Early-stage financing and firm growth in new industries

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Abstract

This paper shows that active investors, such as venture capitalists, can affect the speed at which new ventures grow. In the absence of product market competition, new ventures financed by active investors grow faster initially, though in the long run those financed by passive investors are able to catch up. By contrast, in a competitive product market, new ventures financed by active investors may prey on rivals that are financed by passive investors by “strategically overinvesting” early on, resulting in long-run differences in investment, profits, and firm growth. The value of active investors is greater in highly competitive industries as well as in industries with learning curves, economies of scope, and network effects, as is typical for many “new economy” industries. For such industries, our model predicts that start-ups with access to venture capital may dominate their industry peers in the long run.

1. Introduction

Agency problems between entrepreneurs and investors can impair the financial viability of new risky ventures. Actively involved, hands-on investors, such as venture capitalists (VCs), can mitigate these inefficiencies. This paper investigates how active investors affect not so much the financial viability of new ventures, but rather the speed at which they grow. In particular, it examines to what extent higher initial investment and faster early-stage growth of new ventures financed by active investors leads to a long-run competitive advantage vis-à-vis rivals who are financed by passive investors.

Active investors, who through their close involvement can better bridge the informational gap vis-à-vis entrepreneurs, can respond quicker to new information than passive investors, leading to an earlier shut-down of less promising ventures and a faster growth of promising ventures. A key insight of our model is that access to active investors can constitute a competitive advantage by allowing firms to “strategically overinvest” early on, thus forestalling their rivals’ future investment and growth.

We model a dynamic investment game in which early investments have a persistent effect on product quality. Our results are reinforced if early investments have additional long-run benefits, e.g., due to learning curves, economies of scope, and network effects. In our baseline model, where we abstract from product market competition, promising ventures financed by active investors receive more funding and make higher investments early on. By contrast, if new ventures are financed by passive investors, then growth proceeds more gradually, and less promising ventures are also kept alive longer.
If new ventures compete with each other on the product market, then those financed by active investors may "prey" on their rivals by "strategically overinvesting" early on. We show that strategic overinvestment is more likely in highly competitive industries. For such industries, our model predicts that new ventures financed by active investors dominate their industry peers in terms of investment, growth, and market shares. In less competitive industries, on the other hand, the source of financing does not matter in the long run, as firms who are financed by passive investors will eventually catch up.

While long-run differences in investment, growth, and profits can arise in our model even if firms have symmetric access to active investors, since in equilibrium some firms may endogenously choose passive investors, the case in which some firms have superior access to active investors is of particular interest, e.g., to understand differences between Europe and the U.S. Though the availability of VC financing has increased in Europe over the last decade (Da Rin, Nicodano, Sembenelli, 2006), "U.S.-style" VCs with specialized industry expertise who are actively involved in the firm's decision-making appear to be (still) relatively scarce on the ground. Using European data, Bottazzi, Da Rin, Hellmann (2008) find that it is primarily partners with prior business experience that become more actively involved. Likewise, Hege, Palomino, Schwienbacher (2007) find that VCs in the U.S. are more active and sophisticated than European VCs, while Schwienbacher (2005) finds that European VCs monitor less than their U.S. counterparts.

As for the Europe–U.S. comparison, our results regarding the size of VC investments and the speed at which firms grow are consistent with findings by Hege, Palomino, Schwienbacher (2007) that VC investments in the U.S. are on average twice as large as in Europe, and that this translates into long-run differences in performance. The authors also find that VC investments in the U.S. have a shorter average length than in Europe—which is consistent with our results that active investors are faster to pull the plug on bad investments—and that VCs in the U.S. "react with an increased funding flow upon good early performance, in contrast to Europeans" (p. 31). Similarly, and also consistent with our results, Puri and Zarutskie (2007) show that, within in the U.S., VC-backed firms make larger investments than their non-VC-backed counterparts.

Our results suggest that in newly developing industries, in particular those with little horizontal differentiation and substantial first-mover advantages, e.g., due to learning curves, economies of scope, and network effects, the presence of active investors can remove barriers to growth in the industry's early phase. Industries that would satisfy these criteria are, for example, the communication and information technology industries.

In our model, financial contracts between firms and active investors must ensure that the active investor acquires information and subsequently implements the efficient investment path, which may include speeding up the investment. Interestingly, this incentive problem only imposes a binding constraint on the contract if the investor's information is sufficiently precise. In this case, incentives can be either provided by limiting the active investor’s discretion over investment decisions or by "front-loading" his compensation by giving him a sufficiently large share of the firm's early-stage profits.

Our model is related to the literature on VC contracting, especially that on stage financing, with which it shares the dynamic perspective on investments. Given our focus on the interaction between outside financing and product market competition, our model is also related to the literature on the strategic use of internal versus external financing and debt versus equity financing (Brander and Lewis, 1986; Maksimovic, 1988). Finally, it is related to models studying the role of corporate venturing (Hellmann, 2002) and strategic alliances (Mathews, 2006) in a competitive context.

Our model is also related to Ueda (2004) and Winton and Yerramilli (2008), both of which examine the endogenous choice between active and passive investors. In Ueda's model, VCs are better at screening projects ex ante, but they are also more likely to steal the entrepreneur's idea. Winton and Yerramilli examine, among other things, the trade-off between VCs' higher funding costs (i.e., liquidity costs) and their superior monitoring ability. In our model, active investors are beneficial only if they can be induced to acquire information, which is costly. While the cost-benefit analysis of banks versus VCs is richer in Winton and Yerramilli's model, our model considers the interaction between outside financing, investment, and product market competition.

The rest of this paper is organized as follows. Sections 2 and 3 examine the baseline model without competition. In Section 4, we embed our model in a competitive product market. Section 5 considers various extensions.

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2 While Europe has its fair share among the 300 global leaders in terms of R&D expenditures, only two of the European firms among the top 300 were created after 1990, including Amazon, eBay, and Google.

3 See also Bartelsman, Haltiwanger, Scarpetta (2006) and Aghion, Fally, Scarpetta (2007), who show that while exit and entry rates are similar in the U.S. and Europe, successful new ventures grow faster and expand more rapidly in the U.S. Aghion, Fally, Scarpetta conclude that "the analysis of firm dynamics and its links with financial development and other institutional factors cannot only focus on entry, but should also explore the development of new ventures in the first years of their life." (p. 8, emphasis added).

4 The authors show that this result is not demand-driven in the sense that firms with larger investment opportunities might seek more VC financing.

5 "Front-loading" in our model can also be interpreted as the retention of early-stage profits and using them towards future investments, thereby reducing the active investor’s future capital injections.

6 For contributions to the VC contracting literature, see Hellmann (1998), Casamatta (2003), Inderst and Mueller (2003), and Repullo and Suarez (2004). In the stage financing literature, staging is typically interpreted as a short-term financial contract giving the VC control over the continuation decision, which alleviates agency problems (Neher, 1999; Cornelli and Yoshia, 2003).

7 Cestone and White (2003) consider the financing of competing ventures through a single investor. Inderst and Mueller (2003) consider competition among start-ups for VC financing in the capital market, while Kainz and Knuechel (2003), Fulghieri and Svolik (2005), and Inderst, Mu¨nnich, Mueller (2007) consider competition among portfolio companies of the same VC for the VC's scarce resources.
2. Investment and the value of information

As a benchmark, we consider first the investment decision of a single, wealthy, and risk-neutral entrepreneur. In Section 3, we relax the assumption that the entrepreneur is wealthy. In Section 4, we relax the assumption that there is a single entrepreneur by considering a strategic financing game between two start-ups. The entrepreneur has a new venture that requires an initial investment of \( I_0 \) in \( t = 0 \). The venture’s product is sold on the market both in \( t = 1 \) and \( 2 \). At these dates, the firm can make additional investments of \( I_1 \) and \( I_2 \), respectively.

The venture’s success depends, next to \( I_1 \) and \( I_2 \), on the state of nature \( \theta \), which can be either “bad” \( (\theta = b) \) or “good” \( (\theta = g) \). Prior beliefs about \( \theta \) are given by \( \mu_0 = \text{Pr}(\theta = g) \), where \( 0 < \mu_0 < 1 \). In \( t = 1 \), before making the investment \( I_1 \), the entrepreneur receives a signal \( s \) in \{g, b\} about \( \theta \). The signal is only informative with probability \( \psi > 0 \), while with probability \( 1 - \psi \) it constitutes pure noise. Prior beliefs about \( \theta \) after observing \( s \) in \{g, b\} are given by

\[
\mu_g := \frac{\mu_0(1 + \psi)}{\mu_0(1 + \psi) + (1 - \mu_0)(1 - \psi)} \quad (1)
\]

and

\[
\mu_b := \frac{\mu_0(1 - \psi)}{\mu_0(1 - \psi) + (1 - \mu_0)(1 + \psi)}. \quad (2)
\]

The investments \( I_1 \) and \( I_2 \) determine the product’s quality, which for the time being can be either “low” \((q = 1)\) or “high” \((q = h)\). Ignoring competition for the moment, we assume that quality \( q \) gives rise to a (representative) consumer’s utility of \( u_q \), where \( u_q > u_1 > 0 \). To simplify the notation, we set \( u := u_0 = u_1 = u_q \), where \( u \) is a constant utility increment. Positive utility is realized only if \( \theta = g \). If \( \theta = b \), the product fails, e.g., because it is technologically infeasible.\(^8\) The parameter \( A_1 > 0 \) denotes the market size given that \( \theta = g \). The firm’s profits (gross of investment costs) are \( V_1 := u_1 A_1 \). Importantly, as \( V_1 \) is observable and \( V_1 > 0 \) only if \( \theta = g \), the state of nature \( \theta \) is perfectly known after \( t = 1 \) and thus before the second-period investment \( I_2 \) is made.

To produce quality \( q_1 \), the firm must invest \( I_1 = K_{q_1} \), where \( K_{q_1} > K > 0 \). Incremental investment costs are denoted by \( K_q := K - K_{q_1} \) and \( K_q > 0 \). Given that utility increments are constant, we assume (weakly) increasing incremental investment costs: \( K > K_q > 0 \). We also assume that product quality does not deteriorate over time, capturing the “persistency” of early investments. For example, if the firm invests \( I_1 = K_{q_1} = K_{q_2} = K \) and \( I_2 = 0 \), the quality is \( q_1 = q_2 = h \) both in \( t = 1 \) and \( 2 \).

\(^8\) Ex ante uncertainty about the market’s potential might allow for a different interpretation. However, interpreting the state of nature in terms of the product’s technological feasibility allows us to assume later that competing ventures face the same (technological) uncertainty.

The firm’s choices in \( t = 1 \) and \( 2 \) are thus as follows: (i) discontinue the venture in \( t = 1 \) by investing zero both in \( t = 1 \) and \( 2 \); (ii) invest \( I_1 = K_{q_1} \) and \( I_2 = 0 \), thus producing quality \( q_1 = q_2 = l \) both in \( t = 1 \) and \( 2 \); (iii) pursue a gradual investment path by investing \( I_1 = K_{q_1} \) and \( I_2 = K_{q_2} \), thus producing quality \( q_1 = l \) in \( t = 1 \) and \( q_2 = h \) in \( t = 2 \); (iv) speed up the investment by investing \( I_1 = K_{q_1} + K_{q_2} \) and \( I_2 = 0 \), thus producing quality \( q_1 = q_2 = h \) both in \( t = 1 \) and \( 2 \).

We first characterize the efficient investment path if the signal is uninformative (\( \psi = 0 \)). Clearly, if \( \theta = g \), then it is ex ante efficient to invest \( I_0 \). If \( \theta = b \), then it must also be efficient to continue the venture in \( t = 1 \) by investing at least \( I_1 = K_{q_1} \).

To make the subsequent analysis when the signal is informative interesting, we assume that it is efficient to pursue a gradual investment path when the signal is uninformative. The conditions for this are as follows. Investing \( I_2 = K_{q_2} \) is efficient if

\[
A_2 u > K_{q_2}, \quad (3)
\]

while, provided that condition (3) holds, investing \( I_1 = K_{q_1} \) is efficient if

\[
\mu_0 (A_1 u + 2 A_2 u - K_{q_2}) - K_{q_1} > \mu_0^2 (A_1 + A_2) u - K_{q_1} - K_{q_2}. \quad (4)
\]

This can be rearranged as

\[
\frac{\mu_0}{1 - \mu_0} < \frac{K_{q_2}}{A_1 u}. \quad (5)
\]

To characterize the efficient investment path for general \( \psi \), we first determine the efficient decision rule based on the updated belief \( \mu_1 \) in \( t = 1 \).

**Lemma 1.** There are two thresholds \( 0 < \mu' < \mu'' < 1 \) for posterior beliefs \( \mu_1 \) such that:

(i) If \( \mu_1 < \mu' \) it is efficient to discontinue the venture in \( t = 1 \).

(ii) If \( \mu' < \mu_1 < \mu'' \) it is efficient to pursue a gradual investment path by investing \( K_{q_1} \) in \( t = 1 \) and, provided that \( \theta = g \) is realized, \( K_{q_2} \) in \( t = 2 \).

(iii) If \( \mu_1 > \mu'' \) it is efficient to speed up the investment by investing \( K_{q_1} + K_{q_2} \) in \( t = 1 \), and zero in \( t = 2 \).

Throughout this paper, the disclaimer “provided that \( \theta = g \) is realized” implies a zero investment in \( t = 2 \) if \( \theta = g \) is not realized, i.e., if instead \( \theta = b \) is realized. If the signal’s precision \( \psi \) is sufficiently high, posterior beliefs satisfy \( \mu_1 < \mu' \) and \( \mu_1 > \mu'' \). By Lemma 1, it is then optimal to either discontinue the venture in \( t = 1 \) (if \( s = g \) is observed) or invest \( I_1 = K_{q_1} + K_{q_2} \) (if \( s = g \) is observed). Compared to the case where \( \psi = 0 \), a sufficiently precise signal thus allows to improve the investment decision both by discontinuing the venture after bad news and speeding up the investment after good news. Moreover, when \( \mu_0 \) is not too large (see the threshold derived in the Proof of Proposition 1), then, for intermediate values of \( \psi \), only a bad signal changes the investment path relative to...
the benchmark case in which the signal is uninformative. As our primary interest lies with risky ventures that have relatively little chance of success ex ante, as is typically the case in venture capital finance, we shall henceforth focus on this case.

Proposition 1. There are two thresholds \( 0 < \psi' < \psi'' < 1 \) for the signal’s precision \( \psi \) such that:

(i) If \( \psi \leq \psi' \) it is efficient to pursue a gradual investment path by investing \( k_1 \) in \( t = 1 \) and, provided that \( \theta = g \) is realized, \( k_h \) in \( t = 2 \).

(ii) If \( \psi' < \psi < \psi'' \) it is efficient to discontinue the venture in \( t = 1 \) after observing \( s = b \) and to pursue a gradual investment path after observing \( s = g \).

(iii) If \( \psi \geq \psi'' \) it is efficient to discontinue the venture in \( t = 1 \) after observing \( s = b \) and to speed up the investment after observing \( s = g \) by investing \( k_1 + k_h \) in \( t = 1 \).

Based on Proposition 1, we can characterize the ex ante value of information.

Corollary 1. The value of information (in the form of the signal) is as follows. If \( \psi \leq \psi' \) the value of information is zero, if \( \psi' < \psi < \psi'' \) the value from discontinuing the venture after observing \( s = b \) is

\[
1 \frac{(1 - \mu_0)(1 + \psi)}{} (k_1 - \frac{\mu_0}{1 - \psi} (A_1 u + 2 A_2 u - k_l - k_h)),
\]

and if \( \psi \geq \psi'' \) the value from speeding up the investment after observing \( s = g \) is

\[
1 \frac{\mu_0(1 + \psi)}{} (A_1 u - \frac{\mu_0}{1 + \psi} k_h).
\]

Note that the respective conditions \( \psi' < \psi < \psi'' \) and \( \psi \geq \psi'' \) ensure that (6) and (7) are both positive. Note also that the total value of information if \( \psi \geq \psi'' \) is the sum of (6) and (7). Intuitively, from (6) we have that the value from discontinuing the venture in \( t = 1 \) is higher if the larger is the (otherwise lost) capital outlay \( k_h \), while from (7) we have that the value from speeding up the investment is higher if the larger is the firm’s incremental first-period profit \( A_1 u \).

3. Outside financing: active versus passive investors

3.1. Extension of the model

To provide a role for outside financing, we now assume that the entrepreneur is penniless. Outside financing is provided by competitive risk-neutral investors, whose cost of capital is normalized to zero.

In \( t = 1 \), if the investment \( I_1 \) is sunk, some investors can obtain information about the state of nature \( \theta \) at private (monitoring) cost of \( k > 0 \). We refer to such investors as “active investors” and denote their signals by \( S_A \in \{b, g\} \), which are obtained with precision \( \psi_A \). To avoid confusion, we denote the entrepreneur’s signal by \( S_E \), which is obtained with precision \( \psi_E \). Investors who cannot obtain information about \( \theta \), e.g., because they lack expertise, are called “passive investors.” (Alternatively, passive investors could be viewed as having a completely uninformative signal.)

Venture capitalists can provide active support in numerous ways (see Section 1). In our setting, besides providing capital infusions at different stages, active investors can obtain valuable information. Even if this information is less precise than that of the entrepreneur, it is valuable as the entrepreneur cannot be trusted to make an unbiased decision once he receives outside funding. This is because we assume that he derives private benefits from larger investments. Precisely, we assume that for every dollar invested, the entrepreneur receives arbitrarily small private benefits of \( \gamma \).

Assuming that \( \gamma \) is arbitrarily small allows us to conveniently ignore the entrepreneur’s private benefits both when determining the efficient investment path and when deriving the firm’s optimal choice of financing. If \( \gamma \) was non-negligible, then this would affect the specific threshold values in Proposition 1 as well as the value of information in Corollary 1. However, it would not qualitatively affect our analysis. Note, in particular, that since the private benefits cannot be shared with the investor, they would not relax the investor’s break-even constraint.

As is standard in the financial contracting literature, we assume that payments to the (penniless) entrepreneur can only be made if the venture is successful. It is also obvious that payments exceeding the venture’s profits are never optimal. A financial contract thus stipulates that the entrepreneur receives a share \( 0 < \sigma_1 < 1 \) of the venture’s profits \( A_1 u \). As investments are verifiable, a financial contract could, in principle, also specify an investment path, possibly contingent on \( \theta \) and the entrepreneur’s signal (precisely, his message). As we will show below, there is no need to spell out the mechanism-design problem in

\[11\] Recall that the state of nature \( \theta \) becomes perfectly known after \( t = 1 \). The benefit of having information about \( \theta \) already in \( t = 1 \) is that it can be used to improve the decision regarding \( I_1 \).

12 If \( \gamma = 0 \), then condition (5) would be relaxed given that investing \( I_2 = k_h \) would now be efficient if \( A_1 u > k_h (1 - \gamma) \). Furthermore, even though the entrepreneur does not derive larger private benefits if a given investment is undertaken earlier, if the decision to speed up the investment in \( t = 1 \) must be made under uncertainty (if \( \psi < 1 \)), then assuming a non-negligible value of \( \gamma \) would also affect the choice between \( k_1 \) and \( k_1 + k_h \) in \( t = 1 \) and thus condition (5) as well as the threshold \( \psi' \) in Proposition 1. Precisely, replacing \( k_h \) by \( k_h (1 - \gamma) \) would result in a lower value of \( \psi' \). By contrast, the other threshold in Proposition 1, \( \psi'' \), would increase, given that the cost of (wrongly) sinking \( k_1 \) to keep the venture alive is lower if the entrepreneur derives private benefits from investing.

13 The common justification for this assumption is that non-state contingent payments would attract “fake” entrepreneurs who have no real projects (so-called “fly-by-night” operators).

14 Likewise, the sharing rules \( \sigma_1 \) could also condition on the entrepreneur’s message, next to \( \theta \) and \( I_1 \).
3.2. Analysis

We first consider the case in which the venture is financed by a passive investor. In principle, investment decisions could be made contingent on the entrepreneur’s private signal (precisely, his message). This is, however, not feasible. In order to elicit truthful information from the entrepreneur that would change the firm’s investment path (relative to the gradual investment path that is efficient if no signal is available), the entrepreneur would have to be rewarded for revealing bad news, since he obtains private benefits from larger investments. As his compensation can only be tied to the venture’s success, however, no such reward is incentive compatible, because it would also be preferred by an entrepreneur with a good signal. If \( s_A = 0 \), a gradual investment path is thus the most efficient outcome that can be achieved. In this case, any set of sharing rules \( \{\sigma_1, \sigma_2\} \) that satisfies the passive investor’s break-even constraint

\[
\mu_0(\sigma_1 A_1 u + 2\sigma_2 A_2 u - \kappa_h) - \kappa_l > I_0
\]

with equality is optimal. We assume that the venture is sufficiently profitable such that (8) holds strictly for \( \sigma_1 = \sigma_2 = 1 \).

**Proposition 2.** A firm financed by a passive investor pursues a gradual investment path.

We next consider the case in which the venture is financed by an active investor. As in the case above, the entrepreneur’s signal cannot be relied upon. Recall from Corollary 1 that the value of information is zero if the signal’s precision is low \( (\psi_A < \psi) \). On the other hand, if \( \psi_A > \psi \), it is efficient to induce the active investor to acquire information if the associated cost \( k \) is not too large. By Corollary 1, if \( \psi < \psi_A < \psi'' \), this is the case if \( k \) is less than (6), while if \( \psi \geq \psi'' \), it is the case if \( k \) is less than the sum of (6) and (7).

The case where \( \psi < \psi_A < \psi'' \) mirrors that with a passive investor in that any set of sharing rules \( \{\sigma_1, \sigma_2\} \) that satisfies with equality the active investor’s break-even constraint, which is now

\[
\mu_0 \frac{1 + \psi_A}{2} (\sigma_1 A_1 u + \sigma_2 2A_2 u - \kappa_l - \kappa_h) - \kappa_l > I_0 + k
\]

is also optimal. Any such contract induces the active investor to acquire information at private cost \( k \) and to implement the efficient investment path. As for intuition, recall from Case (ii) of Proposition 1 that efficiency dictates that the venture should be discontinued if \( s_A = b \) is observed. Given that the investor fully funds the investment out of his own pocket, he has no incentives to continue unless this is also efficient. If \( \psi < \psi_A < \psi'' \), the role of information acquisition is thus primarily protective from the investor’s viewpoint, namely, to avoid sinking \( I_1 = \kappa_l \) of his own funds if the venture is unlikely to succeed, which is also why he has adequate incentives to acquire information in the first place.

If \( \psi_A > \psi'' \), efficiency dictates that the active investor should speed up the investment after observing \( s_A = g \). There are two ways to make this privately optimal for the active investor. The first is to limit the investor’s discretion by requiring that he invests either \( I_1 = 0 \) or \( I_1 = \kappa_l + \kappa_h \) but not \( I_1 = \kappa_l \). As can be shown (see Proof of Proposition 3), investing only \( I_1 = \kappa_l \) would be the active investor’s preferred choice had he not acquired information. Intuitively, it is easier to induce the active investor to acquire information if his subsequent choice set is limited to precisely those values of \( I_1 \) that are optimal if and only if he acquired information. Given this limitation on the active investor’s discretion, any set of sharing rules \( \{\sigma_1, \sigma_2\} \) that satisfies his break-even constraint

\[
\mu_0 \frac{1 + \psi_A}{2} (\sigma_1 2A_1 u + \sigma_2 2A_2 u - \kappa_l - \kappa_h) - (1 - \mu_0) \frac{1 - \psi_A}{2} (\kappa_l + \kappa_h) > I_0 + k
\]

also induces the active investor to both acquire information and implement the efficient investment path from Case (iii) of Proposition 1.

The second way is to give the active investor full discretion over the investment decision while making a judicious choice of the sharing rules. To make it privately optimal for the active investor to speed up the investment after observing \( s_A = g \), he has to be given a sufficiently large fraction \( \sigma_1 \) of the firm’s first-period profits \( A_1 u \). Formally, it is shown in the Proof of Proposition 3 that \( \sigma_1 \) must satisfy

\[
\sigma_1 > \kappa_h \frac{1 - \mu_0}{\mu_0} \frac{1 - \psi_A}{1 + \psi_A}.
\]

Incidentally, increasing \( \sigma_1 \) while reducing \( \sigma_2 \) to satisfy (10) with equality also relaxes the active investor’s incentive constraint to acquire information in the first place. As is shown in the Proof of Proposition 3, the active investor acquires information if

\[
\mu_0 \sigma_1 \psi_A A_1 u - \sigma_2 (1 - \psi_A) A_2 u > \kappa_l + \kappa_h (1 - \psi_A) - (1 - 2\mu_0) - \kappa_l \sigma_2 (1 + \psi_A - 2\mu_0 \psi_A).
\]

where the left-hand side is increasing in \( \sigma_1 \) and decreasing in \( \sigma_2 \).

**Proposition 3.** Inducing information acquisition by an active investor is optimal if either \( \psi < \psi_A < \psi'' \) and \( k \) is less than (6) or if \( \psi \geq \psi'' \) and \( k \) is less than the sum of (6) and (7).

In the first case, any set of sharing rules that allows the active investor to break even also ensures that he acquires information and implements the efficient investment path. In the second case, it is furthermore necessary to either limit the active investor’s discretion to investments \( I_1 \in [0, \kappa_l + \kappa_h] \) or to “front-load” his compensation by increasing \( \sigma_1 \) and decreasing \( \sigma_2 \) so that (11) and (12) are jointly satisfied.
Throughout this paper, we assume that if active investors remain equally uninformed as passive investors (e.g., because \( k \) is too high), then the entrepreneur turns to a passive investor. This assumption could be endogenized by assuming that active investors, such as venture capitalists, have marginally higher funding costs (e.g., Winton and Yerramilli, 2008). Likewise, active investors could be more scarce than passive investors, allowing them to require a higher rate of return.

4. Strategic financing and investment when firms compete with each other in the product market

4.1. Extension of the model

We now extend our model by introducing a competitive product market in \( t = 2 \). We specify that at most two ventures \( n = a, b \) can be started in \( t = 0 \). Both ventures require the same initial investment \( I_0 \) and the same follow-up investments \( I_1 \) and \( I_2 \) to produce a given product quality \( q_0 \). Likewise, technological uncertainty, as captured by the state of nature \( \theta \), affects both ventures in the same way.

To capture the idea that markets evolve gradually, we assume that initially, in \( t = 1 \), firms act as monopolists in their own local markets, generating profits of \( \pi^0_n \) in case \( \theta = g \) is realized, where we abbreviate a (representative) consumer’s utility from quality \( q_0 \) by \( \pi^0_n \). Subsequently, in \( t = 2 \), firms compete in a “global” market, where we model competition using a standard Hotelling framework, although we only make use of properties of the competition game that also hold more generally (see below).

With regard to the competition game, suppose that in \( t = 2 \) the mass \( 2A_2 \) of consumers is uniformly distributed over a unit interval, with the two firms \( n = a, b \) being located at the respective endpoints. By specifying a market of size \( 2A_2 \), we make our analysis directly comparable to the case without competition, where the market size was \( A_2 \) for each firm. A consumer with “location” \( 0 < x < 1 \), which is either in geographic space or in the space of preferences over product characteristics, derives net utility \( u^2_n - p^2_n - \tau x \) from purchasing a good from firm \( a \) at price \( p^2_n \) in \( t = 2 \). Here, \( \tau > 0 \) is a measure of horizontal product differentiation. If the same consumer purchases from firm \( b \), he derives net utility of \( u^2_b - p^2_b - \tau(1 - x) \).

If both firms have positive market shares, then it is well known that in \( t = 2 \) firm \( n \) realizes equilibrium profits of

\[
\pi^n_n = \frac{A_2}{2} \left( \tau + \frac{u^2_n - u^2_b}{3} \right)^2. \tag{13}
\]

Differentiating (13) shows that the benefits to firm \( n \) from a marginal increase in \( u^2_n \) are

\[
\frac{2}{3\tau} A_2 \left( \tau + \frac{u^2_n - u^2_b}{3} \right), \tag{14}
\]

which is increasing in \( u^2_n \) and decreasing in \( u^2_b \). Hence, a firm’s profits in \( t = 2 \) are convex in the quality of its own products, while the marginal benefits from producing higher quality by making larger investments are decreasing in the quality of its rival’s products. These features are key for our analysis and hold for most standard models of product differentiation (see Athey and Schmutzler, 2001). Note also that as firms’ products become less horizontally differentiated (i.e., \( |u^2_n - u^2_b| \) decreases), product market competition intensifies, resulting in lower total industry profits.

We enrich our model further by introducing an additional investment level, and thus an additional product quality. By investing \( k_H \) in addition to \( k_L + k_H \), a firm can produce quality \( q = H \) with consumer surplus \( 3u \). (Recall that if a firm produces quality \( q = h \) (\( q = l \)) by investing \( K_1 + k_H (k_L) \), the consumer surplus is \( 2u (u) \).) We assume that

\[
k_H > (A_1 + A_2)u, \tag{15}
\]

which ensures that quality \( q = H \) would never be optimal in our previously analyzed setting without competition. We also assume that

\[
2u < 3\tau \tag{16}
\]

to ensure that both firms have positive market shares for all investment levels \( \ell^2 > 0. \)

4.2. Analysis

We first specify exogenously whether a firm is financed by an active or passive investor. In Section 4.2.2, we endogenize the choice of outside financing. We assume that financial contracts are not observable by competitors, thus ruling out their use as a strategic commitment device. To keep the analysis simple, we first assume that the active investor’s signal is fully informative (\( \psi_A = 1 \)). In Section 4.2.1, we extend our results to the case with \( \psi_A < 1 \). Finally, we replace condition (3) with the stronger condition

\[
\frac{1}{2} A_2 u > k_h. \tag{17}
\]

4.2.1. Exogenous choice of outside financing

Given that we specify exogenously whether a firm is financed by an active investor, we must set \( k \) sufficiently small to ensure that it is optimal to induce the active investor to acquire information. For simplicity, we set \( k = 0 \). When we endogenize the choice of outside financing below, we will naturally assume that \( k > 0 \).

If both firms are financed by active investors, the investment game unfolds in \( t = 1 \). Analogous to the case without competition, provided that \( s_A = g \) is observed, there always exists a symmetric equilibrium in which both firms invest \( K_1 + k_h \) in \( t = 1 \) and zero in \( t = 2 \), thus producing quality \( q_1 = q_2 = h \) both in \( t = 1 \) and \( 2 \). There exist no other symmetric equilibria. However, for some

\[16\] Both firms have strictly positive market shares if and only if \( |u^2_n - u^2_b| < 3\tau \). Given that \( u^2_n, u^2_b \in [u, 2u, 3u] \), this transforms to (16).

\[17\] That (17) is stronger than (3) follows intuitively from the observation that under competition a higher quality choice is less profitable if a firm expects its rival to also choose a higher quality. On the other hand, we need not strengthen condition (5), as it refers to payoffs in \( t = 1 \), where firms still operate in their own local markets.
parameter values, there additionally exist two asymmetric equilibria.

**Lemma 2.** Suppose both firms are financed by active investors. There always exists a symmetric equilibrium in which, provided that $s_A = g$ is observed, both firms invest $\kappa_1 + \kappa_h$ in $t = 1$ and zero in $t = 2$. If

$$\kappa_h \geq A_1 u + A_2 u \frac{1}{3\tau} (2\tau - u) \quad (18)$$

and

$$\kappa_H \leq A_1 u + A_2 u \frac{4}{9\tau} (u + 3\tau) \quad (19)$$

there additionally exist two asymmetric equilibria in which, provided that $s_A = g$ is observed, one firm invests $\kappa_1 + \kappa_h + \kappa_H$ and the other firm invests $\kappa_1$ in $t = 1$, while both firms invest zero in $t = 2$.

As for the two asymmetric equilibria, conditions (18) and (19) ensure that neither the “investment leader,” who invests $\kappa_1 + \kappa_h + \kappa_H$ in $t = 1$, nor its rival, who invests only $\kappa_1$, want to deviate to the symmetric equilibrium level of $\kappa_1 + \kappa_h$. Intuitively, this imposes both a lower boundary on $\kappa_H$ and an upper boundary on $\kappa_H$. However, picking one of the firms as the “investment leader,” whose profits are strictly larger than those of its rival, seems arbitrary given that both firms face identical financing conditions. In what follows, we thus impose as a refinement the requirement that if both firms face identical financing conditions, then the equilibrium outcome should also be symmetric. Note also that when we endogenize the choice of outside financing below, assuming that $k > 0$, the case in which identical financing conditions result in an asymmetric equilibrium would never arise even for all but very small values of $k$.

Consider next the case in which only one firm is financed by an active investor. Given the reluctance of the passive investor to commit more capital early on than what is absolutely necessary (because he does not observe a signal), the firm financed by an active investor has an endogenous first-mover advantage. It will strategically exploit this advantage if investing $\kappa_1 + \kappa_h + \kappa_H$ early on makes it unprofitable for its rival to step up its investment later, implying the outcome remains asymmetric also in the long run. While such an “overinvestment strategy” would not pay if the rival were to invest $\kappa_1 + \kappa_h$ early on (as in Lemma 2), the fact that the rival (who is financed by a passive investor) invests only $\kappa_1$ renders this strategy profitable. The outcome is a long-run asymmetry between the two firms in terms of total investment, market shares, and profits.

Formally, recall from (14) that the benefits from producing high quality are smaller if the other firm also produces high quality. By committing to the highest quality $q = H$ early on, a firm that is financed by an active investor can forestall any future investment by its rival if

$$\kappa_h \geq A_2 u \frac{1}{3\tau} (2\tau - u). \quad (20)$$

If (20) does not hold, the overinvestment strategy does not work, as the rival would then invest $\kappa_h$ in $t = 2$ despite the high initial investment of $\kappa_1 + \kappa_h + \kappa_H$ by the “investment leader,” and despite the fact that the additional investment of $\kappa_0$ only bears fruit in the second period. But if (20) holds and $\kappa_H$ is not too large so that (19) is satisfied, then an equilibrium exists that features a long-run asymmetric outcome.

**Lemma 3.** Suppose firm $n$ is financed by an active investor, while its rival, firm $n'$, is financed by a passive investor.

Case (i): If either (19) or (20) does not hold, then there exists an equilibrium in which, provided that $\theta = g$, both firms end up with the same total investment $\kappa_1 + \kappa_h$, product quality $q = h$, and market share in the long run, though firm $n$ makes all of its investments in $t = 1$, while firm $n'$ pursues a gradual investment path.

Case (ii): If both (19) and (20) hold, then there exists an equilibrium in which, provided that $s_A = g$ is observed, firm $n$ “strategically overinvests” early on by investing $\kappa_1 + \kappa_h + \kappa_H$ in $t = 1$ and zero in $t = 2$, while firm $n'$ invests $\kappa_1$ in $t = 1$ and zero in $t = 2$.

We show in the Proof of Lemma 3 that there may also exist other equilibria in which the rival firm invests more than $\kappa_1$ in $t = 1$. However, the range of parameters for which such equilibria exist is small. A sufficient set of conditions to rule out these equilibria is that

$$\kappa_H > A_2 u \frac{1}{3\tau} (u + 2\tau) \quad (21)$$

and

$$\mu_0 < \frac{3}{4} \frac{2\tau + u}{3\tau + u}. \quad (22)$$

If these conditions hold, then there exist no equilibria besides those characterized in Lemma 3. We will assume throughout that both conditions hold. Note that condition (21) is relatively mild, given that a lower boundary on $\kappa_H$ is already obtained from (15). Likewise, condition (22) conforms well with our previous restriction to investments that have little chance of success ex ante, as is reflected in our assumption that $\mu_0$ is small. (If $\mu_0 < \frac{1}{2}$, condition (22) is always satisfied.) Intuitively, conditions (21) and (22) ensure that it is too costly for a firm financed by a passive investor to make a high investment early on, given that the passive investor (who does not observe a signal) must make this investment under a considerable degree of uncertainty.

We finally consider the case in which both firms are financed by passive investors. In this case, there exists a unique symmetric equilibrium that mirrors the case without competition.

**Lemma 4.** If both firms are financed by passive investors, then they both pursue a gradual investment path.
4.2.2. Endogenous choice of outside financing

With Lemmas 2–4 at hand, we can now, in analogy to the case without competition, determine the benefits of financing by an active investor. If firms compete in the product market, the active investor’s information entails an additional benefit, namely, it may allow a firm to credibly commit to strategically overinvest early on to forestall a rival’s future investment or to protect itself from a similar strategic move by a rival.

4.2.2.1. Asymmetric access to outside financing. We first consider the case in which only one firm has access to active investors, while the other firm only has access to passive investors. For example, active investors may be regionally clustered, while at the same time local proximity may be key for the active investor’s close involvement with the firm.

If either (19) or (20) does not hold, then financing by an active investor has no strategic value. Consequently, the value of choosing an active investor is the same as without competition and thus, by Corollary 1 (using \( \psi = 1 \)), given by

\[
(1 - \mu_0)k_1 + \mu_0 A_1 u. \tag{23}
\]

Conversely, if both (19) and (20) hold, then financing by an active investor has an additional strategic value.

**Proposition 4.** Suppose firm a has access to active investors, while firm b only has access to passive investors. If either (19) or (20) does not hold, then firm a chooses an active investor if the value of information in (23) exceeds \( k \). In the long run, both firms have then the same total investment, market shares, and profits. If instead both (19) and (20) hold, then firm a chooses an active investor if

\[
(1 - \mu_0)k_1 + \mu_0 A_1 u - \frac{2A_1 u - \kappa_H}{9\tau}(u + 3\tau) \tag{24}
\]

exceeds \( k \), in which case firm a has a higher total investment, market share, and profits in the long run.

By inspection, an increase in the utility increment \( u \) increases both (23) and (24). Hence, regardless of whether (19) and (20) hold, an increase in \( u \) makes it more likely that firm a chooses financing by an active investor. Intuitively, given that product quality is persistent, an increase in \( u \) increases the foregone profits from investing late. In addition, an increase in \( u \) reinforces the additional strategic value of financing by an active investor, which is reflected in the fact that the difference between (23) and (24) is increasing in \( u \). Recall also that the other (non-strategic) benefit of early information is to avoid sinking \( \kappa_H \) if the venture is unpromising. This benefit is increasing in \( \kappa_H \). Furthermore, the benefit of early information is also increasing in the first-period market size \( A_1 \). In contrast to an increase in \( u \), however, this effect is the same regardless of whether the long-run outcome is symmetric or asymmetric. An increase in the second-period market size \( A_2 \) or a decrease in \( \kappa_H \), on the other hand, only increase the value of financing by an active investor if this creates a long-run strategic advantage, in which case it allows the firm to seize at lower costs a larger share of what has become a larger market.

**Corollary 2.** Suppose firm a has access to active investors, while firm b only has access to passive investors. Regardless of whether the long-run outcome is asymmetric or symmetric, firm a is more likely to choose an active investor (i.e., for higher values of \( k \)) if either \( u \), \( A_1 \), or \( \kappa_H \) increases, albeit the effect of an increase in \( u \) stronger if the long-run outcome is asymmetric. In the latter case, firm a is also more likely to choose an active investor if either \( A_2 \) increases or \( \kappa_H \) decreases.

The results in Corollary 2 come with the caveat that an asymmetric long-run outcome becomes itself less likely as either \( u \) or \( A_1 \) increases (see condition (20)). On the other hand, an asymmetric long-run outcome is more likely if competition in the product market is more intense (lower \( \tau \)). This makes it more likely that both (19) and (20) hold and thus that financing by an active investor has a strategic value. Moreover, an increase in competition also makes it more likely that (24) exceeds \( k \).

**Corollary 3.** Suppose firm a has access to active investors, while firm b only has access to passive investors. As product market competition becomes more intense, it becomes more likely that firm a has a long-run advantage over firm b.

This comparative statics result, which also holds if both firms have access to active investors (see below), is one of the main results of this paper. If product market competition becomes more intense, the firms’ products become less horizontally differentiated, and it becomes more likely that one firm has an (endogenous) first-mover advantage by strategically overinvesting early on, thus forestalling the other firm’s future investment, growth, and market share.

4.2.2.2. Symmetric access to outside financing. We next consider the case in which both firms have access to active investors. As in Proposition 4, there are again two cases. The first case is:

**Proposition 5.** Suppose both firms have access to active investors. If either (19) or (20) does not hold, then both firms choose active investors if the value of information in (23) exceeds \( k \). The outcome is then symmetric both in the short and long run.

The second case is that in which (19) and (20) hold, so that financing by an active investor has an additional strategic value. As in Proposition 4, if only one firm chooses an active investor, then the value added is given by (24). Unlike Proposition 4, however, where only one firm has access to active investors, choosing an active investor may now entail an additional (“defensive”) value if a firm anticipates that its rival would otherwise “strategically overinvest.” Formally, the value of choosing an active investor given that the other firm also chooses an

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20 A change in \( \kappa_H \) does not affect the value of information, and thus the choice between active and passive investors, albeit an increase in \( \kappa_H \) relaxes condition (20), making an asymmetric outcome more likely if firms a and b choose different investor types.
active investor is

\[
(1 - \mu_0)k_i + \mu_0 \left[ A_1 u - \kappa_h + A_2 u \frac{4}{9\tau}(3\tau - u) \right].
\]  

(25)

Comparing (24) with (25), we have that (24) is larger than (25) if

\[
\kappa_H - \kappa_h \leq u \left( A_1 + A_2 \frac{8u}{9\tau} \right).
\]  

(26)

Condition (26) is necessary for an asymmetric equilibrium to exist, both in terms of financing choices and long-run outcomes. The following result is then immediate.

**Proposition 6.** Suppose both firms have access to active investors. If (19), (20), and (26) hold, then both firms choose either passive investors (if \(k\) is higher than (24)) or active investors (if \(k\) is lower than (25)). In either case, the outcome is symmetric both in the short and long run.

**Corollary 4.** Suppose both firms have access to active investors. As product market competition becomes more intense, it becomes more likely that one firm has a long-run advantage over its rival.

Corollary 4 mirrors the result in Corollary 2 in that an asymmetric outcome becomes unambiguously more likely as product market competition becomes more intense. The same is not true for the comparative statics results in Corollary 3. Precisely, as the value of financing by an active investor increases, because either \(u\), \(A_1\), or \(\kappa_I\) increases, the outcome in which both firms choose passive investors becomes less likely, while the outcome in which both firms choose active investors becomes more likely.

It remains to analyze the case in which (19) and (20) hold but (26) does not hold, which implies that (24) is smaller than (25). Intuitively, any equilibrium must be symmetric. What is perhaps not so obvious is that there may exist multiple equilibria if \(k\) lies between (24) and (25). In this case, both firms would prefer not to be financed by an active investor, having to compensate him for his information acquisition cost. However, if one firm is expected to choose an active investor, then it becomes optimal for the other firm to do the same. These equilibria can be ruled out using the standard equilibrium selection criterion of Pareto dominance.

**Proposition 7.** Suppose both firms have access to active investors. If (19) and (20) hold but (26) does not hold, then both firms choose either passive investors (if \(k\) is higher than (24)) or active investors (if \(k\) is lower than (24)). In either case, the outcome is symmetric both in the short and long run.

### 4.3. Imperfectly informative signals

We finally consider the case in which the active investors’ signal is only imperfectly informative \((\psi_A < 1)\).\(^{22}\) We restrict attention to the case in which it is efficient to speed up investment after observing a good signal. This is the case if \(\psi_A \geq \psi^*\), where \(\psi^*\) is characterized in Eq. (35) in the Proof of Proposition 1. Accordingly, we assume that

\[
\psi_A \geq \psi^* \quad \text{where} \quad \frac{1 + \psi^*}{1 - \psi^*} = \frac{1 - \mu_0}{\mu_0} \frac{\kappa_H}{A_1 u + A_2 u \frac{4}{9\tau}(u + 3\tau) - \kappa_H}.
\]  

(27)

Analogous to Section 4, we first specify exogenously whether a firm receives financing by an active or passive investor. Suppose first that, as in Lemma 2, both firms are financed by active investors. As before, the outcome then mirrors that without competition, in the sense that both firms invest \(l_1^A = \kappa_i + \kappa_h\) if \(s_A = g\) is observed and zero otherwise.\(^{23}\) Since this holds for all \(\psi_A \geq \psi^*\), the signal’s precision (conditional on \(\psi_A \geq \psi^*\)) plays no role.\(^{24}\) Suppose next that, as in Lemma 3, only one firm, \(n\), is financed by an active investor. To support a long-run asymmetric outcome, (20) must still hold, as otherwise an early investment of \(l_1^A = \kappa_i + \kappa_h\) would not forestall future investment by its rival, firm \(n\). As condition (20) applies only to continuation profits from \(t = 2\) onwards, it is not affected by the signal’s precision. However, to make it profitable for firm \(n\) to strategically overinvest early on, it must additionally hold that\(^{25}\)

\[
\psi_A \geq \psi^* \quad \text{where} \quad \frac{1 + \psi^*}{1 - \psi^*} = \frac{1 - \mu_0}{\mu_0} \frac{\kappa_H}{A_1 u + A_2 u \frac{4}{9\tau}(u + 3\tau) - \kappa_H}.
\]  

(28)

Hence, if only firm \(n\) is financed by an active investor, then, for a long-run asymmetric outcome to obtain, both (20) and (28) must hold, where the latter condition replaces (19).\(^{26}\) Given that (28) is more likely to be satisfied the higher is \(\psi_A\), we have:

**Lemma 5.** Suppose that \(\psi_A \geq \psi^*\). If only one firm is financed by an active investor, then an increase in the signal's

\(^{22}\) Note that the signal \(s_A\) is the same for both firms, given our assumption in Section 2 that the state of nature reflects technological uncertainty which applies equally to all firms.

\(^{23}\) Recall our previous requirement that if both firms face identical financing conditions, then the equilibrium outcome should also be symmetric.

\(^{24}\) This is admittedly an artifact of our restriction to discrete investment levels. If investment levels were continuous, an increase in \(\psi_A\) would likely lead to a higher investment after observing a good signal, even without competition.

\(^{25}\) This condition is obtained by substituting for \(\mu_0\) in

\[
\kappa_H - \kappa_h \leq u \left( A_1 + A_2 \frac{8u}{9\tau} \right).
\]

Also, note that generally \(\psi^*\) and \(\psi^*\) cannot be compared, implying that our restriction to \(\psi_A \geq \psi^*\) neither precludes nor implies that \(\psi_A \geq \psi^*\).

\(^{26}\) Conditions (21) and (22) are still sufficient to rule out cases where the rival firm \(n\) overinvests early on, or where there is a long-run asymmetric outcome despite symmetric investment strategies in \(t = 1\).
Whether it is optimal for firm \( a \) to choose an active investor depends on how the cost of information acquisition \( k \) compares with the value of (early) information. Regardless of whether the long-run outcome is symmetric or asymmetric (the two possible cases in Proposition 4), the value of information to firm \( a \) is increasing in the signal’s precision \( \psi_a \). If either (20) or (28) does not hold, so that the long-run outcome is symmetric, then the value of information is the same as without competition and thus given by the sum of (6) and (7). By inspection, both terms are increasing in \( \psi_a \). On the other hand, if both (20) and (28) hold, so that the long-run outcome is asymmetric, then the value of information is given by the sum of (6) and

\[
\begin{align*}
\frac{1 + \psi_a}{2} & \left[ 2A_1u + A_2u \frac{4}{9} (u + 3t) \right] - (1 - \mu_0) \frac{1 - \psi_a}{2} \kappa_h \\
- \left[ \mu_0 \frac{1 + \psi_a}{2} + (1 - \mu_0) \frac{1 - \psi_a}{2} \right] \kappa_h,
\end{align*}
\]

which is again increasing in \( \psi_a \).

**Proposition 8.** Suppose that \( \psi_a > \psi^* \). If only one firm has access to active investors, then the value of information to that firm is strictly increasing in the signal’s precision \( \psi_a \), implying that the firm is more likely to choose an active investor the higher \( \psi_a \) is.

Together, Lemma 5 and Proposition 8 imply that if only one firm has access to active investors, then, as the signal’s precision \( \psi_a \) increases, it becomes more likely that (i) the firm indeed chooses an active investor, and (ii) this forestalls future investment by the firm’s rival, leading to a long-run asymmetric outcome.

Suppose finally that both firms have access to active investors. In this case, the effect of an increase in \( \psi_a \) on the long-run outcome is ambiguous. To see this, note first that, as is immediate from our previous discussion, the higher \( \psi_a \) is, the less likely it is that both firms choose passive investors. On the other hand, if one firm chooses an active investor, then the value to the other firm from also choosing an active investor is increasing in \( \psi_a \). Formally, this value is given by the sum of (6), which captures the value from discontinuing the venture after observing \( s_1 = b \), and

\[
\begin{align*}
\frac{1 + \psi_a}{2} & \left[ 2A_1u + A_2u \frac{4}{9} (3\tau - u) \right] - (1 - \mu_0) \frac{1 - \psi_a}{2} \kappa_h \\
- \left[ \mu_0 \frac{1 + \psi_a}{2} + (1 - \mu_0) \frac{1 - \psi_a}{2} \right] \kappa_h,
\end{align*}
\]

where both (6) and (30) are increasing in \( \psi_a \).

5. **Discussion**

5.1. **Heterogeneity across firms**

Whether a firm chooses an active investor depends, next to the value of information, on the costs of information acquisition \( k \). In reality, these costs may vary across firms if they depend on the geographic proximity between firms and their investors. Alternatively, some firms may be more opaque than others, making it more costly to obtain information. It is straightforward to extend our model to heterogeneous information acquisition costs \( k^n \) for firms \( n = a, b \). The case analyzed in Proposition 4, in which only firm \( a \) has access to active investors, can then be viewed as a special case with \( k^a = \infty \). Given Propositions 5–7, the following result is immediate.

**Proposition 9.** Suppose both firms have access to active investors, but firm \( a \) has a lower information acquisition cost than firm \( b \), i.e., \( k^a < k^b \). If (19) and (20) hold but (26) does not hold, then either firm chooses an active investor if \( k^a \) is smaller than (23). If either (19) or (20) does not hold, then either firm chooses an active investor if \( k^a \) is smaller than (24). Finally, if (19), (20), and (26) jointly hold, then:

(i) both firms choose active investors if \( k^a \) is smaller than (24);
(ii) both firms choose passive investors if \( k^a \) is larger than (25);
(iii) firm \( a \) chooses an active investor while firm \( b \) chooses a passive investor if either \( k^a \) is smaller than (24) and \( k^b \) is larger than (24), or if \( k^a \) lies between (24) and (25) while \( k^b \) is larger than (25).

Introducing heterogeneity in the costs of information acquisition enlarges the scope for asymmetric outcomes. Previously, if either (19), (20), or (26) did not hold, then the outcome was necessarily symmetric in that both firms either chose active investors or passive investors (Propositions 5 and 7, respectively). As Proposition 9 shows, if there is heterogeneity in information acquisition costs, then the outcome may well be asymmetric in these cases.

Heterogeneity across firms may also result from timing differences. Even if both firms have potential access to active investors, if firm \( a \) was founded prior to firm \( b \), it has a first-mover advantage by being the first to choose an active investor, thus forestalling firm \( b \)’s future investment and growth. Given Propositions 5–7, this is the case whenever there are asymmetric equilibria in the corresponding simultaneous-move game.

**Proposition 10.** If firm \( a \) can choose an active investor before firm \( b \), and if this choice is observable, then being the first benefits firm \( a \) if and only if there exist asymmetric equilibria in the corresponding simultaneous-move financing game.

5.2. **Learning curves, economies of scale, and network effects**

There are natural circumstances in which we would expect that the long-run benefits from strategically
overinvesting early on are even higher than described here, reinforcing our main results and underscoring the strategic importance of active investors. For brevity’s sake, we will confine ourselves to three examples.

Learning curves: Suppose firms have marginal production costs \( c^n_2 \), where the production cost \( c^n_2 \) in \( t = 2 \) is decreasing in the amount produced in \( t = 1 \). To enrich the model further, we could think of a non-degenerate (but realistic) pricing problem in which the quantity \( x^n_1 \) sold in \( t = 1 \) depends not only on the price but also (positively) on the good’s quality, which in turn depends on the investment \( I^n_1 \). By investing more early on, a firm can therefore move down the “manufacturing learning curve” faster, resulting in lower marginal costs in future periods and reinforcing the long-run benefits from strategically overinvesting early on.

When we introduce time-dependent marginal costs \( c^n_2 \) into our Hotelling model, we have that firm \( n \) realizes equilibrium profits in \( t = 2 \) of

\[
\pi^n = \frac{A_2}{\tau} \left( \tau + \left( \frac{u^n_2 - c^n_2}{3} \right) - \left( \frac{u^n_2 - c^n_2}{3} \right) \right)^2.
\] (31)

(Compare this expression to Eq. (13).) Likewise, in analogy to (14), the benefits to firm \( n \) from a marginal increase in \( u^n_2 \) are

\[
2 \frac{3A_2}{\tau} \left( \tau + \left( \frac{u^n_2 - c^n_2}{3} \right) - \left( \frac{u^n_2 - c^n_2}{3} \right) \right).
\] (32)

which is decreasing in the firm’s own marginal cost \( c^n_2 \) and increasing in the rival’s marginal cost \( c^n_2 \).

Installed base: A similar insight obtains if we allow firms to invest not only in the quality of their products but also in the production capacity and technology. In the industrial organization literature, a common way of modelling this is to assume that firms have quadratic production costs \( c^2/k \), where \( k \) denotes previously invested capital. Given this specification, firm \( n \)’s marginal costs in \( t = 2 \) are then \( 2c^2/k^n_2 \), where \( k^n_2 = I^n_1 + I^n_2 \). Like above, marginal costs in \( t = 2 \) are thus decreasing in the amount invested in \( t = 1 \), reinforcing the long-run benefits from strategically overinvesting early on.

Network externalities: If there are network effects, a consumer’s utility in a given period depends on the number of all other buyers of the same product. If the good is durable or, in the case of services, if there are switching costs (exogenous or endogenous via contractual lock-in), then a firm that makes a higher investment early on (and therefore has more customers early on) can raise the value of its goods in future periods, reinforcing the long-run benefits from strategically overinvesting early on.

As these examples suggest, the mechanism analyzed in this paper may be particularly important for newly developing, high-innovation industries such as the information technology and communication industries. For instance, steep learning curves and intense competition due to lack of horizontal differentiation (despite ongoing branding efforts) are often described as being typical of the chip industry. In a similar vein, internet trading platforms appear to exhibit considerable network externalities, while internet browsers are often associated with consumer lock-in effects and switching inertia.

6. Empirical implications

Our model is best descriptive of new risky ventures that have relatively little chance of success ex ante, as is reflected in our basic assumption that the ex ante success probability \( \mu_0 \) is sufficiently low. For such ventures, our model shows that there are benefits to being financed by active investors, such as venture capitalists. The following implication summarizes benefits that materialize even if new ventures do not compete with each other on the product market.

Implication 1. New ventures financed by active investors are more likely to receive higher funding and to make higher investments early on, but they are also more likely to be terminated earlier than new ventures financed by passive investors.

Note that the investment gap in Implication 1 pertains only to early investments. In the absence of strategic overinvestment, which occurs only in a competitive context, new ventures financed by passive investors will eventually catch up. If new ventures compete with each other on the product market, however, then a new venture financed by an active investor may be able to credibly commit to strategically overinvest early on, thus foretell its rivals’ future investment and growth.

Implication 2. If new ventures compete with each other on the product market, then those financed by active investors are likely to make even higher investments early on, as well as higher total long-run investments, compared to the case without competition, while those financed by passive investors are likely to invest even less in the long run.

In a recent empirical study, Hege, Palomino, Schwenbacher (2007) find that VCs in the U.S. play a more active role than their European counterparts. Consistent with our results, the authors find that VC investments in the U.S. are on average twice as large as in Europe, while VCs in the U.S. appear to “react with an increased funding flow upon good early performance, in contrast to Europeans” (p. 31). In another empirical study, Puri and Zarutskie (2007) compare VC- and non-VC-backed investments in the U.S. Consistent with our results, the authors find that VC-backed ventures make larger investments than their non-VC-backed counterparts, although prior to receiving funding, VC-financed firms do not look different from non-VC-financed firms.

In our model, investments are made to improve the product quality, which in turn leads to higher market shares and firm growth. The following implication is a corollary to Implication 2.

Implication 3. If new ventures compete with each other on the product market, then those financed by active investors are likely to have higher growth, higher market shares, and higher profits in the long run than those financed by passive investors.
Implication 3 has interesting cross-country implications. If new ventures in one country have better access to active, well-informed investors than new ventures in another country, and if they all compete on a global product market, then, over the long pull, those from the “better-access” country are likely to dominate their rivals in terms of investment, growth, and global market shares. In Section 1, we have already alluded to the commonly held perception that the supply of active, well-informed VCs is better in the U.S. than in Europe. (See, e.g., Schwienbacher, 2005, who finds that European VCs are less actively involved and monitor less than their U.S. counterparts.) In this vein, Implication 3 also sheds light on some recent findings by Bartelsman, Haltiwanger, and Scarpetta (2009), who find that, while entry and exit rates are similar in the U.S. and Europe, post-entry growth is much higher in the U.S. (see also Aghion, Fally, and Scarpetta, 2007).

A key feature of our model is that an increase in product market competition increases the benefits from strategically overinvesting early on.

Implication 4. New ventures financed by active investors are more likely to have a long-run advantage in terms of total investment, market shares, and profits if competition in the product market is more intense.

As discussed in Section 5.2, the incentives to make a strategically high investment early on are reinforced if investing early entails additional benefits.

Implication 5. The potential long-run advantage of new ventures financed by active investors is more pronounced in the presence of learning curves, economies of scale, and network externalities.

Our model also provides conditions for when we should observe that a given firm chooses an active investor, provided that it has access to such an investor pool.

Implication 6. A new venture is more likely to choose an active investor if the investor’s information is more precise (ψA), if the loss from wrongly continuing a bad venture is higher (KJ), and if the immediate profits from early investments are higher (A1u). If choosing an active investor creates a long-run competitive advantage, then a new venture is additionally more likely to choose an active investor if the long-run market size is bigger (A2) and if the costs of upgrading to the highest quality level are lower (Ki/k).

Our model also has implications for new ventures that face identical access to active investors. Hence, it also applies to new ventures within the same county or geographic region. Precisely, our model shows that despite facing identical access conditions, some new ventures may (endogenously) end up with active investors, while others may end up with passive investors. Importantly, the former will have an advantage over the latter in the long run. Hence, even if all new ventures have the same access to active investors, there may be dispersion in long-run outcomes.

Implication 7. Even if all new ventures have the same access to active investors, there may be long-run dispersion in investment, market shares, and profits.

As is shown in Corollary 4 and Proposition 9, long-run dispersion in outcomes is more likely if competition in the product market is more intense and if new ventures exhibit heterogenous information acquisition costs, e.g., because some new ventures are more opaque than others.

Implication 8. A long-run asymmetric outcome, even if all new ventures have the same access to active investors, is more likely if competition in the product market is more intense and if new ventures have heterogeneous costs of information acquisition.

7. Concluding remarks

We model a dynamic investment game to examine the interaction between outside financing and product market competition. We show that the lack of access to actively involved, hands-on investors such as VCs can constitute an obstacle to firm growth, especially if other firms that are being financed by such investors “prey” on their rivals by “strategically overinvesting” early on. Our model predicts that new ventures financed by active investors will dominate their industry peers in the long run. Industries in which such strategic overinvestment is more likely to be profitable are highly competitive industries as well as industries in which early investments have persistent effects, e.g., due to learning curves, economies of scope, and network effects.

An interesting avenue for future research is to explore what alternatives new ventures without access to VC financing might have to mitigate their strategic disadvantage. One alternative might be to seek financing from corporate venture capitalists, as in Hellmann (2002). Another alternative might be to change the firm’s organizational form, e.g., through vertical integration or strategic alliances, as in Fulghieri and Sevilir (2004).

We would like to conclude with a caveat. If business creation in knowledge-intensive industries involves local externalities, e.g., through knowledge spillover and the spawing of new firms, then this might provide a justification for policy intervention. In the area of risk capital, the pressure on governments to intervene has been particularly strong in Europe, given the many success stories of VC-backed companies in the U.S. Responding to this pressure, European governments have launched a number of programs to stimulate the provision of risk capital.28 However, our model implies that even a large subsidy to passive investors will not change the slower

28 Following the example of the Small Business Innovation Research program in the U.S., which awards grants to technology-intensive small firms, several European countries have implemented similar schemes, e.g., the UK High Technology Fund in 2003, the Danish Growth Fund in 2001, or the French OSEO in 2005. Measures targeted directly at VCs include the use of tax-exempt investment vehicles such as the Fonds Commun de Placement Innovation (1997) in France or the Venture Capital Trust (1995) in the UK. Moreover, lower capital gains tax rates were introduced, for instance, in Germany in 1998 and 2000.
pace at which firms financed by these investors grow, unless the subsidy is so large that the passive investors indiscriminately make higher investments early on. That is, even if there is only a small likelihood that the venture is promising, passive investors would always have to make a high investment early on. Clearly, the flip side of this is massive investments into unpromising ventures.

Appendix

Proof of Lemma 1. From rewriting (5) we have that choosing \( I_1 = k_I \) and \( I_2 = k_h \) if \( \theta = g \) is (weakly) more profitable than choosing \( I_1 = k_J + k_h \) (and thus also \( I_2 = 0 \)) if \( \mu_c \) satisfies
\[
\mu_c \geq \mu'' := \frac{k_h}{k_h + A_1u}. \tag{33}
\]
If the converse of (33) holds strictly, then \( I_1 = k_I + k_h \) is instead strictly optimal.

Next, investing \( I_1 = k_I \) instead of discontinuing the venture \((I_1 = 0)\) is in turn (weakly) more profitable if 
\[
\mu_c[I_1u + 2A_2u - k_h] - k_I > 0,
\]
which transforms to
\[
\mu_c > \mu' := \frac{k_I}{A_1u + 2A_2u - k_h}. \tag{34}
\]
(Note that the denominator is necessarily strictly positive if it was ex ante efficient to invest \( I_0 \geq 0 \) in \( t = 0 \).) That \( \mu'' > \mu' \) follows finally as \( k_h \geq k_I \) and as \( A_2u > k_h \) holds from (3). \( \square \)

Proof of Proposition 1. We first rewrite condition (33) from the proof of Lemma 1 for \( s = g \). Substituting from the definition of \( \mu_c \), investing \( I_1 = k_I + k_h \) is then more profitable than investing first only \( I_1 = k_I \) if
\[
\frac{1 + \psi}{1 - \psi} \geq \frac{1 - \mu_c k_h}{\mu_c A_1u}. \tag{35}
\]
Imposing equality in (35) yields a threshold \( 0 < \psi'' < 1. \)

For \( s = b \) we have from (34) and after substituting from the definition of \( \mu_c \), that \( I_1 = 0 \) is (weakly) more profitable than \( I_1 = k_I \) if
\[
\frac{1 + \psi}{1 - \psi} \geq \frac{\mu_c}{1 - \mu_c} \left( \frac{A_1u + 2A_2u - k_I - k_h}{k_I} \right). \tag{36}
\]
Imposing equality in (36) yields a threshold \( 0 < \psi' < 1. \)

We finally compare the two derived thresholds \( \psi' \) and \( \psi'' \). For \( \psi' > \psi'' \) to be satisfied it must hold that
\[
\left( \frac{\mu_c}{1 - \mu_c} \right)^2 \leq \frac{k_h}{k_I} \left( \frac{k_I}{A_1u + 2A_2u - k_I - k_h} \right), \tag{37}
\]
which imposes an upper boundary on \( \mu_c \). \( \square \)

Proof of Corollary 1. Using from (37) that \( \psi' > \psi'' \) holds for low \( \mu_c \), take first the case where \( \psi < \psi' \). From Proposition 1, the additional information allows to (optimally) discontinue the venture after observing \( s = b \). If \( s = b \) is generated by \( \theta = b \), which happens with probability \((1 + \psi')/2\), then the additional value adds a value equal to the otherwise incurred investment cost \( k_c \). Otherwise, i.e., if \( s = b \) is generated by \( \theta = g \), which happens with probability \((1 - \psi)/2\), then the erroneous shut-down of the project leads to a (relative) destruction of value \( A_1u + 2A_2u - k_I - k_h \). In expectation, the value of information is thus
\[
(1 - \mu_c) \frac{1 + \psi}{2} k_I - \mu_c \frac{1 - \psi}{2} (A_1u + 2A_2u - k_I - k_h), \tag{38}
\]
which transforms into (6).

For \( \psi > \psi'' \) the more precise information leads, in addition, to a reversal of the decision after observing \( s = g \). If \( s = g \) is generated by \( \theta = g \), the added value from investing \( I_1 = k_I + k_h \) instead of only \( I_1 = k_I \) equals \( A_1u \). If \( s = g \) is generated, instead, by \( \theta = b \), then the additional investment cost \( k_h \) is incurred erroneously. In expectation, the additional value of information in the case of \( \psi > \psi'' \) is then
\[
\mu_c \frac{1 + \psi}{2} A_1u - (1 - \mu_c) \frac{1 + \psi}{2} k_h, \tag{39}
\]
which transforms into (7). \( \square \)

Proof of Proposition 2. Without a strictly positive payment to the entrepreneur in case no cash flow is generated, it is clearly not possible to truthfully extract information such that \( I_1 = 0 \) is only chosen for \( s_E = b \). We show next that it is also not possible to ensure that \( I_1 = k_I + k_h \) is chosen if and only if \( s_E = g \).

We argue to a contradiction. Consider thus a message game where \( s_E = g \) induces \( I_1 = k_I + k_h \), while \( s_E = b \) leads to \( I_1 = k_I \). The message also pins down the sharing rules for the subsequent payoffs. For the purpose of this proof, we simplify the notation by denoting the total expected payoff of the entrepreneur if \( \theta = g \) by \( R(s_E) \).

Under truth-telling, ”type” \( s_E = b \) thus realizes the payoff \( \mu_cR(b) + \gamma(k) + \mu_cK_h \). To ensure incentive compatibility, this payoff must not be smaller than the payoff obtained when sending instead the message \( s_E = g \), which equals \( \mu_cR(g) + \gamma(k) + k_h \). We can transform this condition into the requirement that
\[
k_h \leq \frac{1}{\gamma} \frac{\mu_c}{1 - \mu_c} [R(b) - R(g)]. \tag{40}
\]

Proceeding likewise for \( s_E = g \), we have in this case the incentive compatibility constraint
\[
k_h \geq \frac{1}{\gamma} \frac{\mu_c}{1 - \mu_c} [R(b) - R(g)]. \tag{41}
\]

Clearly, whenever the signal is informative as \( \psi_E > 0 \), implying that \( \mu_c > \mu_c \), the two conditions (40) and (41) cannot be jointly satisfied.

Note next that from (5) investing \( I_1 = k_I + k_h \) is not efficient given the prior \( \pi_0 \), while from (3) it is optimal to invest \( I_2 = k_h \) if \( q_1 = 1 \) and \( \theta = g \). As by optimality for the entrepreneur the investor’s break-even constraint (8) will be satisfied just with equality, making the entrepreneur the full residual claimant, it is thus clearly also optimal to choose the efficient investment path (though only based on the prior beliefs \( \mu_c \)). \( \square \)
Proof of Proposition 3. Note first that it is not efficient that the active investor acquires information if either $\psi_A < \psi'$, or $\psi < \psi_A < \psi''$ and not
\[
(1 - \mu_0) \frac{1 + \psi_A}{2} \kappa_l - \mu_0 \frac{1 - \psi_A}{2} (A_1 u + 2A_2 u - \kappa_l - \kappa_h) \geq k,
\]
(42)
or if $\psi_A > \psi''$ and not
\[
\mu_0 [\psi_A - A_1 u - (1 - \psi_A)] A_2 u] + \kappa_2 \left[ (1 + \psi_A - 2\mu_0 \psi_A) - \kappa_2 \frac{(1 - \psi_A)}{1 - 2\mu_0} \right] \geq k,
\]
(43)
where we made use of Corollary 1, while summing up (38) and (39) to obtain (43).

If the active investor does not acquire information, then the analysis is identical to that in Proposition 2. In particular, the contract could then prescribe $I_1 = \kappa_l$ as well as any $\sigma_l$, so as to satisfy (8). (Note also that $I_2 = \kappa_h$ can simply be contractually stipulated as the realization of $\theta = g$ is verifiable in $t = 1$.)

We next assume that $\psi_A \leq \psi''$ and that (42) holds strictly. If the investor acquires information and if the efficient investment decision $I_1$ as characterized in Proposition 1 is followed, then the investor's break-even constraint is given by (9) in the main text. Note next that in this case the investor indeed prefers the efficient choice of $I_1$. This follows from the following two observations.

First, for $s_A = b$ it is efficient not to continue and as the investor would, otherwise, have to bear all investment costs, $I_1 = 0$ is clearly also privately optimal. Second, at $s_A = g$ it is likewise not privately optimal to invest $\kappa_l + \kappa_h$ given that this is not efficient and as the additional costs $\kappa_h$ would be born by the investor. Finally, if it was then privately optimal to choose $I_1 = 0$, then the break-even constraint (9) could clearly not be satisfied.

For $\psi_A \leq \psi''$ it thus remains to consider the investor's incentives to acquire information in the first place. Shifting can clearly only be optimal if subsequently $I_1 = \kappa_l$ is chosen, in which case the investor realizes
\[
\mu_0 [\sigma_1 A_1 u + \sigma_2 A_2 u - \kappa_l - \kappa_h] - (1 - \mu_0) \kappa_l.
\]
(44)
Comparing this to (9), we thus have after rearranging terms the incentive constraint
\[
(1 - \mu_0) \frac{1 + \psi_A}{2} \kappa_l - \mu_0 \frac{1 - \psi_A}{2} (A_1 u + 2A_2 u - \kappa_l - \kappa_h) \geq k,
\]
(45)
which is implied by condition (42) as $\sigma_l \leq 1$. Summing up, we have thus found that if $\psi_A \leq \psi''$ and if (42) holds, then any contract satisfying (9) also induces information acquisition and the efficient investment choice. From optimality for the firm, (9) is then satisfied with equality, which if (42) holds strictly also implies that the firm strictly prefers to induce information acquisition.

The final case is where $\psi_A > \psi''$ and where (43) holds strictly. Note here that we can from the arguments in the main text restrict consideration to the analysis of the case where the investor's discretion over the investment in $t = 1$ is not restricted. Provided that information is used to implement the efficient investment path, the break-even constraint for the investor is then given by (10). As in the case of $\psi_A \leq \psi''$, we can next conclude that, first, the investor prefers $I_1 = 0$ to any other investment level when observing $s_A = b$ and that, second, he does not prefer $I_1 = 0$ when observing $s_A = g$.

For $s_A = g$, the investor prefers $I_1 = 2\kappa$ over $I_1 = \kappa$ if
\[
\mu_g 2u(\sigma_1 A_1 + \sigma_2 A_2) - \kappa_l - \kappa_h
\]
\[
\geq \mu_g [u(\sigma_1 A_1 + 2\sigma_2 A_2) - \kappa_l] - \kappa_h
\]
(46)
which after substituting for $\mu_g$ transforms to (11). Note that from (5) it follows that condition (11) holds surely if $\sigma_1$ is sufficiently close to one.

To consider the incentives to acquire information, note first that the investor prefers $I_1 = \kappa_l$ if he receives no information. Consequently, he exerts effort only if (10) does not fall short of (44) minus $I_0$, which yields condition (12). It is also useful to note that constraint (12) is implied by condition (43) if
\[
(1 - \psi_A)(1 - \sigma_2) A_2 u > \psi_A A_1 u (1 - \sigma_1).
\]
(47)
We conclude the analysis by showing that it is indeed possible to find sharing rules such that all three (remaining) constraints are satisfied simultaneously, i.e., (10)–(12). As information acquisition is efficient and as any increase in $\sigma_1$ or $\sigma_2$ relaxes (10), this would only be the case if (10) does not hold if we substitute $\sigma_1 = 1$ and the highest value for $\sigma_2 > 0$ for which (12) would still be satisfied. But this case cannot arise as we know from (47) that (43) implies (12) in case $\sigma_1 = 1$. □

Proof of Lemma 2. Note first that from $\psi_A = 1$ and $A_1 > 0$ we can restrict consideration to investments in $t = 1$, while also $I^*_0 = 0$ holds if $s_A = b$. If firms end up with symmetric qualities, then in the case of $\theta = g$ they realize in $t = 2$ profits of $\tau A_2$. To support an equilibrium with $I^*_0 = \kappa_l + \kappa_h$ for both firms, note that a deviation to a lower investment of $I^*_0 = \kappa_l$ is not profitable if the sum of the thereby saved investment cost $\kappa_h$ and of the new, lower revenues $A_1 u + (A_2/\tau)(\tau - u/3)^2$ does not exceed $2A_1 u + \tau A_2$. This obtains the condition
\[
\kappa_h \leq A_1 u + A_2 u \frac{1}{9\tau} [6\tau - u],
\]
(48)
which given $u < \frac{2}{3}\tau$ from (16) must hold from (17) even if $A_1 = 0$. Next, a deviation to a higher investment level by spending, in addition, $\kappa_H$ is also not profitable if the new revenues of $3A_1 u + (A_2/\tau)(\tau + u/3)^2$ minus the additional investment cost $\kappa_H$ do not exceed $2A_1 u + \tau A_2$. This transforms to the requirement that
\[
\kappa_H \geq A_1 u + A_2 u \frac{1}{9\tau} [6\tau + u].
\]
(49)
\footnote{Note that as this is out of equilibrium, it need not be the case that the investor's expected payoff is then strictly positive. Also, note that from (5), which implies that $(\mu_g/(1 - \mu_0)) \sigma_1 - \kappa_h/A_1 u$, it is immediate that the investor prefers $I_1 = \kappa_l$ to $I_1 = \kappa_h + \kappa_l$.}
To see that (49) is implied by (15) we can again use that $u < \frac{1}{2} \tau$ holds from (16).

We next rule out an asymmetric equilibrium where only one firm, $n'$, invests $k_1 + k_{n'}$. If the other firm, $n$, chooses $l_1^n = k_1$ and thus realizes profits of $(A_2/2)(\tau - u/3)^2 - k_1$, a deviation to $l_1^n = k_1 + k_{n'}$ is strictly profitable if (48) holds, which we already showed to be the case. If instead $n$ is supposed to choose $l_1^n = k_1 + k_{n'}$, then a reduction by $k_{n'}$ must be strictly profitable from (49).

We finally derive the conditions for when we can support an asymmetric equilibrium with $q_1^n = l$ and $q_{n'}^n = h$. If $n'$ wants to deviate, then from the previous observations the best alternative choice is to choose $l_1^n = k_1 + k_{n'}$. To render this unprofitable, the saved costs $k_{n'}$ must not exceed the revenues gained, i.e., the difference of $3A_1u + (A_2/2)(\tau - u/3)^2$ and $A_1u + A_2\tau$, which yields condition (19). (Note that after the deviation both firms end up with $q_2^2 = q_2^3 = h$.)

Turning to firm $n$, by the previous observations the next best alternative to choosing $l_1^n = k_1$ is to choose instead $l_1^n = k_1 + k_{n'}$. To render this unprofitable, the additionally incurred costs $k_{n'}$ must not be smaller than the revenues gained, i.e., the difference of $2A_1u + (A_2/2)(\tau - u/3)^2$ and $A_1u + (A_2/2)(\tau - 2u/3)^2$, which yields condition (18). □

**Proof of Lemma 3.** We turn first to the strategies in $t = 2$, provided $\theta = g$. We know from Lemma 2 that in an equilibrium with $q_2^3 = h$ it must likewise hold that $q_2^1 = h$. Suppose next that $q_2^1 = H$ and $q_1^2 = L$. For the optimal choice of $l_2^2$ note first that we can again rule out optimality of $l_2^2 = k_1 + k_{n'}$, while $l_2^2 = k_2$ is only (weakly) optimal if $k_{n'} \leq A_2u(1/3\tau(2\tau - u))$. As the converse of this must hold weakly to support an asymmetric outcome in the long run, we obtain from this condition (20).

Suppose now first that (20) holds. In this case, if firm $n$ with an active investor chooses $q_1^2 = H$, then it is indeed optimal for firm $n'$ to choose $q_1^2 = q_2^2 = l$. (Note that we use from (5) that the firm would optimally choose a higher investment not before $t = 2$, which by (20) is, however, not profitable.) To support the asymmetric equilibrium, it thus only remains to show that the strategy of firm $n$ is optimal. As in the proof of Lemma 2, the optimal deviating strategy would be to $q_1^n = h$, which is not optimal if (19) holds.

Suppose next that either (19) or (20) do not hold, in which case we cannot support the previously constructed asymmetric outcome. In this case, firm $n$ with an active investor would thus not find it profitable to deviate from $l_1^n = k_1 + k_{n'}$ and $l_2^n = 0$, provided that firm $n'$ does not end up with higher quality than $q_2^3 = h$. Given the strategy of firm $n$, from our previous results it thus only remains to determine whether firm $n'$ invests gradually with $l_1^n = k_1$ and $l_2^n = k_{n'}$, which holds from (5).

Finally, conditions (21) and (22) rule out any other pure-strategy equilibria. A proof of this result is contained in an earlier working paper version and is available from the authors upon request. □

**Proof of Lemma 4.** We turn first to the equilibrium candidate where both firms invest gradually. In this case, the expected profit for either firm equals $\mu_0[A_1u + \tau - k_1] - k_1$.

To check when we can support this equilibrium, note that we need only consider deviations in $t = 1$. Moreover, if some firm $n$ deviates to $l_1^n = k_1 + k_{n'}$, recall that we can still support an equilibrium of the continuation game where $l_2^n = 0$ and $l_2^n = k_{n'}$, implying from (5) that profits of the deviating firm $n = 1$ would be lower. Consequently, it remains to check for a deviation to $l_1^n = k_1 + k_{n'}$, which in turn can only be profitable if $l_2^n = 0$ and thus if (20) holds. In this case, firm $n$ will still not deviate if

$$k_{n'} \geq \mu_0[A_1 + A_2(2\tau/3 + u) - (1 - \mu_0)k_{n'}].$$

This condition is implied by (21) and (22), implying that a deviation is unprofitable for firm $n$.

Finally, we can rule out any other pure-strategy equilibria. A proof of this result is contained in an earlier working paper version and is available from the authors upon request. □

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