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Competitive search markets for durable goods*

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Summary. This paper considers a dynamic version of Akerlof's (1970) lemons problem where buyers and sellers must engage in search to find a trading partner. We show that if goods are durable, the market itself may provide a natural sorting mechanism. In equilibrium, high-quality goods sell at a higher price than low-quality goods but also circulate longer. This accords with the common wisdom that sellers who want to sell fast may have to accept a lower price. We then compare the equilibrium outcomes under private information with those under complete information. Surprisingly, we find that for a large range of parameter values the quilibrium outcomes under the two information regimes coincide, despite the fact that circulation time is used to achieve separation.

Keywords and Phrases: Lemons problem, Durable goods, Search markets.
JEL Classification Numbers: D82, D83, L15.

1 Introduction

Since the seminal work by Akerlof (1970), there has been an abundance of contributions dealing with the lemons problem. Typically, these models involve the use of a sorting variable such as, e.g., education (Spence, 1973), insurance deductibles (Rothschild and Stiglitz, 1976), or warranties (Grossman, 1980).[1] If

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[1] More recently, several models have emphasized the role of institutional innovations in ameliorating the lemons problem. Papers falling into this category are, e.g., Taylor (1999) (on quality

the marginal rate of substitution between money and the sorting variable differs for different types of informed agents ("single-crossing property"), the sorting variable can be used to separate informed agents according to their type. In many practical situations of interest, however, explicit sorting variables either do not exist or are not used for whatever reasons. For instance, in the market for second-hand cars, warranties typically play no role, except perhaps when the seller is a professional car dealer.

In this paper we show that for certain goods the market itself may provide a natural sorting mechanism, thus making an explicit sorting variable unnecessary. We do this by considering a model of decentralized trade in a search market environment. Following recent developments in the theory of search and matching (see, for instance, Mortensen and Pissarides, 1998), we allow for the co-existence of different trading environments or submarkets where different prices prevail.[2] Moreover, we restrict consideration to durable goods which, by definition, allow the potential seller to derive (flow) utility while searching for a buyer. For instance, in the case of second-hand cars, car owners can usually drive their car until it is sold.

We show that the trade of durable goods in a search market environment naturally induces separation. In equilibrium, high-quality goods sell at a higher price than low-quality goods but also circulate longer. As a consequence, we find that the major inefficiency commonly associated with the lemons problem, viz., that the number of trades may be less than the number of agents on the short side of the market, disappears in a durable goods market. More precisely, if search frictions are sufficiently low, the short side of the market is always fully served. Surprisingly, we find that the equilibrium under private information may coincide with that where quality is observable, despite the fact that search time is used to achieve separation. This leads us to conclude that decentralized markets for (used) durable goods may perform surprisingly well once a dynamic perspective is taken.

Technically, our paper adds to the growing literature on the co-existence of different trading environments. The primitives of our model are the time invariant flows of prospective buyers and sellers arriving at the market fringe. Prospective buyers and sellers entering the (grand) market subsequently choose between one of several submarkets. Similar settings have been examined by, e.g., Shimer (1996), Moen (1997), Mortensen and Wright (1997), and Mortensen and Pissarides (1998).[3] In contrast to our model, however, these models do not consider private information.[4]

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inspections), Hendel and Lizerri (1999a) (on the interaction between the new and used car market), and Waldman (1999) and Hendel and Lizerri (1999b) (on leasing contracts).

[2] For a recent overview of search theory see McMillan and Rothschild (1994).

[3] While Gale (1992) and Peters (1997) also allow for the co-existence of different submarkets, their approach differs from both our approach and the approach in the above papers in that Gale and Peters consider a one-shot setting where all trade must take place instantaneously.

[4] Albrecht and Vroman (1992) also study a search market with private information. In their model, however, there is only a single market.

The idea that delay or the probability of trade can be used to sort between different types of sellers has been recognized earlier. However, existing contributions consider only consumption goods. For instance, Evans (1989) and Vincent (1989) consider a potential seller of a consumption good that must yet be produced. The important assumption is that high-quality goods are more costly to produce than low-quality goods, which makes low-quality sellers more impatient to trade.[5] Clearly, delay loses its sorting power with respect to consumption goods that have already been produced (all sellers then have the same marginal rate of substitution between time and price). In this case, the probability of trade may act as a sorting device if the number of trading rounds is limited (see, e.g., Wilson 1980). If all goods are traded eventually, however (e.g., because there are more buyers than sellers and the number of trading rounds is *not* limited), the probability of trade also loses its power as a sorting device. As is shown in our model, this is different if we consider durable goods.

Our paper is also related to recent work by Janssen and Roy (1999). In this paper, the authors show that in a Walrasian market sellers may sort themselves by selling in different time periods. All sellers enter the market at time zero. Over time, the market gradually empties. From a cross-sectional perspective, this implies that in any given point in time there exists at most a single price in the market.[6] By contrast, in our model sellers of different quality sort themselves by trading in different search market environments, implying that in each period different prices naturally coexist. Even more important, we consider a search market environment where delay is basically unavoidable. Indeed, for a given search or matching technology, speeding up trade for one side (e.g., sellers) naturally increases the expected delay for the other side (e.g., buyers). We demonstrate that this feature is essential for our efficiency results, which, incidentally, are absent in the paper by Janssen and Roy.

The rest of the paper is organized as follows. Section 2 presents the model and defines the equilibrium concept. As a first benchmark, Section 3 derives the set of competitive equilibria if trade takes place instantaneously, while Section 4 solves the search market model. Section 5 compares the derived equilibria with the benchmark of complete information. Section 6 concludes with possible extensions.

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[5] A similar setup also underlies the dynamic auction game by Vincent (1990) where buyers compete for a good of unknown quality. Delay also plays an important role in the literature on private values bargaining with incomplete information. See, e.g., Fudenberg and Tirole (1983), Sobel and Takahashi (1983), Cramton (1984), Fudenberg, Levine, and Tirole (1985), and Gul, Sonnenschein, and Wilson (1986). In the present paper, however, we only consider common values environments.

[6] There may be also periods where no good is offered at all; Janssen and Roy refer to this as “break in trading”. In a follow-up paper, Janssen and Roy (2000) allow for cohorts of new sellers arriving in later periods. Still, it remains true that in any given period there is at most a single price in the market.

2 The model

2.1 Agents and preferences

Agents are endowed with either zero or one unit of an indivisible good. We frequently refer to agents who are endowed with zero units as potential buyers and agents who are endowed with one unit as potential sellers. The good comes in two qualities $q \in Q := \{l, h\}$. All agents are risk neutral and discount future payoffs at the same rate $r > 0$. The model is set in continuous time with an infinite time horizon. Potential sellers derive a constant flow utility v_q from using the good. If the good is sold, the seller's flow utility from using the good is zero. Likewise, the flow utility of a potential buyer before and after the purchase is zero and u_q, respectively. We assume that for both types of agents the flow utility from a high-quality good is greater than the flow utility from a low-quality good, i.e., $v_h > v_l > 0$ and $u_h > u_l > 0$. Moreover, we assume that for both types of goods there are strictly positive gains from trade, i.e., $u_h > v_h$ and $u_l > v_l$. Finally, any agent can derive utility from at most one unit of the good at any point in time. For instance, if the good is a car, this means that agents can derive utility from driving the car, but not from owning it per se.

The *asset value* of a good is defined as the discounted stream of utilities from using the good until the indefinite future. Accordingly, the asset value of a good of quality q for a potential seller is $V_q := v_q/r$, and the corresponding asset value for a potential buyer is $U_q := u_q/r$.

During one unit of time (what constitutes a time unit is implicitly defined by the discount rate r), the measure one of agents appears at the market fringe and may possibly enter the market.[7] Of these agents, a fraction $b \in (0, 1)$ are potential buyers. The fraction $1 - b = s$ of potential sellers is divided further into a fraction $s_h > 0$ of owners of high-quality goods and a fraction $s_l > 0$ of owners of low-quality goods. The quality of a good is private information. For simplicity, we restrict attention to generic parameter values $b \neq s$, $b \neq s_q$ for all $q \in Q$, and $v_h \neq u_l$. Agents arriving at the market fringe can choose whether to enter the market or stay outside. If they do not enter, their utility is determined by their initial endowment.

2.2 Matching technology and asset value equations

The good is traded in a search market where potential buyers are matched with potential sellers. While searching, agents incur a time-invariant search cost of $c > 0$. The market consists of a continuum of potential buyers and sellers, which implies that the probability of trade is determined by the respective measures of agents engaging in search. We restrict attention to stationary equilibria where the stock of agents is constant over time (see condition (E.2) below). The measure of potential buyers in the market is denoted by β, and the measure of potential

[7] This is similar to Mortensen and Wright (1997).

sellers of type q is denoted by σ_q. The measure of buyer-seller matches per unit of time is expressed by the matching function $x(\beta,\sigma)$, where $\sigma = \sigma_l + \sigma_h$. Following standard assumptions, the matching function is continuous and homogeneous of degree one in both arguments. The transition rates for potential buyers and sellers are then $f(k) := x(\beta,\sigma)/\beta = x(1,1/k)$ and $x(\beta,\sigma)/\sigma = f(k)k$, respectively, where $k := \beta/\sigma$ denotes the "market tightness" from the perspective of potential buyers.[8] Define $g(k) := f(k)k$. Following again standard assumptions, $f(k)$ is strictly decreasing in k with limits $\lim_{k\to 0} f(k) = \infty$ and $\lim_{k\to\infty} f(k) = 0$, and $g(k)$ is strictly increasing in k with limits $\lim_{k\to 0} g(k) = 0$ and $\lim_{k\to\infty} g(k) = \infty$.[9] The search market is fully characterized by the tightness k, the distribution of offered qualities $\pi(q) := \sigma_q/\sigma$, and the prevailing price p. Note that by restricting attention to prices, we implicitly rule out more complicated contractual arrangements where, e.g., buyers are given an option to sell back the good after they have observed its quality.

Denote by $V_q^M(k,p)$ the utility of a potential seller of a good of quality q in a market with tightness k and price p. The asset value (or continuous-time Bellman) equation for $V_q^M(k,p)$ is then

$$rV_q^M(k,p) = -c + v_q + g(k)\left[p - V_q^M(k,p)\right]. \tag{1}$$

Equation (1) is intuitive. While searching for a potential buyer, a potential seller of type q incurs search cost of c and derives flow utility of v_q from using the good. With probability rate $g(k)$, which depends on the market tightness k, a buyer is found, in which case the good is exchanged at the price p.[10] Rearranging (1) yields

$$V_q^M(k,p) = \frac{v_q - c + g(k)p}{r + g(k)}. \tag{2}$$

Similarly, denote by $U^M(\pi,k,p)$ the utility of a potential buyer. As the good comes in different qualities, $U^M(\pi,k,p)$ depends on the distribution of qualities in the market. The asset value equation for $U^M(\pi,k,p)$ is

$$rU^M(\pi,k,p) = -c + f(k)\left[\sum_{q\in Q}\pi(q)U_q - p - U^M(\pi,k,p)\right],$$

which can be rearranged as

$$U^M(\pi,k,p) = \frac{-c + f(k)\left[\sum_{q\in Q}\pi(q)U_q - p\right]}{r + f(k)}. \tag{3}$$

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[8] The *market tightness* is an indirect measure of the expected time that goods must circulate before trade takes place. As the tightness is defined from the perspective of potential buyers, a greater tightness implies a shorter circulation time.

[9] On occasion, it will be convenient to assume additionally that f and g are differentiable.

[10] Agents will find it optimal to trade if and only if p satisfies $\sum_{q\in Q}\pi(q)U_q - p \geq 0$ and $p \leq V_q$ for all q with $\pi(q) > 0$.

2.3 Equilibrium conditions

The “grand” market consists of several submarkets indexed by natural numbers $n \in N = \{1, 2, ..., \overline{n}\}$. Each submarket constitutes an independent search environment with corresponding submarket price p^n.[11] As an illustration, consider the case of used cars referred to in the Introduction. Used cars are frequently offered via classified ads in newspapers or on the internet. There, cars of a given type are typically grouped into different clusters according to their respective bid or ask price.

We denote the time invariant measures of agents in marke n by σ_q^n and β^n respectively. Without loss of generality, we restrict ourselves to “active” submarkets populated by both buyers and sellers. An immediate implication of this is that $k^n = \beta^n/\sigma^n$ and $\pi^n(q) = \sigma_q^n/\sigma^n$, where $\sigma^n = \sigma_l^n + \sigma_h^n$. Both potential sellers and buyers must decide i) whether or not to enter the grand market, and ii) if entry occurs, which of the n submarkets to enter.[12]

We now present the equilibrium concept. The equilibrium conditions closely resemble those in Moen (1997), Mortensen and Wright (1997), and Mortensen and Pissarides (1998), except that our conditions apply to markets with private information. Given some set of active submarkets N, define by

$$U^* := \max\left\{0, \max_{n\in N} U^M(\pi^n, k^n, p^n)\right\}$$

and

$$V_q^* := \max\left\{V_q, \max_{n\in N} V_q^M(k^n, p^n)\right\}$$

the utility of potential buyers and sellers of type q, respectively, when choosing optimally between their outside option and the submarket with the highest utility.

Similarly, for all $n \in N$ define by

$$U^*_{N\setminus\{n\}} := \max\left\{0, \max_{n'\in N\setminus\{n\}} U^M(\pi^{n'}, k^{n'}, p^{n'})\right\}$$

and

$$V^*_{q,N\setminus\{n\}} := \max\left\{V_q, \max_{n'\in N\setminus\{n\}} V_q^M(k^{n'}, p^{n'})\right\}$$

the utility of potential buyers and sellers of type q, respectively, when choosing optimally between their outside option and the submarket with the highest utility which is different from submarket n. As agents can choose between different

[11] By requiring that $p^n \neq p^{n'}$ for any pair $n, n' \in N$, $n \neq n'$, we may rule out the possibility of redundant markets. However, unless one imposes additional restrictions on the matching technology, this will typically not suffice to ensure uniqueness.

[12] We implicitly assume that once a submarket is entered, agents must stay in this submarket until trade occurs. This assumption is only restrictive if agents are indifferent between different submarkets or between entering and not entering the grand market, in which case they may want to switch back and forth between submarkets or between the grand market and their outside option. Allowing for this possibility is straightforward but does not generate any new results.

search environments but face frictions to find a trading partner, this type of model is often referred to as one of "directed search".

As we restrict ourselves to stationary equilibria, the measures of potential buyers and sellers of type q in submarket n are time-invariant. Denote these measures by b^n and s_q^n, respectively. The first equilibrium condition requires that entry be optimal.

(E.1) Optimality. The decision to enter a submarket must be optimal. Hence the measure of potential sellers of type q entering submarket $n \in N$ must satisfy

$$s_q^n = \begin{cases} 0 & \text{if } V_q^M(k^n, p^n) < V^*_{q,N\setminus\{n\}} \\ s_q & \text{if } V_q^M(k^n, p^n) > V^*_{q,N\setminus\{n\}} \\ \in \left[0, s_q\right] & \text{if } V_q^M(k^n, p^n) = V^*_{q,N\setminus\{n\}}, \end{cases}$$

where $\sum_{n\in N} s_q^n \leq s_q$.

Likewise, the measure of potential buyers entering submarket n must satisfy

$$b^n = \begin{cases} 0 & \text{if } U^M(\pi^n, k^n, p^n) < U^*_{N\setminus\{n\}} \\ b & \text{if } U^M(\pi^n, k^n, p^n) > U^*_{N\setminus\{n\}} \\ \in [0, b] & \text{if } U^M(\pi^n, k^n, p^n) = U^*_{N\setminus\{n\}} \end{cases}$$

for all $n \in N$, where $\sum_{n\in N} b^n \leq b$.

Note that (E.1) takes care of the fact that the measure of agents entering the grand market cannot exceed the measure of agents arriving at the market fringe. We now come to the second equilibrium condition: stationarity.

(E.2) Stationarity. In each submarket, the flow of entries must equal the flow of exits, i.e., $s_q^n = \sigma_q^n g(k^n)$ for all $q \in Q$ and $b^n = \beta^n f(k^n)$ for all $n \in N$.

Our third and last requirement is that markets be competitive. The equilibrium condition is adopted from Mortensen and Wright (1997) and Mortensen and Pissarides (1998). Basically, it says that it must not be profitable for middlemen to open up new submarkets. Middlemen can open up new submarkets by announcing a pair (p, k) which potential entrants then compare with the price and tightness prevailing in existing submarkets.[13,14] As Mortensen and Wright (1997) and Mortensen and Pissarides (1998) consider complete information environments, we augment their condition by adding the requirement that agents deciding whether to enter a particular submarket must form rational expectations about the distribution of qualities in that submarket.

[13] A crucial assumption underlying this concept of competitiveness is that deviating submarkets are negligible in size. As a consequence, opening up a new submarket does not affect the equilibrium utilities prevailing in the existing set of submarkets.

[14] Note that by offering pairs (p, k), middlemen can avoid the well-known coordination failure according to which profitable new submarkets do not open up as agents from either side have (rational) beliefs that they will not find a counterparty to trade with. An alternative approach would be to assume that each price is associated with a unique value of k. For more on this, see Gale (1996).

(E.3) Competitiveness. There must not exist a pair $(p,k) \neq (p^n,k^n)$ such that $V_q^M(p,k) > V_q^*$ for some $q \in Q$ and $U^M(\pi,k,p) > U^*$ for all $\pi \in \Delta_{Q'}$, where $\Delta_{Q'}$ is the set of probability distributions satisfying $\pi(q) = 0$ for all $q \notin Q'$, where Q' is defined as $Q' := \left\{q \in Q \mid V_q^M(p,k) > V_q^*\right\}$.[15]

Alternatively, (E.3) could be replaced by the following requirement adopted from Moen (1997). Suppose buyers could open up new submarkets by posting a new price p. For any such price, one can then determine the set of pairs (k,π) that are consistent with p in the following sense. If $\pi(q) > 0$ for some q, sellers of this type must enter the new submarket until the tightness k satisfies $V_q^M(k,p) = V_q^*$. On the other side, if $\pi(q) = 0$, the tightness must adjust to ensure that sellers of this type weakly prefer to stay out of the submarket, i.e., $V_q^M(k,p) \leq V_q^*$. It can be shown that this alternative requirement yields identical results.[16]

Observe that the informational requirements underlying (E.3) are minimal as only a negligible fraction of market participants needs to know about the existence of new submarkets. By contrast, for (E.1) to be satisfied all agents must have perfect foresight about the price and market tightness in each of the $\overline{n}$ submarkets.

To summarize, a *search market equilibrium* consists of a set N of active submarkets with characteristics $(p^n, \beta^n, \left\{\sigma_q^n\right\}_{q\in Q})$ satisfying conditions (E.1)-(E.3). As a benchmark, we first derive the set of competitive equilibria for a static economy where prices are set by a Walrasian auctioneer to equate supply and demand. In Section 4, we then continue with our analysis of search market equilibria.

3 Standard competitive analysis

Consider a static version of the model in which all trade takes place instantaneously. The economy is populated by a measure one of agents, of which a fraction $b \in (0,1)$ constitutes potential buyers. The fraction $1-b = s$ of potential sellers is divided further into a fraction $s_h > 0$ of owners of high-quality goods and a fraction $s_l > 0$ of owners of low-quality goods. The utilities of potential sellers and buyers from the good are v_q and u_q, respectively, where $q \in Q$. The supply correspondence is then

$$S(p) = \begin{cases} 0 & \text{if } p < v_l \\ \in [0, s_l] & \text{if } p = v_l \\ s_l & \text{if } v_l < p < v_h \\ \in [s_l, s] & \text{if } p = v_h \\ s & \text{if } p > v_h. \end{cases}$$

[15] In fact, it makes no difference whether the inequality in the definition of Q' is strict or weak.

[16] For a formal analysis, we refer the reader to the working paper version (Inderst and Müller 1999). The analysis there is similar in spirit to the (refinement) approach by Gale (1996).

Clearly, whenever a positive fraction of the high-quality good is supplied, the entire fraction of the low-quality good must be supplied as well. For a given value $S > 0$, the distribution of qualities is therefore $\pi(h,S) = \max\{0, S - s_l\}/S$ and $\pi(l,S) = 1 - \pi(h,S)$. Hence the demand correspondence is

$$D(p,\pi) = \begin{cases} 0 & \text{if } \pi(l)u_l + \pi(h)u_h < p \\ \in [0,b] & \text{if } \pi(l)u_l + \pi(h)u_h = p \\ b & \text{if } \pi(l)u_l + \pi(h)u_h > p. \end{cases}$$

A *competitive equilibrium* is a triple (D, S, p) such that $D = S \geq 0$, $S \in S(p)$, and $D \in D(p, \pi(S))$. Recall that we restrict attention to generic parameter values. We can then distinguish between three cases. The first two cases represent buyer markets ($s > b$), whereas the third case represents a seller market ($b > s$). Since the analysis is standard, we confine ourselves to summarizing the results.

Case 1 ($s_l > b$). There exists a unique equilibrium where all potential buyers purchase the low-quality good at the price $p = v_l$.

Case 2 ($s > b > s_l$). We can distinguish between three subcases. If $v_h < u_l$, there are gains from trade regardless of the distribution of qualities. In this case, there exists a unique equilibrium where both low- and high-quality goods are traded at $p = v_h$. In this equilibrium, the measure s_l of buyers purchases the low-quality good, and the measure $b - s_l$ of buyers purchases the high-quality good. If $v_h > u_l$ and $s_l u_l + (b - s_l)u_h < bv_h$, there exists a unique equilibrium where only the low-quality good is traded at the price $p = u_l$. The measure $b - s_l$ of potential buyers does not trade despite the presence of high-quality sellers in the market. Finally, if $v_h > u_l$ and $s_l u_l + (b - s_l)u_h \geq bv_h$, there exist two equilibria. Either only the low-quality good is traded at $p = u_l$, or both the low- and high-quality good are traded at the price $p = v_h$.

Case 3 ($b > s$). The analysis is similar to Case 2. We can again distinguish between three subcases. If $v_h < u_l$, there exists a unique equilibrium where all goods are traded at the price $p = (u_l s_l + s_h u_h)/s$, which implies that potential buyers are forced down to their reservation utility of zero. Second, if $v_h > u_l$ and $s_l u_l + s_h u_h < s v_h$, there exists a unique equilibrium where only the low-quality good is traded at the price $p = u_l$. In this case, the measure s_h of high-quality sellers does not trade despite the presence of potential buyers in the market. Finally, if $v_h > u_l$ and $s_l u_l + s_h u_h \geq s v_h$, there exist three equilibria. Either only the low-quality good is traded at $p = u_l$, or both the low- and high-quality good are traded at the price $p = (u_l s_l + s_h u_h)/s$, or $p = v_h$, in which case high-type sellers are indifferent. In the last case, we can then specify that only the measure $0 < \overline{s}_h < s_h$ of high-type sellers trades. To make buyers indifferent, it must then be true that $(s_l u_l + \overline{s}_h u_h)/(s_l + \overline{s}_h) = v_h$. It can be checked that this equation has a solution $\overline{s}_h < s_h$ if $s_l u_l + s_h u_h > s v_h$.

To summarize, for a large range of parameter values there are gains from trade which remain unexhausted in equilibrium. In particular, if $v_h > u_l$, a strictly positive fraction of the short side of the market does not trade (Cases 2 and 3). This is different if we consider a search market environment. As we show in the following section, all members of the short side of the market engage in trade if search costs are sufficiently low.

4 Competitive search market equilibria

We begin with a characterization of the equilibrium utilities for the long side of the market (in terms of potential entrants). Suppose first that potential buyers outnumber potential sellers ($b > s$). Since buyers and sellers must exit the market in pairs, potential buyers must be indifferent between entering and not entering the grand market to ensure that the stock of agents in the market remains constant (this follows immediately from combining (E.1) with (E.2)). Consequently, all potential buyers must receive their reservation utility of zero.

Next, consider the case where sellers outnumber buyers ($s > b$). By (2), we have

$$\left(V_l^M(k,p) - V_l\right) - \left(V_h^M(k,p) - V_h\right) = \frac{g(k)\,(v_h - v_l)}{r(r + g(k))} > 0, \tag{4}$$

which implies that if high-quality sellers weakly prefer to enter the market, low-quality sellers must strictly prefer to enter. Accordingly, if $s_l > b$, potential sellers of the low-quality good must be indifferent between entering and not entering the market, whereas potential sellers of the high-quality good must strictly prefer not to enter. Consequently, all potential sellers must receive their reservation utility of V_q, where $q \in Q$. On the other hand, if $s > b > s_l$, potential sellers of the high-quality good must be indifferent between entering and not entering the market, which implies that they must receive their reservation utility of V_h. The following lemma summarizes these results.

Lemma 1. *In equilibrium, the following must hold:*

Case 1 ($s_l > b$). Potential sellers obtain their reservation utility of V_q, where $q \in Q$.
Case 2 ($s > b > s_l$). Potential sellers of the high-quality good obtain their reservation utility of V_h.
Case 3 ($b > s$). Potential buyers obtain their reservation utility of zero.

Given a transition rate $g(k)$, the expected search time of potential sellers is $T := 1/g(k)$. The expected utility from search $V_q^M(k,p)$ is then $\left(T(v_q - c) + p\right) / (Tr + 1)$, which implies that the marginal rate of substitution between search time and sales price depends on the good's quality and equals

$$\frac{dp}{dT} = -\frac{v_q - c - rp}{Tr + 1}. \tag{5}$$

Hence, for a given increase in expected search time, high-quality sellers must be compensated with a smaller increase in price than low-quality sellers. Recall from our introductory remarks that this condition holds only with respect to durable goods. If the good is a consumption good, potential sellers derive no flow utility from the good while engaging in search. Consequently, the marginal rate of substitution between search time and price is the same for both types of sellers.

It is now straightforward to show that there cannot exist a submarket which attracts more than one type of seller.

Lemma 2. *In equilibrium, there cannot exist a pooling submarket. Moreover, in any separating equilibrium where submarket n attracts high-quality sellers and submarket m attracts low-quality sellers, it must hold that* $p^n > p^m$ *and* $k^n < k^m$.

Proof. Consider first the claim that there cannot exist a pooling submarket. Suppose to the contrary that submarket $n \in N$ is pooling, implying that $\pi^n(q) > 0$ for $q \in Q$. We show that this violates (E.3). Define for all $V \geq V_q$ and $p > V$ the tightness $k_q(p, V)$ which ensures that potential sellers of type q stay on their indifference curve $V_q^M(k, p) = V$, implying that

$$g\left(k_q(p, V)\right) = \frac{rV + c - v_q}{p - V}.$$

Observe that $k_q(p, V)$ is unique and continuous with respect to both arguments. Next, take some price $p = p^n + \varepsilon$, where $\varepsilon > 0$, and set $k_\varepsilon := k_l(p^n + \varepsilon, V_h^*)$. From (5) and the fact that $V_q^M(k^n, p^n) = V_q^*$ for $q \in Q$, it follows that $V_h^M(k_\varepsilon, p^n + \varepsilon) > V_h^*$. Moreover, since $\pi(l) > 0$ and $u_h > u_l$, it must also hold that $U^M(\pi^n, k^n, p^n) < U^M(\pi, k^n, p^n) = U^*$, where $\pi(l) = 0$. Hence, by continuity there must exist sufficiently small values $\varepsilon, \varepsilon' > 0$ such that $p' = p^n + \varepsilon - \varepsilon'$, $k = k_\varepsilon$, and $\pi(l) = 0$ together imply that $V_h^M(k, p') > V_h^*$, $V_l^M(k, p') < V_l^*$, and $U^M(\pi, k, p') > U^*$, contradicting (E.3).

Consider next the second claim. We argue to a contradiction and assume that $p^n \leq p^m$. By buyer optimality, this implies that $k^n < k^m$. To ensure seller optimality, (E.2) requires that $V_h^M(k^n, p^n) \geq V_h^M(k^m, p^m)$ and $V_l^M(k^m, p^m) \geq V_l^M(k^n, p^n)$. However, given the sorting condition (5) and $k^n \geq k^m$, these two inequalities cannot be jointly satisfied. Hence it must be true that $p^n > p^m$. But this implies that $k^n < k^m$ as otherwise *all* sellers would strictly prefer submarket n to submarket m. □

Lemma 2 follows immediately from condition (5). In particular, it states that in any equilibrium where both high- and low-quality goods are traded, high-quality goods must sell at a higher price but also circulate longer than low-quality goods. Whether it is actually true that both types of goods are traded depends on the initial distribution of buyers and sellers in the population. In what follows, we provide a full characterization of the set of competitive search equilibria for each of the three cases analyzed in Section 3. For expositional clarity, the results are stated in three separate propositions. If search is too costly, there may be no trade at all. In all three propositions, we therefore assume that search costs are sufficiently low.

We begin with Case 1. We obtain the intuitive result that high-quality goods are never traded if the supply of low-quality goods already exceeds the potential demand. The outcome is thus the same as in the standard analysis in Section 3.[17]

Proposition 1. *If $s_l > b$ and search costs are sufficiently low, an equilibrium exists. Moreover, in any equilibrium only low-quality goods are traded. More precisely,* $\sum_{n \in N} b^n = b$, $\sum_{n \in N} s_l^n = b < s_l$, *and* $\sum_{n \in N} s_h^n = 0$.

Proposition 1 is proved in the Appendix. Note that the proposition is silent about the precise values of p^n and k^n prevailing in equilibrium. In the existence proof in the Appendix, we show that any pair (p^n, k^n) must solve a particular program. In that program, (p^n, k^n) is chosen to maximize the utility of potential buyers $U^M(\pi, k, p)$, where $\pi(l) = 1$, subject to the binding participation constraint of the low-quality seller $V_l^M(k, p) = V_l$. We will have more to say about equilibrium prices and transition rates in Section 5 where we discuss welfare issues.

Next, consider Case 2. If there are more potential buyers than low-quality sellers, the equilibrium outcome changes dramatically. In particular, it is then always true that i) both low- and high-quality goods are traded, and ii) all agents on the short side of the market (here: potential buyers) engage in trade.

Proposition 2. *If $s > b > s_l$ and search costs are sufficiently low, an equilibrium exists. Moreover, any equilibrium exhibits the following characteristics:*

i) Both low- and high-quality goods are traded. More precisely, $\sum_{n \in N} b^n = b$, $\sum_{n \in N} s_l^n$ $= s_l$, *and* $\sum_{n \in N} s_h^n = b - s_l < s_h$.
ii) The set of submarkets is fully separating.
iii) High-quality sellers obtain their reservation utility of $V_h^* = V_h$. The equilibrium utilities of buyers and low-quality sellers are uniquely determined and satisfy $U^* > 0$ and $V_l^* > V_l$, respectively.

The proof of Proposition 2 is relegated to the Appendix. While Proposition 2 suggests that embedding the lemons problem in a dynamic framework may resolve the problem that some agents on the short side of the market are rationed, the solution *may* come at a cost: to achieve separation high-quality goods must circulate longer than low-quality goods. By condition (E.3), which states that markets be competitive, separation must now be achieved at least costs for sellers of high-quality goods. Proposition 2 is illustrated in the following diagram.

For an interpretation of Figure 1, consider first the indifference curve of high-quality sellers. Recall that a seller prefers both a higher price and a higher tightness $k = \beta/\sigma$, as this reduces expected search costs. Hence, his indifference curve must be strictly decreasing. It is denoted by $V_h^M(k, p) = V_h^*$. For the

[17] Recall that the gains from trading either type of good are strictly positive. To ensure that the market operates, these gains must not be fully offset by the total expected search costs incurred by the buyer and seller. While this is clearly the case for $c = 0$, it also holds for all sufficiently low values of $c > 0$ by a standard continuity argument.

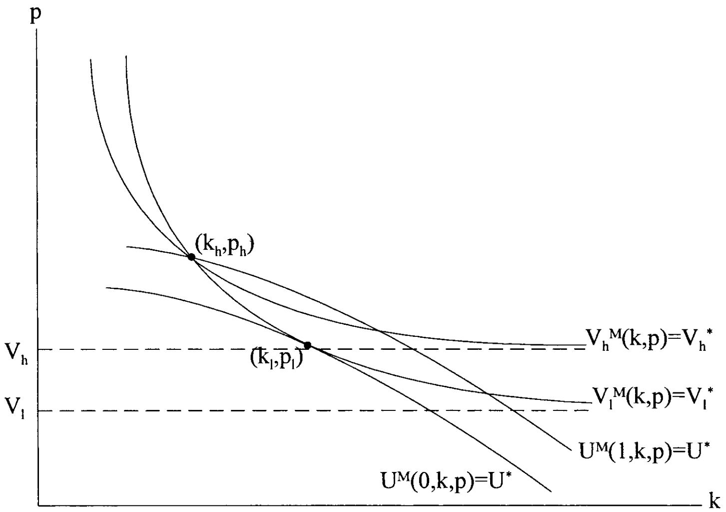

Figure 1

purpose of illustration, we assume that this curve is strictly convex. By Proposition 2 the assumption $s > b > s_l$ ensures that high-quality sellers realize just their reservation utility $V_h^* = V_h$ in the market. Clearly, this implies that their indifference curve converges to the price level $p = V_h$ as the tightness k and thus the seller's probability rate of finding a buyer goes to infinity. The indifference curve of a low-quality seller is denoted by $V_l^M(k,p) = V_l^*$. While this curve is also strictly decreasing, there are two differences relative to the indifference curve of a high-quality seller. First, we know from Proposition 2 that low-quality sellers realize strictly more than their reservation value V_l in the market. Second, by condition (5), which was used in Lemma 2, the indifference curve of a low-quality seller must cross that of a high-quality seller from above. Intuitively, high-quality sellers are more willing to accept a greater delay in return for an increase in the price than low-quality sellers.

Consider next a representative buyer who prefers both a lower price and a lower tightness k. For the purpose of illustration, we assume that his indifference curves in Figure 1 are strictly concave. If a buyer trades in the low-quality market, his indifference curve is denoted by $U^M(\pi,k,p)m = U^*$, where we set $\pi = 0$. By our assumptions concerning the shape of indifference curves, there exists at most one point of tangency with the indifference curve of a low-quality seller, which is denoted by (k_l,p_l). Condition (E.3) ensures that in this example there then opens at most one submarket for low-quality goods which has precisely these characteristics. Consider next buyers trading in a market for high-quality goods. Clearly, trade in a high-quality market shifts their indifference curve to the north-east. It is denoted by $U^M(\pi,k,p)m = U^*$, where we now set $\pi = 1$. If there were only high-quality goods in the economy, (E.3) would again ensure that the equilibrium utility for high-quality sellers V_h^* and the respective characteristics (k_h,p_h) are determined by a tangency condition. With two goods in the economy and private information about quality, this may, however, not be incentive compatible. As a consequence, the characteristics in markets for high-quality goods must be chosen appropriately to ensure separation. In the example, this is achieved at least costs exactly at the intersection of the indifference curves of high- and low-quality sellers.

We finally come to Case 3, which is very similar to Case 2. If potential buyers outnumber potential sellers, the major inefficiencies associated with the lemons problem disappear, i.e., both types of goods are traded in equilibrium and the short side of the market (here: potential sellers) is never rationed. The proof of Proposition 3 is provided in the Appendix.

Proposition 3. *If $b > s$ and search costs are sufficiently low, an equilibrium exists. Moreover, any equilibrium exhibits the following characteristics:*

i) *Both low- and high-quality goods are traded. More precisely, $\sum_{n \in N} b^n = s < b$ and $\sum_{n \in N} s_q^n = s_q$, where $q \in Q$.*

ii) *The set of submarkets is fully separating.*

iii) *Buyers obtain their reservation utility of $U^* = 0$. The equilibrium utilities of both low- and high-quality sellers are uniquely determined and satisfy $V_q^* > V_q$ for $q \in Q$.*

5 Market performance

In this section we assess the performance of competitive search markets under adverse selection. We do so by comparing the equilibrium outcomes in Section 4 with those under complete information. Surprisingly, we find that for certain parameter values the presence of private information entails no efficiency loss. Hence, both the choice of traded goods and market characteristics may not depend on the information regime. As the allocation under complete information is efficient, this implies that markets with private information may perform surprisingly well even if there exists no explicit (contractual) sorting variable.

5.1 The benchmark of complete information

Under complete information, the sellers' types are publicly known, which implies that it is possible to regulate the access of particular types to submarkets. By common values, it is then obvious that in a properly specified equilibrium goods of different qualities cannot sell at the same price. Without loss of generality, we therefore restrict attention to submarkets which are fully separating. For any given set of submarkets N, define $N_q := \{n \in N \mid \pi^n(q) > 0\}$ and $V_q^* := \max\{V_q, \max_{n \in N_q} V_q^M(k^n, p^n)\}$. For potential buyers, U^* is defined as in Section 2. Analogous to (E.1)-(E.3), we then have the following equilibrium conditions for the case of complete information.

(E.1') Optimality. For all $n \in N_q$, the measure of potential sellers of type q in submarket n must satisfy

$$s_q^n = \begin{cases} 0 & \text{if } V_q^M(k^n,p^n) < \max\{V_q, \max_{n' \in N_q \setminus \{n\}} V_q^M(k^{n'},p^{n'})\} \\ s_q & \text{if } V_q^M(k^n,p^n) > \max\{V_q, \max_{n' \in N_q \setminus \{n\}} V_q^M(k^{n'},p^{n'})\} \\ \in [0,s_q] & \text{if } V_q^M(k^n,p^n) = \max\{V_q, \max_{n' \in N_q \setminus \{n\}} V_q^M(k^{n'},p^{n'})\}, \end{cases}$$

where $\sum_{n \in N} s_q^n \leq s_q$. The condition for potential buyers is identical to (E.1).

(E.2') Stationarity. The condition is identical to (E.2).

(E.3') Competitiveness. There must not exist a pair $(p,k) \neq (p^n,k^n)$ and a type $q \in Q$ such that $V_q^M(p,k) > V_q^*$ and $U^M(\pi,k,p) > U^*$, where $\pi(q) = 1$.

To characterize the equilibrium outcomes under complete information and confront them with the outcomes under private information, we must compare the gains from trade for goods of different qualities. For this purpose, define for $q \in Q$

$$\Delta_q := u_q - v_q .$$

The following proposition provides a full characterization of the equilibrium outcomes under complete information. The proof is provided in the Appendix.

Proposition 4. *If search costs are sufficiently low, an equilibrium under complete information exists. Moreover, any equilibrium exhibits the following characteristics:*

i) $b > s$: $\sum_{n \in N} s_q^n = s_q$, $V_q^* > V_q$ *for* $q \in Q$, *and* $U^* = 0$.
ii) $b < s$: $\sum_{n \in N} b^n = b$ *and* $U^* > 0$. *As for sellers, the following holds:*
$\Delta_h < \Delta_l$: $\sum_{n \in N} s_l^n = \min[b, s_l]$, $\sum_{n \in N} s_h^n = b - \sum_{n \in N} s_l^n$, *and* $V_h^* = V_h$.
$\Delta_h > \Delta_l$: $\sum_{n \in N} s_h^n = \min[b, s_h]$, $\sum_{n \in N} s_l^n = b - \sum_{n \in N} s_h^n$, *and* $V_l^* = V_l$.
$\Delta_h = \Delta_l$: $V_q^* = V_q$.
iii) Submarkets must solve the following program: if $\pi^n(q) > 0$, k *maximizes* $V_q^M(k,p)$, *where* p *is determined by* $U^M(\pi,k,p) = U^*$ *(given* $\pi(q) = 1$*).*
iv) If both goods are traded and $\Delta_q > \Delta_{q'}$, *it must hold for any pair of submarkets* (n,n') *with* $\pi^n(q) = 1$ *and* $\pi^{n'}(q') = 1$ *that* $k^n > k^{n'}$.

By Claims i) and ii), goods with lower gains from trade are only traded if all goods with higher gains from trade are traded. Claim iii) follows from (E.3') and requires that submarket characteristics (i.e., price and circulation time) be chosen efficiently. In the present context, this means that the marginal rates of substitution between price and delay must be the same for buyers and sellers. This gives rise to Claim iv), according to which goods with higher gains from trade must sell faster. Applying the arguments of Moen (1997), it can now be shown that under complete information the equilibrium is efficient.

5.2 Comparison of information regimes

Recall from Section 4 that in a competitive search market all agents on the short side of the market trade even under private information. While this already eliminates a major inefficiency associated with the static setting, we should still expect that the outcomes under symmetric and private information differ for two reasons. First, if sellers outnumber buyers, the "wrong" goods (i.e., those with lower gains from trade) may be traded. Second, even if the right set of goods is traded, market characteristics in (by Lemma 2 separating) submarkets may be chosen inefficiently to ensure separation. This intuition can be confirmed whenever the gains from trade are higher for low-quality goods. On the other side, if this is not the case and buyers are the short side of the market, we can show that the set of equilibria is independent of the information regime. Before going into details, observe first that the inefficiencies arising under the static analysis of Section 3 do not depend on the sign of the difference $\Delta_h - \Delta_l$. Moreover, given that all players derive more utility from goods with higher quality, there is a priori no reason how the gains from trade should change with quality, which makes the cases $\Delta_h < \Delta_l$ and $\Delta_h > \Delta_l$ equally realistic.

By comparison with Proposition 4, we obtain the following result stating conditions under which equilibria with private information are still efficient.

Proposition 5. *If search costs are sufficiently low, an equilibrium under private information is also an equilibrium under complete information if and only if*

i) $\Delta_h \leq \Delta_l$ *and* $b < s_l$, *or*
ii) $\Delta_h < \Delta_l$ *and* $s_l < b < s$.

Proposition 5 is proved in the Appendix. The result is rather immediate for Case i) where we know from Proposition 1 that only low-quality goods are traded. This implies no loss of efficiency as the respective gains from trade are not smaller than for high-quality goods. Moreover, condition (E.3) ensures that the characteristics in (sub-) markets for low-quality goods are chosen efficiently. Both arguments regarding the set of traded goods and the conditions under which low-quality sellers trade carry over to Case ii). In addition, we must now consider trade of high-quality goods. Recall from Lemma 2 that under private information high-quality goods always trade more slowly but at a higher price than low-quality goods, which was proved by appealing to incentive compatibility. We argue now that in Case ii) the differences in circulation times between high- and low-quality goods are chosen efficiently even under private information.

To see this, observe first that the requirements $s_l < b < s$ and $\Delta_h < \Delta_l$ imply intuitively that $V_h^* = V_h$ and $V_l^* > V_l$. Hence, while sellers of high-quality goods only realize their respective reservation value, sellers of low-quality goods are strictly better off when entering the market. In other words, the entire surplus from trading high-quality goods is appropriated by potential buyers, while the surplus is shared (more equally) when trading low-quality goods. Suppose now that there is complete information regarding quality, which from Section 5.1 (and Proposition 4) implies that the characteristics in the respective submarkets must be chosen efficiently. Consider any submarket where high-quality goods are traded. Recall next that searching and delay involves two types of inefficiencies due to the direct costs $c > 0$ and discounting with $r > 0$. The efficient choice of submarket characteristics for high-quality goods must thus balance two conflicting objectives. To minimize aggregate search costs due to $c > 0$, it is optimal to choose “intermediate” values for the tightness k. On the other side, as players discount future utilities and as the entire surplus is realized by buyers due to $V_h^* = V_h$, the expected waiting time of buyers should be much lower than that of sellers. As $c \to 0$, the second objective dominates the first, implying that the tightness k converges to zero such that the transition rate of high-quality sellers $f(k)$ converges to zero.[18] Consider now for these (low) choices of k the payoff of a (deviating) low-quality seller. Clearly, as $r > 0$ and as trade is extremely delayed, his payoff becomes arbitrarily close to his reservation value V_l as $c \to 0$, which again is strictly less than his equilibrium payoff V_l^*. Summing up, as search costs c become sufficiently low, the efficient characteristics chosen under complete information ensure incentive compatibility if quality is private information. Hence, even though goods of different quality are traded in equilibrium at different prices, we find also for Case ii) that private information does not imply inefficiencies.

The key to the previous argument is the existence of a search market environment, where frictions due to delay are unavoidable. Indeed, if trade is speeded up for one side, expected delay must increase for the other side. In such an environment it may hold for reasonable parameter choices that the efficient characteristics chosen under complete information already ensure incentive compatibility under private information.[19]

6 Conclusion

In this paper we consider a search market for durable goods where sellers have private information about the good’s quality. In sharp contrast to the standard (static) analysis, we show that in equilibrium goods of different qualities sell at different prices. To ensure incentive compatibility, high-quality goods must circulate longer than low-quality goods. This accords with the common wisdom that sellers who want to sell fast may have to accept a lower price. Moreover, we show that if search costs are sufficiently low, all agents on the short side of the market trade. This is again in sharp contrast to the static analysis where part of the short side is typically rationed.

To analyze the welfare properties of competitive search markets, we compare the market outcome with the benchmark of complete information. Surprisingly, we find that for a large range of parameter values the outcomes under complete and private information coincide. This is despite the fact that under private information high-quality goods must circulate longer than low-quality goods to ensure incentive compatibility. In the working paper version (Inderst and Müller 1999) we also allow for heterogeneous buyers who value goods differently. The

[18] Hence the argument relies on the fact that c becomes very small *relative* to the interest rate r.

[19] Instead of choosing a benchmark where information is complete, we could have considered the case of a regulator who controls the characteristics prevailing in the different submarkets. The corresponding analysis is found in the working paper version (Inderst and Müller, 1999).

standard static analysis where a single price clears the market then leads to an inefficient allocation of goods across buyers. This inefficiency can again be resolved in a market with directed search. Overall, these results suggest that markets with private information may function much better than what is commonly thought.

As we argued repeatedly in the paper, our results depend crucially on the fact that goods are durable. If the good is a consumption good, potential sellers derive no utility from the good while engaging in search. Consequently, delay (or circulation time) cannot act as a sorting device, and the lemons problem remains present in full force. Having said this, we should emphasize that this clear-cut distinction between durable goods and consumption goods vanishes if quality has other (external) effects, e.g., if low-quality consumption goods depreciate faster than high-quality consumption goods. In this case, the considerations underlying our model may apply as well to consumption goods.

7 Appendix: Proofs

Proof of Propositions 1–3. Though Propositions 1–3 do not make any claims regarding the characteristics of submarkets (besides the fact that they must be separating by Lemma 2), we will prove more here. We will identify programs which characterize submarkets. These results will also be used in Propositions 4–5 below.

We begin by defining the following programs.

Unconstrained programs

Program $P_q^S(U)$. For given $0 \leq U \leq U_h$ the program $P_q^S(U)$ chooses $k \in [0,\infty)$ to maximize $V_q^M(k,p)$, where p is uniquely defined by the requirement $U^M(\pi,k,p) = U$ with $\pi(q) = 1$. By $c > 0$, which ensures that an optimal k is bounded, and by the continuity of payoffs, a solution exists. Denote the realized utility by $\overline{V}_q^S(U)$, which is continuous by an application of the maximum theorem. Moreover, $\overline{V}_q^S(U)$ is strictly decreasing in U. The correspondence of solutions is denoted by $K_q^S(U)$.

Program $P_q^B(V)$. For given $0 \leq V \leq U_h$ the program $P_q^B(V)$ chooses $k \in [0,\infty)$ to maximize $U^M(\pi,k,p)$, where p is uniquely defined by the requirement $V_q^M(k,p) = V$ and where $\pi(q) = 1$. Again a solution exists and the maximum, which is denoted by $\overline{U}_q^B(V)$, is continuous and strictly decreasing in V. The correspondence of solutions is denoted by $K_q^B(V)$.

Actually, one of the two programs $P_q^S(U)$ and $P_q^B(V)$ is redundant as $\overline{V}_q^S(\overline{U}_q^B(V)) = V$ and $\overline{U}_q^B(\overline{V}_q^S(U)) = U$, while for the respective utility levels it holds that $K_q^S(U) = K_q^B(V)$. The proof is straightforward and omitted.

Constrained program

Program $P^C(U,V)$. For given $0 \leq U \leq U_h$ and $V_l \leq V \leq U_h$ the program P^C chooses $k \in [0,\infty)$ to maximize $V_h^M(k,p)$ subject to $V_l^M(k,p) \leq V$, where p is uniquely defined by the requirement $U^M(\pi,k,p) = U$ with $\pi(h) = 1$. Observe first that the set of feasible values k, which is denoted by $K(U,V)$, is non-empty as by $c > 0$ incentive compatibility is ensured by choosing k sufficiently low. Denote the realized utility by $\overline{V}^C(U,V)$. Observe that $\overline{V}^C(U,V)$ is strictly decreasing in U and nondecreasing in V.[20] We show next that it is also continuous. As $K(U,V)$ may not change continuously, this is not immediate. However, $K(\cdot)$ is surely upper semi-continuous in both arguments, while for $U > U'$ it holds (strictly) that $K(U,V) \supset K(U',V)$ for all V. Given the strict monotonicity of $\overline{V}^C(U,V)$ in U this indeed implies continuity in U. Consider next the behavior in V. The previous argument does not apply in this case as $\overline{V}^C(U,V)$ is nondecreasing in V, while $V > V'$ may well imply $K(U,V) \supset K(U,V')$. However, if $\overline{V}^C(U,V)$ is not continuous at V for some given U, this must imply existence of a pair $k > k'$ with $k \in K(U,V)$, $k' \in K(U,V)$, $V_l^M(k,p) = V_l^M(k',p') = V$ (where p and p' are given by $U^M(\pi,k,p) = U^M(\pi,k',p') = U$), and $V_h^M(k,p) > V_h^M(k',p')$. But this contradicts the single-crossing property of (5). Hence, $\overline{V}^C(U,V)$ must be also continuous in V. Denote finally the arg-max correspondence of $P^C(U,V)$ by $K^C(U,V)$.

We proceed by proving a series of claims relating to the characteristics of active submarkets. Observe that we use throughout the analysis that submarkets must be separating from Lemma 2.

Claim 1. *If $b < s_l$ and the market opens up, then in any submarket n it holds that $k^n \in K_l^B(V_l)$, while p^n is uniquely defined by $V_l^M(k^n,p^n) = V_l$. This implies $U^* = \overline{U}^B(V_l)$.*

Proof. By Lemma 1 and (E.2) only low-quality goods may be traded, while it holds that $V_l^* = V_l$. Suppose next that $k^n \notin K_l^B(V_l)$. By the definition of the program P_l^B this implies existence of a pair (k,p) with p defined by $V_l^M(k,p) = V_l$ where $U^M(\pi^n,k,p) > U^M(\pi^n,k^n,p^n) = U^*$. By continuity we can thus choose some (k',p') implying $V_l^M(k',p') > V_l^*$ and $U^M(\pi,k',p') > U^*$ for all $\pi \in \Delta_Q$, which contradicts (E.3). □

Claim 2. *If $s_l < b < s$ and if all buyers enter the market, the characteristics in two submarkets n and m where $\pi^n(h) = 1$ and $\pi^m(l) = 1$ must satisfy the following requirements:*

i) $k^m \in K_l^B(V_l^*)$, *while p^m is uniquely defined by* $V^M(k^m,p^m) = V_l^*$.

[20] One way to see the strict monotonicity in U is to consider the modified programs where (k,p) are chosen jointly to maximize $V_h^M(k,p)$ subject to $V_l^M(k,p) \leq V$ and $U^M(\pi,k,p) \geq U$. Using the single-crossing condition (5) it can be shown that the buyers' participation constraint must bind at an optimum, implying that the optimal pairs (k,p) coincide with those derived under $P^C(U,V)$. Moreover, this implies that the set of solutions differs for any pair $U' > U''$, which proves the strict monotonicity.

ii) $k^n \in K^C(U^*, V_l^*)$, *while* p^n *is uniquely defined by* $V^M(k^n, p^n) = V_h^*$.
iii) $V_h^* = V_h$, *while* U^* *and* V_l^**are uniquely defined by*

$$\begin{aligned} U^* &= \overline{U}_l^B(V_l^*), \\ \overline{V}^C(U^*, V_l^*) &= V_h. \end{aligned} \tag{6}$$

Proof. Observe first that by assumption $\sum_{i \in N} b^i = b$, implying from (E.2) and Lemma 1 that $\sum_{i \in N} s_l^i = s_l$ and $\sum_{i \in N} s_h^i = b - s_l > 0$. Given equilibrium utilities, assertion i) for markets with low-quality goods is immediate from the argument in Claim 1. Turn therefore to assertion ii) regarding high-quality goods, where we suppose that $k^n \notin K^C(U^*, V_l^*)$. By construction of $P^C(U^*, V_l^*)$ this implies existence of some $k \in K^C(U^*, V_l^*)$ realizing $V_h^M(k, p) = \overline{V}^C(U^*, V_l^*) > V_h^*$, where p solves $U^M(\pi^n, k, p) = U^*$. By continuity we can slightly adjust the price p downwards to some $p' < p$ such that $V_h^M(k, p') > V_h^*$, $U^M(\pi^n, k, p') > U^*$, and $V_l^M(k, p') < V_l^*$, where we use that $V_l^M(k, p) \le V_l^*$ by construction of $P^C(U^*, V_l^*)$. Hence, the original set of submarkets does not satisfy (E.3).

Turn next to assertion iii). By Lemma 1 it must hold that $V_h^* = V_h$. Moreover, by assertions i) and ii) and due to (E.1) the pair (U^*, V_l^*) must indeed satisfy (6). It remains to prove that a solution is unique. By substitution we obtain the requirement $\overline{V}^C(\overline{U}_l^B(V_l^*), V_l^*) = V_h$, where the left side is strictly decreasing in V_l^*. (Observe that we do not assert existence, which will be proved below for sufficiently low values of c.). □

Claim 3. *If* $b > s$ *and if high-quality goods are traded, the characteristics in two separating submarkets* n *and* m *with* $\pi^n(h) = 1$ *and* $\pi^m(l) = 1$ *must satisfy the following requirements:*

i) $k^m \in K_l^S(0)$, *while* p^m *is uniquely defined by* $U^M(\pi^m, k^m, p^m) = 0$.
ii) $k^n \in K^C(0, V_l^*)$, *while* p^n *is uniquely defined by* $U^M(\pi^n, k^n, p^n) = 0$.
iii) $U^* = 0$, $V_l^* = \overline{V}_l^S(0)$, *and* $V_h^* = \overline{V}^C(0, V_l^*)$.

Proof. Recall that $K_l^S(U) = K_l^B(V)$ if $U = \overline{U}_l^B(V)$. The proof is now analogous to that in Claim 2. □

For the following claims, it is convenient to analyze how the solutions to the programs defined above change in c. In a slight abuse of notation, we will sometimes denote the solutions by $\overline{U}_q^B(V, c)$, $\overline{V}_q^S(U, c)$, and $\overline{V}^C(U, V, c)$ respectively. By the arguments following the definitions of the respective programs and by continuity of (market) payoffs in c, the realized utilities are all continuous in c. The following results follow then immediately from inspection of the objective functions in the respective programs.

Claim 4. $\lim_{c \to 0} \overline{U}_q^B(V_q, c) = U_q - V_q$ *and* $\lim_{c \to 0} \overline{V}_q^S(0, c) = U_q$.

We use now Claim 4 to derive the characterization of entry flows.

Claim 5. *If* c *is sufficiently small, all agents of the short side must enter, i.e.* $\sum_{n \in N} b^n = b$ *if* $b < s$ *and* $\sum_{q \in Q} \sum_{n \in N} s_q^n = s$ *if* $b > s$. *Moreover, (6) has a solution.*

Proof. We can now use the characteristics derived in Claims 1-3. We consider all three parameter constellations (discussed in Propositions 1-3) in turn. For $b < s_l$ observe from Claim 4 that $\overline{U}_l^B(V_l) > 0$ holds for c sufficiently small. As $U^* = \overline{U}_l^B(V_l)$ by Claim 1, all buyers must indeed enter by (E.1).

Suppose next $s_l < b < s$, where we show again that $U^* > 0$, which is now defined implicitly by (6). We start by rewriting this condition. Observe first that $U^* = \overline{U}_l^B(V_l^*)$ implies $V_l^* = \overline{V}_l^S(U^*)$, which allows to rewrite the second requirement in (6) as $\overline{V}^C(U^*, \overline{V}_l^S(U^*)) = V_h$. Observe that the left-side is continuous and strictly decreasing in U^*. We will prove that it strictly exceeds V_h for $U^* = 0$ if c is sufficiently small. The objective function of $P^C(U^*, V_l^*)$ transforms for $U^* = 0$ to

$$\frac{v_h - c + g(k)(U_h - c/f(k))}{r + g(k)}, \tag{7}$$

while the (incentive compatibility) constraint becomes

$$\frac{v_l - c + g(k)(U_h - c/f(k))}{r + g(k)} \leq \overline{V}_l^S(0). \tag{8}$$

Given $\lim_{c \to 0} \overline{V}_l^S(0, c) = U_l \geq V_l$ by Claim 4 and the properties of $f(k)$ and $g(k)$, we can find a pair $\overline{c} > 0$, $\overline{k} > 0$ such that (8) is satisfied for all $c < \overline{c}$ and $k < \overline{k}$. Moreover, by (7) we can find for any $k > 0$ a threshold $\overline{\overline{c}}(k) > 0$ such that the objective function strictly exceeds V_h for all $c < \overline{\overline{c}}(k)$. As a consequence, $\overline{V}^C(0, \overline{V}_l^S(0)) > V_h$ holds for all $c < \min\left\{\overline{c}, \overline{\overline{c}}(\overline{k})\right\}$, which completes the proof for $s_l < b < s$.

Suppose finally $b > s$. By $U^* = 0$ and Claim 4, $\overline{V}_l^S(0) > V_l$ holds for sufficiently low values of c. As $V_h^* = \overline{V}^C(0, \overline{V}_l^S(0))$ by Claim 3, the further argument is analogous to that for $s_l < b < s$. □

We are now in a position to complete the proof of Propositions 1–3. The characterization follows directly from combining Lemmas 1-2 with Claim 5 such that it remains to prove existence. This is done by construction, where we use the characterization in Claims 1-3. By the derivation of equilibrium utilities (E.1) and (E.2) are satisfied, while the programs defining the characteristics of submarkets ensure that (E.3) holds. □

Proof of Proposition 4. We prove first the following auxiliary result where we use the programs defined in the proof of Propositions 1-3.

Claim. *If $\Delta_q > \Delta_{q'}$, $k \in K_q^S(U)$ and $k' \in K_{q'}^S(U)$imply $k > k'$.*

Proof. Considering the programs $P_q^S(U)$ for $q \in Q$, it follows by optimality that

$$\frac{c(1+k) + \Delta_q + rU(k-1)}{r + kf(k)} \leq \frac{c(1+k') + \Delta_q + rU(k'-1)}{r + k'f(k')},$$
$$\frac{c(1+k') + \Delta_{q'} + rU(k'-1)}{r + k'f(k')} \leq \frac{c(1+k) + \Delta_{q'} + rU(k-1)}{r + kf(k)}.$$

These requirements imply

$$\frac{\Delta_q - \Delta_{q'}}{r + kf(k)} \leq \frac{\Delta_q - \Delta_{q'}}{r + k'f(k')},$$

which proves that $k > k'$. □

It is next immediate from (E.3') that k^n must solve $P_q^S(U^*)$ in case goods of quality q are traded in submarket n, i.e. if $\pi^n(q) = 1$. Together with the Claim this proves assertions iii)-iv). Regarding the characterization in assertion i), observe that (E.1')-(E.2') determine $U^* = 0$. By (E.3') this implies $V_q^* = \overline{V}_q^S(0)$, which by Claim 4 in the proof of Propositions 1-3 strictly exceeds V_q for sufficiently low c.

Regarding assertion ii), we choose first the case where $\Delta_h > \Delta_l$ and $b > s_l$. For low c both qualities must be traded. Otherwise, (E.2') implies $U^* = 0$ and $V_q^* = V_q$ for some q. As $\overline{V}_q(0) > V_q$ holds for low c, (E.3') would not be satisfied. Moreover, given some equilibrium utility U^* for buyers, it must hold for both types that $V_q^* = \overline{V}_q^S(U^*)$. Inspection of the objective functions of the programs $P_q^S(U^*)$ reveals that this implies $V_h^* - V_h > V_l^* - V_l$ due to $\Delta_h > \Delta_l$. By (E.1')-(E.2') this implies next $V_l^* = V_l$ and $V_h^* \geq V_h$. Finally, using the results in the proof of Propositions 1-3 we can solve $\overline{V}_l^S(U^*) = V_l$ for a unique value $U^* > 0$ if c is sufficiently low. The arguments for the remaining cases in assertion ii) are analogous. This completes the proof of Proposition 4. □

Proof of Proposition 5. We derive first additional properties of the equilibria characterized in Propositions 2-3 where goods of both qualities are traded under asymmetric information.

Claim 1. *Suppose $s_l < b < s$ and $\Delta_h < \Delta_l$. If c is sufficiently low, then in any equilibrium under asymmetric information it holds that $k^n \in K_h^B(V_h)$ if $\pi^n(h) = 1$.*

Proof. By the proof of Propositions 1-3, it holds that (U^*, V_l^*) are jointly determined by (6), while $V_h^* = V_h$. Moreover, it holds that $k^n \in K^C(U^*, V_l^*)$. By construction of the program P^C, the following result is immediate. If there exists some $k \in K_h^S(U^*)$ such that additionally $V_l^M(k,p) \leq V_l^*$, where p uniquely solves $U^M(\pi, k, p) = U^*$ with $\pi(h) = 1$, then it holds that $K^C(U^*, V_l^*) \subseteq K_h^S(U^*)$. Recall also that $K_h^S(U^*) = K_h^B(V_h^*)$. Consider next $P^B(V_h^*)$, where from $V_h^* = V_h$ the objective function becomes

$$U_h - V_h - \frac{1}{k}\frac{c(1+k)}{r + f(k)} - \frac{\Delta_h}{r + f(k)}.$$

Denote $\overline{k}(c) = \sup K_h^B(V_h)$ and observe that $lim_{c \to 0}\overline{k}(c) = 0$ by the properties of $f(\cdot)$. Observe next that $U^* = \overline{U}_h^B(V_h) < \Delta_h / r$, which by $\Delta_h < \Delta_l$ and $V_l^* = \overline{V}_l^S(U^*)$ implies existence of some lower boundary $\underline{V}_l(c)$ such that $V_l^* \geq \underline{V}_l(c)$ for given c and $lim_{c \to 0}\underline{V}_l(c) > V_l$. By $lim_{c \to 0}\overline{k}(c) = 0$ and $lim_{c \to 0}\underline{V}_l(c) > V_l$ we can now choose c sufficiently small such that some $k \in K_h^B(V_h)$ solves also $P^C(U^*, V_l^*)$ as incentive compatibility is satisfied. By the previous remarks this proves the claim. □

Claim 2. *Suppose $b > s$. If c is sufficiently small, then in any equilibrium under asymmetric information $\pi^n(h) = 1$ implies $k^n < \inf K_h^S(0)$.*

Proof. Recall from Proposition 3 that in any equilibrium it holds that $U^* = 0$. With this specification, the objective function for the program $P_h^S(0)$ transforms to

$$U_h - \frac{c(1+k) + \Delta_h}{r + kf(k)}.$$

In analogy to Claim 1, denote $\underline{k}(c) = \inf K_h^S(0)$ and observe that by optimality and the properties of $f(\cdot)$ it holds that $\underline{k}(c) \to \infty$ for $c \to 0$, while $\lim_{c\to 0} \overline{V}_h^S(0) = U_h$. The expected utility of a low-quality seller entering a submarket where (k, p) are determined by a solution to $P_h^S(0)$ must therefore converge to U_h for $c \to 0$. As the equilibrium utility satisfies $V_l^* = \overline{V}_l(0)$, which converges to U_l for $c \to 0$, this is not incentive compatible. □

We are now in a position to prove Proposition 5. We treat four cases in turn. First, for $b > s$ the assertion follows from Claim 2. Second, for $b < s_l$ the assertion follows directly from inspection of the proof of Propositions 1-3 and from Proposition 4. Third, for $s_l < b < s_h$ and $\Delta_h < \Delta_l$, the assertion follows from Claim 1 and Proposition 4. Finally, we discuss the case where $s_l < b < s_h$ and $\Delta_h \geq \Delta_l$. If the latter inequality is strict, the assertion follows immediately from comparing entries under both information regimes as specified in Propositions 2 and 4. In case of equality, it is straightforward to show that any equilibrium outcome under symmetric information would not be incentive compatible under asymmetric information for any c. □

References

Akerlof, G.A.: The market for lemons: quality uncertainty and the market mechanism. Quarterly Journal of Economics **84**, 488–500 (1970)

Albrecht, J.W., Vroman, S.B.: Non-existence of single-wage equilibria in search models with adverse selection. Review of Economic Studies **59**, 617–624 (1992)

Cramton, P.C.: Bargaining with incomplete information: an infinite-horizon model with continuous uncertainty. Review of Economic Studies **51**, 579–593 (1984)

Evans, R.: Sequential bargaining with correlated values. Review of Economic Studies **56**, 499–510 (1989)

Fudenberg, D., Tirole, J.: Sequential bargaining with incomplete information. Review of Economic Studies **50**, 221–247 (1983)

Fudenberg, D., Levine, D., Tirole, J.: Infinite-horizon models of bargaining with one-sided incomplete information. In: Roth, A. (ed.) Game-theoretic models of bargaining. Cambridge: Cambridge University Press 1985

Gale, D.: A Walrasian theory of markets with adverse selection. Review of Economic Studies **59**, 229–255 (1992)

Gale, D.: Equilibria and Pareto optima of markets with adverse selection. Economic Theory **7**, 207–235 (1996)

Greenwald, B., Stiglitz, J.E.: Pareto inefficiency of market economies: search and efficiency wage models. American Economic Review Papers and Proceedings **78**, 351–356 (1988)

Grossman, S.J.: The role of warranties and private disclosure about product quality. Journal of Law and Economics **24**, 461–483 (1980)

Gul, F., Sonnenschein, H., Wilson, R.: Foundations of dynamic monopoly and the coase conjecture. Journal of Economic Theory **39**, 155–190 (1986)

Hendel, I., Lizzeri, A.: Adverse selection in durable goods markets. American Economic Review **89**, 1097–1115 (1999a)

Hendel, I., Lizzeri, A.: The role of leasing under adverse selection. Mimeo, Princeton University (1999b)
Inderst, R., Müller, H.M: Why peaches must circulate longer than lemons. Mimeo, University of Mannheim (1999)
Janssen, M.C.W., Roy, S.: Dynamic trading in a durable good market with asymetric information. Mimeo, Florida International University (1999)
Janssen, M.C.W., Roy, S.: On the nature of the 'lemons problem' in durable goods markets. Mimeo, Florida International University (2000)
McMillan, J. and Rothschild, M.: Search. In: Aumann, R., Hart, S. (eds.) Handbook of game theory. Amsterdam: North-Holland 1994
Moen, E.R.: Competitive search equilibrium. Journal of Political Economy **105**, 385–411 (1997)
Mortensen, D.T., Pissarides, C.A.: New developments in models of search in the labor market. Forthcoming in: Ashenfelter, O., Card, D. (eds.) Handbook of labor economics, Vol. 3. Amsterdam: North-Holland 1998
Mortensen, D.T., Wright, R.: Competitive pricing and efficiency in search equilibrium. Mimeo, Northwestern University (1997)
Peters, M.: On the equivalence of Walrasian and non-Walrasian equilibria in contract markets: the case of complete contracts. Review of Economic Studies **64**, 241–264 (1997)
Shimer, R.J.: Contracts in a frictional labor market. Mimeo, Massachussetts Institute of Technology (1996)
Sobel, J., Takahashi, I.: A multi-stage model of bargaining. Review of Economic Studies **50**, 411–426 (1983)
Spence, A.M.: Job market signaling. Quarterly Journal of Economics **87**, 355–374 (1973)
Taylor, C.R.: Time-on-the-market as a sign of quality. Review of Economic Studies **66**, 555–578 (1999)
Vincent, D.R.: Bargaining with common values. Journal of Economic Theory **48**, 47–62 (1989)
Vincent, D.R.: Dynamic auctions. Review of Economic Studies **57**, 49–61 (1990)
Waldman, M.: Leasing, lemons, and moral hazard. Mimeo, Cornell University (1999)
Wilson, C.: The nature of equilibrium in markets with adverse selection. Bell Journal of Economics **11**, 108–130 (1980)