Bubbles Go Viral:
Theory and Evidence from China and Emerging Markets

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Based on Ongoing Projects

- Bubbles Go Viral w/ Haoshu Tian
- Contagion and the Commodity Bubble: Hoarding and the Rice Crisis of 2008 w/Aureo De Paula and Vishal Singh
Accounts of speculative bubble episodes point to a surge of new entrants: households who never invested joining the bubble.

- South Sea Bubble of 1720, significant number of English households including Isaac Newton.
- Similar surge in other historical bubble episodes such as the 1920’s boom in the US stock market before the Great Depression.
- US Housing Bubble of 2002-2007: surge of investment home buyers from 17% to 28% in peak year.
Epidemic or Viral Element

- Surge of interest tied to word-of-mouth communication and contagious optimism spreading via friends and neighbors during the run-up in prices.
- For many economists, the epidemic element is synonymous with essence of speculative bubbles.
- Robert Shiller’s Nobel Lecture: “A situation in which news of price increases spurs investor enthusiasm which spreads by psychological contagion from person to person, in the process amplifying stories that might justify the price increase and bringing in a larger and larger class of investors, who, despite doubts about the real value of the investment, are drawn to it partly through envy of others successes and partly through a gamblers excitement.”
Anecdotal and Systematic Evidence for Word-of-Mouth

- Isaac Newton in 1720 drawn in by stories of relatives getting rich and lost his life savings: allegedly said ”I can calculate the movement of stars, but not the madness of men.”

- Twitter shares, and Mrs. Watkins, a 56-year old administrative assistant and avid Mother Jones reader with opened a TD Ameritrade account so she could buy about 50 shares of Twitter. Doesn’t trust the market and says its like gambling, but with the addition of lies and subterfuge. I’m just buying because everybodys talking about Twitter, she said. I’m just gonna take a chance.
Standard Theories Cannot Explain Price-Volume Dynamics During Bubble Run-up

*** Trending Prices and Volume Peaks Before Prices
- Internet bubble
  - Volume: monthly turnover (%)
  - Price: Equal weighted return index (1996=100)
- China stock bubble
  - Price: Equal weighted cumulative return
  - Volume: monthly trading volume (unit: share)
  - Number of accounts (unit: 10,000 accounts)
Chinese Stock Market Bubble 2006-2008

- Non-tradable shares reform done; A healthier market
- Retail investors: Friends made money in stock market
- Government tried to make money flow into stock market to get rid of the sudden increase of supply due to non-tradable share reform
- Belief: government won’t let the market down before 2008 Olympics
Empirical Evidence

Chinese Stock Market Bubble 2006-2008

China's Shanghai A Market

- Cumulative Return
- Trading Volume
- Number of Accounts

Month

2005 2006 2007 2008
Empirical Evidence

Chinese Stock Market Bubble 2006-2008

China’s Shanghai B Market

- Cumulative Return
- Trading Volume
- Number of Accounts


Y-axis: 0.0, 0.5, 1.0, 1.5, 2.0, 100, 110, 120, 130, 140
X-axis: 0.0, 2.0, 4.0, 6.0, 8.0, 10.0, 12.0, 14.0, 16.0

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April 16, 2014
Empirical Evidence

Chinese Stock Market Bubble 2006-2008

![China's Shenzhen A Market graph]

- Cumulative Return
- Trading Volume
- Number of Accounts

Month:
- 2005
- 2006
- 2007
- 2008
Empirical Evidence

Chinese Stock Market Bubble 2006-2008

China's Shenzhen.B Market

- Cumulative Return
- Trading Volume
- Number of Accounts

Month

2005 2006 2007 2008
Our Model

- Introduce two new ingredients to an otherwise standard rational bubbles set-up with forward-looking agent
  - Optimism spreads across population via epidemic process
  - Insiders are pessimistic but have different holding costs and sell when price is high enough
- Retail investors buy but don’t sell since they are all forward-looking and expect prices will rise
- Insider selling moderates the price growth, resulting in trending prices or positive serial correlation.
- Volume at time $t$ depends on new entrants and new supply from insiders.
Key Predictions

- **Prediction 1: Volume peaks before trending prices**
  - S-shaped epidemic leads to spike in participation while supply moderates price growth
  - Show that our model fits Chinese Stock Market Bubble given account opening data and realistic insider liquidity needs
The Model

- Trading occurs during $[0, T]$
- The asset pays dividend $D$ afterwards
- Two kinds of risk-neutral agents
  - Retail investors who buy
  - Insiders who sell
Retail investors come into the market with a process $k(t)$ in an epidemical process:

$$\begin{align*}
k'(t) &= s \cdot k(t) \cdot [1 - k(t)] \\
k(0) &= k_0
\end{align*}$$

- The total population is 1; initially $k_0$ of them are in the market.
- $s$ measures how fast the enthusiasm spreads.
- Solution: $k(t) = \left[1 + (k_0^{-1} - 1) \cdot e^{-st}\right]^{-1}$

1 dollar in total: $k(t)$ dollars in market at $t$.

- Retail investors believe that $D = 1$.
- If the current price is below 1, they will buy the available shares with all their money when they come into the market.
The Model

Insiders

- Insiders own the asset before trading and believe that $D = 1$
- Insiders have different holding costs
- Thus each insider would like to sell their asset whenever the price hits his own level $x \in (0, 1)$
- The population of insiders is small as fraction of retail population.
- They initially own all the shares and are willing to sell at $x \in (0, 1)$ is $\int f(x) \, dx$

\[ \int_a^b f(x) \, dx \]

\[ \int_0^1 xf(x) \, dx \]

\[ 1 \]For example: the population of insiders willing to sell between price $a$ and $b$ is $\int_a^b f(x) \, dx$. The value of the shares is $\int_a^b xf(x) \, dx$. To normalize the model, we require the total amount of cash hold by retail investors equals to the total value of the shares hold by insiders $1 = \int_0^1 xf(x) \, dx$. We require $f(x)$ to be continuous for technical purpose.
The Model

Price

- During the time interval \((t, t + \Delta t)\), the population of retail investors (or the amount of cash) comes into the market is \(k(t + \Delta t) - k(t)\).
- Given the price \(P(t)\), the new price \(P(t + \Delta t)\) should satisfy the market clearing condition:

\[
k(t + \Delta t) - k(t) = \int_{P(t)}^{P(t+\Delta t)} xf(x)dx
\]

- When \(P(t)\) and \(f(x)\) are continuous, there exists \(t' \in (t, t + \Delta t)\) such that

\[
k(t+\Delta t) - k(t) = \int_{P(t)}^{P(t+\Delta t)} xf(x)dx = P(t')f[P(t')]P(t+\Delta t) - P(t)]
\]

- Divide both sides by \(\Delta t\) and let \(\Delta t \to 0\), we have

\[
k'(t) = P(t)f[P(t)]P'(t)
\]
Price (continued)

Thus we have

\[ P'(t) = \frac{k'(t)}{P(t)f[P(t)]} \]

- \( P'(t) \): rate at which price increases
- \( k'(t) \): the retail investor coming rate (or the money flow-in rate)
- \( f[P(t)] \): the density of insiders willing to sell at current price (or the density of shares available at current price)
- \( P(t)f[P(t)] \): the value of shares available at current price

The price increase rate is proportional to the money flow-in rate and is inversely proportional to the value of shares available at current price.

For certain group of density functions \( f(x) \), one can solve this ODE and get the close form solution of price.\(^2\)

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\(^2\)For example, we will solve price for power distributions on the interval \((0, 1)\). When solving the ODE, the initial condition for price is determined by \( k_0 = \int_0^P 0.1 \cdot f(x)dx \).
The Model

Trading volume

- Then during the time interval \((t, t + \Delta t)\), we have trading volume of

\[
\int_{t}^{t+\Delta t} V(t)dt = \int_{P(t)}^{P(t+\Delta t)} f(x)dx
\]

- When \(V(t)\) and \(f(x)\) are continuous, there exist \(t', t'' \in (t, t + \Delta t)\) such that \(V(t'')\Delta t = f[P(t')][P(t + \Delta t) - P(t)]\)

- Divide both sides by \(\Delta t\) and let \(\Delta t \to 0\), we have

\[
V(t) = f[P(t)]P'(t)
\]

- Since we have derived that \(P'(t) = \frac{k'(t)}{P(t)f[P(t)]}\), we know \(V(t) = \frac{k'(t)}{P(t)}\)
The Model

Solve model assuming power function for $f(x)$

- For a given constant $\alpha \in (-1, +\infty)$, assume that $f(x) = (\alpha + 2)x^\alpha$ for $x \in (0, 1)$

- The ODE for price $k'(t) = P(t)f[P(t)]P'(t)$ now becomes $k'(t) = (\alpha + 2)P(t)^{\alpha+1}P'(t)$

- Integrate both sides, we have $P(t) = [k(t) - k_0 + P_0^{\alpha+2}] \frac{1}{\alpha+2}$

- Since $P_0 = k_0^{\alpha+2}$, we have $P(t) = k(t) \frac{1}{\alpha+2}$

- Thus heuristically call "trading volume"

$$V(t) = \frac{k'(t)}{P(t)} = \frac{sk(t)[1 - k(t)]}{k(t)\frac{1}{\alpha+2}} = sk(t)\frac{\alpha+1}{\alpha+2}[1 - k(t)]$$

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3 The coefficient $(\alpha + 2)$ is selected to normalize the total value of the asset hold by insiders equal to the total amount of cash hold by retail investors $1 = \int_0^1 xf(x)dx$

4 $P_0$ can be determined by the initial requirement $k_0 = \int_0^{P_0} xf(x)dx$. 
The Model

The general shape of the solution

Price and Volume Dynamics (alpha = 0, s = 0.1, k_0 = 0.001)
Bubble Dynamics

Understand different dynamics of bubbles

- In the example, for a given constant $\alpha \in (-1, +\infty)$, assume that $f(x) = (\alpha + 2)x^\alpha$ for $x \in (0, 1)$
- We can use $\alpha$ to control the shape of this density function
  - When $\alpha$ goes to -1, most insiders extremely high holding costs are willing to sell at a very low price
  - When $\alpha$ goes to infinity, most insiders are do not have high holding costs and only willing to sell at a very high price
  - Empirically, $\alpha$ is between 0 and .3 from empirical studies regressing insider selling on price changes (unless there is a lot of VCs)
When more insiders are optimistic about the number of retail investors that will be attracted into the market, $\alpha$ increases.

When $\alpha$ gets smaller, volume peaks earlier.

When $\alpha$ gets smaller, price curve gets closer to the population curve. More "S-shaped".
Bubble Dynamics

Volume peak

**Theorem**

*Volume peaks when the* \( k(t) = \frac{\alpha+1}{2\alpha+3} \).

When \( \alpha \) decreases, \( \frac{\alpha+1}{2\alpha+3} \) decreases. Thus

**Corollary (1)**

*When \( \alpha \) gets smaller, volume peaks earlier.*

Since \( \frac{\alpha+1}{2\alpha+3} < \frac{1}{2} \) for all \( \alpha \in (-1, +\infty) \), volume peaks no later than the time when population reaches half. On the other hand, for any finite \( \alpha \), price peaks when population reaches 1. Thus

**Corollary (2)**

*Volume always peaks earlier than price peaks*

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\(^5\)When \( \alpha \to +\infty \), \( P(t) \to 1 \) for all \( t \). There is no way to define "peak" of the price.
S-shape

Measure S-shape by the difference of price and population
\[ \theta = P(t^*) - k(t^*) \] when population is half \( k(t^*) = \frac{1}{2} \)

**Theorem**

*When \( \alpha \) gets smaller, \( \theta \) gets smaller. Price curve gets more S-shaped.*

Together with Corollary (1) we have

**Corollary (3)**

*When volume peaks earlier, price curve gets more S-shaped.*
Empirical Evidence

Chinese Stock Market Bubble 2006-2008

- Monthly equal-weighted market return and monthly total shares traded in each market.\(^6\)
- Total number of accounts by the end of each month.\(^7\)\(^8\)\(^9\)

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\(^6\) CRSP/CSMAR/Monthly Trading Data/Market Returns

\(^7\) Source: China Securities Registration and Settlement Statistical Yearbooks. A and B share individual accounts for both Shanghai and Shenzhen Exchange. Unit: 10,000

\(^8\) According to Liu et. al. (2013), an individual investor can only open two accounts at most using her person identification with the two stock exchanges respectively. This policy ensures that the account number is a good proxy for participant number.

\(^9\) Liu et. al. use the market participation data to distinguish word-of-mouth communication from observational learning in their paper.
Instead of generating \( k(t) \) by epidemic model, find \( k(t) \) by the real number of accounts.

\( k(t) \): Normalize the number of accounts\(^\text{10}\) such that it starts at 0 and ends at 1.

Pick \( \alpha = 0 \) based on the discussion for different bubbles.

\(^{10}\)Let \( a(t) \) be the number of accounts in Shanghai A share market during bubble formation: 2005-Jan to 2008-Apr. Then \( k(t) = \frac{a(t) - a(0)}{a(T)} \) for \( t \in (0, T) \).
Not only predicts the shapes correctly, but also predicts the detailed fluctuations of volumes correctly.
Regression: \( V_t^{real} = a + b \cdot V_t^{predict} \)

- \( b \) different from zero at significant level 0.001
- \( R^2 = 0.77 \)
Regression: \( P_{t}^{\text{real}} = a + b \cdot P_{t}^{\text{predict}} \)

- \( b \) different from zero at significant level 0.001
- \( R^2 = 0.93 \)
Reject $\alpha = -0.8$ because there is no big volume at the beginning in China’s data.

Reject $\alpha = 2$ because there is no big initial price increase in China’s data.

$\alpha = 0$ describes china’s bubble the best. This consists with analysis across countries.
Bubble Dynamics

- Predicts volumes correctly
- Price does not match (B-share market is influenced by A-share market)
  - B-share companies account for 4.4% of A-share companies on 2012-8-16.
  - About 80% B-share companies also listed on A-share market.
If a population scheme $k(t)$ follows epidemic model, we have

$$k'(t) = sk(t)[1 - k(t)]$$

For one month $\Delta t = 1$, we have

$$\frac{k(t + \Delta t) - k(t)}{\Delta t} = sk(t)[1 - k(t)]$$

Thus for each month, we can use $k(t)$ and $k(t + \Delta t)$ to form a point estimation of $s$

$$s_t = \frac{k(t + \Delta t) - k(t)}{k(t)[1 - k(t)]}$$

Delete top and bottom 10% outliers, estimate $s$ by the mean of $s_t$ and median of $s_t$. 
Bubble Dynamics

Monthly Estimation of $s$

- Mean($s$) = 0.27
- Median($s$) = 0.22

(monthly estimation of $s$ vs month)
Model $k(t)$ by epidemic model

- **Real $k(t)$**
- **Epidemic $k(t)$ with $s=0.27$**
- **Epidemic $k(t)$ with $s=0.22$**

$k$ vs. month
Conclusion

- Adding an epidemic element into bubble models helps rationalize previously difficult to explain findings, including why volume peaks before prices.
- Further work on transmission mechanisms of bubbles.