^*(t), s^*(t^*)) \), but also if \( s > s^*(t^*) \) (see the Appendix). Hence, the lender would always claim that the signal is \( s \leq s^*(t^*) \) even when in reality it is \( s > s^*(t^*) \). Consequently, the optimal debt contract \( t^* \) from Proposition 1 will not be renegotiated.

When the lender is too aggressive, she inefficiently accepts the borrower at signals \( s \in (s^*, s_{FB}] \). The argument in this case is trivial. For renegotiations to result in a higher and more efficient cutoff, the lender would have to accept a lower expected payoff, that is, without getting anything in return. This implies renegotiations must necessarily fail.

We consider the following renegotiation game. After the lender observes the signal, either she or the borrower can propose a new contract. If either the borrower or the lender rejects the proposal, the optimal contract \( t^* \) remains in place. We have the following result.

**Proposition 4:** The uniquely optimal contracts from Propositions 1 and 2 are not renegotiated in any (perfect Bayesian) equilibrium of the renegotiation game. This holds irrespective of whether the borrower or the lender can propose a new contract after the project screening.

**Proof:** See Appendix.

### IV. Lending Market Competition

#### A. The Lending Market

Section II shows that, depending on the relative magnitudes of the borrower’s reservation utility \( \bar{V} \) and his wage \( w \) from alternative employment, the lender may be either too conservative or too aggressive. Section III shows that in the first case debt is uniquely optimal, while in the second case levered equity is uniquely optimal. We now close our model by embedding it into a competitive lending market environment. This endogenizes the borrower’s reservation utility \( \bar{V} \) and allows us to characterize, based on ex ante observable project characteristics, under what circumstances the lender is too conservative and too aggressive, respectively, and thus when she holds debt and when she holds equity.

In Section I, we assume that the lender can make a more accurate prediction of the project’s expected cash flow than the borrower due to her specialized expertise. However, other lenders may also have specialized expertise. To endow “our” lender with a competitive advantage over other lenders, we shall assume that she has access to additional information that other lenders do not have. For simplicity, we refer to “our” lender as the “informed lender” and to the other lenders as the “competitive credit market.” Formally, we assume that

---

13 Extending the argument to the case in which the borrower and lender can make menu offers is straightforward.
the competitive credit market has access to all publicly available and easily verifiable information. The expected project cash flow given this information is $\mu := \int_0^1 \mu_s f(s) ds$. In contrast, the informed lender, besides having access to all publicly available information, has access to additional valuable information about the borrower, which provides her the additional informative signal $s$ and hence the more accurate expected cash flow estimate $\mu_s$.\(^{14}\)

There are various reasons why certain lenders may have better information about borrowers than others. First, some lenders may have better access to “local information” due to their proximity to the borrower. Petersen and Rajan (2002) (for the United States) and Degryse and Ongena (2005) (for Belgium) find that the median distance between banks and small-firm borrowers is only 4 and 1.4 miles, respectively, suggesting that local lenders have an advantage over distant lenders. In a similar vein, Petersen and Rajan (1995, p. 418) argue that “credit markets for small firms are local,” and Guiso, Sapienza, and Zingales (2004, p. 936) allude to “direct evidence of the informational disadvantage of distant lenders.” Past lending relationships constitute another potential source of informational advantage, particularly with regard to soft information about the borrower.\(^{15}\) Borrowers commonly also maintain their checking and savings accounts with their relationship lender, and relationship lenders commonly factor borrowers’ accounts receivables—additional channels that may further “increase the precision of the lender’s information about the borrower” (Petersen and Rajan (1994, p. 6)).\(^{16}\)

The timing is as follows. In $\tau = 0$ the informed lender and the competitive credit market make competing offers.\(^{17}\) The informed lender’s offer is conditional on the screening outcome in $\tau = 1$. Hence, if the borrower accepts the informed lender’s offer, the project is financed by the informed lender if and only if $s > s^*$. If the borrower is rejected, he can still go to the credit market in $\tau = 1$, albeit this will not happen in equilibrium as we assume that the credit market can distinguish between “fresh” borrowers and borrowers who have previously sought financing from the informed lender.\(^{18}\) Cash flows are realized in $\tau = 2$.

\(^{14}\) “[P]rivate corporate ratings (internal ratings) reflect the core business of commercial banks, whose superior information as compared to an external assessment by the market allows a more precise estimate of the POD [probability of default]” (Brunner, Krahnen, and Weber (2000, p. 5)).

\(^{15}\) “[T]he [original] lender learns more about the success of the firm’s operation than do outside banks. As a result, the original lender should be in a better position to evaluate the firm’s future performance” (Sharpe (1990, p. 1072)).

\(^{16}\) In related models of lending market competition, Sharpe (1990), Rajan (1992), Hauswald and Marquez (2003), and von Thadden (2004) also consider competition between an informed lender with superior information about a borrower and an uninformed arm’s-length credit market.

\(^{17}\) To ensure that the credit market does not fall prey to the “fly-by-night operators” (see Section I.B), we assume that it (optimally) makes no transfer payments.

\(^{18}\) This assumption ensures the existence of a pure-strategy equilibrium (see Broecker (1990) for details). In practice, lenders usually check a borrower’s credit history prior to approving a loan. In many countries, including the United States, credit bureaus provide this information in the form of credit reports. Credit reports commonly indicate if other lenders have made credit inquiries in the past, including the date of the inquiry and the identity of the inquirer (Jappelli and Pagano (2002)). If the credit market can access the borrower’s credit report, it can thus observe if the borrower has previously sought financing from the informed lender.
B. Equilibrium Analysis

The most the competitive credit market can offer the borrower in $\tau = 0$ is $\max \{\mu - k, 0\}$, where $\mu$ is the expected project cash flow based on publicly available information. If $\mu - k < w$, the borrower therefore prefers his employment opportunity over going to the credit market, which implies his reservation utility in (2) is $\bar{V} = \max\{\mu - k, w\}$. We have the following proposition.

**Proposition 5:** The lending market has a unique equilibrium.$^{19}$

*Case 1:* If $w \geq \mu - k$, the borrower always goes to the informed lender. If $w = 0$ the informed lender can implement the first-best with $t(x) = x$, while if $w > 0$, she is too aggressive.

*Case 2:* If $w < \mu - k$, the borrower either always goes to the informed lender, or there exists a cutoff value $w_1 > 0$ such that he goes to the competitive credit market if $w \leq w_1$ and to the informed lender if $w \geq w_1$. If the borrower goes to the informed lender, there exist cutoff values $0 < w_2 < w_3$ such that if $w < w_2$, the informed lender is too conservative, if $w_2 \leq w \leq w_3$, the first-best is attainable, and if $w > w_3$, the informed lender is too aggressive.

*Proof:* See Appendix.

Proposition 5 confirms our intuition from Section II.C that the informed lender is too conservative if $\bar{V} - w$ is large relative to $w$ and too aggressive if $\bar{V} - w$ is small relative to $w$. We can express this intuition alternatively in terms of the project’s ex ante NPV, $\mu - k - w$. Conditional on the borrower going to the informed lender in $\tau = 0$, the informed lender is too conservative if the ex ante NPV is large (Case 2, where $w < w_2$). By Proposition 1, the uniquely optimal security in this case is debt. Conversely, if the ex ante NPV is either negative (Case 1) or positive but small (Case 2, where $w > w_3$) the informed lender is too aggressive. (There is one exception, see below.) By Proposition 2, the uniquely optimal security in this case is levered equity.

There are two subcases in which the first-best is attainable. One is the exception mentioned above: If $\mu - k \leq w = \bar{V} = 0$, the ex ante NPV is nonpositive yet the informed lender is not too aggressive (Case 1, where $w = 0$). This case is special, because the informed lender internalizes the entire project cost and she can extract the full surplus. The uniquely (first-best) optimal security in this case is $t(x) = x$. The other subcase is the intermediate case in Case 2, in which the ex ante NPV is neither too small nor too large ($w_2 \leq w \leq w_3$). The optimal security in this case is not unique.

In summary, the borrower goes to the informed lender in all cases but one. Intuitively, the informed lender can generally outbid the credit market as her

$^{19}$The equilibrium is unique, except in nongeneric cases such as $w = w_1$. For instance, if $w = w_1$, there is both an equilibrium in which the borrower goes to the informed lender and where he pursues his employment opportunity.
superior information allows her to sort between good and bad projects. Except in one case to be discussed below, this creates additional surplus relative to what is created by the credit market, which provides the informed lender with a competitive advantage.\footnote{In Case 1, the credit market creates no surplus since $\mu - k \leq w$. In this case, the informed lender effectively competes against the borrower’s outside employment opportunity.}

The exception is the case in which the borrower goes to the credit market when the ex ante NPV is large (Case 2, where $w \leq w_1$). Intuitively, the competitive credit market is the “most aggressive” of all lenders: It accepts all positive-but also all negative-NPV projects, which is equivalent to using a cutoff signal of zero. When the informed lender’s cutoff signal is $0 < s^* \leq s_{FB}$, she therefore creates additional surplus relative to the credit market. (She accepts fewer negative-NPV projects.) When the informed lender is too conservative, however, that is, when $s^* > s_{FB}$, this comparison is not obvious: While the informed lender now rejects all negative-NPV projects, she also rejects some positive-NPV projects. For instance, suppose $s_{FB}$ is close to zero. In this case, there are virtually no negative-NPV projects. Accordingly, financing by the competitive credit market is “almost” first-best efficient. If the informed lender is sufficiently conservative, it is therefore possible that she creates less surplus than the credit market, which means she loses her competitive edge.

Generally, if the borrower goes to the competitive credit market for low values of $w$, there exists a cutoff value $w_1$ such that he switches to the informed lender when $w$ reaches this cutoff. For instance, suppose that $w = 0$, $s^* > s_{FB}$, and $s_{FB}$ is close to zero, so that by our previous argument the borrower goes to the competitive credit market. As $w$ increases, the first-best cutoff $s_{FB}$ also increases, while the informed lender’s cutoff $s^*$ remains constant. Hence, the competitive credit market becomes less efficient while the informed lender becomes more efficient. At some point ($w \geq w_2$) the informed lender becomes more efficient than the credit market, which implies she is able to attract the borrower. But it is perfectly possible that the borrower goes to the informed lender for all—even for the lowest—values of $w$, in which case he will naturally also go to the informed lender for all higher $w$.\footnote{Even when $\mu - k$ is large relative to $w$, it need not be the case that $s_{FB}$ is small. Hence, the case in which the borrower goes to the credit market for small $w$ may not arise. Whether or not this case arises depends on how precisely $\mu$ maps into $\mu_*$, and thus on the distributional assumptions of the model. A previous version of this paper (Inderst and Mueller (2004)) contains a numerical example in which $G_t(x)$ is exponential, $F(s)$ is uniform, and $w = 0$ that analyzes this issue in detail.}

\section*{C. Empirical Implications}

Our model provides testable predictions linking financing and contract choices to observable project characteristics. One implication is that uninformed arm’s-length lenders finance relatively safer projects whose NPV based on public information is sufficiently large. Intuitively, these are projects for which screening provides the least value added.
Petersen and Rajan (2002) document that relatively safer firms with a good credit quality are indeed more likely to borrow at arm’s length. Similarly, Denis and Mihov (2003) find that profitable firms with a high credit quality are more likely to tap public debt markets, while less profitable firms tend to borrow more from banks. Relatedly, Cole, Goldberg, and White (1999) document that large banks approve loans primarily on the basis of hard information while small banks are more likely to use soft information. Accordingly, large banks act more like arm’s-length lenders while small banks act more like our informed lender. Consistent with our empirical implications, Haynes, Ou, and Berney (1999) find that large banks are more likely to lend to financially secure firms.

As for the choice between debt and equity, our model predicts that safer projects that are quite likely to break even based on verifiable information alone are financed with debt. In contrast, opaque projects that are less likely to break even based on verifiable information alone are financed with equity. Ceteris paribus, it is easier to document (using only verifiable information) that an investment is profitable if it is transparent, for example, as in the case of investments in tangibles or property, plant, and equipment. In this regard, our implications are consistent with the observation that “high risk–high growth enterprises whose assets are mostly intangible more often obtain external equity, whereas relatively low risk-low growth firms whose assets are mostly tangible more often receive external debt” (Berger and Udell (1998, p. 615)). Similarly, Brewer et al. (1996) document that small firms finance 84% of their plant modernizations and 91% of their land acquisitions with debt. In contrast, 91% of marketing activities and 93% of research and development projects, arguably more opaque investments, are financed with equity.

The primary sources of debt finance for small firms are commercial banks and credit unions. In contrast, the primary sources of external equity finance for small firms are angel and venture capital finance (Berger and Udell (1998)). Like banks, venture capitalists also screen projects prior to making investments. Moreover, venture capital contracts also appear to have sparse menu characteristics: The project is either accepted under the terms specified in the initial term sheet, or it is rejected (Bernhardt and Krasa (2004)). (As for bank financing, recall the quote by Saunders and Thomas (2001) in the Introduction.) One difference between venture capital and bank finance suggested by our model concerns the types of projects being financed: Venture capital (i.e., equity) finance should be used if the project’s NPV based on publicly available information is small or negative, while bank (i.e., debt) finance should be used if the project’s NPV based on publicly available information is large.

**D. Informed Lending: A Monopoly?**

We assume there is a single informed lender who competes with an uninformed credit market. In relationship lending models, this information monopoly derives from the assumption that firms initially borrow from a single lender. Due to the soft information acquired in the initial period, the first-period lender retains an informational advantage in all following periods. The finding
by Petersen and Rajan (1994) that 82% of the firms in their sample continue to borrow from a single bank supports this notion. Alternatively, the informed lender’s informational advantage might derive from her geographical proximity to the borrower (see Section IV.A). In this case, a local monopoly may arise from a fixed cost of establishing a local presence (e.g., Hauswald and Marquez (2003)).

The analog to lending relationships in venture capital finance is the continued presence of venture capitalists in different financing rounds. Anand and Galetovic (2000) document that the likelihood of a given first-round venture capitalist being present in the second-round syndicate is 82%.

V. Concluding Remarks

When lenders base their accept or reject decisions on their personal, and hence private, project assessments, this can lead to inefficiencies. There are two kinds of inefficiencies in our model. The lender may be “too conservative,” in which case she rejects too many projects, or “too aggressive,” in which case she accepts too many projects. Depending on which of the two cases applies, the lender offers a different security. If the lender is too conservative she offers standard debt. Standard debt maximizes the lender’s payoff from marginally profitable projects that put a high probability mass on low cash flows, and thus precisely from those projects that are inefficiently rejected. Conversely, if the lender is too aggressive she offers levered equity. Levered equity minimizes the lender’s payoff from projects that put a high probability mass on low cash flows, and thus precisely from those projects that are inefficiently accepted.

While the type of optimal security depends on the nature of the problem, the general role of security design in our model is to minimize the efficiency loss that arises when lenders make their accept or reject decisions under private information. A fairly robust insight from our analysis is that the optimal security design will be deliberately sparse, limiting the ex post self-dealing problem that privately informed lenders will always propose loan terms that are ex post optimal for them given their information. In our specific setting, this implies

---

22 Consistent with our result that a rejected borrower cannot obtain financing elsewhere, Bruno and Tyebjee (1983) report that being denied follow-up financing by a previous-round venture capitalist reduces a firm’s chances of obtaining financing from outsiders by 74%.

23 See also Bygrave and Timmons (1992, p. 187): “There is a great deal of cronyism among venture capital firms and one venture capitalist would be considered greedy to hog an especially attractive investment.”
that allowing the lender to choose from a menu of contracts after screening the project is not optimal. However, if the lender optimally commits to a single contract, she has only two options: Accept the project under the terms specified in the initial contract, or reject it. This is consistent with the notion that loans are often either granted at standardized terms or not at all.

In the United States, banks are generally forbidden to hold significant equity stakes in nonfinancial firms. In this context, Berlin (2000, p. 4) asks: “Do restrictions against U.S. banks holding equity make a difference for banks’ behavior? Are U.S. banks’ borrowers at a disadvantage because their lenders are too cautious when evaluating project risks and too harsh when a borrower experiences financial difficulties?” Our model suggests that whether the current U.S. restrictions are binding depends on the problem in question, namely, are lenders too conservative or too aggressive? In the first case the optimal security is debt, implying the restrictions are not binding. In the second case the restrictions are binding. Incidentally, while the above quote seems to suggest that lenders are too cautious because they hold debt, our model suggests that the reverse causality may be true: Given that lenders are too cautious, it is optimal for them to hold debt.

Appendix

Proof of Lemma 1: Suppose that some contract $t$, where $V(t) = \bar{V} + \Delta$ with $\Delta > 0$, is optimal. Define $\tilde{t}(x) := t(x) + \gamma[x - t(x)]$, where $0 \leq \gamma \leq 1$. Note that $\tilde{t}(x)$ is a feasible contract as i) $0 \leq \tilde{t}(x) \leq x$, ii) $\tilde{t}(x)$ is nondecreasing given that $t(x)$ is nondecreasing, and iii) $x - \tilde{t}(x)$ is nondecreasing as $\tilde{t}(x') - \tilde{t}(x) \leq x' - x$ from $t(x') - t(x) \leq x' - x$ for $x' > x$. Next, note that $u_s(\tilde{t})$ is strictly increasing and continuous in $\gamma$. Since $u_s(\tilde{t})$ is also continuous in $s$, this implies that the unique cutoff $s^*(\tilde{t})$ is continuous in $\gamma$, which finally implies that $V(\tilde{t})$ is continuous in $\gamma$. From $\Delta > 0$ it then follows that there exists some $\gamma > 0$ such that $U(\tilde{t}) > U(t)$ and $V(\tilde{t}) > \bar{V}$, contradicting the optimality of $t$. Q.E.D.

Proof of Proposition 1: Suppose that some nondebt contract $t = t(x)$ is optimal. We can then show that there exists a debt contract $\tilde{t} = \tilde{t}(x)$ that satisfies (2) and is strictly preferred by the lender, contradicting the optimality of $t$. Holding the cutoff signal fixed at $s^*(t)$, we choose $\tilde{t}$ such that it provides the lender with the same expected payoff as $t$. Accordingly, $\tilde{t}$ is uniquely defined by

$$\int_{s^*(t)}^{1} \int_{X} z(x)g_s(x)dx \ f(s)ds = 0,$$

(A1)

where $z(x) := \tilde{t}(x) - t(x)$. Given that the cutoff signal is fixed and the lender’s expected payoff is the same under $t$ and $\tilde{t}$, the borrower’s expected payoff is also the same under $t$ and $\tilde{t}$.

Because $\tilde{t} \neq t$ and the lender’s conditional expected payoff $u_s$ is continuous in $s$ under both $t$ and $\tilde{t}$, (A1) implies that there exists at least one signal $s'$ satisfying $s^*(t) < s' < 1$ such that $\int_{X} z(x)g_{s'}(x)dx = 0$. Moreover, because $t$ is
nondecreasing and $\bar{t}$ is a debt contract, this implies that there exists a value $\hat{x} \in (x, \bar{x})$ such that $z(x) \geq 0$ for all $x < \hat{x}$ and $z(x) \leq 0$ for all $x > \hat{x}$, where both inequalities are strict over sets of positive measure. As $G_s(x)$ satisfies MLRP and $s^*(t) < s'$, it follows that $g_{s^*(t)}(x)/g_{s'}(x)$ is strictly decreasing in $x$. Accordingly, since

$$
\int_X z(x)g_{s^*(t)}(x)dx = \int_X z(x)\frac{g_{s^*(t)}(x)}{g_{s'}(x)} dx + \int_{\hat{x}}^x z(x)g_{s^*}(x)\frac{g_{s^*(t)}(x)}{g_{s'}(x)} dx,
$$

(A2)

if follows that

$$
\int_X z(x)g_{s^*(t)}(x)dx = \frac{g_{s^*(t)}(\hat{x})}{g_{s'}(\hat{x})} \left[ \int_X z(x)g_{s^*}(x)dx \right] = 0,
$$

(A3)

where the last equality follows from the definition of $s'$. From $\int_X z(x)g_{s^*(t)}(x)dx > 0$ we have that $u_{s^*(t)}(\bar{t}) > u_{s^*(t)}(t) = k$, where the equality follows from the definition of $s^*(t)$. As $u_{\bar{s}}(\bar{t})$ is strictly increasing in $s$, this implies that $u_{s^*(t)}(\bar{t}) = k$ for some $s^*(\bar{t})$ satisfying $s_{FB} < s^*(\bar{t}) < s^*(t)$, where the first inequality holds as the lender is too conservative for any feasible contract. Given the definition of $\bar{t}$ in (A1) and the fact that the lender prefers $s^*(\bar{t})$ over $s^*(t)$, the lender is strictly better off under $\bar{t}$ provided $\bar{t}$ satisfies (2). This is true if $v_s(\bar{t}) \geq w$ for all $s^*(\bar{t}) \leq s \leq s^*(t)$. Since $u_{s^*(\bar{t})} = k$ by the definition of $s^*(\bar{t})$ and $\mu_{s^*(\bar{t})} > k + w$ from $s^*(\bar{t}) > s_{FB}$, it holds that $v_s(\bar{t}) > w$. Together with the fact that $v_s(\bar{t})$ is strictly increasing in $s$, this implies that $\bar{t}$ satisfies (2), which completes the argument.

Uniqueness is straightforward. Any standard debt contract is uniquely defined by its repayment value $R$. As the borrower's expected payoff is continuous in $R$, there exists for each $\bar{V}$ a compact set of $R$-values at which the borrower's participation constraint binds. Given that the lender's expected payoff is increasing in $R$, the largest value in this set (denoted by $R^*$ in Proposition 1) defines the uniquely optimal debt contract $t^*$. Q.E.D.

Proof of Proposition 2: As in the proof of Proposition 1, we again argue by contradiction and assume that some contract $t = t(x)$ that is not levered equity is optimal. We can now construct a levered equity contract $\bar{t} := \bar{t}(x)$ satisfying (A1), which again implies the existence of at least one signal $s^*$ satisfying $s^*(t) < s' < 1$ such that $\int_X z(x)g_{s^*}(x)dx = 0$. Moreover, because $x - t(x)$ is non-decreasing with $t(x) \geq 0$ and $\bar{t}$ is levered equity, there exists a value $\hat{x} \in (x, \bar{x})$ such that $z(x) \leq 0$ for all $x < \hat{x}$ and $z(x) \geq 0$ for all $x > \hat{x}$, where both inequalities are strict over sets of positive measure. Analogous to (A2)–(A3), MLRP of $G_s(x)$ now implies that $\int_X z(x)g_{s^*(t)}(x)dx < 0$. Hence, we have that $u_{s^*(\bar{t})}(\bar{t}) = k$ for some $s^*(t) < s^*(\bar{t}) < s_{FB}$, where the last inequality holds as the lender is too aggressive for any feasible contract. Further, from the argument in the proof of Proposition 1, it follows immediately that $v_s(\bar{t}) < w$ for all $s < s_{FB}$ and thus for all $s \in [s^*(t), s^*(\bar{t})]$, implying that $\bar{t}$ satisfies (2). Given (A1) and the fact that the lender strictly prefers $s^*(\bar{t})$ over $s^*(t)$, the lender is strictly better off under $\bar{t}$, contradicting the optimality of $t$. The argument that the optimal
levered equity contract is unique is analogous to that for standard debt in Proposition 1. Q.E.D.

Proof of Proposition 3: Suppose the lender can offer a menu of contracts $T := \{t_i\}_{i \in I}$, where $I$ is some index set. As the lender makes an optimal choice from the menu, her conditional expected gross payoff from accepting the borrower after observing the signal $s$ is $u_s(T) := \max_{t \in T} u_s(t)$. Since $u_s(T)$ is the maximum over a set of continuous and nonincreasing functions, it is continuous and nonincreasing in $s$. By the same argument as for single contracts, the optimal decision rule is thus again characterized by some unique cutoff $s^* = s^*(T)$, where $u_{s^*}(T) = k$.

We will make use of the following auxiliary result.

Lemma A1: If $t$ is debt and $\mu_s(\hat{t}) \geq \mu_s(t)$ for some $\hat{s} < 1$ and some $\hat{t} \neq t$, then $\mu_s(\hat{t}) > \mu_s(t)$ for all $s > \hat{s}$. If $t$ is levered equity and $\mu_s(t) \geq \mu_s(\hat{t})$ for some $\hat{s} < 1$ and some $\hat{t} \neq t$, then $\mu_s(t) > \mu_s(\hat{t})$ for all $s > \hat{s}$.

Proof of Lemma A1: We prove the lemma for the case in which $t$ is debt. The proof for levered equity is analogous. The argument follows that in the proof of Proposition 1. Define $z(x) := t(x) - \hat{t}(x)$. Since $t$ is debt and $\hat{t}$ is nondecreasing, there exists some $\hat{x} \in (\hat{x}, x)$ such that $z(x) \geq 0$ for all $x < \hat{x}$ and $z(x) \leq 0$ for all $x > \hat{x}$, where the inequalities are strict over sets of positive measure. By MLRP of $G_s(x)$, we obtain for all $s > \hat{s}$ that

$$\int_X z(x)g_s(x) \, dx < g_{\hat{s}}(\hat{x}) \left[ \int_X z(x)g_{\hat{s}}(x) \, dx \right] \leq 0,$$

which completes the argument. Q.E.D.

We now consider the case in which the lender is too conservative and show that the uniquely optimal menu is degenerate in the sense that, conditional on acceptance, the uniquely optimal debt contract from Proposition 1 is chosen with probability one. We argue by contradiction and assume that some nondegenerate menu $T$ satisfying (2) is optimal. Consider the “cutoff contract” $\hat{t} \in T$ defined by $s^*(T) = s^*(\hat{t})$. (If there are several such contracts in the menu, take one of them.) Next, delete all contracts $t \neq \hat{t}$ from the menu. By construction, the cutoff signal $s^*(\hat{t})$ remains unchanged. Moreover, as the lender (weakly) prefers the deleted contracts over $\hat{t}$ for some $s \geq s^*(\hat{t})$, but the cutoff signal remains unchanged, the borrower is not worse off. That is, (2) remains satisfied after the deletion.

If $\hat{t}$ is not equal to $t^*$, we can use the argument in the proof of Proposition 1 and replace the remaining contract $\hat{t}$ with the optimal debt contract $t^*$. This lowers the cutoff signal while (2) binds under $t^*$. The lender is consequently strictly better off by replacing $T$ with $t^*$, contradicting the optimality of $T$. It remains to rule out the case in which $\hat{t} = t^*$ and $s^*(T) = s^*(t^*)$. As $T$ is nondegenerate, this implies that $u_s(\hat{t}) \geq u_s(t^*)$ for some $\hat{s} < 1$ and $\hat{t} \in T$ with $\hat{t} \neq t^*$. By Lemma A1 and the fact that the lender makes an optimal choice from the menu, we thus
have that $u_s(T) > u_s(t^*)$ for all $s > \bar{s}$. Because this implies that $v_s(T) < v_s(t^*)$ for all $s > \bar{s}$, and because $s^*(T) = s^*(t^*)$, it follows that $T$ violates (2), which yields a contradiction.

Consider next the case in which the lender is too aggressive. If the “cut-off contract” $\hat{t} \in T$ satisfies $\hat{t} = t^*$, we have from Lemma A1 that the menu is degenerate: If the lender (weakly) prefers $t^*$ at $s = s^*(T)$, she strictly prefers $t^*$ to all other contracts in the menu for all $s > s^*(T)$. Suppose therefore that $\hat{t} \neq t^*$. In this case, we can delete all contracts $t \neq \hat{t}$ from the menu. This leaves the cutoff unchanged while (2) remains satisfied after the deletion. As in the proof of Proposition 2, we can then replace $\hat{t}$ with the optimal levered equity contract $t^*$. By construction, (2) binds under $t^*$ and the cutoff $s^*$ is maximized. Q.E.D.

**Proof of Proposition 4:** Consider first the case in which the lender is too conservative. Suppose the borrower offers some new contract $\tilde{t}$ to replace $t^*$. If the lender prefers $\tilde{t}$ over $t^*$ for some $\tilde{s}$, then she strictly prefers $\tilde{t}$ for all $s > \tilde{s}$ by Lemma A1 in the proof of Proposition 3. Consequently, the borrower can only be better off under $\tilde{t}$ if $s^*(\tilde{t}) < s^*(t^*)$. In this case, the project is accepted under the new contract for all $s \geq s^*(\tilde{t})$, implying that the new contract is profitable for the borrower only if $V(\tilde{t}) \geq V(t^*)$. However, existence of some contract $\tilde{t}$ in which $V(\tilde{t}) \geq V(t^*)$ and $s^*(\tilde{t}) < s^*(t^*)$ contradicts the optimality of $t^*$ in the lender’s original problem.

If the lender offers a new contract, we have a signaling game. For the sake of brevity, we restrict attention to equilibria such that in case of indifference the borrower accepts the lender’s offer. We argue by contradiction and suppose that in a given equilibrium there is a nonempty set of accepted new contracts denoted by $T$. Denote by $\hat{T}$ the union of $T$ and $t^*$. In this equilibrium, the project is accepted for all $s \geq s^*(\hat{T})$. Denote by $\tilde{t}$ the “cutoff contract” that is chosen at $s = s^*(\hat{T})$. By our previous arguments, we know that unless $s^*(\hat{T}) < s^*(t)$, we cannot have an equilibrium in which the borrower accepts the lender’s offer. By Lemma A1 in the proof of Proposition 3, it then follows that the lender strictly prefers offering a new contract for all higher signals $s > s^*(\hat{T})$. Moreover, by optimality the lender will offer the contract that maximizes her expected payoff subject to being accepted by the borrower. Consequently, the upper bound for the borrower’s expected payoff in the equilibrium with renegotiations is $V(T) \leq V(\tilde{t})$. But we already know that this is strictly smaller than the borrower’s expected payoff in the absence of renegotiations, $V(t)$, which yields a contradiction.

Next, consider the case in which the lender is too aggressive. Analogous to the case in which the lender is too conservative, the borrower will only be better off if the new contract increases the cutoff signal. But this means at signals for which under the initial contract $t^*$ the lender would have obtained a positive profit by accepting the project, she now (i.e., under the new contract) gets nothing if the project is rejected. As the borrower has no means to compensate the lender, $t^*$ will not be renegotiated. Q.E.D.
Proof of Proposition 5: Consider first Case 1, where the competitive credit market offer is less than or equal to \( w \). (This also means that the break-even offer to rejected borrowers will be strictly less than \( w \), which in turn means that rejected borrowers will pursue their employment opportunity.) If \( \bar{V} = w = 0 \), offering \( t(x) = x \) is uniquely optimal and leads to the first-best decision rule \( s^* = s_{FB} \). To see that for \( w > 0 \) the informed lender is too aggressive, recall that from \( w > 0 \), \( t(0) \geq 0 \), the fact that \( x - t(x) \) is nondecreasing, and MLRP, we have that \( v_s(t) \) is strictly increasing in \( s \). Moreover, from (2) with equality and \( \bar{V} = w \) it follows that

\[
\int_{s^*}^{1} v_s(t) \frac{f(s)}{1 - F(s^*)} \, ds = w. \tag{A5}
\]

In conjunction with the fact that \( v_s(t) \) is strictly increasing, it thus holds that \( v_s(t) < w \), which in turn implies that \( \mu_s := u_s(t) + v_s(t) < k + w \) and therefore that \( s^* < s_{FB} \). Finally, from \( \mu_1 > w + k \) it follows that the informed lender is able to attract the borrower.\(^{24}\)

Consider next Case 2. We first show that in any equilibrium in which the borrower goes to the informed lender, rejected borrowers will not be financed by the market in \( \tau = 1 \). As \( u_s(t) \) is strictly increasing, we have for \( s^* < 1 \) that the informed lender makes a strictly positive expected profit. If rejected borrowers were financed by the market in \( \tau = 1 \), the overall generated surplus would be \( \mu - k \). (The project would then always be financed, either by the informed lender or by the market.) As the informed lender makes a positive expected profit, the expected payoff of a borrower who accepts the informed lender’s offer must then be strictly less than \( \bar{V} = \mu - k \), violating (2). Note that this also implies that as the informed lender’s optimal offer \( t^* \) satisfies (2), the expected NPV of borrower who is rejected under \( t^* \) is strictly negative.\(^{25}\)

Next, set \( \bar{V} = \mu - k > w \) and define

\[
V_{\text{max}} := \max_t \left[ \int_{s^*(t)}^{1} v_s(t) f(s) \, ds + F(s^*(t))w \right], \tag{A6}
\]

where \( t \) must be a feasible contract. If \( V_{\text{max}} \geq \bar{V} \), the informed lender can make an offer that is at least as attractive as the credit market’s offer in \( \tau = 0 \). If \( V_{\text{max}} > \bar{V} \), this implies that in any equilibrium the borrower will go to the informed lender. In contrast, if \( V_{\text{max}} < \bar{V} \), the borrower will go to the credit market. Hence, whether the borrower goes to the credit market or to the informed lender depends on the relative magnitudes of \( V_{\text{max}} \) and \( \bar{V} = \mu - k \).

\(^{24}\) Note that while \( s^* < s_{FB} \) holds under the optimal contract, it does not necessary hold for all feasible contracts. Put differently, by \( \mu_1 > w + k \) there exist contracts \( t \) with \( s_{FB} < s^* < 1 \) such that by \( v_s(t) > w \), the borrower’s participation constraint (2) is satisfied (though not with equality) under \( t \).

\(^{25}\) Formally, (2) in conjunction with \( \int_{0}^{1} [u_s(t) - k] f(s) \, ds > 0 \) and \( \bar{V} = \mu - k > w \) implies that the expected NPV of a rejected borrower equals \( \int_{0}^{s^*} \mu_s(t) \frac{f(s)}{1 - F(s^*)} \, ds - w - k < 0 \).
There are two subcases. In the first subcase (Case 2i), it holds for \( w = 0 \) that \( V_{\max} \geq \bar{V} \). As \( V_{\max} \) is strictly increasing in \( w \) by (A6), this implies that \( V_{\max} > \bar{V} \) also for all \( w > 0 \). In the second subcase (Case 2ii), it holds for \( w = 0 \) that \( V_{\max} < \bar{V} \). By continuity of \( V_{\max} \) in \( w \) and as \( V_{\max} > \bar{V} \) if \( w \) is close to \( \mu_1 \), there exists a unique threshold \( w_1 > 0 \) such that \( V_{\max} = \bar{V} \) for \( w = w_1 \), \( V_{\max} < \bar{V} \) for \( w < w_1 \), and \( V_{\max} > \bar{V} \) for \( w > w_1 \).

Consider now Case 2i, where the borrower goes to the informed lender for all \( w \geq 0 \). At \( w = 0 \) we have from \( \bar{V} = \mu - k > 0 \) that the informed lender is too conservative. Denote by \( t_D \) the unique debt contract with the highest repayment \( R \) such that (2) holds with equality. (By Proposition 1, \( t^* = t_D \) if the lender is too conservative.) As we increase \( w \), (2) will be relaxed. (The borrower’s outside option, which is realized with probability \( F(s^*) \), increases while the credit market’s offer in \( \tau = 0 \) remains unchanged at \( \mu - k \).) This allows us to increase \( R \) in \( t_D \) such that \( s^*(t_D) \) decreases. On the other side, \( s_{FB} \) is continuous and strictly increasing in \( w \). If we increase \( w \) by a sufficient amount, we obtain \( s^*(t_D) < s_{FB} \). (We know that at \( w = \mu - k > 0 \), any feasible contract for which (2) binds must make the lender too aggressive.) Consequently, there exists a unique threshold \( w_2 > 0 \) such that \( s^*(t_D) > s_{FB} \) for \( w < w_2 \) and \( s^*(t_D) < s_{FB} \) for \( w > w_2 \). Next, denote by \( t_E \) the unique levered equity contract with the lowest “strike price” \( R \) such that (2) holds with equality. Using an argument analogous to that for \( t_D \), we obtain a unique threshold \( w_3 \) such that \( s^*(t_E) > s_{FB} \) for \( w < w_3 \) and \( s^*(t_E) < s_{FB} \) for \( w > w_E \). Finally, from the arguments in the proofs of Propositions 1 and 2 it follows that \( w_1 < w_3 \).

We have shown that in Case 2i the informed lender is too conservative if \( w < w_2 \) and too aggressive if \( w > w_3 \). It remains to show that if \( w_2 \leq w \leq w_3 \), the optimal contract implements \( s^* = s_{FB} \). Define \( t(x) = R \) for \( x \leq R \) and \( t(x) = R + \gamma(x - R) \) with \( 0 \leq \gamma \leq 1 \) for \( x > R \). Starting from \( t_D \) at \( w = w_2 \), we can gradually lower \( R \) while adjusting \( \gamma \) accordingly so that (2) remains binding while \( s^* \) increases until it equals \( s^* = s_{FB} \). (As \( t_D \) and \( t(x) \) intersect only once, we can fully rely on the argument in the proof of Proposition 1.)

As for Case 2ii, where the borrower goes to the credit market for \( w \leq w_1 \), we can proceed in the same way as in Case 2i to obtain two unique thresholds \( w_2 \) and \( w_3 \), where \( w_2 > w_1 \) holds by construction of the two thresholds. Q.E.D.

REFERENCES
Axelson, Ulf, 2003, Security design with investor private information, Mimeo, University of Chicago.


Degryse, Hans, and Steven Ongena, 2005, Distance, lending relationships, and competition, Journal of Finance 60, 231–266.


DeMarzo, Peter M., and Michael J. Fishman, 2003, Optimal long-term financial contracting with privately observed cash flows, Mimeo, Stanford University.


Garmane, Mark, 2001, Informed investors and the financing of entrepreneurial projects, Mimeo, University of Chicago.


