Safeguarding Specific Human Capital Investments

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Abstract

When a firm changes its unique mix of products, markets, and technologies, some of employees’ previously acquired human capital depreciates. As a result, the firm can credibly renegotiate down employees’ quasi rents, which it had promised them as a reward for their human capital investments. We show that linking employees’ pay to total firm value, a widely observed phenomenon in practice, can mitigate the firm’s ex post opportunism. Consistent with available empirical evidence, our model implies that firms which link the pay of their rank-and-file employees to total firm value should be, in particular, firms with a high degree of human capital intensity, young firms and firms in dynamic, high-growth industries, such as new economy firms, and firms in risky, volatile industries.

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1 Introduction

Even if human capital is a bundle of general skills it may be specific to a firm. Other firms may demand a different mix of skills or put different weights on each skill. As a result, an employee who switches firms may experience that the new firm puts a lower value on some of the skills he acquired at the previous firm, even though all the skills are general in nature. However, acquired human capital may also depreciate if an employee does not switch firms. The mix of skills that is “optimal” inside a firm is not arbitrary; it depends on the firm’s unique mix of products, markets, and technologies (henceforth “business mix”). If a firm changes its business mix, the optimal skill mix inside the firm also changes. Some previously acquired skills may become obsolete, while others may simply become less important.¹

That acquired skills may become obsolete (or less important) as the firm changes its business mix has repercussions on employees’ incentives to invest in human capital. When human capital investments are observable but non-verifiable, firms can provide employees with incentives by promising them an above-market wage (a “quasi rent”).² An employee who does not invest in human capital will not receive the above-market wage as the firm can credibly renegotiate down his wage under the threat of firing. As Kahn and Huberman (1988) and Prendergast (1993) point out, the firm will also be tempted to renegotiate down the wage of an employee who has invested in human capital, creating a dual moral hazard problem.³ However, as long as the

¹See Section 2 for an example. The notion of human capital as a mix of general skills is due to Lazear (2003). As Lazear points out, besides cliched examples such as “learning who does what at the firm and to whom to go to get something done,” human capital is seldom truly firm-specific. What is firm-specific, however, is the mix of (general) skills demanded by firms, where the weights on each skill vary across firms depending on their business mix. Lazear argues that this “skill-weights view” is more successful at explaining the large tenure coefficients found in the data than are traditional notions of specific human capital. Gibbons and Waldman (2004, 2006) propose a related notion of “task-specific human capital” whereby human capital may also depreciate if an employee switches tasks within a firm. Our notion of human capital depreciation is also consistent with task-specific human capital if a change in the firm’s business mix requires employees to perform different tasks.

²The assumption that human capital investments are non-verifiable is common in the literature (see, e.g., Prendergast (1993), Malcomson (1997)). See also Gibbons (1998, p. 125), who argues that “contracts based on the worker’s potential contribution cannot be enforced”. Providing employees with incentives through explicit incentive schemes is too costly in our model if the free-rider problem among employees is sufficiently severe. See Section 5.1 for a formal analysis.

³The problem of ex post renegotiation, which undermines the firm’s credibility to reward employees for specific human capital investments, is absent from early work on firm-specific human capital (e.g., Becker (1962, 1964).
promised above-market wage lies below the employee’s marginal contribution to the firm, the employee can resist such pressure as the firm’s threat of firing is not credible.\footnote{While modifying an existing wage contract requires mutual consent, the firm can always fire the employee, providing it with a strong bargaining position in the renegotiations: “[E]mployers and employees can avoid a possible charge of breach if they stick to the practice of modifying terms only by mutual consent. ... Mutual agreement on modifications of terms does not preclude wage changes—employees may agree to a wage cut if the alternative is being laid off” (Malcomson, 1997, p. 1921). Thus, if the employee rejects the firm’s offer to renegotiate down his wage the firm has two options. It can either continue employment or fire the employee. If employment is continued, the existing wage contract remains in place: “[R]efusal of an offer by either party followed by continued employment leaves the contract unchanged” (Malcomson, 1997, p. 1933). Hence, the threat of firing is only credible if the firm is better off firing the employee than employing him under the existing wage contract. Otherwise the employee will optimally reject the firm’s offer, knowing that the fallback option is continued employment under the existing contract. If the employee does not invest in human capital his value to the firm is below the promised above-market wage, implying that the threat of firing is credible.}

Incentive provision becomes more difficult when there is a possibility that the firm will change its business mix, say, from $A$ to $B$, while the employee’s human capital investment is only valuable under business mix $A$. (For example, the employee may acquire skills that are no longer needed under business mix $B$.) If the firm switches to business mix $B$ the value of the employee’s human capital to the firm depreciates, which implies that the firm can renegotiate down his above-market wage because the threat of firing is now credible. As the employee anticipates this, the firm must promise him an even higher quasi rent from the beginning to induce him to invest in human capital. Since de facto the employee receives this quasi rent only if the firm continues with business mix $A$, the quasi rent must be higher the more likely it becomes that the firm will switch to business mix $B$.

While the discussion implies that firms which are more likely to change their business mix must promise their employees higher quasi rents, there is no real inefficiency. The inefficiency comes into play if the firm switches to business mix $B$ for the sole purpose of appropriating employees’ quasi rents, implying that the dual moral hazard problem analyzed by Kahn and Huberman (1988) and Prendergast (1993) has real implications for firms’ business decisions. Arguably, if business mix $A$ is much better than mix $B$ it would seem implausible that the firm will switch to the less efficient mix only to appropriate quasi rents. But if business mix $A$ is only marginally better than mix $B$, the prospect of appropriating employees’ quasi rents may
tilt the firm’s decision in favor of the less efficient mix $B$.

The discussion so far has ignored wage contracts. Indeed, the quasi rents that must be promised to employees only pin down expected future wages payments, not the structure of wage contracts. However, the structure of wage contracts matters for the firm’s decision to (inefficiently) switch to business mix $B$. For example, suppose that employees receive a fixed-wage contract. The promised quasi rent is then simply the difference between the fixed wage and the employees’ market wage. Most important, the quasi rent is a fixed amount. As a result, the firm will inefficiently switch to business mix $B$, thus depreciating employees’ specific human capital, whenever the efficiency loss from switching is less than the (fixed) quasi rent which the firm can appropriate.

Suppose now instead that employees receive pay that is contingent on total firm value. For example, the firm may have a broad-based stock or stock option plan or employees receive a bonus that depends on the firm’s performance. While employees must receive a given quasi rent in expectation to invest in specific human capital, the quasi rent that they will actually receive is now variable. It will be large when the firm’s profitability under business mix $A$ is high and small when the firm’s profitability under business mix $A$ is low. Most important, the quasi rent will be small when business mix $A$ is only marginally better than mix $B$, which is precisely when the firm has the biggest incentives to appropriate employees’ quasi rents. Thus, minimizing employees’ quasi rents when business mix $A$ is only marginally better than mix $B$ minimizes the firm’s incentive to inefficiently switch to mix $B$, protecting employees’ specific human capital investments. Whether the first best is attainable—that is, the firm switches to business mix $B$ if and only if mix $B$ is more profitable than mix $A$—depends on how low the firm can set the base wage. Either way, under the optimal wage contract employees’ pay must be contingent on total firm value.

Our model implies that firms which link the pay of their rank-and-file employees to total firm value should be, in particular, firms with a high degree of human capital intensity and firms that frequently undergo changes in their business mix, such as young firms that are still experimenting with their business mix, firms in risky, volatile industries, and firms in dynamic, high-growth industries, such as new economy firms. As we will show in Section 4, the available

\footnote{In Section 3 we show that the firm’s decision to switch to business mix $B$ will not be renegotiated in equilibrium because the firm makes the decision under private information.}
empirical evidence is broadly consistent with these implications.

Some scholars, especially Blair (1995), have argued that employees should have a stronger say in the firm’s decisions to ensure that they earn adequate returns on their specific human capital investments. For example, employee stock ownership plans allow employees (or their trustees) to cast votes as shareholders in favor of employee-friendly proposals. Our model shows that ceding control rights to employees may not be necessary—the cash-flow rights that come with broad-based equity plans may already be sufficient to safeguard their specific human capital investments. Our model also offers a novel take on the (informal) argument that broad-based equity pay causes a cultural change in organizations (e.g., Kruse and Blasi (1997)). In our model, an employee’s decision to invest in human capital does not depend on whether he receives broad-based equity pay but on whether all of the firm’s employees do. This is because the firm’s decision to switch to an alternative business mix to appropriate employees’ quasi rents depends on the firm’s total wage bill, not that of an individual employee.

While many firms link the pay of their rank-and-file employees to total firm value—through broad-based stock and stock option plans or by making bonus payments contingent on the firm’s performance—it is less clear what the benefits of such arrangements are, given that free-rider problems among employees quickly thwart any direct incentive effects. Our model is immune against the “free-riding critique” by locating the incentive problem that broad-based equity pay addresses not with individual employees, but with the firm. Broad-based equity pay minimizes the firm’s incentives to appropriate employees’ quasi rents, thus protecting their specific human capital investments. Another important benefit has been documented by Oyer (2004), who shows that broad-based equity pay allows firms to successfully retain employees while saving on renegotiation costs. On the other hand, Bergman and Jenter (2006) argue that firms use broad-based equity pay to extract rents from overly optimistic employees, while Hall and Murphy (2003) point to the (potentially wrongfully) perceived tax and accounting benefits of, in particular, employee stock options.

The dual moral hazard problem that exists if employees invest in specific human capital has been previously analyzed by Kahn and Huberman (1988) and Prendergast (1993). Kahn and Huberman propose “up-or-out” contracts of the sort found in law firms to govern promotion to partnerships as a solution to the problem. Prendergast’s solution is to promote workers to different jobs with different wages attached, where jobs are designed such that the assignment of
workers who have invested in human capital is ex post efficient. Tournaments where firms commit
to specific prizes ex ante (e.g., Carmichael (1983)) may also address the problem, although these
are rarely observed in practice (see Prendergast (1999)).

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 shows
that linking employees' pay to total firm value protects their specific human capital. Section
4 provides comparative static analyses and discusses the empirical literature. Section 5 models
the free-rider problem among employees that exists if the firm uses explicit incentive contracts
and examines the role of severance pay. Section 6 concludes.

2 The Model

Agents and Timing of Events

A firm that is run by a manager maximizing firm value employs a representative employee.
As will become clear shortly, the role of linking employee pay to firm value is to mitigate a
time inconsistency problem on the part of the firm, which in turn affects the value of employees'
specific human capital. Thus, our argument is immune to free-riding problems among employees,
which is why our base model has a single representative employee.6

There are three dates. In \( t = 0 \) the employee is hired. He can subsequently decide whether
to invest in human capital. In \( t = 1 \) the firm makes two decisions. The first decision is whether
to continue with the existing mix of products, technologies, and markets, which we referred to
as “business mix” in the Introduction. The second decision is whether to retain the employee,
and if so, under what terms. The firm’s value is realized in \( t = 2 \).

Human Capital Investment

Investing in human capital comes at private cost \( c \) to the employee. As is standard in the
literature, we assume that the human capital investment is observable but non-verifiable. We
can think of the human capital investment as an investment in general skills that affects the
employee’s skill mix. It may entail acquiring new skills or simply acquiring more of some existing
skills, thus changing the relative weights in the employee’s skill mix (see Introduction). If the
employee does not invest in human capital, his value both to the firm and to the market is
\( m \). If the employee invests in human capital, his market value is \( M \geq m \). (His value to the

6In Section 5.1 we explicitly model the free-rider problem among employees.
firm depends on the firm’s choice of business mix, see below.) The difference $M - m$ captures the extent to which the human capital investment has value outside the firm. We assume that $M - m < c$. Otherwise there would be no incentive problem as the difference in market values alone would be sufficient to induce the employee to invest in human capital.

**Firm’s Choice of Business Mix**

We assume that it is optimal to induce the employee to invest in human capital. To what extent the acquired human capital will be ultimately of value to the firm, however, depends on its choice of business mix. We denote the firm’s existing business mix by $A$. In $t = 1$ the firm can either continue with business mix $A$ or switch to some alternative mix $B$. All that is important for our analysis is that the switch from mix $A$ to $B$ affects the firm’s profitability, in a sense made precise below, and that it depreciates the employee’s specific human capital. Precisely, if the firm switches to business mix $B$ the value of the employee’s human capital to the firm depreciates down to its market value of $M$.\(^7\)

**Example**

Consider Lazear’s (2003) example of a Silicon Valley start-up that produces enterprise software for tax optimization (business mix $A$). The typical employee in this firm must know something about tax laws, something about economics, and something about Java programming. None of these skills, if taken alone, is firm-specific. Suppose that a new employee knows about Java programming, which commands a (market) wage of $m$, but knows little about economics or taxes. The employee must now decide whether to also learn about economics and taxes, which is optimal given the firm’s business mix. Suppose further that no other firm in the region values all three skills (programming, economics, taxes) but there are firms that value two of the three skills.\(^8\) Precisely, suppose that the current market wage for an individual with any two of the three skills is $M \geq m$. If the employee also learns about economics and taxes, his

\(^7\)The assumption that the employee’s human capital under business mix $B$ depreciates down to its market value is only for expositional convenience. The model can be extended to the case where the employee’s human capital under business mix $B$ depreciates relative to its value under mix $A$ but remains above its market value of $M$.

\(^8\)Alternatively, we could assume that other firms also value all three skills but put different weights on each skill, with the effect that the employee’s current employer still assigns the highest value to his skill mix while other firms assign a lower value, namely, $M$. 

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market wage consequently increases to $M$, which is more than $m$ but less than his value to his current employer, who values all three skills. Finally, if the firm switches to some other business mix $B$, say, to produce software for valuing employee stock options, then it no longer values the employee’s knowledge of taxes, but it still values his programming and economics skills. Just like any other firm that values two of the three skills, the value of the employee to his current employer—in case it switches to business mix $B$—is then only $M$.

**Payoffs and Information Set**

We now describe the firm’s payoffs if it either continues with business mix $A$ or switches to mix $B$. Which of the two alternatives is more profitable depends on the “state of nature” $\theta \in \{a, b\}$, which is realized in $t = 1$. The probabilities of states $a$ and $b$ are $\pi$ and $1 - \pi$, respectively. If the state of nature is $a$, then continuing with business mix $A$ generates a high payoff $x_h$ with probability $p_H$ and a low payoff $x_l < x_h$ with probability $1 - p_H$ while switching to mix $B$ reduces the probability of a high payoff to $p_L < p_H$. Likewise, if the state of nature is $b$, then switching to business mix $B$ generates a high payoff $x_h$ with probability $p_H$ and a low payoff $x_l$ with probability $1 - p_H$ while continuing with mix $A$ reduces the probability of a high payoff to $p_L$.\(^9\) Ex ante, business mix $A$ is thus more profitable with probability $\pi$ while mix $B$ is more profitable with probability $1 - \pi$.

In $t = 1$ the manager, who chooses the firm’s business mix, imperfectly observes the state of nature while the employee has no information. Precisely, the manager observes a signal $s \in S = [s_a, s_b]$ which is informative in the following sense. If the state of nature is $\theta$, then the signal $s$ is drawn from the distribution function $F_\theta(s)$, which is atomless with continuous density $f_\theta(s) > 0$ for all $s \in S$. The unconditional density and distribution functions are $f(s) := \pi f_a(s) + (1 - \pi) f_b(s)$ and $F(s)$, respectively. Signals are ordered such that a high signal makes it more likely that the state of nature is $a$ in the sense of the Monotone Likelihood Ratio Property (MLRP). Formally, $f_a(s)/f_b(s)$ is increasing in $s$.

The fact that the manager only imperfectly observes the state of nature makes his decision problem continuous—even though his choice set is discrete (mix $A$ or $B$)—which ensures that the

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\(^9\)These probabilities hold if the employee invests in human capital, which will be the case in equilibrium. For our main analysis it is not necessary to specify probabilities for the event that the employee does not invest in human capital. (See Section 5.1, however, where we model the free-rider problem among employees.) All we need to assume is that the difference in success probabilities is sufficiently large to make it optimal to induce the employee to invest in human capital.
optimal wage contract is unique. By Bayes’ rule, after observing signal \( s \) the manager believes that the state of nature is \( a \) with probability
\[
\pi(s) := \frac{\pi f_a(s)}{\pi f_a(s) + (1 - \pi) f_b(s)},
\]
which is increasing in \( s \) by MLRP. Conditional on observing \( s \), the probability that business mix \( A \) generates a high payoff \( x_h \) is thus \( p_A(s) := \pi(s)p_H + [1 - \pi(s)]p_L \) while the conditional probability that mix \( B \) generates a high payoff is \( p_B(s) := \pi(s)p_L + [1 - \pi(s)]p_H \). Accordingly, the conditional expected payoff under mix \( A \) and \( B \) is
\[
\mu_A(s) := p_A(s)x_h + [1 - p_A(s)]x_l
\]
and
\[
\mu_B(s) := p_B(s)x_h + [1 - p_B(s)]x_l,
\]
respectively. Note that \( \mu_A(s) \) and \( \mu_B(s) \) are gross of wages.

Employment Contracts

We consider two types of employment contracts: at-will contracts and long-term (commitment) contracts. The main difference between the two is that under a long-term contract the firm can commit itself not to renegotiate down the employee’s wage. In contrast, under an at-will contract the firm may renegotiate down the employee’s wage under the threat of firing.\(^\text{10}\) An employment contract specifies a base wage \( w \geq \underline{w} \) and a bonus \( b \) if the high payoff \( x_h \) is realized. In what follows, we refer to \( w \geq \underline{w} \) as “base-wage constraint”. If \( \underline{w} = 0 \) the base-wage constraint becomes a standard limited-liability constraint. If the employee has private wealth, then setting \( \underline{w} < 0 \) allows for positive transfers from the employee to the firm. Finally, if \( \underline{w} > 0 \) the base wage is bounded away from zero, for example, to ensure a minimum consumption level. To rule out trivial solutions where the base wage is mechanically determined we assume that \( w \leq x_l \)

\(^{10}\)The threat of firing is weaker if the employment contract stipulates severance pay. See Section 5.2 for a formal analysis. We always allow the employee to leave the firm (“non-slavery” condition), though this imposes no binding constraint on the optimal contract.
does not bind. A sufficient condition for this is that $x_I \geq M$. In line with previous literature on specific human capital we assume risk neutrality, allowing us to obtain a closed-form solution for the optimal wage contract.

As we will see shortly, the optimal bonus under an at-will contract is not driven by the problem of inducing the employee to invest in human capital but rather by the problem of inducing the firm to make an efficient choice in $t = 1$.\footnote{Incentives to invest in human capital under an at-will contract do not derive from the bonus but from the difference between the employee’s expected wage if he invests in human capital and his market wage, similar to efficiency-wage models. Thus, any contract that provides the employee a given expected wage, including a fixed-wage contract, can induce him to invest in human capital.} By contrast, under a long-term contract the model becomes a standard principal-agent model where the bonus plays the usual role of providing the employee with incentives. Using a standard free-riding argument, we will show in Section 5.1 that if the number of employees becomes sufficiently large the at-will contract is uniquely optimal. For this reason, we will focus on at-will contracts in the main part of our analysis.

3 Tying Employee Pay to Total Firm Value

3.1 Preliminary Analysis

From the employee’s perspective, what matters for his decision to invest in human capital is his future wage taking into account renegotiations. While modifying an existing wage contract requires mutual consent, the firm has always the option to fire the employee. Thus, if the employee rejects the firm’s offer to renegotiate down his wage the firm has two options. It can either continue employment or fire the employee. If employment is continued, however, the existing wage contract remains in place (see footnote 4). Thus, the threat of firing is only credible if the firm is better off firing the employee than employing him under the existing wage contract. If the threat of firing is not credible, the employee will optimally reject the firm’s offer, knowing that the fallback option is continued employment under the existing wage contract.\footnote{This would be the unique equilibrium outcome of a standard bargaining game. If bargaining frictions take the form of a risk of breakdown, the existing contract constitutes the relevant “outside option”. If bargaining frictions take the form of delay, the existing contract constitutes the relevant “inside option”.} Accordingly, if the employee invests in human capital and the firm continues with
business mix A, then no renegotiations will occur as long as the employee’s expected wage lies above his market value $M$ but below his value to the firm. This is because neither side can then credibly threaten to terminate the relationship if the other side refuses to modify the existing wage contract. As we will show below, the employee’s expected wage under business mix A always lies above his market value $M$ (see equation (6)).

The outcome is different if the employee does not invest in human capital. In this case, his value both to the firm and to the market is $m$, implying that the firm has a credible threat to fire him if his wage exceeds $m$. Thus, the employee’s wage if he does not invest in human capital will be renegotiated down to $m$. Likewise, if the firm switches to business mix $B$, the employee’s value both to the firm and to the market is $M$, implying that his wage taking into account renegotiations will be $M$. Note that in either case no turnover need occur in equilibrium as the employee is indifferent between staying and leaving. Also, it does not matter if the firm renegotiates down the employee’s wage immediately after it switches to business mix $B$ or at some later time. What matters is only that if the firm switches to business mix $B$ the expected value of the employee’s future wage payments is reduced. Moreover, our model is agnostic as to which part of the employee’s wage contract is renegotiated—the base wage $w$, the bonus $b$, or both. In particular, our model does not assume that the structure of the wage contract (i.e., base wage cum bonus) changes after the renegotiations. Finally, renegotiations may also occur before the firm decides to switch to business mix $B$. In this case, we will show below that the existing wage contract will not be renegotiated as the signal $s$ is private information (see Proof of Proposition 1).

Consider the firm’s choice of business mix given that the employee has invested in human capital. The firm’s expected net payoff (i.e., net of wages) under business mix $A$ is $\mu_A(s) - w(s)$, where $w(s) := w + p_A(s)b$. Likewise, the firm’s expected net payoff under mix $B$ is $\mu_B(s) - M$. Hence, the firm continues with business mix $A$ if and only if

$$\mu_A(s) - w(s) \geq \mu_B(s) - M. \quad (3)$$

We can safely rule out the trivial case where the firm never continues with business mix $A$; otherwise it would not be optimal to induce the employee to invest in human capital in the first place. Likewise, if $\mu_A(s) - \mu_B(s)$ is sufficiently negative at low signals we can also rule out that the firm always continues with business mix $A$.\footnote{A sufficient, but not necessary, condition to rule out that the firm always continues with business mix $A$ is...} Finally, we can rule out the case where the
bonus $b$ is so large that the firm prefers a low payoff to a high payoff, that is, the firm prefers $x_l$ over $x_h$. If this were the case, then the firm would like to switch to business mix $B$ at high signals and continue with mix $A$ at low signals, which is the opposite of what efficiency dictates. Given that $p_A(s)$ is increasing in $s$, we thus have that $\mu_A(s) - w(s)$ must be increasing in $s$. Since $\mu_B(s)$ is decreasing in $s$ and $p_A(s)$ and $p_B(s)$ are both continuous in $s$, this implies that the firm’s optimal decision rule is characterized by a unique interior cutoff signal $\underline{s} < s^* < \overline{s}$ given by

$$\mu_A(s^*) - \mu_B(s^*) = w(s^*) - M, \quad (4)$$

such that the firm continues with business mix $A$ if and only if $s \geq s^*$.

When deciding whether to invest in human capital the employee rationally anticipates the firm’s decision whether to switch to business mix $B$. The ex-ante likelihood that the firm will switch to mix $B$ is $F(s^*)$, in which case the employee’s wage will be renegotiated down to $M$ (see above). Likewise, if the employee does not invest in human capital his renegotiated wage will be $m$. Thus, the employee invests in human capital if and only if

$$\int_{s^*}^{\overline{s}} w(s)f(s)ds + F(s^*)M - c \geq m, \quad (5)$$

where the firm’s ex-post optimal cutoff signal $s^*$ is uniquely defined by (4). Rearranging terms, we can rewrite (5) more conveniently as

$$\int_{s^*}^{\overline{s}} w(s)\frac{f(s)}{1 - F(s^*)}ds \geq M + \frac{c - (M - m)}{1 - F(s^*)}. \quad (6)$$

The left-hand side represents the employee’s expected future wage payment conditional on business mix $A$. Hence, to induce the employee to invest in human capital his expected wage payment under business mix $A$ must sufficiently exceed his market value $M$. If the firm continues with business mix $A$ the employee thus earns a quasi rent equal to $[c - (M - m)]/[1 - F(s^*)]$. As we will show below, the presence of this quasi rent, which is necessary to induce the employee to invest in human capital, may cause the firm to switch too often to business mix $B$, thus aggravating the very problem of human capital depreciation that it is supposed to compensate the employee for.
3.2 The Optimal Base Wage and Bonus

When designing the employee’s wage contract the firm chooses the base wage $w$ and bonus $b$ to maximize its expected net payoff

$$V := \int_a^{s^*} [\mu_B(s) - M] f(s)ds + \int_{s^*}^T [\mu_A(s) - w(s)] f(s)ds,$$

where the ex-post optimal cutoff signal $s^*$ is given by (4), subject to the employee’s incentive constraint (5) and the base-wage constraint $w \geq w$. Hence, besides the fact that the base wage $w$ must equal or exceed $w$, the firm must ensure that the employee’s expected wage payment under business mix $A$ sufficiently exceeds his market wage $M$.

At the optimal solution, the employee’s incentive constraint (5) must bind (see Proof of Proposition 1). Inserting the binding incentive constraint into (7), the firm’s objective function becomes

$$V = \int_a^{s^*} \mu_B(s) f(s)ds + \int_{s^*}^T \mu_A(s) f(s)ds - (c + m).$$

The first two terms in (8) represent the firm’s expected gross payoff under the ex-post optimal decision rule $s^*$. The third term represents the cost of inducing the employee to invest in human capital: He must be compensated for his private cost $c$ plus his wage $m$ in case he does not invest.

By inspection, the firm’s objective function is maximized at $s^* = s_{FB}$. From an ex-ante perspective, the firm would thus like to commit to the first-best decision rule. However, the firm’s ex-post optimal decision rule $s^*$ may not coincide with the first-best decision rule $s_{FB}$, creating a time inconsistency problem. Comparing (2) and (4) shows that the two decision rules coincide if and only if at the first-best cutoff signal $s = s_{FB}$ it holds that $w + b_pA(s_{FB}) = M$. That is, the two decision rules coincide if and only if at $s = s_{FB}$ the firm’s expected wage payments are the same under business mix $A$ and $B$.

How can the firm design the employee’s wage contract so as to make its ex-post optimal decision rule $s^*$ as efficient as possible? Suppose first that the employee receives a fixed-wage contract $w(s) = w$. From the incentive constraint (5) it then follows that $w = M + [c - (M - m)]/[1 - F(s^*)]$, implying that $w > M$ holds at all signals $s \in S$ and thus also at the first-best cutoff signal $s = s_{FB}$. Since at $s = s_{FB}$ it must hold that $\mu_A(s_{FB}) = \mu_B(s_{FB})$ (see equation (2)), this in turn implies that $\mu_A(s_{FB}) - w < \mu_B(s_{FB}) - M$. Hence, although at $s = s_{FB}$ business mix $A$ and $B$ have the same expected gross payoffs, the firm strictly prefers to switch
to business mix $B$. As for the firm’s ex-post optimal cutoff signal $s^*$, recall that it must hold that $\mu_A(s^*) - \mu_B(s^*) = w - M$ (see equation (4)). Since $\mu_A(s_{FB}) - \mu_B(s_{FB}) = 0 < w - M$, and since $\mu_A(s) - \mu_B(s)$ is increasing in $s$, this implies that $s^* > s_{FB}$. Hence, there exists a range of signals $(s_{FB}, s^*)$ at which the firm switches to business mix $B$ even though mix $A$ has a higher expected gross payoff. The reason is that under business mix $A$ the employee earns a quasi rent equal to $[c - (M - m)]/[1 - F(s^*)]$ (see equation (6)). By switching to business mix $B$ the firm can appropriate this quasi rent. Of course, the employee must be compensated for this possibility ex ante. The more likely it becomes that the firm switches to business mix $B$—i.e., the higher is the firm’s ex-post optimal cutoff signal $s^*$—the higher is the quasi rent that the firm must pay the employee under business mix $A$.15

Arguably, the firm would never switch to business mix $B$ merely to appropriate the employee’s quasi rent if mix $A$ had a much higher expected gross payoff. Indeed, this is why at sufficiently high signals $s \geq s^*$ the firm continues with business mix $A$. But if business mix $A$ is only marginally better than mix $B$, as is the case if $s \in (s_{FB}, s^*)$, then the prospect of appropriating the quasi rent will induce the firm to switch to mix $B$, driving a wedge between ex-ante and ex-post efficiency.

Given that a fixed-wage contract $w(s) = w$ induces the firm to switch too often to business mix $B$, the next question is how can the firm make its ex-post optimal decision rule $s^*$ more efficient? From the above discussion we know that what causes the firm to switch too often to business mix $B$ is that the employee’s expected wage at $s = s_{FB}$ is higher under business mix $A$ than it is under mix $B$. By increasing the bonus $b$ and reducing the base wage $w$, thus making the employee’s expected wage $w(s) = w + bp_A(s)$ “steeper”, the firm can increase $w(s)$ at high signals and reduce it at low signals without violating the employee’s incentive constraint (5).16 The point of making $w(s)$ steeper is that at $s = s_{FB}$ the difference between $w(s_{FB})$ and $M$ becomes smaller, thus pushing the firm’s ex-post optimal cutoff signal $s^*$ down and closer toward the first-best optimal cutoff signal $s_{FB}$.

15While the employee earns a quasi rent if the firm continues with business mix $A$, he earns no rent ex ante. From an ex-ante perspective, the employee is just compensated for his human capital investment.

16Precisely, by increasing $b$ and reducing $w$ the firm can increase $w(s) - M$ at high signals and reduce it at low signals such that, in expectation, the wedge between $w(s)$ and $M$ required by (5) is preserved.
Graphical Illustration. The basic principle of making the expected wage $w(s)$ steeper, thus tying the employee’s pay closer to firm value, is illustrated in Figure 1. In the picture, the firm’s ex-post optimal cutoff signal $s^*$ is where the difference between $\mu_A(s)$ and $\mu_B(s)$ equals the difference between $w(s)$ and $M$. The fixed-wage contract discussed above is denoted by $w_1(s) = w$, while the corresponding ex-post optimal cutoff signal is denoted by $s^*_1$. The second wage contract $w_2(s)$ has a lower base wage but a higher bonus than $w_1(s)$. As can be seen, switching from $w_1(s)$ to $w_2(s)$ pushes the firm’s ex-post optimal cutoff signal $s^*$ to the left and closer toward $s_{FB}$, thus improving efficiency. The third wage contract $w_{FB}(s)$ has an even lower base wage and an even higher bonus than $w_2(s)$. If this wage contract is feasible, then the first best is attainable. Finally, note that reducing $w$ and increasing $b$ further—that is, beyond $w_{FB}(s)$—is not desirable. Doing so would only push $s^*$ to the left of $s_{FB}$, creating the opposite inefficiency whereby the firm continues too often with business mix A.

\[17\] The functions do not have to be linear (except for $w_1(s)$). The linear appearance in Figure 1 is only for simplicity.
Let us summarize. To induce the employee to invest in human capital his expected wage payment under business mix $A$ must exceed his market wage $M$. If the firm continues with business mix $A$ the employee thus earns a quasi rent. By switching to business mix $B$ the firm can appropriate this quasi rent because it can renegotiate down the employee’s wage to $M$ under the threat of firing. What is more, if business mix $A$ is only marginally better than mix $B$, then the prospect of appropriating the quasi rent may induce the firm to inefficiently switch to mix $B$. The firm can mitigate this inefficiency by tying the employee’s pay to total firm value. Effectively, the employee’s quasi rent then becomes variable. It becomes large when business mix $A$ is much better than mix $B$ and small when mix $A$ is only marginally better. This is feasible because the employee’s incentive constraint only pins down an expected value for the quasi rent; it imposes no constraint as to how this quasi rent should vary with firm value. But if the quasi rent is small when business mix $A$ is only marginally better than mix $B$, then the firm gains only little from appropriating the quasi rent. Indeed, if the gain is less than the efficiency loss from switching to business mix $B$, then the firm has no incentive to switch at all, in which case the first best can be attained.

We can explicitly solve for the base wage and bonus associated with the first-best optimal wage contract $w_{FB}(s)$. Under this wage contract the employee’s expected wage at $s = s_{FB}$ must be the same under business mix $A$ and $B$, i.e., it must hold that $w + bp_A(s_{FB}) = M$. Moreover, the employee’s incentive constraint (5) must hold with equality with $s^* = s_{FB}$. Inserting $w = M/bp_A(s_{FB})$ into the binding incentive constraint, we obtain for the first-best optimal bonus

$$b_{FB} = \frac{c - (M - m)}{\int_{s_{FB}}^s [p_A(s) - p_A(s_{FB})] f(s)ds} > 0,$$

where the inequality follows from $c - (M - m) > 0$ and the fact that $p_A(s)$ is increasing in $s$. Inserting $b_{FB}$ back into $w + bp_A(s_{FB}) = M$ we obtain for the first-best optimal base wage

$$w_{FB} = M - b_{FB}p_A(s_{FB}),$$

which is strictly less than the employee’s market wage $M$.

Before we state our findings in a proposition, let us add some remarks.

First, the first-best optimal wage contract in (9) and (10) is unique. While there are other wage contracts under which the firm’s ex-post optimal decision rule $s^*$ coincides with the first-best decision rule $s_{FB}$, such wage contracts either violate the employee’s incentive constraint (5) or leave it slack, leaving the employee an (unnecessary) ex-ante rent.
Second, while reducing \( w \) and increasing \( b \) pushes the firm’s ex-post optimal cutoff signal \( s^* \) down and closer toward \( s_{FB} \), the firm will only be able to attain the first best if the base-wage constraint \( w \geq w \) does not bind. If the constraint binds, then the second-best optimal base wage is uniquely determined by \( w = w \) while the second-best optimal bonus is uniquely determined by the employee’s binding incentive constraint (5). Naturally, this base wage will be larger and the bonus will be smaller relative to the first-best optimal wage contract, implying that the second-best optimal wage contract will be “flatter” than \( w_{FB}(s) \). Thus, if the base-wage constraint \( w \geq w \) binds the firm switches too often to business mix \( B \), although the inefficiency is smaller than it would be under a fixed-wage contract.

Third, it is important to recall that the reason for why the employee receives a bonus is not to induce him to invest in human capital. The incentive constraint (5) does not imply that the employee must receive a bonus; it only requires that his expected wage payment under business mix \( A \) must be sufficiently large. To see this most clearly, suppose that the signal \( s \) was contractible so that the firm could commit to the first-best decision rule \( s_{FB} \). In this case, any wage contract satisfying the binding incentive constraint would be optimal, including a fixed-wage contract with \( w(s) = w = M + [c - (M - m)]/[1 - F(s_{FB})] \). However, as the firm cannot commit to a particular decision rule, it must design the wage contract so as to make its ex-post optimal decision rule \( s^* \) as efficient as possible, which rules out a fixed-wage contract.

Fourth, there is the issue of renegotiations. We have already shown that there will be no renegotiations if the firm continues with business mix \( A \), while if the firm switches to \( B \) the employee’s wage will be renegotiated down to \( M \) under the threat of firing.\(^{18} \) However, renegotiations may also occur before the firm makes its decision. In particular, if the signal is \( s \in (s_{FB}, s^*) \) the employee might be willing to accept a pay cut to prevent a switch to business mix \( B \). As we will show in the Proof of Proposition 1, no such renegotiations will occur in equilibrium because the signal \( s \) is private information. Precisely, the employee does not know if the signal is \( s \in (s_{FB}, s^*) \), in which case he might be willing to accept a pay cut to prevent a switch to \( B \), or if the signal is \( s \geq s^* \), in which case a pay cut would merely constitute a windfall gain to the firm as it was planning to continue with business mix \( A \) anyway.

Proposition 1 summarizes the results of this section.

\textbf{Proposition 1.} \textit{There exists a uniquely optimal wage contract. Under this wage contract the

\(^{18}\text{Likewise, if the employee does not invest in human capital his wage will be renegotiated down to } m.\)
employee receives a positive bonus $b$, implying that his pay is tied to total firm value. 

i) If the base-wage constraint $w \geq w_0$ does not bind, the optimal wage contract is uniquely determined by (9) and (10). In this case, the firm’s ex-post optimal decision rule $s^*$ coincides with the first-best optimal decision rule $s_{FB}$. 

ii) If the base-wage constraint $w \geq w_0$ binds, the optimal base wage is uniquely determined by $w = w_0$ while the optimal bonus is uniquely determined by the employee’s binding incentive constraint (5). In this case, the firm’s ex-post optimal decision rule $s^*$ is inefficient. The firm switches too often to business mix $B$, thus depreciating the employee’s specific human capital.

**Proof.** See Appendix.

4 Comparative Statics

**Specificity of Human Capital Investment**

By investing in human capital, the employee can increase his market wage from $m$ to $M \geq m$. Thus, if $M = m$ the human capital investment is fully specific to the firm’s business mix $A$ while if $M > m$ it is only partly specific. Generally, the smaller is $M$ relative to $m$, the more specific is the employee’s human capital investment, the bigger is the loss from human capital depreciation if the firm switches to business mix $B$, and the higher is the quasi rent that it must pay the employee under business mix $A$. This quasi rent, in turn, creates a wedge between ex-ante and ex-post efficiency, making it ex-post optimal for the firm to switch to business mix $B$ even though mix $A$ is marginally more profitable. As we have shown above, the firm can mitigate this inefficiency by adjusting the employee’s wage contract, namely, by increasing the bonus and, if possible, lowering the base wage.

**Proposition 2.** The more specific is the employee’s human capital investment, the more will his pay be tied to total firm value. Formally, the smaller is $M$ relative to $m$, the higher will be the bonus $b$.

**Proof.** See Appendix.

By tying the employee’s pay to total firm value, the firm can alleviate the inefficiency that it may switch too often business mix $B$, in which case the employee’s specific human capital
depreciates. Thus, tying the employee’s pay to firm value is a means to protect his specific human capital investment and the quasi rent associated with it.

**Strategy Risk**

From the employee’s perspective there are two types of risk. First, there is the risk that business mix $B$ may become more profitable than mix $A$, inducing the firm to switch to mix $B$, which in turn depreciates the employee’s specific human capital. Second, even if the firm continues with business mix $A$ there is the risk related to the firm’s payoff, i.e., whether the payoff is $x_l$ or $x_h$. We refer to the first type of risk as “strategy risk” and to the second type as “payoff risk”. We first address the effect of strategy risk on the employee’s wage contract. Subsequently we address the effect of payoff risk.

Consider an increase in the likelihood $1 - \pi$ that business mix $B$ will become more profitable than mix $A$. The more likely it becomes that the firm switches to business mix $B$, thus depreciating the employee’s specific human capital, the higher is the quasi rent it must pay the employee under mix $A$. By the same argument as above, the firm then optimally adjusts the employee’s wage contract, namely, by increasing the bonus and, if possible, lowering the base wage.

**Proposition 3.** The more likely it becomes that switching to business mix $B$ is efficient, the more will the employee’s pay be tied to total firm value. Formally, the smaller is $\pi$, the higher will be the bonus $b$.

**Proof.** See Appendix.

**Payoff Risk.**

Under state $a$ and business mix $A$ or under state $b$ and business mix $B$ the firm’s success probability—i.e., the probability that $x = x_h$—is $p_H$. Likewise, under state $b$ and business mix $A$ or under state $a$ and business mix $B$ the firm’s success probability is $p_L$. In a slight abuse of notation, let us denote the corresponding expected payoffs by $\mu_H := p_H x_h + (1 - p_H) x_l$ and $\mu_L := p_L x_h + (1 - p_L) x_l$. We model an increase in payoff risk as a mean-preserving spread where the high payoff $x_h$ increases, thus widening the spread between the two payoffs, while the probabilities $p_H$ and $p_L$ must decrease accordingly to ensure that $\mu_H$ and $\mu_L$ remain unchanged.\(^{19}\)

Note that with this specification the first-best cutoff signal $s_{FB}$ remains unchanged.

\(^{19}\)Given the constraint that $x_l \geq M$ (see Section 2) a decrease in $x_l$ may not be feasible. For this reason,
Consider the case where the firm’s ex-post optimal cutoff signal \( s^* \) coincides with the first-best cutoff signal \( s_{FB} \). (The case where the first best is not attainable is discussed in the Proof of Proposition 4.) Since \( s_{FB} \) remains unchanged following an increase in payoff risk, we have from \( s^* = s_{FB} \) that \( s^* \) also remains unchanged. As the employee’s incentive constraint (5) binds, this implies that the firm’s net expected payoff—both at \( s^* = s_{FB} \) and conditional on \( s \geq s^* = s_{FB} \)—remains unchanged, which finally implies that the bonus \( b \) must increase.\(^{20} \)

Intuitively, holding the wage contract fixed, the firm’s expected net payoff becomes “steeper” following an increase in payoff risk: the high payoff \( x_h - w - b \) increases while the low payoff \( x_l - w \) either decreases or remains constant (see footnote 19). To remain at the first-best optimum, the firm must consequently “flatten” its expected net payoff, namely, by increasing the bonus and, if possible, lowering the base wage. Formally, inserting \( p_A(s) = p_L + \pi(s)(p_H - p_L) \) and \( \mu_H - \mu_L = (p_H - p_L)(x_h - x_l) \) into equation (9) yields

\[
b_{FB} = \frac{x_h - x_l}{\mu_H - \mu_L} \int_{s_{FB}}^{x_h} \left[ \pi(s) - \pi(s_{FB}) \right] f(s) ds, \tag{11}\]

which is increasing in the payoff spread \( x_h - x_l \).

**Proposition 4.** The riskier is the firm’s payoff, the more will the employee’s pay be tied to total firm value. Formally, the larger is the increase in payoff risk in the sense of a mean-preserving spread, the higher will be the bonus \( b \).

**Proof.** See Appendix.

It is important to recall that Propositions 3 and 4 are derived under risk neutrality, allowing us to explicitly solve for the optimal wage contract. If employees are risk averse there is an obvious countervailing effect: Firms that tie their employees’ pay more to total firm value must offer a larger risk premium. Despite this countervailing effect, firms in practice do tie employees’ pay to total firm value—through broad-based stock and stock option plans and by making bonus payments contingent on the firm’s performance—implying that there must exist benefits that we have defined payoff risk in a way that is independent of whether a decrease in \( x_l \) is feasible. In the Proof of Proposition 4 we allow for \( x_l \) to both decrease and remain constant.

\(^{20}\)That the firm’s expected payoff at \( s^* = s_{FB} \) remains unchanged is implied by the fact that the decision rule is first-best optimal. That the firm’s expected payoff conditional on \( s \geq s^* = s_{FB} \) remains unchanged is implied by the fact that the employee’s incentive constraint binds.
outweigh the costs of risk aversion. This paper offers an explanation what these benefits might be and for what types of firms they might be particularly important.

**Empirical Evidence**

The above results suggest that firms that tie their employees’ pay to total firm value should be, in particular, firms with a high degree of human capital intensity, firms that frequently change their “business mix” (e.g., products, markets, technologies), and firms with highly volatile cash flows. The latter two features, which we referred to as “strategy risk” and “payoff risk”, respectively, seem particularly important for young firms that are still experimenting with their business mix, for firms in dynamic high-growth industries, such as new economy firms, and for firms in risky, volatile industries.

The available empirical evidence is broadly consistent with these implications. Anderson, Banker, and Ravindran (2000), Ittner, Lambert, and Larcker (2003), and Oyer and Schafer (2005) all find that broad-based stock and stock option plans are more common in new economy firms than in old economy firms.21 Likewise, Core and Guay (2001) and Kroumova and Sesil (2006) find that broad-based stock and stock option plans are more prevalent in high-growth firms, measured by market-to-book ratios, while Ittner, Lambert, and Larcker (2003) find that broad-based stock and stock option plans are more likely in firms with high investment opportunities.22

As for firms’ riskiness, Kroumova and Sesil (2006) and Oyer and Schafer (2005) document that broad-based option plans are more prevalent in firms with high sales volatility and in firms in high-volatility industries, respectively. For Japan, Nagaoka (2005) finds that broad-based option plans are more likely in young firms, even after controlling for firm size, liquidity constraints, and growth opportunities. In addition, Nagaoka shows that for such firms broad-based option plans are positively related to firms’ stock return volatility. Similarly, for Finland, Jones, Kalmi, and Mäkinen (2006) document that broad-based option plans are more

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21 It is unlikely that this result is driven by new economy firms being financially constrained. Ittner, Lambert, and Larcker (2003) find that cash levels and cash flows in new economy firms are positively related to broad-based equity grants, “providing no support for claims that new economy firms under cash constraints use equity grants in place of cash compensation” (p. 113). Kroumova and Sesil (2006) and Bergman and Jenter (2006) find similar results. Oyer and Schaefer (2005) find mixed results while Core and Guay (2001) find evidence in support of the cash-constraints hypothesis.

22 Ittner, Lambert, and Larcker (2003) measure investment opportunities as a weighted average of five measures, including market-to-book and R&D.
prevalent in firms with high stock return volatility.

Several studies find a positive relation between human capital intensity and employees’ pay being tied to total firm value. Core and Guay (2001) and Kroumova and Sesil (2006) find that broad-based stock and stock option plans are more prevalent in high R&D firms. For Finland, Jones, Kalmi, and Mäkinen (2006) find a positive relation between broad-based stock option plans and the ratio of intangible to total assets, which the authors use as a measure of human capital intensity. While consistent with our theoretical arguments, these studies do not distinguish between general and specific human capital. The only paper that gets closer to the issue is Robinson and Zhang (2005), who use data from the British Employee Relations Survey, which covers 2191 workplaces across Britain. The survey includes various measures of human capital intensity which the authors divide into measures of general and specific human capital. Consistent with our theoretical arguments, the authors find a positive relation between measures of specific human capital and the prevalence of employee stock ownership plans.

5 Extensions

5.1 Long-Term Contracts and the Free-Rider Problem

We now depart from our base model and explicitly model the free-rider problem that exists if there are many employees. Doing so allows us to compare at-will contracts with long-term (commitment) contracts, which we have neglected so far. Under a long-term contract the firm can commit itself not to renegotiate down the employee’s wage. Consequently, the firm can commit to a wage that exceeds the employee’s value to the firm. As we will show below, the problem of inducing the employee to invest in human capital then becomes a standard principal-agent problem. In particular, as the number of employees becomes sufficiently large, the free-rider problem among employees becomes increasingly severe, rendering the long-term contract too costly and making the at-will contract uniquely optimal.

Suppose that the firm has \( N \) employees. As we increase the number of employees we must scale up the firm’s operations accordingly. We assume that the firm’s payoff \( x \in \{x_l, x_h\} \) increases proportionately by some factor \( \gamma(N) \). As the firm’s total wage bill must grow in line with \( N \) we require that \( \gamma(N)/N \) must be non-decreasing. Though stronger than needed,  

\footnote{Bergman and Jenter (2006) find a similar result in most, but not all, of their regressions.}
we further require that $\gamma(N)/N$ does not grow beyond bounds. Formally, there exists some (arbitrarily large but finite) value $K$ such that $\gamma(N)/N \leq K$ for all $N$. In line with our base model, we assume that it is optimal to induce all $N$ employees to invest in human capital. Also in line with our base model, we assume that the human capital investment is valuable to the firm under business mix $A$ but not under mix $B$.

Recall that under state $a$ and business mix $A$ or under state $b$ and business mix $B$ the firm’s success probability—i.e., the probability that $x = x_h$—is $p_H$. Likewise, under state $b$ and business mix $A$ or under state $a$ and business mix $B$ the firm’s success probability is $p_L$. We stipulate that these probabilities remain unchanged if all $N$ employees invest in human capital. If only $N - 1$ employees invest in human capital the respective probabilities under business mix $A$ decrease to $\tilde{p}_H$ and $\tilde{p}_L$. For simplicity, we assume that both probabilities decrease by the same amount $\Delta$ such that $\tilde{p}_L = p_L - \Delta$ and $\tilde{p}_H = p_H - \Delta$. (The assumption that both probabilities decrease by the same amount can be relaxed.) Consequently, if only $N - 1$ employees invest in human capital business mix $A$ generates a high payoff $x_h$ with probability

$$\tilde{p}_A(s) = \pi(s)(p_H - \Delta) + [1 - \pi(s)](p_L - \Delta) = p_A(s) - \Delta.$$ 

To capture the notion that an individual employee’s incentives to free ride become stronger as the number of employees $N$ increases, we stipulate that $\Delta$ is decreasing in $N$ with $\Delta \to 0$ as $N \to \infty$.\textsuperscript{24} One possible specification is that $\Delta = \overline{\Delta}/N$ where $0 < \overline{\Delta} < p_L$.

Note first that our previous analysis of at-will contracts still applies. In particular, the first-best optimal wage contract in (9) and (10) holds irrespective of the number of employees $N$ (see Proof of Proposition 5). This is no longer true under a long-term contract. Employees’ incentives to invest in human capital must then derive from the bonus $b$ as the firm loses its ability to renegotiate down the wage of an employee who has not invested in human capital.

To examine an individual employee’s incentive to invest in human capital under a long-term contract, we first consider the firm’s ex-post optimal decision rule $s^*$. Under a long-term contract the firm can no longer appropriate the employee’s quasi rent by switching to business mix $B$. Irrespective of its business mix the firm’s total (i.e., aggregate) wage bill is therefore $Nw$ if $x = x_l$ and $N(w + b)$ if $x = x_h$. This implies that the firm always chooses the business mix with the highest expected payoff, which in turn implies that the ex-post optimal decision rule

\textsuperscript{24}Since all payoffs are multiplied by $\gamma(N)$, an individual employee’s marginal contribution to the firm value, provided that he invests in human capital, does not have to be decreasing in $N$. 

23
\( s^* \) is first-best efficient. However, the first-best optimal cutoff signal now depends on how many employees invest in human capital. If all \( N \) employees invest in human capital, the first-best optimal cutoff signal is given by \( \mu_A(s_{FB}) = \mu_B(s_{FB}) \) (see equation (2)), which can be rewritten as \( p_A(s_{FB}) = p_B(s_{FB}) \). However, if only \( N - 1 \) employees invest in human capital the first-best optimal cutoff signal is given by \( p_A(s_{FB}) - \Delta = p_B(s_{FB}) \), where \( s_{FB} > s_{FB} \) since \( p_A(s) \) is increasing in \( s \) and \( p_B(s) \) is decreasing in \( s \). Intuitively, if only \( N - 1 \) employees invest in human capital the success probability under business mix \( A \) becomes smaller, pushing the first-best optimal cutoff signal upward.

Given that the remaining \( N - 1 \) employees invest in human capital, an individual employee who invests in human capital receives the bonus \( b \) with probability

\[
\int_{s}^{s_{FB}} p_B(s)f(s)ds + \int_{s_{FB}}^{\pi} p_A(s)f(s)ds,
\]

while if the employee does not invest in human capital he receives the bonus only with probability

\[
\int_{s}^{s_{FB}} p_B(s)f(s)ds + \int_{s_{FB}}^{\pi} [p_A(s) - \Delta] f(s)ds,
\]

which is less than (12) since \( s_{FB} > s_{FB} \) and \( p_A(s) > p_B(s) \) for all \( s > s_{FB} \). Because investing in human capital involves private costs of \( c > 0 \), the employee’s incentive constraint becomes

\[
b \geq \frac{c}{\int_{s_{FB}}^{s_{FB}} [p_A(s) - p_B(s)] f(s)ds + [1 - F(s_{FB})] \Delta}.
\]

The denominator in (14) equals the difference between (12) and (13), which is the difference in the likelihood with which the employee receives the bonus if he invests and does not invest in human capital, respectively. As the number of employees \( N \) increases, the denominator in (14) becomes smaller, implying that free-riding becomes less costly for an individual employee. (Note that \( s_{FB} - s_{FB} \) is decreasing in \( \Delta \), which in turn is decreasing in \( N \)) Consequently, as the number of employees \( N \) increases, it becomes increasingly difficult to incentivize an individual employee, implying that the bonus \( b \) must increase along with \( N \) to keep employees incentivized.

The final step is to write down an individual employee’s utility at the ex-ante stage:

\[
U := w + b \left[ \int_{s}^{s_{FB}} p_B(s)f(s)ds + \int_{s_{FB}}^{\pi} p_A(s)f(s)ds \right].
\]

If the bonus required by the employee’s incentive constraint (14) is small, then the firm can find a base wage \( w \geq w \) such that the employee’s participation constraint binds, implying that \( U = m \). However, as the number of employees increases, the bonus required in (14) increases
as well. Since the firm cannot go below \( w = \underline{w} \), this implies that at some point the employee’s participation constraint must become slack, which in turn implies that he will earn a rent equal to \( R := U - m > 0 \). Substituting (14) with equality into (15) we have

\[
R := c \left[ \frac{\int_{s}^{s_{FB}} p_{B}(s)f(s)ds + \int_{s_{FB}}^{s} p_{A}(s)f(s)ds - (m - w)}{\int_{s_{FB}}^{s_{FB}} [p_{A}(s) - p_{B}(s)] f(s)ds + [1 - F(s_{FB})] \Delta} \right],
\]

(16)

which is increasing in \( N \) as the bonus \( b \) that is necessary to induce an individual employee to invest in human capital becomes larger the less he is in a position to affect the likelihood of receiving the bonus. Given that employees receive no (ex-ante) rent under an at-will contract, the following result is immediate.

**Proposition 5.** There exists a finite number \( \bar{N} \geq 1 \) such that the at-will contract described in Proposition 1 is uniquely optimal if the firm has at least \( \bar{N} \) employees.

**Proof.** See Appendix.

### 5.2 Severance Pay

The use of the threat of firing to appropriate quasi rents is inefficient as it causes the firm to switch too often to business mix \( B \). This suggests that one possibility to improve efficiency is to weaken the threat of firing. (However, see Section 5.1: Absent any threat of firing employees’ incentives must derive from the bonus, which causes the free-rider problem.) The firm can weaken the threat of firing by stipulating severance pay if the employee gets fired. While no firing need occur in equilibrium, the stipulation of severance pay will affect renegotiations and thus the firm’s choice between business mix \( A \) and \( B \).

Suppose that the firm stipulates a severance payment \( P > 0 \) which the employee receives if he gets fired. If the employee does not invest in human capital, the most that the firm can renegotiate down his wage to is therefore \( m + P \). Likewise, if the employee invests in human capital but the firm switches to business mix \( B \), the most that it can renegotiate down the employee’s wage to is \( M + P \). The employee’s incentive constraint (6) thus becomes

\[
\int_{s^{*}}^{s} w(s) \frac{f(s)}{1 - F(s^{*})} ds \geq M + P + \frac{c - (M - m)}{1 - F(s^{*})}.
\]

(17)

Importantly, since the employee can always guarantee himself a payoff of \( m + P \), the severance payment \( P \) constitutes a pure rent. This is in contrast to our base model where the employee
received no (ex-ante) rent. An immediate implication of this is that if the first best is attainable with \( P = 0 \), then it remains uniquely optimal to set \( P = 0 \) and use the wage contract in Case i) of Proposition 1. However, if the first best is not attainable with \( P = 0 \), then it might be optimal to set \( P > 0 \). While this leaves the employee a rent, it also pushes \( s^* \) down and closer to \( s_{FB} \). Formally, if \( P > 0 \) the firm’s ex-post optimal cutoff signal \( s^* \) is given by

\[
\mu_A(s^*) - \mu_B(s^*) = w(s^*) - (M + P). \tag{18}
\]

Looking at Figure 1, one might be tempted to conclude that if the firm wishes to push \( s^* \) all the way down to \( s_{FB} \), all it needs to do is raise \( P \) by enough so that the expected wage payment \( w(s) \) intersects with the horizontal line \( M + P \) at \( s = s_{FB} \). However, this ignores that \( w(s) \) will not remain constant. The employee’s incentive constraint (17) requires that his expected wage payment under business mix \( A \) must increase correspondingly or else the constraint is violated. Fortunately, this constraint only concerns the expectation of \( w(s) \) conditional on \( s \geq s^* \), not the functional form of \( w(s) \), leaving the firm some freedom how to adjust \( w(s) \) while raising \( P \).

Suppose, for example, that along with raising \( P \) the firm increases the base wage \( w \) (but not the bonus \( b \)) such that the incentive constraint remains satisfied. To see the effect this has on the firm’s ex-post optimal cutoff signal \( s^* \), we can rewrite (17) as

\[
w + \int_{s^*}^{s_{FB}} bp_A(s) \frac{f(s)}{1 - F(s^*)} ds \geq M + P + \frac{c - (M - m)}{1 - F(s^*)}.
\]

Thus, to satisfy the employee’s incentive constraint (17) the firm must raise the base wage \( w \) by the same amount as \( P \). However, this would leave \( w(s) - P \) unchanged for all \( s \in S \), thus leaving the firm’s ex-post optimal cutoff signal \( s^* \) unchanged, which cannot be optimal. (The firm would only leave the employee a rent without lowering the cutoff signal.) Accordingly, the optimal way to accommodate an increase in \( P \) is not to raise the base wage but instead to raise the bonus \( b \) while leaving the base wage fixed at \( w = w \). As a result, the expected wage payment \( w(s) \) increases relatively more at high signals and less at low signals, thus reducing \( w(s^*) - P \) and pushing the firm’s ex-post optimal cutoff signal \( s^* \) down and closer toward \( s_{FB} \). The following proposition summarizes this discussion. (The proof follows directly from the above analysis and the Proofs of Propositions 1 and 2.)

**Proposition 6.** **Introducing severance pay** \( P \) **does not affect the key result in Proposition 1 that the employee receives a positive bonus** \( b \), **implying that his pay is tied to total firm value.**
i) If the first best is attainable with $P = 0$, then the wage contract described in Case i) of Proposition 1 remains uniquely optimal.

ii) If the first best is not attainable with $P = 0$, then it may be optimal to set $P > 0$. In this case, the firm will raise the bonus $b$ along with $P$, implying that the employee’s pay will be even more tied to total firm value than it is in Case ii) of Proposition 1.

Whether the firm will actually set $P > 0$ in Case ii) depends on the tradeoff between improving efficiency—raising $P$ (and increasing $b$ at the same time) pushes the firm’s ex-post optimal cutoff signal $s^*$ closer toward $s_{FB}$—and leaving the employee rents. In either case, the employee will receive a bonus, implying that his pay will be tied to total firm value. In fact, the higher is the severance payment $P$, the higher will be the bonus. Incidentally, the firm will never set $P$ so high as to implement the first-best cutoff signal $s^* = s_{FB}$. Inserting the binding incentive constraint (17) into the firm’s objective function (7) (with $M$ being replaced by $M + P$) yields

$$V := \int_{s^*}^{s_{FB}} \mu_B(s)f(s)ds + \int_{s^*}^{s_{FB}} \mu_A(s)f(s)ds - (c + m + P),$$

implying that

$$\frac{dV}{dP} \bigg|_{s^* = s_{FB}} = -1 + \left[ \frac{ds^*}{dP} \right] \left[ \mu_B(s^*) - \mu_A(s^*) \right] f(s^*) \bigg|_{s^* = s_{FB}} = -1$$

where the last equality follows from $\mu_B(s_{FB}) = \mu_A(s_{FB})$ (see equation (2)). At the first-best optimum, a small decrease in $P$ has a negligible effect on efficiency while the associated reduction in the employee’s rent constitutes a first-order cost savings for the firm. Hence, severance pay does not lead to the first best. If the first best is not attainable with $P = 0$, then it will also not be attained if we introduce severance pay.

6 Conclusion

Specific human capital investments may depreciate even if employees do not switch firms. When a firm changes its unique mix of products, markets, and technologies (“business mix”) the skill mix that is optimal inside the firm also changes. Some previously acquired skills may become obsolete while others may become less important. This in turn has repercussions on employees’ incentives to invest in specific human capital. If such investments are observable but non-verifiable, firms can provide incentives by promising employees an above-market wage (a “quasi
rent”). However, if employees’ specific human capital depreciates following a change in the business mix the firm will renegotiate down the promised quasi rent under the threat of firing, undermining its credibility to make promises in the first place. What is more, the firm may be tempted to switch to an alternative business mix even if the current mix is marginally better only to appropriate employees’ quasi rents.

This paper shows that firms can mitigate the ex-post opportunism problem associated with human capital investments by tying employees’ pay to total firm value. While employees must receive a given quasi rent in expectation to invest in human capital, the quasi rent that they will actually receive becomes variable. In particular, the quasi rent will be small when the firm’s current business mix is only marginally better than the best alternative business mix, minimizing the firm’s payoff from switching to a less efficient business mix only to appropriate quasi rents. Thus, the widely observed practice of linking employees’ pay to total firm value—through broad-based stock or stock option plans or by making bonus payments contingent on the firm’s performance—can help to safeguard employees’ specific human capital investments. Consistent with the available empirical evidence, our model implies that firms which link the pay of their rank-and-file employees to total firm value should be, in particular, firms with a high degree of human capital intensity, firms that frequently undergo changes in their business mix, such as young firms that are still experimenting with their business mix, firms in risky, volatile industries, and firms in dynamic, high-growth industries, such as new economy firms.

7 Proofs

Proof of Proposition 1. We first show that the incentive constraint (5) must bind. If this was not the case, the firm could increase its expected net payoff by lowering the base wage \( w \) (if \( w \geq w \) does not bind) or the bonus \( b \) or both. Formally, differentiating (7) with respect to \( w \) and \( b \) and using \( w(s) := w + bp_A(s) \) it follows from Leibniz’s rule and (4) that \( dV/dw = -[1 - F(s^*)] < 0 \) and \( dV/db = -\int_{s^*}^\infty p_A(s)f(s)ds < 0 \). Hence, given that the firm’s cutoff signal \( s^* \) is chosen optimally, the “indirect effect” of a decrease in \( w \) or \( b \) via \( s^* \) is negligible. What remains is only the “direct effect” that a decrease in \( w \) or \( b \) reduces the firm’s wage costs.

The rest of the proof is straightforward. If the base-wage constraint \( w \geq \underline{w} \) does not bind, the contract in (9) and (10) is uniquely (first-best) optimal as \( b_{FB} \) and \( w_{FB} \) depend only on primitives and \( s_{FB} \), which is uniquely defined by (2). If the base-wage constraint binds, the
(second-best) optimal base wage $w$ is uniquely determined by $w = \underline{w}$ while the uniquely optimal bonus is the smallest value of $b$ at which (5) binds. (There may be several values of $b$ that solve (5) since the left-hand side may not be everywhere monotonic in $b$, which is due to the fact that increasing $b$ pushes up $s^*$; see Figure 1 for a graphical illustration.)

We turn next to the issue of renegotiations. The only remaining case to be discussed is that where the signal $s$ has been realized but the firm has not yet decided whether to switch to business mix $B$. (The case where the firm has already decided is discussed in the main text.) There is only scope for mutually beneficial renegotiations if $s^* > s_{FB}$, which implies that the base-wage constraint must bind, i.e., $w = \underline{w}$.

Consider first the case in which the uninformed employee offers a new wage contract $(\tilde{w}, \tilde{b})$ to replace the existing optimal contract $(w, b)$. There are two relevant cases. The first case is where $\tilde{b} < b$ and $\tilde{w} = w = \underline{w}$. The second case is where $\tilde{b} < b$ and $\tilde{w} > w = \underline{w}$. Denote the firm’s ex-post optimal cutoff signal under $(w, b)$ and $(\tilde{w}, \tilde{b})$ by $s^*$ and $\tilde{s}^*$, respectively. Consider first the case where $\tilde{b} < b$ and $\tilde{w} = w = \underline{w}$. Since $(\tilde{w}, \tilde{b})$ has the same base wage but a lower bonus than $(w, b)$ the employee will be better off under $(\tilde{w}, \tilde{b})$ only if $\tilde{s}^* < s^*$. Formally, the employee will be better off under $(\tilde{w}, \tilde{b})$ if and only if

$$\int_{\tilde{s}^*}^{\tilde{w}(s)f(s)ds + F(\tilde{s}^*)M \geq \int_{s^*}^{\tilde{w}(s)f(s)ds + F(s^*)M,}$$

(21)

where $\tilde{s}^* < s^*$. By inspection, if $(\tilde{w}, \tilde{b})$ satisfies (21) then it also satisfies the incentive constraint (5). But there cannot exist a contract $(\tilde{w}, \tilde{b})$ satisfying (5) and $\tilde{s}^* < s^*$, for this would contradict the optimality of $(w, b)$. Consider next the case where $\tilde{b} < b$ and $\tilde{w} > w = \underline{w}$. By construction, $\tilde{w}(s)$ is flatter than $w(s)$, implying that there exists a critical signal $\tilde{s}$ with $s^* < \tilde{s} < \bar{s}$ such that the firm prefers $(\tilde{w}, \tilde{b})$ to $(w, b)$ if and only if $s \geq \tilde{s}$. Consequently, the firm accepts $(\tilde{w}, \tilde{b})$ if $s \geq \tilde{s}$ and rejects it—implying that $(w, b)$ remains in place—if $s < \tilde{s}$. Note that since $\tilde{s} > s^*$ the firm’s ex-post optimal cutoff signal $s^*$ remains unchanged, i.e., the firm still chooses business mix $A$ if and only if $s \geq s^*$. However, if $s^*$ remains unchanged while the firm is better off at all signals $s \geq \tilde{s}$, then the employee must be worse off, implying that he will never offer to replace $(w, b)$ with $(\tilde{w}, \tilde{b})$ in the first place.

Consider next the case in which the firm offers a new wage contract $(\tilde{w}, \tilde{b})$ after observing $s$. We show that there exists no perfect Bayesian equilibrium of the signaling game in which $(w, b)$ is successfully renegotiated with positive probability. Though the argument is more general, we restrict attention to equilibria in which both the firm (with “type” $s$) and the employee, who
must either accept or reject the firm’s offer, play pure strategies. For any given equilibrium of the renegotiation game, denote the set of new wage contracts that the employee accepts by $\Omega$. Also, define $\tilde{\Omega} := \Omega \cup \{w, b\}$. The outcome of the renegotiation game is equivalent to that of a game in which the firm offers a menu of contracts $\tilde{\Omega}$ in $t = 0$ from which it chooses after observing $s$. For any given equilibrium of the renegotiation game, denote the set of new wage contracts that the employee accepts by $\Omega$. Also, define $e := \Omega \cup \{w, b\}$. The outcome of the renegotiation game is equivalent to that of a game in which the firm offers a menu of contracts $\tilde{\Omega}$ in $t = 0$ from which it chooses after observing $s$. For any given menu $e$ there is again a unique cutoff signal $e^*$ such that the firm chooses business mix $A$ if and only if $s \geq e^*$. Denote by $(\bar{w}, \bar{b}) \in e^*$ the contract that the firm chooses at the cutoff signal $e^*$. By optimality, for all $s > e^*$ the firm chooses a contract that maximizes its expected net payoff, i.e., the firm chooses $(w, b) = \arg \max_{(w, b) \in e} [\mu_A(s) - \bar{w} - p_A(s)\bar{b}]$. Thus, if we delete all contracts from $\tilde{\Omega}$ except $(\bar{w}, \bar{b})$, implying that the cutoff signal $e^*$ remains unchanged, then the employee’s expected payoff must (weakly) increase. But this implies that the employee’s expected payoff under the single contract $(\bar{w}, \bar{b})$ constitutes an upper bound for his expected payoff in the renegotiation game in which the firm makes the contract offer. However, we have shown that there cannot exist a single contract $(\bar{w}, \bar{b}) \neq (w, b)$ satisfying both the incentive constraint (5) and $e^* < s^*$, for this would contradict the optimality of $(w, b)$ among all single contracts. Q.E.D.

Proof of Proposition 2. If the first best is attainable, implying that the base-wage constraint $w \geq w$ does not bind, then the result follows immediately from differentiating (9) with respect to $M$. (The other result mentioned in the main text that the firm reduces $w$ follows immediately from inserting (9) into (10) and differentiating with respect to $M$.)

If the first best is not attainable, implying that $s^* > s_{FB}$ and $w = w$, then we know from the Proof of Proposition 1 that the optimal bonus is uniquely determined as the smallest value of $b$ at which the incentive constraint (5) binds. Holding $b$ constant, the left-hand side of (5) is increasing in $M$ and decreasing in $s^*$, where the latter result follows from $w(s^*) > M$, which is in turn implied by $s^* > s_{FB}$. Moreover, again holding $b$ constant, we have from (4) that $s^*$ is decreasing in $M$. Taken together, this implies that, holding $b$ constant, the incentive constraint (5) becomes slack as $M$ increases, which in turn implies that the firm will optimally decrease $b$ until (5) binds again. Q.E.D.

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25 Existence of $e^*$ follows from the fact that $\max_{(w, b) \in \tilde{\Omega}} E[w + bp_A(s)]$ is nondecreasing and continuous in $s$.

26 A decrease in $M$ works the opposite way: it violates the (previously binding) incentive constraint (5), implying that the firm must increase $b$ until (5) binds again. Note that we only need that the firm’s objective function is decreasing in $b$ (see Proof of Proposition 1) and that the left-hand side of (5) is continuous in $b$, taking into
**Proof of Proposition 3.** Consider first the case where the first best is attainable, implying that the base-wage constraint \( w \geq w_0 \) does not bind. Inserting \( p_A(s) = p_L + \pi(s)(p_H - p_L) \) into (9) and rearranging we obtain

\[
b_{FB} = \frac{1}{p_H - p_L} \frac{1}{\pi(s_{FB})} \int_{s_{FB}}^{\pi} \frac{c - (M - m)}{\pi(s) - 1} f(s) ds.
\]

We first prove an auxiliary result, namely, that

\[
1 - \frac{1}{\pi} = \frac{f_a(s_{FB})}{f_b(s_{FB})},
\]

implying that \( ds_{FB}/d\pi < 0 \) since \( f_a(s)/f_b(s) \) is increasing in \( s \) by MLRP. To see that (23) is true, note from (2) that first-best optimality implies that \( p_A(s_{FB}) = p_B(s_{FB}) \). Substituting \( p_A(s_{FB}) := p_L + \pi(s_{FB})(p_H - p_L) \), \( p_B(s_{FB}) := p_H - \pi(s_{FB})(p_H - p_L) \), \( \pi(s_{FB}) := \pi f_a(s_{FB})/\pi f_a(s_{FB}) + (1 - \pi)f_b(s_{FB}) \) and rearranging yields (23).

Note next that

\[
\frac{d}{d\pi} \frac{1}{\pi(s_{FB})} = \frac{1}{p_H - p_L} \frac{1}{\pi(s_{FB})} \left[ \int_{s_{FB}}^{\pi} \frac{c - (M - m)}{\pi(s) - 1} f(s) ds \right] = 0,
\]

where the last equality follows from (23). Differentiating (22) using (24) yields

\[
\frac{db_{FB}}{d\pi} = - \frac{1}{p_H - p_L} \frac{1}{\pi(s_{FB})} \left[ \int_{s_{FB}}^{\pi} \frac{c - (M - m)}{\pi(s) - 1} f(s) ds \right] \frac{d}{d\pi} \left[ \int_{s_{FB}}^{\pi} \frac{\pi(s)}{\pi(s_{FB})} - 1 \right] f(s) ds,
\]

where \( c - (M - m) > 0 \). Note that \( s_{FB} \) depends on \( \pi \). It then follows from Leibniz’s rule that

\[
\frac{d}{d\pi} \left[ \int_{s_{FB}}^{\pi} \frac{\pi(s)}{\pi(s_{FB})} - 1 \right] f(s) ds = - \left( \frac{\pi(s_{FB})}{\pi(s_{FB})} - 1 \right) f(s_{FB}) \frac{ds_{FB}}{d\pi} \frac{d}{d\pi} \left[ \int_{s_{FB}}^{\pi} \frac{\pi(s)}{\pi(s_{FB})} - 1 \right] f(s) ds,
\]

where the first term on the right-hand side is zero. Moreover, by (1), (23), and \( f(s) := \pi f_a(s) + (1 - \pi)f_b(s) \) the integrand on the right-hand side of (26) can be rewritten as

\[
\frac{d}{d\pi} \left[ \frac{\pi(s)}{\pi(s_{FB})} - 1 \right] f(s) = \frac{d}{d\pi} \left[ \pi f_a(s) - \pi f_b(s) \frac{f_a(s_{FB})}{f_b(s_{FB})} \right] = \left[ f_a(s) - f_b(s) \frac{f_a(s_{FB})}{f_b(s_{FB})} \right] - \pi \left[ f_b(s) \frac{d}{d\pi} \left( \frac{f_a(s_{FB})}{f_b(s_{FB})} \right) \frac{ds_{FB}}{d\pi} \right] > 0,
\]

account that \( s^* \) is continuous in \( b \) from (4). In particular, the left-hand side of (5) need not be monotonic in \( b \) (see the remark in the Proof of Proposition 1.)
Proof of Proposition 4. It remains to prove the result for the case in which the first best is not attainable, implying that \( s^* > s_{FB} \) and \( w = \underline{w} \). From the definitions of \( \mu_H \) and \( \mu_L \) we have that \( p_H := (\mu_H - x_l)/(x_h - x_l) \) and \( p_L := (\mu_L - x_l)/(x_h - x_l) \), respectively, implying that

where \( s \geq s_{FB} \) by the integral boundaries in (26). The inequality in (27) follows from the fact that the first term in brackets is positive for \( s > s_{FB} \) by MLRP while the second term in brackets is negative by MLRP and \( ds_{FB}/d\pi < 0 \). In conjunction with (25) and (26) this finally implies that the first-best optimal bonus \( b_{FB} \) is decreasing in \( \pi \).

Consider next the case where the first best is not attainable, implying that \( s^* > s_{FB} \) and \( w = \underline{w} \). Analogous to the Proof of Proposition 2, we will now show that, holding \( b \) constant, an increase in \( \pi \) causes the incentive constraint (5) to be slack, implying that the firm optimally decreases \( b \) until (5) binds again. To prove this, it is convenient to use \( p = p_A(s) = p_L + \pi(s)(p_H - p_L) \) as the random variable instead of \( s \). The cumulative distribution function of \( p \) is

\[
G(\tilde{p}) := \Pr(p \leq \tilde{p}) = \Pr\left(1 + \frac{1}{1 + f_a(s) 1 - \pi \frac{\hat{p} - p_L}{p_H - p_L}}\leq \frac{\tilde{p} - p_L}{p_H - p_L}\right),
\]

(28)

where we used the definition of \( \pi(s) \) from (1). By inspection, for any given \( \tilde{p} \) an increase in \( \pi \) decreases \( G(\tilde{p}) \), implying that it causes a First-Order Stochastic Dominance shift in \( G(p) \). Instead of casting the firm’s decision problem in terms of an ex-post optimal cutoff shift in \( G(p) \), the problem is now cast in terms of an ex-post optimal cutoff probability \( p^* \). Importantly, holding the bonus \( b \) constant, the firm’s ex-post optimal cutoff probability \( p^* \) remains independent of \( \pi \), which is why it is more convenient to work with \( p \) instead of \( s \).27 Given this transformation, the incentive constraint (5) becomes

\[
\int_{p \geq p^*} [pb + w - M]dG(p) \geq c - (M - m).
\]

(29)

Holding \( b \) constant, a First-Order Stochastic Dominance shift in \( G(p) \) relaxes (29), implying that an increase in \( \pi \) renders the (previously binding) incentive constraint (5) slack, which in turn implies that the firm optimally decreases \( b \) until (5) binds again. Like in the Proof of Proposition 2, a decrease in \( \pi \) works the opposite way: it violates the (previously binding) incentive constraint, implying that the firm must increase \( b \) until (5) binds again. Q.E.D.

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27That \( p^* \) is independent of \( \pi \) is easy to show. Substituting \( \pi(s) \) from \( p_A(s) = p_L + (p_H - p_L)\pi(s) \) into \( p_B(s) = p_L + (p_H - p_L)(1 - \pi(s)) \) and using \( w(s) = \underline{w} + bp \) (recall that \( w = \underline{w} \)), equation (4) becomes \( p^* = [(p_H + p_L)(x_h - x_l) + \underline{w} - M]/[2(x_h - x_l) - b] \), which is independent of \( \pi \).
We consider a marginal decrease in payoff risk; the argument for an increase in payoff risk is analogous. We use the notation \( x_l, x_h \), and \( p_A(s) \) for the original, i.e., riskier payoff distribution and \( \tilde{x}_l, \tilde{x}_h \), and \( \tilde{p}_A(s) \) for the new, i.e., safer payoff distribution, where \( \tilde{x}_h < x_h \) and \( \tilde{x}_l \geq x_l \). Let \((w, b)\) be the optimal wage contract under the original payoff distribution. To implement the same cutoff signal \( s^* \) as under the original distribution the bonus must change to \( \tilde{b} = b p_A(s^*) / \tilde{p}_A(s^*) \), where \( \tilde{b} < b \) since \( \tilde{p}_A(s^*) > p_A(s^*) \).

We will now show that the incentive constraint (5) is either satisfied with equality, in which case \( b \) and \( s^* \) are indeed optimal after the decrease in payoff risk, or that (5) is slack, in which case the optimal bonus and cutoff signal are even lower than \( b \) and \( s^* \), implying that the optimal bonus is surely lower than \( b \). Inserting \( \tilde{p}_A(s) \) and \( \tilde{b} \) into the incentive constraint (5) while taking into account that this constraint binds under the original payoff distribution, we obtain

\[
\int_{s^*}^\pi \left( \frac{\mu_A(s^*) - x_l}{\mu_A(s^*) - \tilde{x}_l} - \frac{\mu_A(s) - x_l}{\mu_A(s) - \tilde{x}_l} \right) f(s) ds \geq 0. \tag{30}
\]

If \( \tilde{x}_l = x_l \) the incentive constraint (30) holds with equality, implying that \((w, \tilde{b})\) with \( \tilde{b} < b \) is indeed optimal after the decrease in payoff risk. By contrast, if \( \tilde{x}_l > x_l \) then (30) is slack as \( \mu_A(s) \) is strictly increasing in \( s \). In this case, we know from the Proof of Proposition 2 that the firm optimally decreases \( b \) until (30) binds again, implying that the optimal bonus is even lower than \( \tilde{b} \) and thus surely than \( b \). Q.E.D.

**Proof of Proposition 5.** Consider first the at-will contract. If the first best is attainable, then the optimal wage contract is uniquely determined by (9) and (10), neither of which depends on \( N \). Moreover, the condition determining whether the first best is attainable, namely, the base-wage constraint \( w \geq \underline{w} \), does also not depend on \( N \).

If the first best is not attainable, implying that \( w = \underline{w} \) then the incentive constraint (5) does still not depend on \( N \) but the firm’s ex-post optimal cutoff signal \( s^* \) does. The cutoff signal is now given by

\[
\gamma(N) \left[ \mu_A(s^*) - \mu_B(s^*) \right] = N \left[ w(s^*) - M \right], \tag{31}
\]

where we used the fact that the firm’s scale of operations increases by \( \gamma(N) \). Note that if \( \gamma(N) = N \) then, holding \( b \) constant, the cutoff signal \( s^* \) is again independent of \( N \). However, if \( \gamma(N)/N \) is increasing in \( N \) then, holding \( b \) constant, (31) implies that \( s^* \) is decreasing in \( N \). From our previous arguments we then know that this relaxes the incentive constraint (5), with the result that the firm optimally decreases \( b \), implying that \( s^* \) decreases even further. To
make this dependency explicit, we write the firm’s ex-post optimal cutoff signal as $s^*(\eta)$, where

$$\eta := \gamma(N)/N.$$ 

Also, define

$$\omega(s^*) := x_l + (x_h - x_l) \left[ \int_{s^*}^{s^* + \eta} p_B(s)f(s)ds + \int_{s^*}^{s^* + \eta} p_A(s)f(s)ds \right] - c - m > 0.$$ 

In sum, if the first best is not attainable under an at-will contract, then the firm’s expected net payoff for given $N$ equals $\Pi_1(N) = \gamma(N)\omega(s^*)$, where $s^* = s^*(\eta)$.

Consider finally a long-term contract. From the analysis in the main text we know that for sufficiently large $N$ the firm’s expected net payoff equals $\Pi_2(N) = \gamma(N)\omega(s_{FB}) - NR(N)$, where $R(N)$ is the rent per employee from (16). We also know that $R(N)$ is strictly increasing in $N$ and that it grows beyond bounds as $N$ becomes large. It remains to show that for sufficiently large $N$ it holds that $\Pi_1(N) > \Pi_2(N)$, which translates into the requirement that

$$R(N) > \eta [\omega(s_{FB}) - s^*(\eta)],$$

which holds by the properties of $R(N)$ and as $\eta$ is bounded from above by $K$. Q.E.D.

8 References


