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The effect of capital market characteristics on the value of start-up firms ☆

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Abstract

We develop an equilibrium model of contracting, bargaining, and search in which the relative scarcity of venture capital affects the bargaining power of entrepreneurs and venture capitalists. This in turn affects the pricing, contracting, and value creation in start-ups. The relative scarcity of venture capital is endogenous and depends on the profitability of venture capital investments, entry costs, and transparency of the venture capital market. Supply and demand conditions also affect the incentives of venture capitalists to screen projects ex ante. We characterize both the short- and long-run dynamics of the venture capital industry, which

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provides us with a stylized picture of the Internet boom and bust periods. Our model is consistent with existing evidence and provides a number of new empirical predictions.

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1. Introduction

The venture capital industry is highly cyclical, with periodic changes in supply and demand conditions (Gompers and Lerner, 1999; Lerner, 2002). Between the fourth quarters of 2000 and 2001 alone, for instance, total funds raised dropped by more than 80%. In this paper, we ask whether, and how, such variations in capital supply affect the competitive pricing, contracting, and value creation in start-ups.

Our theory is based on two building blocks. The first is a model of contracting and bargaining in start-ups. Building on Sahlman (1990), Kaplan and Strömberg (2002), and Hellmann and Puri (2001), we model the relationship between the entrepreneur and venture capitalist as a double-sided incentive problem: A greater fraction of the firm owned by the venture capitalist improves the venture capitalist's incentives but weakens the entrepreneur's incentives. Efficiency requires balancing the two incentive problems, or equivalently, balancing ownership shares. Actual ownership shares, however, are determined by bargaining, and thus by the relative strength of the entrepreneur's and venture capitalist's outside options.

The second building block is a search model linking outside options to the relative scarcity of venture capital. An increase in capital supply, for instance, makes it easier for an entrepreneur to obtain financing, thereby increasing his outside option vis-à-vis a venture capitalist. The supply of capital, in turn, is endogenous and depends on primitive market characteristics such as the profitability of investments, entry costs, and capital market transparency.

The end result is an equilibrium model in which capital market characteristics affect the relative supply and demand for capital, which in turn affects bargaining powers and ownership shares, which in turn affects the pricing and value creation in start-ups.

The core of our model is an analysis of the short- and long-run dynamics of the venture capital industry. In the short run, the number of venture capitalists is fixed. In the long run, changes in market conditions lead to entry or exit of venture capitalists, thereby making capital market competition endogenous. Our model predicts that an increase in the return to venture capital investments—whether true or merely perceived—leads to new entry and a long-run increase in capital market competition, coupled with a rise in valuations and a decline in venture capitalists' ownership shares. A decrease in investment returns, on the other hand, leads to exit, a drop in valuations, and better deal terms for those venture capitalists remaining in the market.

While stylized, our model can provide a useful picture of the Internet boom and bust periods. As winners often tend to materialize quicker than losers (poor performers may be able to hold out until their cash is finally burned up), the initial success stories at the beginning of the Internet boom period might not have been representative of the industry as a whole. The general public and investors might have therefore overestimated the true returns to Internet investments. As more and more firms began to fail, investors realized that their initial assessment was wrong. They consequently adjusted their return estimates downward. In our model, the boom and bust of the Internet correspond to an increase and decrease, respectively, in the perceived return to venture capital investments.

We also examine the equilibrium effects of changes in entry costs and capital market transparency. Such changes affect outside options either directly or via their effect on capital market competition. Our model predicts that an increase in transparency improves the value created in start-ups, while a decrease in entry costs can destroy value if the aggregate capital supply is already relatively high. Finally, capital market competition affects the incentives not only after but also prior to the formation of a new venture. In an extension of our model, we show that venture capitalists are more likely to screen projects in "down markets" when competition among investors is weak, and less likely in "hot markets" when competition is strong.

Our search model contrasts with traditional venture capital contracting models, which consider an isolated setting with one entrepreneur and one venture capitalist at a time. These models assume a "competitive capital market" in which entrepreneurs extract all the surplus. In a world with many venture capitalists *and* many entrepreneurs, however, it is not clear why entrepreneurs should extract all the surplus. On the contrary, anecdotal evidence suggests that the ability to extract surplus shifts back and forth between entrepreneurs and investors, depending on who is currently in short supply.[1] In this paper, we explicitly depart from the standard assumption that entrepreneurs have all the bargaining power. As we show, this standard assumption is not innocuous. By giving venture capitalists bargaining power, one can get closer to the sharing rule that maximizes the joint surplus. In fact, if venture capitalists and entrepreneurs each have the "right" amount of bargaining power, efficient surplus sharing is possible.

A central tenet of our model is that changes in capital market competition and bargaining power translate into changes in ownership shares. Supporting evidence is provided by Gompers and Lerner (2000), who find a positive relation between the valuation of new ventures and capital inflows, suggesting that "increases in the supply of venture capital may result in greater competition to finance companies and

[1] The following statement compares the height of the Internet bubble—when "too much money was chasing too few deals" (Gompers and Lerner, 2000)—with the aftermath: "If you went into a ... start-up three to six months ago, you almost certainly got a very bad deal. Companies could ask for anything they wanted [in terms of valuation]. Now entrepreneurs are much more realistic" (Financial Times, "Open Season for Europe's Turkeys," January 11, 2001). Along similar lines, Bartlett (2001b) argues that the burst of the bubble brought about "changes in deal terms ... all of which are designed to enhance returns and the quantum of control enjoyed by nervous investors."

rising valuations.'' (Ceteris paribus, a higher valuation implies a smaller ownership share for venture capitalists).

Another tenet of our model is the link between ownership shares and incentives. Empirical support is provided by Kaplan and Strömberg (2002), who find that equity incentives increase the likelihood that venture capitalists provide value-adding support activities. Similarly, industry observers have expressed concerns that unfavorable deal terms (from the perspective of entrepreneurs) in the recent down market might have had a negative effect on entrepreneurial incentives: ''The terms of current venture financings can be such that founders may well lose interest. ... They start plotting their next career move, perhaps with a competitor, from the date the deal is closed. In short, the VCs, while putting in place extremely favorable terms from their point of view, face the possibility of shooting themselves in the feet'' (Bartlett, 2001a).

Michelacci and Suarez (2002) also have a search model of start-up financing. Unlike this paper, however, Michelacci and Suarez do not consider incentive contracts or the inefficiencies arising from an imbalance of ownership shares. Rather, they focus on search inefficiencies, using an insight from the search literature that entry creates externalities for the matching chances of other market participants.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes the equilibrium when capital market competition is exogenous. Section 4 endogenizes the level of capital market competition. The exogenous variables are primitive market characteristics such as investment returns, entry costs, and capital market transparency. Section 5 considers robustness issues as well as welfare and policy implications. Section 6 examines the incentives of venture capitalists to screen projects ex ante. Section 7 summarizes the empirical implications and compares them with the available evidence. Section 8 concludes. All proofs are in Appendix B.

2. The model

The model has two building blocks: (i) a model of contracting and bargaining in start-ups, and (ii) an equilibrium model of search. We first derive the *contract frontier* characterizing the utilities of the entrepreneur and venture capitalist for all Pareto optimal contracts. We then derive the bargaining solution, which determines a point (i.e., contract) on the contract frontier. In the bargaining problem, we take the entrepreneur's and venture capitalist's outside options as given. We finally embed the bargaining problem in a search market to endogenize outside options as a function of the relation between capital supply and demand, or *degree of capital market competition*. The degree of capital market competition, in turn, is taken as given. It will be endogenized in Section 4 when we introduce free entry of capital.

2.1. Financial contracting

A penniless entrepreneur has a project requiring an investment $I > 0$. Financing is provided by a venture capitalist. The project payoff is $X_l \geqslant 0$ with probability $1 - p$

and $X_h > I$ with probability p. The success probability $p = p(e, a)$ depends on the entrepreneur's and venture capitalist's non-contractible efforts $e \in [0, 1]$ and $a \in [0, 1]$. Effort costs are denoted by $c(e)$ and $g(a)$, respectively, and are strictly convex. All agents are risk neutral, which implies our results continue to hold if X_l and X_h are expected values instead of final cash flows.

In the base model (Sections 2–4) we assume that $X_l = 0$. The division of the project payoff is then fully characterized by a sharing rule $s \in [0, 1]$ representing the venture capitalist's ownership, or *equity share*. The case where $X_l > 0$ is considered in Section 5. In that section, we also consider the possibility that the venture capitalist pays the entrepreneur a wage.

Given some sharing rule s, the entrepreneur's and venture capitalist's utilities are $u(s) \equiv p(1 - s)X_h - c(e)$ and $v(s) \equiv psX_h - g(a)$, respectively. By varying s from zero to one, we can trace out the utility possibility frontier depicting the set of all possible $u - v$ combinations. This frontier need not be decreasing everywhere. For instance, if one side is more productive than the other, both utilities might be increasing over some range (see Fig. 1). The same is true if efforts are complements. We call the undominated, i.e., decreasing, segment of the utility possibility frontier the *equity frontier*. The equity frontier depicts the set of all Pareto-optimal $u - v$ combinations. It is derived from the set of Pareto-optimal equity shares and denoted by $u = \psi(v)$. The domain of the equity frontier is $[\underline{v}, \bar{v}]$, where $\underline{v} \geqslant v(0)$ and $\bar{v} \leqslant v(1)$ are the venture capitalist's utilities under the lowest and highest Pareto-optimal equity share, respectively.[2]

The shape of the equity frontier depends on the production technology $p(e, a)$. We focus on production technologies that are well behaved in the following sense:

(i) the function $\psi(v)$ is decreasing and strictly concave over $[\underline{v}, \bar{v}]$;
(ii) the sum $u(s) + v(s)$ has a unique maximum in the interior of $[\underline{v}, \bar{v}]$; and
(iii) the venture capitalist's utility $v(s)$ is increasing in s over $[\underline{v}, \bar{v}]$.

Properties (i)–(ii) follow naturally from the fact that the incentive problem is two-sided and effort costs are strictly convex. Maximizing the sum of utilities then requires balancing the two incentive problems. In particular, giving one side a very high and the other side a very low equity share does not maximize total utility, as the side with the high equity share will then provide effort at a level where his or her marginal effort cost is extremely high. Consequently, the total utility $u + v = \psi(v) + v$ has a unique maximum in the interior of $[\underline{v}, \bar{v}]$. Define $\hat{s} \equiv \arg\max[u(s) + v(s)]$, $\hat{v} \equiv v(\hat{s})$, and $\hat{u} \equiv u(v(\hat{s}))$. We refer to the surplus-maximizing sharing rule $\hat{s}$ as the *efficient sharing rule* (or *efficient equity share*).[3] Evidently, it holds that $\psi'(\hat{v}) =$

[2] See Appendix A for examples. If the venture capitalist and entrepreneur have different productivities or if efforts are complements, the lowest Pareto-optimal equity share will be positive and the highest Pareto-optimal equity share will be less than one, implying that $\underline{v} > v(0)$ and $\bar{v} < v(1)$. (See Fig. 1 for an illustration.) In this case, it is *not* Pareto-optimal to give the venture capitalist or the entrepreneur all the equity.

[3] Formally, $\hat{s}$ is efficient within the class of budget-balanced mechanisms. Allowing budget-breaking mechanisms à la Holmström (1982) might yield superior outcomes. Hence $\hat{s}$ is constrained (or second-best) efficient.

-1, i.e., the equity frontier has a slope of minus one at the point where total utility is maximized. The third property states that the venture capitalist's utility is increasing in her equity share. This automatically implies that along the equity frontier we have $u'(s)<0$, ruling out situations where a given value of $\psi(v)$ is associated with more than one equity share.

All three assumptions are innocuous and satisfied by many production technologies. In Appendix A we give two examples of technologies used in the venture capital literature that satisfy (i)–(iii): the linear technology $p(e,a)=\gamma a+(1-\gamma)e$ used in, e.g., Casamatta (2003), and the Cobb-Douglas technology $p(e,a)=a^{\gamma}e^{1-\gamma}$ used in, e.g., Repullo and Suarez (2000). Under the linear technology the two efforts are substitutes, while under the Cobb-Douglas technology they are complements. To make the problem nontrivial, we assume that $\hat{v}>I$, i.e., the surplus-maximizing allocation is sufficiently profitable to allow the venture capitalist to break even.

The entrepreneur's and venture capitalist's *deal utilities* are $U(s)\equiv u(s)$ and $V(s)\equiv v(s)-I$, respectively. The deal utilities are the overall utilities from the *contract* (s,I). The utilities derived from equity shares, $u(s)$ and $v(s)$, are thus merely one component of the agents' deal utilities. The investment outlay I is another component. In Section 5 we will introduce two more components: the safe project payoff X_l and wage payments.

The set of all Pareto-optimal $U-V$ combinations is called the *contract frontier*. The contract frontier is derived from the set of Pareto-optimal contracts (s,I) and denoted by $U=\Psi(V)$. In the base model where $X_l=0$, the contract frontier is obtained by shifting the equity frontier $\psi(v)$ to the left by I, implying that $\Psi(V)\equiv\psi(V+I)$; see Fig. 1. In Section 5 when we reintroduce $X_l>0$ and wage payments, the construction of the contract frontier is more complicated. The domain of the contract frontier is $[\underline{V},\bar{V}]$, where $\underline{V}\equiv\max\{\underline{v}-I,0\}$ and $\bar{V}\equiv\bar{v}-I$. (As the venture capitalist and entrepreneur never bargain to a point where the venture capitalist's deal utility is negative, the constraint that $\underline{V}\geqslant 0$ is without loss of generality.)

Fig. 1 depicts the utility possibility frontier characterizing all possible $u-v$ combinations, the equity frontier characterizing all Pareto-optimal $u-v$ combinations, and the contract frontier characterizing all Pareto-optimal $U-V$ combinations for the Cobb-Douglas technology.

2.2. Bargaining

It is reasonable to assume that when bargaining over a contract, the entrepreneur and venture capitalist select a contract that is Pareto efficient. Bargaining thus corresponds to choosing a utility pair (V,U) on the contract frontier. If the bargaining breaks down, the entrepreneur and venture capitalist realize their outside options U^{o} and V^{o}, respectively. For the time being, we shall take these outside options as given. They will be endogenized in Section 4 as a function of relative supply and demand in the capital market. Our bargaining concept is the generalized Nash bargaining solution. Accordingly, the bargaining outcome consists of deal utilities $U=\Psi(V)\geqslant U^{\mathrm{o}}$ and $V\geqslant V^{\mathrm{o}}$ maximizing the Nash product

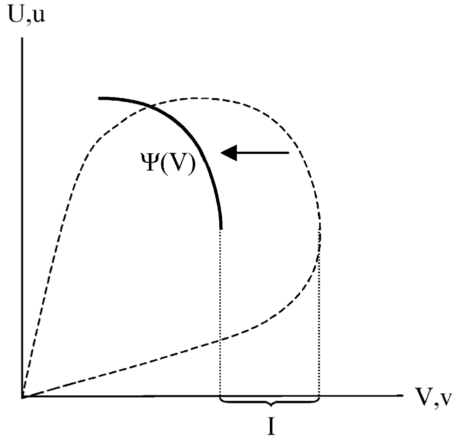

Fig. 1. Equity frontier and contract frontier for the Cobb-Douglas technology. The dashed curve represents the utility possibility frontier (UPF) depicting the entrepreneur's and venture capitalist's utilities from equity shares, u and v, for all possible equity shares $s \in [0,1]$. The decreasing segment of the UPF is the equity frontier. It contains all utilities u and v associated with a Pareto-optimal equity share. Shifting the equity frontier to the left by the investment amount I yields the bold contract frontier $\Psi(V)$, which depicts the entrepreneur's and venture capitalist's utilities from Pareto-optimal contracts, $U = \Psi(V)$ and V, respectively.

$[V - V^{\mathrm{o}}]^{\eta}[\Psi(V) - U^{\mathrm{o}}]^{1-\eta}$, where $\eta \in (0,1)$. For expositional convenience, we define $\beta \equiv \eta/(1-\eta)$.

As the contract frontier is strictly concave, the bargaining problem has a unique solution. In Appendix B we show that this solution must lie in the interior of $[\underline{V}, \bar{V}]$. Denote the bargaining solution by $(V^{\mathrm{d}}, U^{\mathrm{d}})$, where $U^{\mathrm{d}} \equiv \Psi(V^{\mathrm{d}})$. The superscript indicates that V^{d} and U^{d} are the equilibrium deal utilities. Maximizing the Nash product with respect to V, we obtain

Lemma 1. *The equilibrium deal utilities V^{d} and U^{d} are uniquely determined by*

$$\beta = -\Psi'(V^{\mathrm{d}})\frac{V^{\mathrm{d}} - V^{\mathrm{o}}}{\Psi(V^{\mathrm{d}}) - U^{\mathrm{o}}}, \tag{1}$$

where $U^{\mathrm{d}} = \Psi(V^{\mathrm{d}})$.

The venture capitalist's deal utility V^{d} is increasing in her outside option V^{o} and decreasing in the entrepreneur's outside option U^{o}. The reverse is true for the entrepreneur. As is well known, the axiomatic Nash bargaining solution can be derived as the limit of a non-cooperative bargaining game where the two parties bargain with an open time horizon under the risk of breakdown (Binmore et al., 1986). It is worth noting that our results do not depend on the specifics of the Nash bargaining solution. All we need is that an agent's deal utility is positively related to his or her own outside option and negatively related to the counterparty's outside option. Any bargaining solution with this feature yields similar results.

2.3. Search

To endogenize outside options, we embed the bargaining problem in a market environment. We consider a stationary search market populated by entrepreneurs and venture capitalists.[4] The measure of entrepreneurs and venture capitalists in the market is M_e and M_v, respectively. A key variable is the ratio of venture capitalists to entrepreneurs, or *degree of capital market competition*, $M_v/M_e \equiv \theta$. A high value of θ indicates that the capital market is very competitive. Each venture capitalist has capital k, which implies she can finance at most one project. All our results extend to the case where each venture capitalist can finance a finite number of projects. As each venture capitalist has a fixed amount of capital, the ratio $M_v/M_e \equiv \theta$ also indicates the relative magnitude of capital supply and demand. Time is continuous, and the discount rate is $r > 0$.[5]

The measure of deals, or matches, per unit of time is given by the matching function $x(M_e, M_v)$. From the perspective of a venture capitalist, the (Poisson) arrival rate of a deal is $x(M_e, M_v)/M_v$, while the entrepreneur's deal arrival rate is $x(M_e, M_v)/M_e$. We assume that the matching function exhibits constant returns to scale. This has the convenient implication that arrival rates depend solely on the degree of capital market competition θ. (See Section 5.4 for details.) Specifically, the venture capitalist's deal arrival rate is $x(M_e, M_v)/M_v \equiv q_v(\theta)$, which is decreasing in θ with $\lim_{\theta \to 0} q_v(\theta) = \infty$ and $\lim_{\theta \to \infty} q_v(\theta) = 0$. Hence a venture capitalist is more likely to meet an entrepreneur in a given time interval if the ratio of venture capitalists to entrepreneurs is low. As the measure of deals per unit of time is $M_v q_v(\theta)$, the entrepreneur's deal arrival rate is $\theta q_v(\theta) \equiv q_e(\theta)$, which is an increasing function of θ. Hence an entrepreneur is more likely to meet a venture capitalist in a given time interval if the ratio of venture capitalists to entrepreneurs is high.

If the search is successful, the venture capitalist and entrepreneur bargain over a contract. The outside options in the bargaining, U^o and V^o, are the utilities from going back into the market and searching anew. Given that the market is stationary, the utility from going back into the market equals the utility from entering the market in the first place. Hence U^o and V^o represent both the outside options in the bargaining as well as the overall utilities from searching.

We now determine the outside options. Consider first the entrepreneur's outside option U^o. The Poisson arrival rate of a deal for the entrepreneur is $q_e(\theta)$. The probability that a deal occurs in a small time interval Δ is thus $q_e(\theta)\Delta$. With probability $1 - q_e(\theta)\Delta$ no deal occurs, and the entrepreneur continues the search. The expected discounted utility from searching is therefore

$$U^o = q_e(\theta)\Delta \exp(-r\Delta)U^d + (1 - q_e(\theta)\Delta)\exp(-r\Delta)U^o.$$

[4] In the search literature, our framework is commonly known as the Diamond-Mortensen-Pissarides model (Pissarides, 1990).

[5] Frictions are thus expressed as costs of delay. The model can be easily extended to include search costs. For convenience, we assume that both sides use the same discount rate.

Solving for U^{o} and letting $\Delta \to 0$, we obtain

$$U^{\mathrm{o}} = \frac{q_{\mathrm{e}}(\theta)}{q_{\mathrm{e}}(\theta) + r} U^{\mathrm{d}}. \tag{2}$$

Eq. (2) illustrates the relation between the entrepreneur's overall utility U^{o} and his deal utility U^{d}. The overall utility is the discounted expected utility from searching or, alternatively, the utility from a deal minus the expected cost of delay. Accordingly, it holds that $U^{\mathrm{o}} < U^{\mathrm{d}}$. Moreover, by (2) the difference between U^{d} and U^{o} is smaller the smaller is the discount rate r and the greater is the speed of matching $q_{\mathrm{e}}(\theta)$. Rearranging (2) yields the *asset value equation*

$$rU^{\mathrm{o}} = q_{\mathrm{e}}(\theta)(U^{\mathrm{d}} - U^{\mathrm{o}}). \tag{3}$$

Similarly, the venture capitalist's outside option V^{o} is given by the asset value equation

$$rV^{\mathrm{o}} = q_{\mathrm{v}}(\theta)(V^{\mathrm{d}} - V^{\mathrm{o}}), \tag{4}$$

which implies that the venture capitalist can invest funds at the interest rate r while searching for an investment opportunity.

To close the model, we need to specify what the inflows and outflows are. Stationarity requires that the inflow of venture capitalists and entrepreneurs matches their respective outflow. Let m_{v} and m_{e} denote the measure of venture capitalists and entrepreneurs arriving in the market over one unit of time. The inflows m_{v} and m_{e} are fully, and uniquely, determined by the stationarity conditions $m_{\mathrm{v}} = q_{\mathrm{v}}(\theta)M_{\mathrm{v}}$ and $m_{\mathrm{e}} = q_{\mathrm{e}}(\theta)M_{\mathrm{e}}$, respectively. Accordingly, the model is fully pinned down by its stocks: The stocks M_{v} and M_{e} determine (i) the level of capital market competition $\theta \equiv M_{\mathrm{v}}/M_{\mathrm{e}}$, (ii) the size of the market, (iii) the outflows $q_{\mathrm{v}}(\theta)M_{\mathrm{v}}$ and $q_{\mathrm{e}}(\theta)M_{\mathrm{e}}$, (iv) the inflows m_{v} and m_{e}, and (v) the equilibrium values of V^{d}, U^{d}, V^{o}, U^{o}, and s (see Proposition 1 below).

2.4. Equilibrium conditions

The following definition summarizes the equilibrium conditions.

Capital market equilibrium. *An equilibrium is characterized by the following conditions*:

(i) *the deal utilities* $(V^{\mathrm{d}}, U^{\mathrm{d}})$ *maximize the Nash product* $[V - V^{\mathrm{o}}]^{\eta}[\Psi(V) - U^{\mathrm{o}}]^{1-\eta}$;
(ii) *the outside options* $(V^{\mathrm{o}}, U^{\mathrm{o}})$ *satisfy the asset value equations* (3)–(4); *and*
(iii) *the flows and stocks of entrepreneurs and venture capitalists*, $(m_{\mathrm{e}}, m_{\mathrm{v}})$ *and* $(M_{\mathrm{e}}, M_{\mathrm{v}})$, *satisfy the stationarity conditions* $q_{\mathrm{v}}(\theta)M_{\mathrm{v}} = m_{\mathrm{v}}$ *and* $q_{\mathrm{e}}(\theta)M_{\mathrm{e}} = m_{\mathrm{e}}$.

The equilibrium is the solution to a system of four equations: the first-order condition characterizing the bargaining solution (1), the asset value equations (3) and (4), and the identity $U^{\mathrm{d}} \equiv \Psi(V^{\mathrm{d}})$. In the bargaining solution, the deal utilities V^{d} and U^{d} are a function of the outside options V^{o} and U^{o}. Conversely, in the asset value equations, outside options are a function of the deal utilities. Inserting

(3)–(4) into (1), we obtain

$$\beta\frac{r+q_{\mathrm{v}}(\theta)}{r+q_{\mathrm{e}}(\theta)}=-\Psi'(V^{\mathrm{d}})\frac{V^{\mathrm{d}}}{\Psi(V^{\mathrm{d}})}, \tag{5}$$

which implicitly defines the equilibrium value of V^{d} as a function of θ. Inserting the solution in $U^{\mathrm{d}}=\Psi(V^{\mathrm{d}})$ yields the equilibrium value of U^{d}. Finally, inserting V^{d} and U^{d} in the asset value equations (3)–(4) yields the equilibrium values of V^{o} and U^{o}.

3. Value creation in start-ups

3.1. Equity shares, individual utilities, and value creation

The following proposition characterizes how individual utilities and the value created in start-ups depend on the level of capital market competition.

Proposition 1. *For each level of capital market competition θ there exists a unique equilibrium. The venture capitalist*'s *equity share s, deal utility V^{d}, and overall utility V^{o} are all decreasing in θ. The reverse is true for the entrepreneur. The total value created in the start-up $V^{\mathrm{d}}+U^{\mathrm{d}}$ is first increasing and then decreasing in θ.*

Deal utilities and overall utilities move in the same direction. Consider, for instance, the entrepreneur. An increase in capital market competition makes it easier for the entrepreneur to obtain financing, which reduces his cost of delay. The entrepreneur's outside option U^{o} therefore increases (and the venture capitalist's outside option V^{o} decreases), which implies that the bargaining outcome shifts in favor of the entrepreneur. Consequently, the entrepreneur's deal utility U^{d} increases and the venture capitalist's deal utility decreases. The increase in U^{d}, in turn, feeds back into the search market dynamics. As the utility from doing a deal has gone up, searching for a deal becomes more valuable. The overall utility U^{o} therefore increases again, and so on. This process continues until a steady-state equilibrium is reached. Consequently, an increase in θ corresponds to a move along the contract frontier from the right to the left.

The rest follows from the construction of the contract frontier. As we move along the frontier counterclockwise, the venture capitalist's equity share s decreases. This weakens the venture capitalist's incentives and improves the entrepreneur's incentives. The total effect depends on the current level of s and its relation to the efficient sharing rule $\hat{s}$. If $s>\hat{s}$, a reduction in s increases the total value created in the start-up $V^{\mathrm{d}}+U^{\mathrm{d}}$. If $s=\hat{s}$, the value created in the start-up attains its maximum. Finally, if $s<\hat{s}$, a reduction in s decreases the total value created in the start-up.

3.2. Pre- and post-money valuation

A measure of the firm's value commonly used in the industry is the *post-money valuation*. The post-money valuation is an implied value calculated on the basis of the venture capitalist's equity share. If the venture capitalist pays I in return for a share s, the post-money valuation is $\Lambda \equiv I/s$. The*pre-money valuation* is given by $\Gamma \equiv \Lambda - I = I(1-s)/s$. Post- and pre-money valuations are used as measures of the firm's and entrepreneur's NPV, respectively. By Proposition 1, the venture capitalist's equity share is inversely related to the degree of capital market competition θ. This immediately yields the following result.

Proposition 2. *Both the pre-money valuation and the post-money valuation are increasing in the level of capital market competition θ.*

Hellmann (2002) argues that this notion of valuation used in the industry is potentially flawed. Our model supports this critique. First, the formula for the post-money valuation $s\Lambda = I$ is based on the idea that the venture capitalist just breaks even on her investment. By contrast, our model shows that in a competitive market with frictions and moral hazard, venture capitalists receive strictly more than they invest. On the one hand, venture capitalists need to be compensated for their effort costs. Moreover, and more important, depending on the level of capital market competition, venture capitalists typically earn a bargaining premium. In our model, the venture capitalist's deal utility is $V^{\mathrm{d}} = spX_h - g(a) - I > V^{\mathrm{o}}$. Hence the value of the venture capitalist's equity share, spX_h, strictly exceeds the combined effort and investment cost, $g(a) + I$.

Besides, our model suggests that the post-money valuation is a poor indicator of NPV. Holding I fixed, the valuation varies mechanically with the venture capitalist's equity share s. The equity share, in turn, is determined by bargaining. If anything, the valuation thus reflects relative bargaining powers, not NPV. The notion that the valuation reflects relative bargaining powers is consistent with empirical evidence by Gompers and Lerner (2000) documenting a positive relation between the valuation of new ventures and capital market competition.[6]

For the reasons just mentioned, value and valuation practically never coincide. The exact relation between value and valuation depends on the functional and numerical specifications of the model. An example is given in Fig. 2. The example is based on the linear production technology $p(e,a) = \gamma a + (1-\gamma)e$ with effort cost functions $c(e) = e^2/2\alpha_{\mathrm{e}}$ and $g(a) = a^2/2\alpha_{\mathrm{v}}$ and Cobb-Douglas matching technology $x(M_{\mathrm{e}}, M_{\mathrm{v}}) = \xi[M_{\mathrm{e}}M_{\mathrm{v}}]^{0.5}$. The numerical values are $X_h = 0.5$, $I = 0.01$, $\alpha_{\mathrm{e}} = \alpha_{\mathrm{v}} = \beta = 1$, $r = 0.1$ (implying a 10% interest rate), $\gamma = 0.5$, and $\xi = 50$.[7] In the example, the post-money valuation Λ understates the total value $V^{\mathrm{d}} + U^{\mathrm{d}}$ if $\theta < 12.4$

[6] Bartlett (2002) also suggests that the valuation is an indicator of bargaining power rather than firm value: "The discussion starts out with the issue of valuation. The entrepreneur and his or her advisors lay on the table a number, based on art as much as science, and suggest that the venture capitalist agree with it. ... The price, in other words, is usually left to naked negotiations between the buy and the sell side."

[7] If r is the annual interest rate, the expected arrival time of a deal is $q_{\mathrm{e}}(\theta) = \theta^{-0.5}/50$ years for the entrepreneur and $q_{\mathrm{v}}(\theta) = \theta^{0.5}/50$ years for the venture capitalist. For instance, if $\theta = 1$ it takes 7.3 days for either side to find a counterparty. By contrast, if $\theta = 10$ it takes 2.3 days for an entrepreneur but 23 days for a venture capitalist.

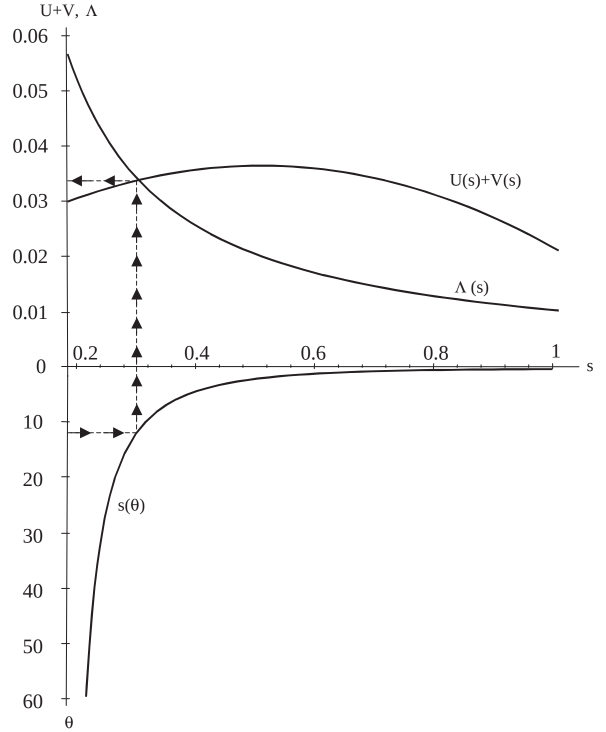

Fig. 2. Total value created in the start-up, $U + V$, and post-money valuation Λ as a function of the venture capitalist's equity share s, which in turn is a function of the level of capital market competition θ. The functional and numerical specifications are given in Section 3.2. The set of Pareto-optimal equity shares is $s \in [0.175, 1]$. $U + V$ and Λ intersect at $s \approx 0.29$, or $\theta \approx 12.4$. If $\theta < 12.4$ the valuation Λ understates the true value created in the start-up, while if $\theta > 12.4$ it overstates it. The graph of $s(\theta)$ has been curtailed at $\theta = 60$ for expositional purposes.

(or $s > 0.29$), i.e., if capital market competition is low, and overstates it if $\theta > 12.4$ (or $s < 0.29$), i.e., if capital market competition is high.

3.3. Market value and success probability

The total value created in the start-up, $V^{\mathrm{d}} + U^{\mathrm{d}}$, takes into account both investment and effort costs. By contrast, the (interim) *market value* pX_h is the expected value of the firm after effort and investment costs are sunk but before cash flows are realized. It is what the entrepreneur and venture capitalist would receive if the firm were sold at an interim date. As X_h is a constant, the functional behavior of pX_h is identical to that of the *success probability* $p(e, a)$.

Unlike the total value $V^{\mathrm{d}} + U^{\mathrm{d}}$, the relation between $p(e, a)$ and s or θ is not necessarily characterized by an inverted U-shape. To be precise, the relation depends on whether the venture capitalist's and entrepreneur's efforts are complements or substitutes. To explore this issue in more detail, we consider the CES production technology $p(e,a) = [\gamma a^\kappa + (1-\gamma)e^\kappa]^{\frac{1}{\kappa}}$, where $\kappa \leqslant 1$ and $\gamma \in (0,1)$. The parameter κ measures the degree of complementarity between the venture capitalist's and entrepreneur's efforts. If $\kappa = 1$, the two efforts are perfect substitutes, while if $\kappa < 1$, the two efforts are complements. To obtain a closed-form solution, we assume quadratic effort costs $c(e) \equiv e^2/2\alpha_e$ and $g(a) \equiv a^2/2\alpha_v$, respectively.

Consider first the case where the two efforts are complements. It is straightforward to show that the success probability is first increasing and then decreasing in s, with interior maximum

$$s_p^* \equiv \frac{1}{1+\varphi}, \quad \text{where } \varphi \equiv \left[\left(\frac{\alpha_e}{\alpha_v}\right)^{\kappa/2}\left(\frac{1-\gamma}{\gamma}\right)\right]^{1/(1-\kappa)}.$$

Due to the inverse relation between s and θ, an increase in θ has a positive effect on the success probability and market value if θ is low and a negative effect if θ is high. Note that the value of s that maximizes the success probability or market value is generally not the same as the value of s that maximizes the total value $V^d + U^d$. The reason is that the total value includes effort costs while market value or success probability do not.

If efforts are perfect substitutes, the success probability and market value are increasing in s (and thus decreasing in θ) if the venture capitalist is more productive ($\varphi < 1$) and decreasing in s if the entrepreneur is more productive ($\varphi > 1$). The following proposition summarizes our results.

Proposition 3. *Unless efforts are perfect substitutes, the market value and success probability are first increasing and then decreasing in the level of capital market competition θ.*

4. Industry dynamics

In the preceding section, we have taken the number of venture capitalists—and hence the level of capital market competition θ—as exogenously given. We believe this is a useful characterization of the short run: "The skills needed for successful venture capital investing are difficult and time-consuming to acquire. During periods when the supply or demand for venture capital has shifted, adjustments in the number of venture capitalists and venture capital organizations appear to take place very slowly" (Gompers and Lerner, 1999). In the long run, however, the number of venture capitalists is endogenous. We now endogenize the entry decision of venture capitalists, thereby endogenizing the level of capital market competition.

4.1. Endogenous entry of venture capitalists

We take as given the flow of new ideas, or projects, that are continuously created in the economy. We hence assume that the supply of venture capital(ists) adjusts more quickly to changes in market characteristics than the supply of new ideas. In Section 5.3 we show that our results continue to hold if both the supply and demand for venture capital is endogenous.

Each idea is associated with an entrepreneur and cannot be traded. We normalize the flow of ideas such that the measure $m_e = 1$ of ideas is created in the economy over one unit of time. The inflow of capital is determined by a zero-profit constraint. We assume that a venture capitalist entering the market incurs a fixed cost $k > 0$. Like any business, setting up a venture capital firm involves fixed costs from setting up a legal structure, hiring experts and support staff, and building a network of lawyers, investment bankers, and clients. Costs also arise from the fact that during the duration of a venture fund, the capital committed to the fund cannot be invested in long-term, illiquid assets. The forgone illiquidity premium represents a fixed cost because it is incurred regardless of whether the capital is invested or not.

The first type of cost is borne by the general partners, while the second type is borne by the limited partners. From the perspective of our model, this is not important, however, as we treat general and limited partners as a homogeneous entity. Moreover, with the exception of Section 4.4, the size of k is irrelevant for our analysis. Hence we are perfectly comfortable with the notion that entry costs are small, as long as they are positive.

Free entry implies that the utility from entry V^o must equal the entry cost k. One implication of this is that $V^o > 0$: To recoup entry costs, the venture capitalist must earn a positive utility in the market. The equilibrium conditions are the same as before, except that $m_e = 1$ and $V^o = k$. Since V^o is monotonic in θ, there is a one-to-one relation between k and θ.

Proposition 4. *For each level of entry cost k, there exists a unique equilibrium associated with a unique level of capital market competition θ.*

Let us briefly state the range of possible equilibrium values. The entry cost k can potentially vary from close (but not equal) to zero to the maximum possible deal utility $\bar{V} \equiv \bar{v} - I$. If $k \to 0$, we have from the venture capitalist's zero-profit constraint that $V^o \to 0$. By (4), this implies that $\theta \to \infty$, i.e., the venture capitalist must wait increasingly long. On the other hand, if $k \to \bar{V}$, we have $\theta \to 0$, i.e., the venture capitalist's waiting time, or cost of delay, must go to zero to ensure that she breaks even. Finally, if $\theta \to \infty$, we have from (5) that $V^d \to \max\{0, \underline{v} - I\}$ and $s \to \max\{s_0, \underline{s}\}$, where s_0 is implicitly defined by $v(s_0) = I$. On the other hand, if $\theta \to 0$, we have from (5) that $V^d \to \bar{V}$ and $s \to \bar{s}$.

4.2. Changes in investment profitability

One possible explanation for the massive entry of venture funds during the Internet bubble is the expectation of high returns. To the extent that the Internet caused a genuine increase in productivity, these expectations might have been justified. To some extent, however, it appears that the market initially overreacted. Firms that perform well have a strong incentive to make their success public. Poor performers, on the other hand, typically keep a low profile. It is possible that the market viewed early winners such as eBay or Amazon as representative of the entire industry, thereby underestimating the true default risk of dotcoms (Hellmann and

Puri, 2002). Moreover, the market might have not fully taken into account the impact of competition on the survival chances of firms. Even if sector growth was predicted correctly, it appears that some firms were valued as if they were the sole competitor in the industry (Lerner, 2002).

In the following, we examine the consequences of a change in investment profitability caused, e.g., by a technological innovation such as the Internet. From the perspective of our model, such a change can be either true or merely perceived. All we require is that investors and entrepreneurs share the same perception, i.e., that they have homogeneous beliefs. Hence, our model applies equally to situations in which entry occurs following a genuine increase in investment profitability as well as to situations in which entry occurs because everybody overestimates the true profitability increase.

4.2.1. Expansion of the equity (and contract) frontier

Since $X_l = 0$, a change in investment profitability translates into a change in X_h. An increase in X_h shifts the equity frontier outward while a decrease in X_h shifts it inward. We assume that along the equity frontier, utilities are separable of the form $u(X_h, s) = \chi(X_h)\omega_e(s)$ and $v(X_h, s) = \chi(X_h)\omega_v(s)$, where $\chi(X_h)$ and $\omega_v(s)$ are increasing functions and $\omega_e(s)$ is a decreasing function. Both the linear technology $p(e, a) = \gamma a + (1-\gamma)e$ and the Cobb-Douglas technology $p(e, a) = a^\gamma e^{1-\gamma}$ introduced earlier satisfy this requirement (see Appendix A for details).

Geometrically, the various equity frontiers are radial expansions of each other.[8] This has two implications. Consider an increase in X_h while holding s constant. First, the ratio of utilities $u(X_h, s)/v(X_h, s) = \omega_e(s)/\omega_v(s)$ remains unchanged. That is, the entrepreneur's and venture capitalist's *relative* utilities from equity shares depend only on the way returns are split (i.e., on the sharing rule s), but not on absolute returns. Second, the slope of the equity frontier (and thus also the slope of the contract frontier Ψ'),

$$\psi'(X_h, s) = \frac{\mathrm{d}u(X_h, s)}{\mathrm{d}v(X_h, s)} = \frac{\partial u(X_h, s)/\partial s}{\partial v(X_h, s)/\partial s} = \frac{\omega_e'(s)}{\omega_v'(s)}, \tag{6}$$

is a function of s only and remains therefore unchanged following an increase in X_h.[9]

[8] The concept of a radial expansion is well known from microeconomic demand and production theory. If a utility or production function is homogeneous (of arbitrary degree), its level sets (i.e., indifference curves or isoquants) are radial expansions of each other.

[9] This implies that our results continue to hold if X_h is a random variable that is realized after the contract is signed but before efforts are made. (If X_h is realized after efforts are made, our results hold trivially.) In principle, the optimal contract can take the form of a menu prescribing a different value of s for each realization of X_h. By standard arguments, the optimal menu will equate the ratio of marginal utilities across different states of nature:

$$\frac{\partial U(X_{hi}, s_i)/\partial s}{\partial V(X_{hi}, s_i)/\partial s} = \frac{\partial U(X_{hj}, s_j)/\partial s}{\partial V(X_{hj}, s_j)/\partial s},$$

where $i \neq j$, implying that $\Psi'(X_{hi}, s_i) = \Psi'(X_{hj}, s_j)$. Given our assumption of separability, the slope of the contract frontier Ψ' is independent of X_h. Moreover, each slope Ψ' is associated with a unique value of s. Together, this implies that $s_i = s_j$. Hence the optimal menu consists of a single sharing rule s, which implies all our results continue to hold. We thank the referee for pointing this out.

The increase in X_h is depicted in Fig. 3 (upper picture). The (dashed) equity frontier shifts outward from EF_1 to EF_2. Holding s constant, we move along the dashed line as indicated by the arrow. Consequently, the utility ratio u/v remains constant. This is the first of the two properties stated in the previous paragraph. The second is that the slopes of EF_1 and EF_2 are identical along the dashed line. This second property has a simple economic interpretation. The slope of the equity frontier, $\psi' = (\partial u/\partial s)/(\partial v/\partial s)$, captures the fundamental tradeoff underlying the double-sided incentive problem: A marginal increase in s increases the venture capitalist's utility by $\partial v/\partial s$ but reduces the entrepreneur's utility by $\partial u/\partial s$. The fact that the slope remains constant following a shift in X_h implies that this tradeoff is independent of absolute return levels.

4.2.2. Short-run analysis

We now consider the equilibrium effect of a change in investment profitability. We begin with the short run where the number of venture capitalists, and hence the level of capital market competition θ, is fixed. Subsequently, we consider the long run where entry is endogenous, and where the supply of capital can adjust freely in response to changes in investment profitability.

Consider again Fig. 3 (upper picture). For expositional convenience, we have broken up the short-run effect into two effects. The first effect (marked "1") depicts the increase in X_h while s is held constant. Along with the equity frontier, the (bold) contract frontier shifts outward from CF_1 to CF_2. As can be easily seen, the two contract frontiers are not radial expansions of each other. That is, if we increase X_h while holding s constant, we do not move along the (dotted) line going through the origin. Instead, we move along the bold line as indicated by the arrow. Along the bold line, the two frontiers CF_1 and CF_2 have the same slope, a property that was defined in (6).

The second effect (marked "2") depicts the adjustment in the venture capitalist's equity share s as a move along the new contract frontier CF_2 from the right to the left, implying a decrease in s. To understand this effect, consider Eq. (5), which characterizes the short-run equilibrium:

$$\beta \frac{r + q_{\mathrm{v}}(\theta)}{r + q_{\mathrm{e}}(\theta)} = -\Psi'(X_h, s)\frac{V^{\mathrm{d}}}{U^{\mathrm{d}}}. \tag{7}$$

Under the first effect, the slope $\Psi'(X_h, s)$ remains unchanged. However, X_h—and therefore also $\chi(X_h)$—has increased, which implies that the utility ratio

$$\frac{V^{\mathrm{d}}}{U^{\mathrm{d}}} = \frac{\omega_{\mathrm{v}}(s)}{\omega_{\mathrm{e}}(s)} - \frac{I}{\chi(X_h)\omega_{\mathrm{e}}(s)} \tag{8}$$

must have also increased. Accordingly, under the first effect the right-hand side in (7) has increased. By contrast, the left-hand side remains constant since θ is fixed in the short run. To restore the equality in (7), the second effect must therefore bring the right-hand side in (7), $-\Psi' V^{\mathrm{d}}/U^{\mathrm{d}}$, back to its original level. But this implies that s must decrease, for both $-\Psi'$ and $V^{\mathrm{d}}/U^{\mathrm{d}}$ are increasing in s. Hence the second effect

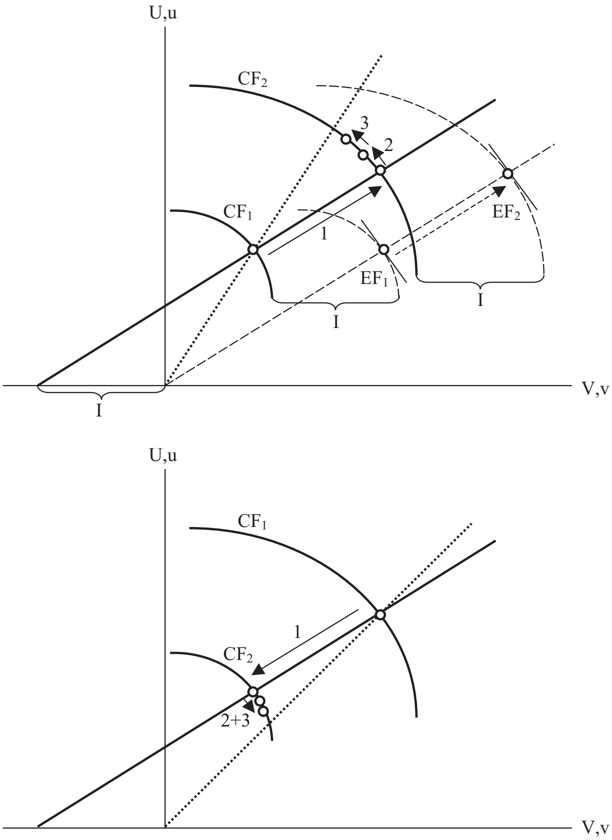

Fig. 3. Short- and long-run effects of an increase (upper picture) and decrease (lower picture) in investment profitability. The upper picture depicts both the change in the (dashed) equity frontier and (bold) contract frontier, while the lower picture depicts only the change in the contract frontier. An increase in investment profitability shifts both the equity and contract frontier outward, from EF_1 to EF_2 and from CF_1 to CF_2, respectively. Effects 1 and 2 together constitute the short-run effect. Effect 3 depicts the additional adjustment in the venture capitalist's equity share that occurs in the long run. Effects 1–3 together constitute the total, i.e., long-run, effect.

must involve a decrease in s corresponding to a leftward move along the new contract frontier CF_2.

This leftward move along CF_2 must end before the dotted line, which is the line along which V/U is constant. In other words, the short-run equilibrium must lie on CF_2 between the bold and dotted line. To see this, suppose to the contrary that the short-run equilibrium lies *on* the dotted line. In this case, V/U remains unchanged while $-\Psi'$ has decreased, violating (7). Similarly, the short-run equilibrium cannot lie to the left of the dotted line, for this implies that both V/U and $-\Psi'$ have decreased, again violating (7). In the short run, both V^{d} and U^{d} must therefore increase, s must decrease, and $V^{\mathrm{d}}/U^{\mathrm{d}}$ must increase.

Let us rephrase the argument for why s must decrease slightly. Suppose, contrary to what is true, s remains constant. (The argument for why s cannot increase is similar.) Since the various equity frontiers are radial expansions of each other, u and v will increase by the same factor. The contract frontiers, on the other hand, are *not* radial expansions of each other: If u and v increase by the same factor—thereby leaving u/v constant—the ratio $V/U = (v - I)/u$ increases. At the same time, the slope of the equity frontier—and thus the slope of the contract frontier, Ψ'—remains unchanged. But this violates the requirement that in the short run the ratio of elasticities

$$-\frac{(\partial U(X_h, s)/\partial s)}{(\partial V(X_h, s)/\partial s)} \frac{V^{\mathrm{d}}}{U^{\mathrm{d}}} = -\Psi' \frac{V^{\mathrm{d}}}{U^{\mathrm{d}}}$$

must remain constant. (This follows directly from combining (7) and (8)). The only way this requirement can be satisfied given an increase in X_h is if V/U increases and $-\Psi'$ decreases at the same time, which corresponds to a decrease in s.

4.2.3. Long-run analysis

In the short run, an increase in X_h increases the venture capitalist's deal utility V^{d}, thereby raising her overall utility V^{o} above the entry cost k. In the long run, the zero-profit constraint $V^{\mathrm{o}} = k$ must bind. To restore this equality, new entry must occur, implying that the degree of capital market competition θ must increase. This increase in θ, in turn, feeds back into the bargaining solution. As the venture capitalist's outside option remains fixed at $V^{\mathrm{o}} = k$, the increase in θ must show up as an increase in the entrepreneur's outside option U^{o}. The end result is a further decline in the venture capitalist's equity share s. On top of the short-run effect described above, we thus have the additional effect that θ increases, coupled with a (further) decrease in s.

In Fig. 3, the additional adjustment effect is marked "3." The total, i.e., long-run effect is obtained by adding effects 1–3. Unlike the short run, we cannot say for sure whether the long-run equilibrium lies to the right or left of the dotted line along which V/U is constant. We do know, however, that (i) s has decreased, and (ii) the venture capitalist's deal utility V^{d} has increased. The second statement follows from the fact that θ—and therefore the venture capitalist's cost of delay—has increased, while at the same time her overall utility V^{o} remains fixed at $V^{\mathrm{o}} = k$. Together, (i) and (ii) imply that the new long-run equilibrium must lie on CF_2 between the bold line and a vertical line going through the old equilibrium point (i.e., the point where the bold and dotted lines intersect). The following proposition summarizes our results. "Short run" refers to effects 1–2 while "long run" refers to effects 1–3.

Proposition 5. *An increase in investment profitability has the following effects.*

(i) *In the short run, the level of capital market competition θ remains constant. The deal and overall utilities V^{d}, U^{d}, V^{o}, and U^{o} all increase, while the venture capitalist's equity share s decreases.*

(ii) *In the long run, new entry occurs until the zero-profit condition $V^{\mathrm{o}} = k$ is restored. As a result, θ, V^{d}, U^{d}, and U^{o} all increase, V^{o} remains constant, and s decreases.*

4.2.4. Value and valuation

Combining Proposition 5 with Propositions 1 and 2, we obtain the following result.

Proposition 6. *An increase in investment profitability increases the total value created in the start-up, $V^{\mathrm{d}} + U^{\mathrm{d}}$, as well as the pre- and post-money valuation, both in the short and long run.*

4.2.5. Internet boom and bust

Fig. 3 provides a useful, albeit stylized, picture of the effects associated with the Internet boom and bust periods of the 1990s and early 2000s, respectively. There is no doubt that the Internet boom involved an increase in venture capital returns, which is captured by the outward shift of the contract frontier (upper picture). This increase in returns, in turn, led to massive entry both by professional venture capital organizations and would-be venture capitalists from Wall Street.[10] The consequence was a rise in valuations, as documented by Gompers and Lerner (2000). As argued earlier, it is possible that the market initially overestimated the true increase in investment returns associated with the Internet. As the first firms went bankrupt, the market corrected its initial assessment, which is captured by the inward shift of the contract frontier (bottom picture). This caused a fall in valuations and led to exit by many investors.

According to this view, the initial outward shift of the contract fontier could reflect both a genuine increase in investment profitability and a possible overreaction by market participants. (Our model is consistent with both interpretations; see our earlier remarks at the beginning of Section 4.2.) The subsequent inward shift, on the other hand, could be viewed a correction of this initial overreaction. As argued by Hellmann and Puri (2002) and others (e.g., *Business Week,* "Innovation Drought," July 9, 2001), this correction might itself constitute an overreaction, however.

4.2.6. Speed of matching

A change in investment profitability also affects the speed of matching and hence the expected delay incurred by market participants. Consider the long-run effect of an increase in investment profitability described in Proposition 5. The associated increase in capital market competition makes it easier for an entrepreneur to obtain financing, which implies that the entrepreneur's deal arrival rate $q_{\mathrm{e}}(\theta)$ increases. Conversely, the increase in competition makes it tougher for a venture capitalist to find a deal, which implies that $q_{\mathrm{v}}(\theta)$ decreases. This is summarized in the following proposition.

[10] *The Economist* ("Money to Burn," May 27, 2000) notes: "A host of new entrants are now dabbling in venture capital, ranging from ad hoc groups of MBAs to blue-blooded investment banks such as J.P. Morgan, to sports stars and even the CIA." For a critical assessment of this trend, see Hellmann and Puri (2002).

Proposition 7. *An increase in investment profitability increases the speed of matching for the entrepreneur and reduces the speed of matching for the venture capitalist*

4.3. Changes in entry costs

Entry costs can change over time. For instance, prior to the Department of Labor's adjustment of the "prudent man rule" in 1979, pension funds were not allowed to invest in venture capital, while after 1979 they were. For a large segment of the U.S. financial market, the cost of providing venture capital thus dropped from infinity to a reasonable number. As a consequence, the supply of venture capital increased sharply (Fenn et al., 1995). Another example is the massive entry of inexperienced players into the venture capital industry during the Internet boom. One interpretation is that entry costs have fallen. After all, it takes less skill to advise a company selling dog food over the Internet than to advise a PC manufacturer.

Consider a decrease in entry costs. By the zero-profit constraint $V^{\mathrm{o}} = k$, a decrease in k implies that the venture capitalist's overall utility V^{o} must decrease by the same amount. Since V^{o} and θ are inversely related by Proposition 1, this in turn implies that the level of capital market competition θ must increase. The effects on individual utilities, equity shares, the total value created in start-ups, market value, success probability, and pre- and post-money valuation then follow immediately from Propositions 1–3.

Proposition 8. *A decrease in entry costs increases the level of capital market competition* θ.

4.4. Changes in capital market transparency

Technological innovations such as the Internet, but also the regional concentration of venture capitalists in Silicon Valley and Boston's Route 128, arguably had a positive effect on the transparency of the venture capital market. To explore the role of transparency, we extend the matching function as follows. The mass of deals per unit of time is $x(M_{\mathrm{e}}, M_{\mathrm{v}}, \xi)$, where ξ is a real-valued transparency parameter, and where $\mathrm{d}x/\mathrm{d}\xi > 0$ implies that an increase in transparency makes matching easier. In particular, it holds that $\partial q_{\mathrm{e}}(\theta, \xi)/\partial \xi > 0$ and $\partial q_{\mathrm{v}}(\theta, \xi)/\partial \xi > 0$, i.e., an increase in transparency speeds up the matching process for both sides.

As an example, consider the Cobb-Douglas matching technology $x(M_{\mathrm{e}}, M_{\mathrm{v}}, \xi) = \xi[M_{\mathrm{e}}M_{\mathrm{v}}]^{0.5}$. Given this specification, arrival rates are $q_{\mathrm{e}}(\theta, \xi) = \xi\theta^{0.5}$ and $q_{\mathrm{v}}(\theta, \xi) = \xi\theta^{-0.5}$, respectively.

4.4.1. Short-run analysis

We begin with the short run, where the degree of capital market competition is fixed. Subsequently, we consider the long-run equilibrium where entry is endogenous.

The short-run effect of an increase in capital market transparency is to amplify the role of relative market power for the bargaining outcome. Inserting the asset value equations (3)–(4) in the first-order condition characterizing the bargaining outcome (1) yields

$$\beta \frac{r + q_v(\theta, \xi)}{r + q_e(\theta, \xi)} = -\Psi'(V^d) \frac{V^d}{\Psi(V^d)}. \tag{9}$$

Implicitly differentiating V^d with respect to ξ shows that if $\theta \equiv M_v/M_e < 1$, i.e., if venture capitalists constitute the short side of the market, an increase in capital market transparency improves the venture capitalist's deal utility.[11] If $\theta > 1$, the opposite is true.

In addition to this amplification effect, there is the direct effect that an increase in transparency speeds up the matching process. Together, these two effects jointly determine the overall utilities V^o and U^o. As an illustration, consider the asset value equation (4) characterizing the equilibrium relation between the venture capitalist's deal utility V^d and her overall utility V^o:

$$V^o = \frac{q_v(\theta, \xi)}{r + q_v(\theta, \xi)} V^d. \tag{10}$$

The direct effect is that $q_v(\theta, \xi)/[r + q_v(\theta, \xi)]$ is increasing in ξ. The amplification effect is that V^d is either increasing or decreasing in ξ, depending on whether venture capitalists constitute the short or the long side of the market. Hence if $\theta < 1$ both effects go in the same direction, and an increase in ξ unambiguously increases V^o. By contrast, if $\theta > 1$ the two effects go in different directions. Similarly, if $\theta > 1$ an increase in ξ increases U^o, while if $\theta < 1$ the effect is ambiguous.

4.4.2. Long-run analysis

In the short run, an increase in capital market transparency either increases or decreases V^d, depending on whether venture capitalists constitute the short or long side of the market. By contrast, in the long run V^d must decrease. Consider again Eq. (10). Since $V^o = k$, the left-hand side is a constant. Moreover, $q_v(\theta, \xi)/[r + q_v(\theta, \xi)]$ is increasing in ξ but decreasing in θ. Therefore, to offset the increase in ξ, either θ must increase or V^d must decrease, or both. But if θ increases, V^d decreases as well. No matter what happens to θ, an increase in transparency therefore decreases the venture capitalist's deal utility V^d. Naturally, this implies that s decreases, while the entrepreneur's deal utility U^d and outside option U^o both increase.

Finally, an increase in transparency improves the speed of matching and reduces the cost of delay. As far as the venture capitalist is concerned, this follows from the fact that V^d decreases while V^o remains constant. As for the entrepreneur, consider again Eq. (9). If V^d decreases and $q_v(\theta, \xi)$ increases, the entrepreneur's speed of matching $q_e(\theta, \xi)$ must increase.

The following proposition summarizes the effect of a change in capital market transparency. The effects on pre- and post-money valuation, market value, and

[11] More precisely, since $\Psi' < 0$ and $\Psi'' < 0$ we have that $\mathrm{d}V^d/\mathrm{d}\xi > 0$ if and only if the left-hand side in (9) is increasing in ξ. Given that $q_e(\theta, \xi) = \theta q_v(\theta, \xi)$ the result is immediate.

success probability are omitted for the sake of brevity. They follow immediately from combining Proposition 9 with Propositions 2–3.

Proposition 9. *An increase in capital market transparency has the following effects.*

(i) *In the short run, the level of capital market competition θ remains constant. If $\theta < 1$, the venture capitalist's deal utility V^d, equity share s, and overall utility V^o all increase, while the entrepreneur's deal utility U^d decreases. The effect on U^o is ambiguous. If $\theta > 1$, the reverse holds.*
(ii) *In the long run, V^o is determined by the zero-profit constraint $V^o = k$. Moreover, V^d and s both decrease while U^d and U^o both increase.*
(iii) *Both in the short and long run, the entrepreneur's and venture capitalist's speed of matching increase.*

5. Discussion and robustness

In this section, we reconsider various assumptions of our model. In Section 5.1, we relax the assumption that the project payoff is zero in the bad state. In Section 5.2, we allow for nondistortionary transfers, or wage payments, from the venture capitalist to the entrepreneur. In Section 5.3, we allow for endogenous entry by both venture capitalists and entrepreneurs. In Section 5.4, we reconsider the assumption that the matching technology exhibits constant returns to scale. Finally, in Section 5.5, we examine the welfare and policy implications of our model.

5.1. Safe project payoff

Adding a safe payoff $X_l > 0$ has no qualitative effects on our model. By definition, the equity frontier $\psi(v)$ remains unchanged. The contract frontier $\Psi(V)$, on the other hand, changes. In the following, we sketch how the contract frontier and our results need to be modified if $X_l > 0$. Formal proofs are found in the working paper version (Inderst and Müller, 2002). The proofs are analogous to those in the base model.

5.1.1. Construction of the contract frontier

If $X_l > 0$, the project payoff can be decomposed into two parts: a safe, state-independent claim X_l and a risky claim whose value is zero with probability $1 - p$ and $X_h - X_l$ with probability p. With the usual degree of caution, we shall label these claims debt and equity, respectively. For our model, such labeling is not important, however. All that matters is that one claim has incentive effects while the other has not. To minimize the amount of notation, we continue to denote the venture capitalist's equity share by $s \in [0, 1]$. The amount of debt held by the venture capitalist is $D \in [0, X_l]$.[12] Deal utilities are $U(s, D) \equiv u(s) + X_l - D$ and $V(s, D) \equiv v(s) + D - I$, respectively.

[12] This rules out that an agent receives a higher payment in the bad state than in the good state. It is easy to show that such contracts are never optimal.

The modified contract frontier is depicted in Fig. 4. It is constructed from the equity frontier by adding the debt in a way that minimizes incentive distortions. Suppose, for instance, that $s > \hat{s}$, implying that the venture capitalist holds too much equity relative to the efficient benchmark. Any Pareto-optimal contract where $s > \hat{s}$ must also have $D = X_l$, i.e., the venture capitalist must hold the entire debt. If this was not the case, a Pareto improvement would be possible whereby the entrepreneur trades in his debt for a greater share of the equity. Similarly, if $s < \hat{s}$ the entrepreneur must hold the entire debt, while if $s = \hat{s}$ any debt allocation is Pareto optimal.

5.1.2. Equilibrium analysis

There are two main differences between the new contract frontier and the one in the base model. First, adding a nonzero payoff in the bad state is like an increase in investment profitability in that it shifts the contract (but not the equity) frontier outward. Second, the new contract frontier has a linear, intermediate segment with slope equal to minus one. On this segment, the sharing rule—or division of equity shares—is efficient, and utility is transferred exclusively by shifting debt. Fig. 4 depicts how the debt/equity mix varies as we move along the contract frontier from the right to the left. In the right segment where $s > \hat{s}$, utility is transferred by reducing s. When the efficient allocation $s = \hat{s}$ is reached, the venture capitalist transfers utility by reducing D, which has no incentive effects. Finally, when $D = 0$ is reached, the left segment begins. The only possible way to transfer utility to the entrepreneur is now to reduce s.

It is straightforward to show that all our results continue to hold—with minor qualifications. (Regarding the shift in investment profitability analyzed in Section 4.2, our results hold regardless of whether the shift occurs as a shift in the safe or risky payment.) All results involving the deal utilities U^{d} and V^{d} (but not the sum $U^{\mathrm{d}} + V^{\mathrm{d}}$), the overall utilities U^{o} and V^{o}, and the speed of matching remain unchanged. By contrast, results involving the total value, success probability, market value, or pre- and post-money valuation need to be adjusted. Whenever the relation between one of these variables and θ was previously monotonic, or first increasing and then decreasing, it now has a flat segment since s is constant for intermediate θ-values. For instance, the total value $U^{\mathrm{d}} + V^{\mathrm{d}}$ is now first increasing, then constant, and then decreasing in θ.

5.2. Wage (or transfer) payments

The benefit of a safe payoff is that it can be used to transfer utility without affecting incentives. A wage, or transfer payment, does exactly the same.

We continue to assume that the entrepreneur has no wealth. Hence the venture capitalist can pay the entrepreneur a wage, but not vice versa. Fig. 5 depicts the effect of a wage payment on the contract frontier. Consider an increase in the level of capital market competition θ, which implies that we move along the contract frontier from the right to the left. In the right, concave segment where $s > \hat{s}$, the optimal way to transfer utility to the entrepreneur is to reduce s. Hence for low values of θ, wages are never used. Wages are only used when the efficient sharing rule $s = \hat{s}$ is reached.

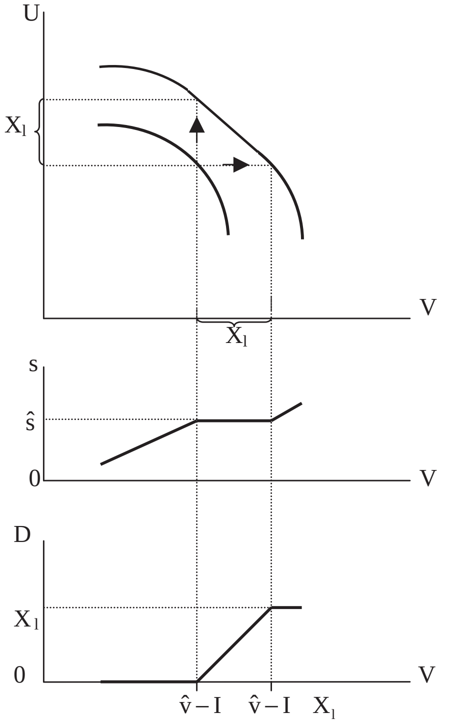

Fig. 4. Contract frontier and optimal contract if there is a safe payoff X_l. Adding a safe payoff shifts the contract frontier outward. The contract frontier with $X_l > 0$ is constructed from that with $X_l = 0$ as follows: The segment where $s < \hat{s}$ is shifted upward, and the segment where $s > \hat{s}$ is shifted rightward by X_l. The shifted segments are connected by a straight line with slope minus one. In the picture, s and $\hat{s}$ denote the venture capitalist's and second-best equity share, respectively, U and V are the entrepreneur's and venture capitalist's utilities from Pareto-optimal contracts, respectively, and $\hat{v} - I$ is the venture capitalist's utility under the contract $(\hat{s}, D = 0)$, where $D \in [0, X_l]$ is the amount of debt held by the venture capitalist. The middle and lower pictures show how s and D vary along the (new) contract frontier.

The optimal way to transfer utility is then either to reduce D or to raise the entrepreneur's wage. (The two are perfect substitutes.) If the venture capitalist can pay a sufficiently high wage—as is assumed in Fig. 5—the linear segment of the contract frontier extends all the way to the left. On the other hand, if wages are limited, the situation is like in Fig. 4. With potentially unlimited wages, some of our results change. All results involving the deal utilities U^d and V^d (but not $U^d + V^d$), the overall utilities U^o and V^o, and the speed of matching remain unchanged. By contrast, the total value, pre- and post-money valuation, success probability, and market value are now constant for high θ.

While venture capital contracts frequently stipulate a wage, these wages are relatively small. The vast bulk of the entrepreneur's compensation comes in the form of financial claims. From an empirical perspective, the most plausible scenario is therefore Fig. 1 or Fig. 4, not Fig. 5. One possible reason why we do not observe large (non-state-contingent) wage payments is that they would attract fraudulent

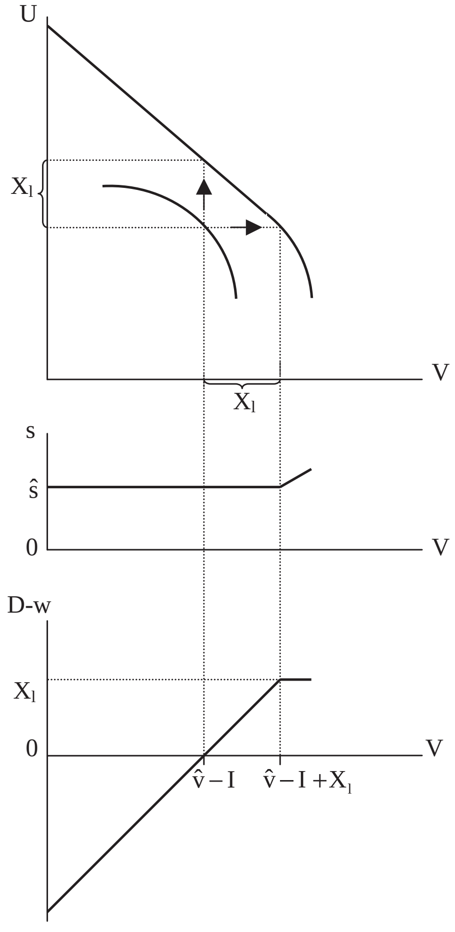

Fig. 5. Contract frontier and optimal contract if there is a safe payoff X_l and wage payments w are potentially unlimited. The construction of the contract frontier with $X_l > 0$ and $w > 0$ is similar to Fig. 4, except that the left segment where $s < \hat{s}$—i.e., where the venture capitalist's equity share is below the second-best equity share—is replaced by a straight line with slope minus one. In the picture, U and V are the entrepreneur's and venture capitalist's utilities from Pareto-optimal contracts, respectively, and $\hat{v} - I$ is the venture capitalist's utility under the contract $(\hat{s}, D - w = 0)$, where $D \in [0, X_l]$ is the amount of debt held by the venture capitalist. (Wages and debt are perfect substitutes, which implies that only the difference $D - w$ is uniquely determined.)

entrepreneurs, or "fly-by-night operators" (Rajan, 1992; von Thadden, 1995; Hellmann, 2002). Another reason are incentive problems between the venture capitalist and limited partners (Holmström and Tirole, 1997). To mitigate the incentive problem, the venture capitalist will generally have to put up a fraction of her own wealth, which naturally puts an upper bound on the wage that she can pay to the entrepreneur.

5.3. Endogenous entry of venture capitalists and entrepreneurs

All our results extend to the case where entry by both venture capitalists and entrepreneurs is endogenous, provided entrepreneurs face heterogeneous entry costs. (If venture capitalists and entrepreneurs both have homogeneous entry costs, the economy is not well defined.) We continue to assume that the mass one of new ideas is created per unit of time. To transform an idea into a viable project, an entrepreneur must now pay an upfront cost of b, however. This includes patent fees as well as the cost of setting up a firm. We assume that b varies from entrepreneur to entrepreneur. For simplicity, we assume that b is the realization of a random draw from the continuous distribution function $G(b)$ with support $[0, \infty)$. Entrepreneurs thus differ only with respect to their entry cost. Once they enter the market, all entrepreneurs are identical.

By (2), the overall utility of an entrepreneur from entering the market is

$$U^{\mathrm{o}} = \frac{q_{\mathrm{e}}(\theta)}{q_{\mathrm{e}}(\theta) + r} U^{\mathrm{d}}. \tag{11}$$

Consequently, there exists a unique threshold $\bar{b} \equiv q_{\mathrm{e}}(\theta)U^{\mathrm{d}}/(q_{\mathrm{e}}(\theta) + r)$ such that all entrepreneurs with entry cost $b \leqslant \bar{b}$ enter while those with entry cost $b > \bar{b}$ stay out. Note that, unlike venture capitalists, entrepreneurs with entry cost $b < \bar{b}$ make a positive profit. Only the marginal entrant whose entry cost is $b = \bar{b}$ makes no profit. To keep the market stationary, the measure of entrepreneurs entering the market must equal the measure of entrepreneurs leaving the market, i.e., $G(\bar{b}) \equiv m_{\mathrm{e}} = q_{\mathrm{e}}(\theta)M_{\mathrm{e}}$.

The inflow of entrepreneurs, $G(\bar{b})$, is increasing in the level of capital market competition θ: A higher value of θ implies a higher value of $q_{\mathrm{e}}(\theta)U^{\mathrm{d}}/(q_{\mathrm{e}}(\theta) + r)$, which in turn implies a higher $\bar{b}$ and therefore a higher $G(\bar{b})$. In other words, the supply of entrepreneurs (or new ideas) is upward sloping in θ. The supply of venture capital(ists), on the other hand, is perfectly elastic: If θ is low such that $V^{\mathrm{o}}(\theta) > k$, new entry of venture capitalists occurs—thereby driving up θ—until the equality $V^{\mathrm{o}}(\theta) = k$ is restored. Similarly, if $V^{\mathrm{o}}(\theta) < k$, exit by venture capitalists drives θ down until $V^{\mathrm{o}}(\theta) = k$ is restored. The entry cost of venture capitalists, k, thus (again) uniquely pins down the equilibrium level of θ. The equilibrium level of θ, in turn, uniquely pins down the inflow of entrepreneurs, $G(\bar{b})$.

Since $G(\bar{b})$ is endogenous, no new constraints have been added. Consequently, all our results remain unchanged. As an example, consider, the increase in investment profitability examined in Section 4.2 and illustrated in Fig. 3. By definition, the short-run effects 1 and 2 remain the same: Both the deal utilities U^{d} and V^{d} as well as the overall utilities U^{o} and V^{o} increase. What is new is that this increase in utilities now triggers new entry by both venture capitalists *and* entrepreneurs. And yet, the overall effect is unambiguous: The level of capital market competition θ must increase by the same amount as in the base model (effect 3). That is, we must arrive at exactly the same end point in Fig. 3, with the same sharing rule s and the same value and valuation. Intuitively, since capital supply is perfectly elastic, the equilibrium is again uniquely (and solely) determined by the venture capitalists' zero-profit constraint $V^{\mathrm{o}}(\theta) = k$. Since k is the same as in the base model, the corresponding increase in θ must also be the same.

5.4. *Constant-returns-to-scale matching technology*

A convenient, but potentially limiting, assumption of our model is that the matching function exhibits constant returns to scale. This implies that $q_{\mathrm{v}}(\theta) \equiv x(M_{\mathrm{e}}, M_{\mathrm{v}})/M_{\mathrm{v}} = x(M_{\mathrm{e}}/M_{\mathrm{v}}, 1) = x(\theta, 1)$, i.e., the arrival rate of a deal for a venture capitalist depends solely on the *relative* numbers of entrepreneurs and venture capitalists in the market, not on the absolute market size. The same is true for the entrepreneur's arrival rate. This has the somewhat counterfactual implication that when M_{e} and M_{v} increase by the same factor, the matching probability remains constant even though the market becomes bigger, and thus potentially more liquid.

If capital market competition is endogenous, the assumption of constant returns to scale is innocuous. That is, all our results continue to hold if the matching function exhibits decreasing or increasing returns to scale. All we need is that $x(M_{\mathrm{e}}, M_{\mathrm{v}})$ is increasing in both arguments. The intuition is simple. With endogenous entry we have $x(M_{\mathrm{e}}, M_{\mathrm{v}}) = m_{\mathrm{e}} = 1$, implying that $\mathrm{d}M_{\mathrm{v}}/\mathrm{d}M_{\mathrm{e}} = -(\partial x/\partial M_{\mathrm{e}})/(\partial x/\partial M_{\mathrm{v}}) < 0$. Hence M_{v} and M_{e} are inversely related, which implies that for every feasible pair $(M_{\mathrm{v}}, M_{\mathrm{e}})$ there is a unique θ. Once again, $q_{\mathrm{v}}(M_{\mathrm{v}}, M_{\mathrm{e}})$ and $q_{\mathrm{e}}(M_{\mathrm{v}}, M_{\mathrm{e}})$ are then fully determined by θ, with $q_{\mathrm{v}}'(\theta) < 0$ and $q_{\mathrm{e}}'(\theta) > 0$.

This is not true if capital market competition is exogenous. As flows and stocks can always be scaled accordingly (see Section 2.3), there exists an additional degree of freedom. This introduces a potentially countervailing effect. For example, suppose M_{e} and M_{v} both increase such that θ increases. With constant returns to scale, q_{e} increases while q_{v} decreases. With increasing returns to scale, however, the increase in both M_{e} and M_{v} improves the matching chances of both agents. While this reinforces the positive effect on q_{e}, it runs counter to the negative effect on q_{v}. The total effect is ambiguous and depends on the matching technology. For a further discussion of the matching function and its microfoundations, see Petrongolo and Pissarides (2001).

5.5. *Welfare and policy implications*

Unless the level of capital market competition θ is such that equity shares are exactly $\hat{s}$ and $1 - \hat{s}$, the total value $V^{\mathrm{d}} + U^{\mathrm{d}}$ is not maximized. Generally, there is no reason why free entry should lead to a value-maximizing competition level. The reason is that an individual venture capitalist entering the market does not take into account the effect of her entry on the overall level of competition, and thus on the bargaining outcome and value creation in other start-ups. Depending on whether the prevailing level of competition is below or above the value-maximizing level, entry entails either a positive or negative *contracting externality*. The policy implication is that a regulator—by affecting the level of capital market competition—can improve welfare.

The surplus created in start-ups is only one facet of the social surplus. The other is the utility loss from search frictions, or cost of delay. A welfare criterion taking into account both aspects is the total gains realized by all market participants minus entry costs, i.e., $V^{\mathrm{o}} + U^{\mathrm{o}} - k$. Free entry implies that $V^{\mathrm{o}} = k$, which in turn implies that social welfare equals the utility realized by a new cohort of entrepreneurs in the market U^{o}.

As a benchmark, consider a situation with search frictions but no moral hazard. The welfare-maximizing competition level is then the one which minimizes search frictions. Straightforward calculations show that the welfare-maximizing level of θ satisfies

$$-\frac{q_{\mathrm{v}}(\theta)}{q'_{\mathrm{v}}(\theta)}\frac{q'_{\mathrm{e}}(\theta)}{q_{\mathrm{e}}(\theta)}\frac{r+q_{\mathrm{v}}(\theta)}{r+q_{\mathrm{e}}(\theta)} = \frac{V^{\mathrm{d}}}{U^{\mathrm{d}}}. \tag{12}$$

By contrast, the equilibrium in our model is characterized by

$$\beta\frac{r+q_{\mathrm{v}}(\theta)}{r+q_{\mathrm{e}}(\theta)} = \frac{-\Psi'(V^{\mathrm{d}})}{\Psi(V^{\mathrm{d}})}V^{\mathrm{d}},$$

where $\Psi(V^{\mathrm{d}}) = U^{\mathrm{d}}$. If there is no moral hazard, the bargaining frontier has slope $\Psi'(V^{\mathrm{d}}) = -1$, implying that the equilibrium coincides with the welfare maximum if and only if $-\beta$ equals the ratio of elasticities of arrival rates, $[q'_{\mathrm{e}}(\theta)/q_{\mathrm{e}}(\theta)]/[q'_{\mathrm{v}}(\theta)/q_{\mathrm{v}}(\theta)]$. In the search literature, this is known as Hosios' condition (Hosios, 1990). Except by pure coincidence, this condition will not be satisfied. However, a regulator can attain the welfare maximum by taxing or subsidizing capital inflows (see Michelacci and Suarez (2002) for details).

If in addition to search frictions there is also moral hazard, the welfare-maximizing level of θ is

$$-\frac{q_{\mathrm{v}}(\theta)}{q'_{\mathrm{v}}(\theta)}\frac{q'_{\mathrm{e}}(\theta)}{q_{\mathrm{e}}(\theta)}\frac{r+q_{\mathrm{v}}(\theta)}{r+q_{\mathrm{e}}(\theta)} = \frac{-1}{\Psi'(V^{\mathrm{d}})}\frac{V^{\mathrm{d}}}{\Psi(V^{\mathrm{d}})}. \tag{13}$$

As is easy to see, unless $\Psi'(V^{\mathrm{d}}) = -1$—i.e., unless the sharing rule is efficient—(12) and (13) do not coincide. Hence, a welfare-maximizing regulator will typically be unable to implement a value of θ that minimizes both search frictions and moral hazard, implying that the welfare maximum cannot be attained. The reason is that the regulator has only one instrument (namely, θ), but two problems to fix.

6. Project screening

Capital market competition affects incentives not only after but also prior to the formation of a new venture. In this section, we consider the incentives of venture capitalists to screen projects, a function that—besides providing capital and coaching projects—is key to the venture capital business. Suppose projects (or entrepreneurs) come in two qualities: high and low. The probability that a project is of high quality is $\pi > 0$. Only high-quality projects are profitable. For simplicity, suppose low-quality projects yield a zero payoff with certainty. Initially, neither the venture capitalist nor the entrepreneur know the project quality. The venture capitalist can find it out, however, by paying a screening cost $C > 0$.

The timing is as follows. The venture capitalist and the entrepreneur bargain over a contract that gives the venture capitalist the right to withdraw if the project quality turns out to be low. Subsequently, the venture capitalist decides whether or not to screen. Screening reveals the project quality for sure. Absent screening, the venture capitalist holds prior beliefs that the project is high quality with probability π. After the investment is sunk, the project quality is revealed. The last assumption ensures that effort choices are made under complete information.

To minimize the amount of new notation, we denote the venture capitalist's expected utility from investing in a high-quality project by V^{d}. If the venture capitalist screens and finds out that the project quality is low, the optimal strategy is to not invest and to search anew. Moreover, an entrepreneur who has been screened and rejected will optimally leave the market. (This rules out negative pool externalities; see Broecker (1990) for details.) The expected utility from screening is thus $\pi V^{\mathrm{d}} + (1-\pi)V^{\mathrm{o}} - C$. By contrast, the expected utility from not screening is $\pi V^{\mathrm{d}} - (1-\pi)I$.[13] Screening is therefore optimal if and only if

$$C \leqslant (1-\pi)(V^{\mathrm{o}} + I). \tag{14}$$

We have the following result.

Proposition 10. *Venture capitalists screen more if* (i) *the cost of screening C is low*, (ii) *the fraction of low-quality projects* $1-\pi$ *is large*, (iii) *the investment outlay I is large*, *and* (iv) *the venture capitalist's utility in the market* V^{o} *is high*, *or equivalently*, *the level of capital market competition* θ *is low*.

The intuition for the last result, namely, that venture capitalists screen more if θ is low, is as follows. Screening gives the venture capitalist an option to invest only if the project quality is high. If the project quality is low, the venture capitalist optimally goes back into the market and realizes V^{o}. A venture capitalist who does not screen she forgoes this option: She does not realize V^{o} if the project quality is low and moreover loses the investment outlay I. Hence the opportunity cost of not screening is $V^{\mathrm{o}} + I$, multiplied by the probability that the project quality is low, $1-\pi$. Screening is therefore most valuable if V^{o} is high, or equivalently, if θ is low.

7. Empirical implications

This section summarizes the empirical implications of our model and compares them to the available evidence. Whether we assume $X_l = 0$ and no wages, or $X_l > 0$

[13] As the market is stationary, there are only two equilibrium strategies: (i) screening and investing if and only if the project quality is high, and (ii) investing without screening. Since utilities are discounted, it is never optimal to search for one more period and then do either (i) or (ii) in the following period. Evidently, it is also not an equilibrium strategy to always search, i.e., to never do either (i) or (ii).

and positive but limited wages, the implications are practically the same. All that changes is that a monotonic relation sometimes becomes a weakly monotonic relation, or a relation that is first increasing and then decreasing can turn into a relation that is first weakly increasing and then weakly decreasing. As these differences are relatively subtle, we ignore wages and assume that $X_l = 0$. The only case where wages would have a qualitative impact is when they are potentially unlimited, as in Fig. 5, but this case is unlikely to be empirically relevant (see Section 5.2). Finally, when considering industry dynamics, we focus on the long-run industry equilibrium.

7.1. Venture capitalists' equity shares

In our model, the venture capitalist's equity, or ownership, share depends on capital market characteristics. Specifically, the venture capitalist's equity share is negatively related to the level of capital supply and capital market competition, investment profitability, and capital market transparency. By contrast, it is positively related to entry costs. Gompers and Lerner (2000) find a positive relation between capital market competition and the pre-money valuation of start-ups, Γ. Since the latter is defined by $\Gamma \equiv I(1-s)/s$, this implies, ceteris parisbus, a negative relation between capital market competition and venture capitalists' equity shares, as predicted by our model.

7.2. Pre- and post-money valuation of start-ups

Our model predicts that pre- and post-money valuations are positively related to the level of capital supply and capital market competition, investment profitability, and capital market transparency. By contrast, they are negatively related to entry costs. As pointed out above, Gompers and Lerner (2000) document a positive relation between capital supply and the valuation of start-ups. See also Bartlett (2001b), who provides anecdotal evidence that valuations have been "slashed" in the aftermath of the dot.com bubble.

7.3. Total value created, market value, and success probability of start-ups

As the venture capitalist's and entrepreneur's efforts are unlikely to be perfect substitutes, we assume they are complements. The market value and success probability of the start-up is then first increasing and then decreasing in the level of capital market competition, entry costs, and capital market transparency. The total value created in the start-up, on the other hand, is first increasing and then decreasing in the level of capital market competition and entry costs, but monotonically increasing in the level of capital market transparency and investment profitability.

Gompers and Lerner (2000) examine the relation between capital market competition and the success of new ventures. The authors find no statistically significant difference for investments made during the late 1980s, a period when capital inflows and competition were relatively strong, and the early 1990s, a period of low inflows and weak competition. This is consistent with the inverted U-shape predicted by our model, which suggests that the success rate of new ventures is low if capital market competition is either weak or strong. It is, however, also consistent with the hypothesis that there is no relation between capital market competition and success. To distinguish between these two hypotheses, more than two periods will be needed.

7.4. Search time

Our model predicts that entrepreneurial search time is decreasing in the level of capital market competition, investment profitability, and capital market transparency, and increasing in entry costs. The venture capitalist's search time, on the other hand, is decreasing in capital market transparency and entry costs and increasing in capital market competition and investment profitability.

7.5. Project screening

Finally, our model predicts that venture capitalists screens more if the cost of screening is low, if the fraction of low-quality projects is high, if investment costs are high, and if the level of capital market competition is low. In a recent paper, Bengtsson et al. (2002) examine if, and how, venture capitalists' screening efforts changed between the height of the dot.com bubble in 1998–2000Q1 (a period when capital market competition was relatively strong) and the bubble's aftermath in 2001 (a period of weak competition). Consistent with our predictions, the authors find that venture capitalists screen less in "hot markets" when capital market competition is strong and more in "down markets" when capital market competition is weak.

8. Concluding remarks

We provide an equilibrium framework linking characteristics of the venture capital market such as investment profitability, entry costs, and capital market transparency to the value, valuation, and success probability of new ventures. An exogenous increase in investment profitability (due to, e.g., a technological shock) leads to new entry and capital inflows, thereby tilting the (im)balance between capital supply and demand in favor of entrepreneurs. The outside option of entrepreneurs—and thus their relative bargaining power—increases, while the outside option of venture capitalists decreases. This affects the division of equity shares, and thus the incentives and value creation in start-ups. If the imbalance between capital supply and demand is sufficiently strong, the value created in the start-up is relatively low. Such inefficiencies can arise even if there is free entry of capital. As an individual venture capitalist entering the market does not take into account the effect of her entry on the overall level of capital supply, entry involves an externality. Policy measures affecting the supply of venture capital or competitiveness of the venture capital market can then improve welfare.

Our model can be extended in several directions. One convenient, but limiting, assumption is that contracts are relatively simple. In the base model, the optimal contract is a simple sharing rule. In the extension with $X_l > 0$, the optimal contract is a combination of debt and equity. Real-world venture capital contracts, on the other hand, are more complex. They include cash-flow rights, voting rights, liquidation rights, board rights, and other instruments (Kaplan and Strömberg, 2003).[14] An interesting question is how the imbalance between capital supply and demand affect the mix of different contractual provisions. Second, our model treats venture capitalists as a homogeneous entity. In practice, venture capital partnerships consist of general and limited partners tied together by a contract. An interesting question is how changes in capital supply and demand simultaneously affect the contract between the venture capitalist and the entrepreneur and the contract between the venture capitalist and the limited partners. Third, in our model the entrepreneur and venture capitalist jointly influence the likelihood of the project's success, but they cannot choose between different projects. Suppose there are some projects that rely heavily on the venture capitalist's support and others that do not. If the venture capitalist's equity share is low, e.g., because capital market competition is strong, it might be optimal to move away from effort-intensive projects such as early-stage seed financing and move toward later-stage projects that require less coaching and value-added support.

Appendix A. (Well-behaved) production technologies

Linear technology. To obtain a closed-form solution, we assume quadratic effort costs: $c(e) = e^2/2\alpha_e$ and $g(a) = a^2/2\alpha_v$, respectively. Moreover, to ensure that the equilibrium success probability has an interior solution, we assume $\max\{\alpha_e(1-\gamma), \alpha_v\gamma\} < 1/X_h$. Given the sharing rule s, the corresponding equilibrium effort choices are $a^*(s) = \alpha_v\gamma s X_h$ and $e^*(s) = \alpha_e(1-\gamma)(1-s)X_h$, respectively. The equilibrium success probability is $p^*(s) \equiv p(e^*(s), a^*(s)) = \alpha_v\gamma^2 s X_h + \alpha_e(1-\gamma)^2(1-s)X_h$. The venture capitalist's and entrepreneur's utilities under the sharing rule s are

$$v(s) = \frac{1}{2}\alpha_v\gamma^2 s^2 X_h^2 + \alpha_e(1-\gamma)^2 s(1-s)X_h^2 \tag{A.1}$$

and

$$u(s) = \frac{1}{2}\alpha_e(1-\gamma)^2(1-s)^2 X_h^2 + \alpha_v\gamma^2 s(1-s)X_h^2, \tag{A.2}$$

respectively.

The equity frontier is derived from the following maximization program: The entrepreneur chooses s to maximize $u(s)$ subject to $v(s) \geqslant v$. The solution is characterized for all feasible reservation values $v \geqslant 0$. (If v is too large, the solution is not feasible).

[14] Berglöf (1994) and Hellmann (1998) analyze control rights aspects of venture capital finance.

We denote the solution by $s^*(v)$. From (A.1) and (A.2), it follows that $v(s)$ and $u(s)$ are both strictly quasiconcave. Accordingly, s^* is a solution to the entrepreneur's problem if and only if $v(s)$ is nondecreasing and $u(s)$ is nonincreasing at s^*. Define $\underline{s} \equiv [\alpha_v\gamma^2 - \alpha_e(1-\gamma)^2]/[2\alpha_v\gamma^2 - \alpha_e(1-\gamma)^2]$ and $\bar{s} \equiv \alpha_e(1-\gamma)^2/[2\alpha_e(1-\gamma)^2 - \alpha_v\gamma^2]$, where $0<\underline{s}<\bar{s}<1$. We obtain the following result:

(i) if $\alpha_e(1-\gamma)^2 > \alpha_v\gamma^2$ the set of Pareto-optimal sharing rules is $[0, \bar{s}]$,
(ii) if $\alpha_e(1-\gamma)^2 = \alpha_v\gamma^2$ the set of Pareto-optimal sharing rules is $[0, 1]$, and
(iii) if $\alpha_e(1-\gamma)^2 < \alpha_v\gamma^2$ the set of Pareto-optimal sharing rules is $[\underline{s}, 1]$.

In case (i), define $\underline{v} \equiv 0$ and $\bar{v} \equiv v(\bar{s})$. In case (ii), define $\underline{v} \equiv 0$ and $\bar{v} \equiv v(1)$. Finally, in case (iii), define $\underline{v} \equiv v(\underline{s})$ and $\bar{v} \equiv v(1)$. For any $v \in [\underline{v}, \bar{v}]$, the solution $s^*(v) \equiv s^*$ satisfies

$$v = \frac{1}{2}\alpha_v\gamma^2(s^*)^2X_h^2 + \alpha_e(1-\gamma)^2s^*(1-s^*)X_h^2. \tag{A.3}$$

Solving (A.3) for s^*, we obtain

$$s^* = \frac{\alpha_e(1-\gamma)^2X_h - \zeta}{(2\alpha_e(1-\gamma)^2 - \alpha_v\gamma^2)X_h}, \tag{A.4}$$

where $\zeta \equiv \sqrt{\alpha_e^2(1-\gamma)^4X_h^2 - 2v(2\alpha_e(1-\gamma)^2 - \alpha_v\gamma^2)}$. Clearly, s^* is strictly increasing in v. Inserting (A.4) in (A.2) yields the equity frontier $\psi(v)$. Differentiating $\psi(v)$ twice with respect to v, we have

$$\frac{\mathrm{d}^2\psi(v)}{\mathrm{d}v^2} = -\zeta^{-3}[(\alpha_e(1-\gamma)^2 - \alpha_v\gamma^2)^2 + \alpha_e(1-\gamma)^2\alpha_v\gamma^2]<0.$$

To show that the sum of utilities $v + \psi(v)$ has a unique maximum in the interior of $[\underline{v}, \bar{v}]$, we compute the derivative of $\psi(v)$ at the boundaries. In case (i), we have $\psi'(\underline{v}) = [\alpha_v\gamma^2 - \alpha_e(1-\gamma)^2]/[\alpha_e(1-\gamma)^2] > -1$ and $\lim_{v\to\bar{v}}\psi'(v) = -\infty$. In case (ii), we have $\psi'(\underline{v}) = 0$ and $\lim_{v\to\bar{v}}\psi'(v) = -\infty$. Finally, in case (iii), we have $\psi'(\underline{v}) = 0$ and $\psi'(\bar{v}) = -\alpha_e\gamma^2/[\alpha_v\gamma^2 - \alpha_e^2(1-\gamma)^2] < -1$. Since $\psi(v)$ is strictly concave, this implies that in each case there exists a unique value $\hat{v} \in (\underline{v}, \bar{v})$ at which $\psi'(\hat{v}) = -1$.

Finally, both in (A.1) and (A.2) the term X_h^2 can be factored out. Hence equilibrium utilities are separable of the form $u(X_h, s) = \chi(X_h)\omega_e(s)$ and $v(X_h, s) = \chi(X_h)\omega_v(s)$, where $\chi(X_h) \equiv X_h^2$ and $\omega_v(s)$ are increasing functions and $\omega_e(s)$ is a decreasing function. $\square$

Cobb-Douglas technology. We assume again that effort costs are quadratic of the form $c(e) = e^2/2\alpha_e$ and $g(a) = a^2/2\alpha_v$. To ensure that the equilibrium success probability has an interior solution, we assume again that $\max\{\alpha_e(1-\gamma), \alpha_v\gamma\} < 1/X_h$. Equilibrium effort choices are then given by $e^*(s) = (\alpha_e(1-\gamma)(1-s)X_h[a^*(s)]^\gamma)^{1/(1+\gamma)}$, and $a^*(s) = (\alpha_v\gamma sX_h[e^*(s)]^{1-\gamma})^{1/(2-\gamma)}$, implying that $p^*(s) = \rho(s)X_h$, where $\rho(s) \equiv [\alpha_v\gamma s]^\gamma[\alpha_e(1-\gamma)(1-s)]^{1-\gamma}$. The venture capitalist's and the entrepreneur's utilities under the sharing rule s are

$$v(s) = \frac{1}{2}(2-\gamma)sX_h^2\rho(s) \tag{A.5}$$

and

$$u(s) = \frac{1}{2}(1+\gamma)(1-s)X_h^2\rho(s), \tag{A.6}$$

respectively.

From (A.5) and (A.6), it follows that $v(s)$ and $u(s)$ are both quasiconcave, implying that s^* solves the entrepreneur's problem for some v if and only if $v(s)$ is nondecreasing and $u(s)$ is nonincreasing at s^*. Differentiating (A.5) with respect to s, we have that $v(s)$ is nondecreasing if and only if $s \leqslant [1+\gamma]/2$. Similarly, differentiating (A.6) with respect to s, we have that $u(s)$ is nonincreasing if and only if $s \geqslant \gamma/2$. Accordingly, the set of Pareto-optimal sharing rules is $[\gamma/2, [1+\gamma]/2]$, implying that $\underline{v} \equiv v(\gamma/2)$ and $\bar{v} \equiv v([1+\gamma]/2)$. Moreover, $v(s)$ is strictly increasing for all $s < [1+\gamma]/2$, implying that $s^*(v) = s^*$ is strictly increasing for all $v < \bar{v}$. We next show that ψ is strictly concave. Differentiating $u(s)$ and $v(s)$ twice, we obtain

$$\begin{aligned}\frac{\mathrm{d}^2\psi(v)}{\mathrm{d}v^2} &= \frac{1}{[v'(s^*)]^2}\left(u''(s^*) - v''(s^*)\frac{\mathrm{d}\psi(v)}{\mathrm{d}v}\right)\\ &= -\frac{1}{2}(1+\gamma)X_h^2\rho(s^*)\frac{1}{[v'(s^*)]^2}\left(\frac{\gamma}{(s^*)^2} + \frac{(1-\gamma)(2s^*-\gamma)}{s^*(1-s^*)(1+\gamma-2s^*)}\right),\end{aligned}$$

which is strictly negative for all $s^* \in [\gamma/2, (1+\gamma)/2]$. To show that the sum of utilities $v + \psi(v)$ attains its maximum in the interior of $[\underline{v}, \bar{v}]$, we compute the derivative of $\psi(v)$ at the boundaries. The derivative of $\psi(v)$ is

$$\frac{\mathrm{d}\psi(v)}{\mathrm{d}v} = \frac{\mathrm{d}u}{\mathrm{d}s^*}\frac{\mathrm{d}s^*}{\mathrm{d}v} = \frac{u'(s^*)}{v'(s^*)} = -\frac{(1+\gamma)(2s^*-\gamma)(1-s^*)}{s^*(2-\gamma)(1+\gamma-2s^*)}. \tag{A.7}$$

Evaluating (A.7) at $\underline{v}$ and $\bar{v}$, we obtain $\psi'(\underline{v}) = 0$ and $\lim_{v\to\bar{v}} \psi'(v) = -\infty$. Since $\psi(v)$ is strictly concave, this implies that there exists a unique value $\hat{v} \in (\underline{v}, \bar{v})$ such that $\psi'(\hat{v}) = -1$.

Finally, it is evident from (A.5) and (A.6) that the equilibrium utilities are separable of the form $u(X_h, s) = \chi(X_h)\omega_e(s)$ and $v(X_h, s) = \chi(X_h)\omega_v(s)$, where $\chi(X_h) = X_h^2$ and $\omega_v(s)$ are increasing functions and $\omega_e(s)$ is a decreasing function. □

Appendix B. Proofs

Proof of Proposition 1. We first show that the bargaining problem has an interior solution $V^d \in (\underline{V}, \bar{V})$ characterized by the first-order condition (1). Since $V^o \geqslant 0$ and $U^o \geqslant 0$, there are only two possible cases where this might not be true: (i) $\underline{v} - I > 0$ and $\Psi'(\underline{v} - I) < 0$, and (ii) $\Psi(\bar{v} - I) > 0$ and $\Psi'(\bar{v} - I) > -\infty$. First consider case (i). If $\underline{v} - I > 0$, then $\underline{V} = \underline{v} - I > 0$. In conjunction with $\Psi'(\underline{V}) < 0$, this implies that there exists a utility pair $V < \underline{V}$ and $U > \Psi(\underline{V})$ that is Pareto-undominated but lies outside the domain of Ψ, contradicting the definition of Ψ. The argument for (ii) is analogous.

We next show that there is a unique capital market equilibrium. Inserting (3)–(4) in (1) yields (5). Uniqueness of the bargaining solution implies that (5) has a unique solution.

Consider finally the comparative statics properties of the equilibrium. Implicitly differentiating (5) with respect to θ yields

$$\frac{\mathrm{d}V^{\mathrm{d}}}{\mathrm{d}\theta}=\beta\frac{(r+q_{\mathrm{e}})q_{\mathrm{v}}'-(r+q_{\mathrm{v}})q_{\mathrm{e}}'}{(r+q_{\mathrm{e}})^2}\frac{\Psi^2}{\Psi(\Psi'+V^{\mathrm{d}}\Psi'')-V^{\mathrm{d}}(\Psi')^2}<0, \tag{B.1}$$

which implies that $\mathrm{d}U^{\mathrm{d}}/\mathrm{d}\theta>0$. In conjunction with $q_{\mathrm{e}}'(\theta)>0$ and $q_{\mathrm{v}}'(\theta)<0$, this implies that $\mathrm{d}V^{\mathrm{o}}/\mathrm{d}\theta<0$ and $\mathrm{d}U^{\mathrm{o}}/\mathrm{d}\theta>0$. The fact that $V^{\mathrm{d}}+U^{\mathrm{d}}$ is first increasing and then decreasing in θ follows from (B.1) and the fact that Ψ is strictly concave with slope $\Psi'(V)=-1$ at some $V\in(\underline{V},\bar{V})$. (Note that, by (5) and the limit properties of q_{v} and q_{e} for $\theta\to 0$ and $\theta\to\infty$, we can indeed trace out the full contract frontier). $\square$

Proof of Propositions 4 and 8. Consider first Proposition 4. The equilibrium is determined by (5) and the zero-profit constraint $V^{\mathrm{o}}=k$. Totally differentiating both equations using (4), we obtain the following equation system:

$$\begin{pmatrix}\beta\dfrac{(r+q_{\mathrm{e}})q_{\mathrm{v}}'-(r+q_{\mathrm{v}})q_{\mathrm{e}}'}{(r+q_{\mathrm{e}})^2} & \dfrac{\Psi(\Psi'+V^{\mathrm{d}}\Psi'')-V^{\mathrm{d}}(\Psi')^2}{\Psi^2}\\ \dfrac{V^{\mathrm{d}}rq_{\mathrm{v}}'}{(r+q_{\mathrm{v}})^2} & \dfrac{q_{\mathrm{v}}}{r+q_{\mathrm{v}}}\end{pmatrix}\begin{pmatrix}\mathrm{d}\theta\\ \mathrm{d}V^{\mathrm{d}}\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}\mathrm{d}k. \tag{B.2}$$

The determinant of this system, Ξ, is negative. In conjunction with the limit properties of q_{v} and q_{e} for $\theta\to 0$ and $\theta\to\infty$, this establishes the existence and uniqueness of a solution to (5) and $V^{\mathrm{o}}=k$.

Consider next Proposition 8. By (B.2) and Cramer's rule, we have that

$$\frac{\mathrm{d}\theta}{\mathrm{d}k}=\frac{-1}{\Xi}\frac{\Psi(\Psi'+V^{\mathrm{d}}\Psi'')-V^{\mathrm{d}}(\Psi')^2}{\Psi^2}<0,$$

where the sign follows the fact that Ξ, q_{v}', Ψ', and Ψ'' are all negative. $\square$

Proof of Proposition 5. Consider first the short-run effect. Combining (7) and (8) we have

$$\beta\frac{r+q_{\mathrm{v}}(\theta)}{r+q_{\mathrm{e}}(\theta)}=-\Psi'(X_h,s)\frac{V^{\mathrm{d}}}{U^{\mathrm{d}}}=-\Psi'(X_h,s)\left(\frac{\omega_{\mathrm{v}}(s)}{\omega_{\mathrm{e}}(s)}-\frac{I}{\chi(X_h)\omega_{\mathrm{e}}(s)}\right), \tag{B.3}$$

where $\omega_{\mathrm{v}}'(s)>0$, $\omega_{\mathrm{e}}'(s)<0$, and $\chi'(X_h)>0$. The left-hand side is constant, while the right-hand side is increasing in s and X_h (Ψ' is constant with respect to X_h but decreasing in s). Accordingly, if X_h increases, s must decrease. This, in turn, increases U^{d} and decreases $-\Psi'(X_h,s)$. By (B.3), this implies that V^{d} must increase. Finally, since θ is constant, the increase in U^{d} and V^{d} implies that U^{o} and V^{o} must increase.

Consider next the long-run effect. Consider two X_h-values, $X_h^1<X_h^2$. We first show that the corresponding competition levels must satisfy $\theta^2>\theta^1$. Suppose to the contrary that $\theta^2\leqslant\theta^1$, implying that $q_{\mathrm{v}}^2(\theta)\geqslant q_{\mathrm{v}}^1(\theta)$. Since $V^{\mathrm{o}}=k$, this implies that $V^{\mathrm{d}2}\leqslant V^{\mathrm{d}1}$. Moreover, $V^{\mathrm{d}2}\leqslant V^{\mathrm{d}1}$ and $X_h^2>X_h^1$ together imply that $s^2<s^1$ and $U^{\mathrm{d}2}>$

U^{d1}, and therefore $V^{\mathrm{d2}}/U^{\mathrm{d2}} < V^{\mathrm{d1}}/U^{\mathrm{d1}}$. But $q_{\mathrm{v}}^2(\theta) \geqslant q_{\mathrm{v}}^1(\theta)$, $q_{\mathrm{e}}^2(\theta) \leqslant q_{\mathrm{e}}^1(\theta)$, and $V^{\mathrm{d2}}/U^{\mathrm{d2}} < V^{\mathrm{d1}}/U^{\mathrm{d1}}$ in connection with (B.3) imply that $-\Psi'(X_h^2, s^2) > -\Psi'(X_h^1, s^1)$, and hence $s^2 > s^1$, which yields a contradiction.

We next show that $s^2 < s^1$. Again, we argue to a contradiction by assuming that $s^2 \geqslant s^1$. Since $X_h^2 > X_h^1$ and $s^2 \geqslant s^1$, we have that $V^{\mathrm{d2}} > V^{\mathrm{d1}}$, $-\Psi'(X_h^2, s^2) > -\Psi'(X_h^1, s^1)$, and $V^{\mathrm{d2}}/U^{\mathrm{d2}} \geqslant V^{\mathrm{d1}}/U^{\mathrm{d1}}$, where the last statement follows from (8). Since the left-hand side in (B.3) is decreasing in θ, this implies that $\theta^2 < \theta^1$ and therefore $q_{\mathrm{v}}^2(\theta) > q_{\mathrm{v}}^1(\theta)$. But $V^{\mathrm{o}} = k$ and $q_{\mathrm{v}}^2(\theta) > q_{\mathrm{v}}^1(\theta)$ together imply that $V^{\mathrm{d2}} > V^{\mathrm{d1}}$, which yields a contradiction.

The rest is straightforward. From $\theta^2 > \theta^1$, we have that $q_{\mathrm{v}}^2(\theta) < q_{\mathrm{v}}^1(\theta)$. In conjunction with $V^{\mathrm{o}} = k$, this implies that $V^{\mathrm{d2}} > V^{\mathrm{d1}}$. Moreover, $X_h^2 > X_h^1$ and $s^2 < s^1$ together imply that $U^{\mathrm{d2}} > U^{\mathrm{d1}}$. In conjunction with $q_{\mathrm{e}}^2(\theta) > q_{\mathrm{e}}^1(\theta)$, this finally implies that $U^{\mathrm{o2}} > U^{\mathrm{o1}}$. $\square$

Proof of Propositions 9 and 10. Consider first Proposition 9. We begin with the short-run effect. Implicitly differentiating (9) and using the fact that $q_{\mathrm{e}}(\theta, \xi) \equiv \theta q_{\mathrm{v}}(\theta, \xi)$, we get

$$\frac{\mathrm{d}V^{\mathrm{d}}}{\mathrm{d}\xi} = \beta \frac{r(1-\theta)}{[r + \theta q_{\mathrm{v}}(\theta, \xi)]^2} \frac{\partial q_{\mathrm{v}}(\theta, \xi)}{\partial \xi} \frac{-\Psi^2}{\Psi(\Psi' + V^{\mathrm{d}}\Psi'') - V^{\mathrm{d}}(\Psi')^2}.$$

Hence $sign(\mathrm{d}V^{\mathrm{d}}/\mathrm{d}\xi) = sign(1-\theta)$. The results regarding s, U^{d}, and U^{o} are then obvious.

Consider next the long run. The equilibrium is determined by (5) and the zero-profit constraint $V^{\mathrm{o}} = k$. Totally differentiating both equations using (4), we obtain the following equation system:

$$\begin{pmatrix} \beta \dfrac{(r+q_{\mathrm{e}})\frac{\partial q_{\mathrm{v}}}{\partial \theta} - (r+q_{\mathrm{v}})\frac{\partial q_{\mathrm{e}}}{\partial \theta}}{(r+q_{\mathrm{e}})^2} & \dfrac{\Psi(\Psi' + V^{\mathrm{d}}\Psi'') - V^{\mathrm{d}}(\Psi')^2}{\Psi^2} \\ \dfrac{V^{\mathrm{d}} r}{(r+q_{\mathrm{v}})^2} \dfrac{\partial q_{\mathrm{v}}}{\partial \theta} & \dfrac{q_{\mathrm{v}}}{r+q_{\mathrm{v}}} \end{pmatrix} \begin{pmatrix} \mathrm{d}\theta \\ \mathrm{d}V^{\mathrm{d}} \end{pmatrix}$$
$$= \begin{pmatrix} -\beta \dfrac{(r+q_{\mathrm{e}})\frac{\partial q_{\mathrm{v}}}{\partial \xi} - (r+q_{\mathrm{v}})\frac{\partial q_{\mathrm{e}}}{\partial \xi}}{(r+q_{\mathrm{e}})^2} \\ \dfrac{-V^{\mathrm{d}} r}{(r+q_{\mathrm{v}})^2} \dfrac{\partial q_{\mathrm{v}}}{\partial \xi} \end{pmatrix} \mathrm{d}\xi.$$

For the sake of brevity, we have omitted the argument in $q_{\mathrm{v}}(\theta, \xi)$. The determinant of this system, Ξ, is negative. By Cramer's rule and $q_{\mathrm{e}}(\theta, \xi) \equiv \theta q_{\mathrm{v}}(\theta, \xi)$, we have

$$\frac{\mathrm{d}V^{\mathrm{d}}}{\mathrm{d}\xi} = \frac{1}{\Xi} \frac{\beta V^{\mathrm{d}} q_{\mathrm{v}}(\theta, \xi)[r + q_{\mathrm{v}}(\theta, \xi)]r}{[r + q_{\mathrm{v}}(\theta, \xi)]^2 [r + q_{\mathrm{e}}(\theta, \xi)]^2} \frac{\partial q_{\mathrm{v}}(\theta, \xi)}{\partial \xi} < 0.$$

The results regarding s, U^{d}, and U^{o} are then obvious.

Consider finally Proposition 10. In the short run, we have $\partial q_{\mathrm{v}}(\theta,\xi)/\partial\xi > 0$ and $\partial q_{\mathrm{e}}(\theta,\xi)/\partial\xi > 0$ by assumption. In the long run, $V^{\mathrm{o}} = k$ and $\mathrm{d}V^{\mathrm{d}}/\mathrm{d}\xi < 0$ imply that the total derivative $\mathrm{d}q_{\mathrm{v}}(\theta,\xi)/\mathrm{d}\xi$ is positive. In conjunction with (5), this implies that $\mathrm{d}q_{\mathrm{e}}(\theta,\xi)/\mathrm{d}\xi > 0$. $\square$

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