Investors diversify, because you get a better return for a given risk.

There is a fully-diversified “market portfolio” that we should all choose.

The risk of an individual asset can be measured by how much risk it adds to the “market portfolio.”
Capital Allocation Possibilities: Treasuries or an Equity Fund?

Expected Return

\[ E(r_p) = 17\% \]
\[ r_f = 7\% \]
\[ \sigma_p = 27\% \]

The Equity Fund

We Can Buy Some T-bills and Some of the Risky Fund...

\[ E(R) = 0.3 \times 7 + 0.7 \times 17 = 14\% \]
\[ SD = 0.7 \times 27 = 18.9\% \]

C.A.L.
SLOPE = 0.37
...Or Buy Two Risky Assets

Diversification
**Portfolio Return...**

To compute the return of a portfolio: use the weighted average of the returns of all assets in the portfolio, with the weight given each asset calculated as (value of asset)/(value of portfolio).

The *portfolio return* $E(R_p)$ is:

$$E(R_p) = (w_1 k_1) + (w_2 k_2) + \ldots + (w_n k_n) = \sum w_j k_j$$

where $w_j = \text{weight of asset } j$, $k_j = \text{return on asset } j$

---

**...and Risk (Standard Deviation)**

- Portfolio return is the weighted average of all assets’ returns,
- But portfolio standard deviation is normally *less* than the weighted average of all assets’ standard deviations!
- The reason: asset returns are imperfectly correlated.
### Risk and Return of Stocks, Bonds and a Diversified Portfolio

<table>
<thead>
<tr>
<th>State</th>
<th>Prob.</th>
<th>Equity</th>
<th>Bond</th>
<th>Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>1/3</td>
<td>-7%</td>
<td>+17%</td>
<td>+5%</td>
</tr>
<tr>
<td>Normal</td>
<td>1/3</td>
<td>+12%</td>
<td>+7%</td>
<td>+9.5%</td>
</tr>
<tr>
<td>Boom</td>
<td>1/3</td>
<td>+28%</td>
<td>-3%</td>
<td>+12.5%</td>
</tr>
<tr>
<td>Expected Return</td>
<td></td>
<td>11%</td>
<td>7.0%</td>
<td>9.0%</td>
</tr>
<tr>
<td>Variance</td>
<td></td>
<td>204.7%</td>
<td>66.7%</td>
<td>9.5%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td></td>
<td>14.3%</td>
<td>8.2%</td>
<td>3.1%</td>
</tr>
</tbody>
</table>

### The Correlation Between Stock and Bond Returns

- **Covariance**
  
  \[ \text{cov}_{e,b} = \sum_{s=1}^{n} p_s [R_{s,e} - E(R_e)] \times [R_{s,b} - E(R_b)] \]
  
  \[ = 0.3333(-7-11)(17-7) + 0.3333(12-11)(7-7) \]
  
  \[ + 0.3333(28-11)(-3-7) \]
  
  \[ = -116.67 \]

- **Correlation**
  
  \[ \text{corr} = \frac{\text{cov}_{e,b}}{\sigma_e \sigma_b} \]
  
  \[ = -116.67 / 14.3(8.2) \]
  
  \[ = -0.99 \]
**Portfolio Return and Standard Deviation**

Given:

- \( W_S = 0.5 \) \[ R_S = 12\% \] \[ \sigma_S = 25\% \]
- \( W_B = 0.5 \) \[ R_B = 9\% \] \[ \sigma_B = 12\% \]

and \( \rho_{S,B} = 0.2 \)

\( R_p = 0.5(12) + 0.5(9) = 10.5\% \)

\[
\sigma_p = \left[ (0.5)^2(25)^2 + (0.5)^2(12)^2 + 2(0.5)(0.5)(25)(12)(0.2) \right]^{1/2}
\]

\[
= (156.25 + 36 + 30)^{1/2}
\]

\[
= (222.25)^{1/2}
\]

\[
= 14.91\%
\]

---

**The Minimum-Variance Frontier of Risky Assets**

- Efficient frontier
- Individual assets
- Global minimum-variance portfolio
The Efficient Frontier of Risky Assets with the Optimal CAL

The Capital Asset Pricing Model (CAPM)

CAPM Says:
- The total risk of a financial asset is made up of two components.
  A. Diversifiable (unsystematic) risk
  B. Nondiversifiable (systematic) risk
- The only relevant risk is nondiversifiable risk.
The Equation for the CAPM

\[ R_j = R_F + \beta_j (R_m - R_F) \]

where:
- \( R_j \) = Required return on asset \( j \);
- \( R_F \) = Risk-free rate of return
- \( \beta_j \) = Beta Coefficient for asset \( j \);
- \( R_m \) = Market return

The term \( \beta_j (R_m - R_F) \) is called the risk premium and \( (R_m - R_F) \) is called the market risk premium.

Beta Estimation in Practice:
Bloomberg
Interpreting Beta

- Market Beta = 1.0 = average level of risk
  - A Beta of 0.5 is half as risky as average
  - A Beta of 2.0 is twice as risky as average
  - A negative Beta asset moves in opposite direction to market

Some Comments on CAPM

- Since Beta coefficients are derived from historical data, they are best viewed as approximations of future expectations of actual risk-return behavior.
- CAPM is based upon an assumed efficient market which, although seemingly unrealistic, is supported empirically in active markets such as the New York Stock Exchange.
- While CAPM is not applicable to all assets, it does provide a conceptual framework that is useful in linking risk and return in financial decisions.
Equity Risk and Return: Summary

- Investors diversify, because you get a better return for a given risk.
- There is a fully-diversified “market portfolio” that we should all choose.
- The risk of an individual asset can be measured by how much risk it adds to the “market portfolio.”

www.giddy.org

Ian Giddy
NYU Stern School of Business
Tel 212-998-0332; Fax 212-995-4233
ian.giddy@nyu.edu
http://www.giddy.org