Floaters and Inverse Floaters

Concepts and Buzzwords

- **Floating Rate Notes**
  - Cash flows
  - Valuation
  - Interest Rate Sensitivity

- **Inverse Floating Rate Notes**
  - Cash flows
  - Valuation
  - Interest Rate Sensitivity

- floater,
- FRN,
- ARN,
- VRN,
- inverse floater,
- benchmark interest rate,
- index
Reading

- Tuckman, chapter 18

Introduction to Floating-Rate Notes

- A floating rate note (FRN) is a bond with a coupon that is adjusted periodically to a benchmark interest rate, or indexed to this rate.
  - Possible benchmark rates: US Treasury rates, LIBOR (London Interbank Offering Rates), prime rate, ....
- Examples of floating-rate notes
  - Corporate (especially financial institutions)
  - Adjustable-rate mortgages (ARMs)
  - Governments (inflation-indexed notes)
Floating Rate Jargon

- Other terms commonly used for floating-rate notes are
  - FRNs
  - Floaters and Inverse Floaters
  - Variable-rate notes (VRNs)
  - Adjustable-rate notes
- FRN usually refers to an instrument whose coupon is based on a short term rate (3-month T-bill, 6-month LIBOR), while VRNs are based on longer-term rates (1-year T-bill, 2-year LIBOR)

Cash Flow Rule for Plain Vanilla Semi-Annual Floater

- The basic semi-annual coupon floating rate note has the coupon indexed to the 6-month interest rate.
- Each coupon date, the coupon is equal to the par value of the note times one-half the 6-month rate quoted 6 months earlier, at the beginning of the coupon period. In other words, the time \( t \) coupon payment as percent of par is \( t \cdot 0.5r \).
- The note pays par value at maturity.
Floating Rate Note Cash Flows

Each coupon is based on the previous 0.5-year rate. Only the next coupon is known at the current date. The later ones are random.

\[
\begin{align*}
_0 r_{0.5} / 2 & \quad _{0.5} \tilde{r}_1 / 2 & \quad _{r-0.5} \tilde{r}_1 / 2 & \quad 1+_{T-0.5} \tilde{r}_T / 2 \\
0 & \quad 0.5 & \quad 1 & \quad \ldots & \quad t & \quad \ldots & \quad T
\end{align*}
\]

Example: Two-Year Semi-Annual Floater

What are the cash flows from $100 par of the note?

- The first coupon on the bond is 100 x 0.0554/2 = 2.77.
- Later coupons set by the future 6-month interest rates.
- For example, suppose the future 6-month interest rates turn out as follows:

\[
\begin{align*}
0 & \quad 0.5 & \quad 1 & \quad 1.5 & \quad 2 \\
5.54\% & \quad 6.00\% & \quad 5.44\% & \quad 6.18\%
\end{align*}
\]

Floater Cash Flows:

\[
\begin{align*}
0 & \quad 0.5 & \quad 1 & \quad 1.5 & \quad 2 \\
2.77 & \quad 3.00 & \quad 2.72 & \quad 103.09
\end{align*}
\]

\[100 \times 0.06/2\]
Replicating a T-Year Floater with Six-Month Par Bonds

- Consider the following trading strategy:
- At time 0, buy a 0.5-year par bond (pay $1)
- At time 0.5, buy another 0.5-year par bond (collect $1 + \frac{r_{0.5}}{2}$, pay $1 = \text{collect } \frac{r_{0.5}}{2}$)
- At time 1, buy another 0.5-year par bond (collect $1 + \frac{r_{1.5}}{2}$, pay $1 = \text{collect } \frac{r_{1.5}}{2}$)
- … and so on, every six months until floater maturity date $T$
- At time $T$, collect $1 + T - 0.5 \frac{r_T}{2}$

\[
\begin{array}{cccccc}
\text{Time 0: Buy 0.5 - yr par bond} \\
-1 & + (1 + 0.5r_{0.5}/2) \\
\text{Time 0.5: Buy 0.5 - yr par bond} \\
-1 & + (1 + 0.5r_{1.5}/2) \\
\text{Time 1: Buy 0.5 - yr par bond} \\
-1 & + (1 + 1.5r_{2.5}/2) \\
\text{...} \\
\text{Time T - 0.5: Buy 0.5 - yr par bond} \\
-1 & + (1 + T - 0.5T/2) \\
\end{array}
\]

Net: 
-1 \[\frac{0.5r_{0.5}}{2}\] \[\frac{0.5r_{1.5}}{2}\] \[r_{1.5}/2\] \[\ldots\] \[1 + T - 0.5T/2\]

0 \[0.5\] 1 \[1.5\] \[\ldots\] \[T\]
A Floater is Equivalent to a Six-Month Par Bond

- A dynamic strategy of rolling six-month par bonds until floater maturity, collecting the coupons along the way, replicates the cash flows of a floater.
- So as semi-annual coupon floater is equivalent to the six-month par bond in its replicating trading strategy.
- Like its replicating trading strategy, a floater is always worth par on the next coupon date with certainty.
- Its coupon is set to make it worth par today.
- The duration of the floater is therefore equal to the duration of a six-month par bond.
- Their convexities are the same, too.

Example: Interest Rate Sensitivity of a Semi-Annual Coupon Floating Rate Note

duration = duration of 6-month bond = \( \frac{0.5}{1 + 0.0554/2} = 0.4865 \)
dollar duration = duration \times price = 0.4865 \times 100 = 48.65

convexity = convexity of 6-month bond = \( \frac{0.5^2 + 0.5/2}{(1 + 0.0554/2)^2} = 0.4734 \)
dollar convexity = convexity \times price = 0.4734 \times 100 = 47.34
Inverse Floating Rate Notes

- Unlike a floating rate note, an inverse floater is a bond with a coupon that varies inversely with a benchmark interest rate.
- Inverse floaters come about through the separation of fixed-rate bonds into two classes:
  - a floater, which moves directly with some interest rate index, and
  - an inverse floater, which represents the residual interest of the fixed-rate bond, net of the floating-rate.

Inverse Floater in Even Split

- Suppose a fixed rate semi-annual coupon bond with par value $N$ is split evenly into a floater and an “inverse floater,” each with par value $N/2$ and maturity the same as the original bond.
- Then the time $t$ coupon of the inverse floater, as a percent of par, is $(k - 0.5r_t)/2$ where $k$ is fixed.
Cash Flows in Even Split

- If the coupon rate on the fixed rate bond is c, then each coupon date, the total payment from the original issue is 
  \( N \times c/2 \).
- Every six months, the floater pays a coupon based on a floating rate, represented by r. The total payment to the floater is 
  \( N/2 \times r/2 \).
- Therefore, total payment to the inverse floater must be 
  \( N \times c/2 - N/2 \times r/2 = N/2 \times (2c-r)/2 \).
- In other words, the rate paid by the inverse floater is \( 2c-r \), or 
  "k-floating" where the fixed portion, k, is twice the coupon rate on the original fixed rate bond.

Example: Two-Year Semi-Annual Inverse Floating Rate Note

- Consider a two year fixed 5.5% bond with $200 par.
- Split it evenly into:
  - $100 floater paying \( r \)
  - $100 paying \( 2c-r = 11\% - r \)
- Consider again the case where \( r_{0.5} = 5.54\% \).
- Total coupon at 0.5 is $200 * 5.5%/2=$5.5
  - the floater holder gets $100*5.54%/2=$2.77
  - the inverse holder gets $100 x (0.11-0.0554)%/2 = $2.73.
- The later coupons will be determined by the future values of the 6-month rate.
Example: Two-Year Semi-Annual Inverse Floating Rate Note...

Suppose future 6-month rates are as follows:

<table>
<thead>
<tr>
<th>Time (Years)</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate (%)</td>
<td>5.54</td>
<td>6.00</td>
<td>5.44</td>
<td>6.18</td>
<td></td>
</tr>
</tbody>
</table>

Inverse Floater Cash Flows:

<table>
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<tr>
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<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow</td>
<td>2.73</td>
<td>2.50</td>
<td>2.78</td>
<td>102.41</td>
<td></td>
</tr>
</tbody>
</table>

Suppose future 6-month rates are as follows:

\[ 100 \times \frac{(0.11 - 0.06)}{2} \]

Example...

Floater + Inverse Floater \((k) = 2 \) Fixed \((k/2)\)

<table>
<thead>
<tr>
<th>Time (Years)</th>
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Floater Cash Flows + Inverse Floater Cash Flows:

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<td>2.78</td>
<td>+102.41</td>
<td></td>
</tr>
<tr>
<td>=</td>
<td>5.50</td>
<td>5.50</td>
<td>5.50</td>
<td>=205.50</td>
<td></td>
</tr>
</tbody>
</table>

Total is cash flow is same as cash flow from 200 par of a 5.50% note.
**Decomposition of an Inverse Floater**

Clearly, the cash flows of an inverse floater with coupon rate "$k \text{ minus floating}"$ are the same as the cash flows of a portfolio consisting of

– long twice the par value of a fixed note with coupon $k/2$, and

– short the same par of a floating rate note.

$\Rightarrow \text{Inverse Floater}(k) = 2 \text{ Fixed}(k/2) - \text{ Floater}$

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**Valuation of an Inverse Floater**

*Price of Inverse Floater*(k) = $2 \times \text{ Price of Fixed}(k/2) - 100$

- For example, the 2-year inverse floater paying 11% minus floating is worth
  
  $2 \times \text{ price of 5.5\% fixed rate bond minus 100}$.

- Using the zero rates from the WSJ,
  
  $r_{0.5} = 5.54\%, r_1 = 5.45\%, r_{1.5} = 5.47\%, r_2 = 5.50\%$,

- to discount each of the cash flows of the 2-year 5.5% fixed coupon bond, the bond is worth $100.0019$.

- Therefore the inverse floater is worth
  
  $2 \times 100.0019 - 100 = 100.0038$
Interest Rate Sensitivity of an Inverse Floating Rate Note

- Dollar Duration of an Inverse Floater\( (k) = \)
  \[ 2 \times \text{Dollar Duration of Fixed}(k/2) - \text{Dollar Duration of Floater} \]

- Dollar Convexity of an Inverse Floater\( (k) = \)
  \[ 2 \times \text{Dollar Convexity of Fixed}(k/2) - \text{Dollar Convexity of Floater} \]

Example: Interest Rate Sensitivity of an Inverse Floating Rate Note

- The dollar duration of $100 par of the 5.5% fixed rate bond is 186.975, while its duration is 1.87.
- This means the dollar duration of the inverse floater paying 11%-floating is \(2 \times 186.975 - 48.65 = 325.30\).
  \[ \text{Its duration is} \frac{325.30}{100.0038} = 3.25! \]
- An inverse floater is roughly twice as sensitive to interest rates as an ordinary fixed rate bond.
Example: Interest Rate Sensitivity of an Inverse Floating Rate Note...

- The dollar convexity of $100 par of the 5.5% fixed rate bond is 448.76, while its convexity is 4.49.
- This means the dollar convexity of the inverse floater paying 11%-floating is $2 \times 448.76 - 47.34 = 850.18$.
- Its convexity is $850.18/100.0038 = 8.50$.

Par Inverse Floater

- What is the value of the fixed component, $k$, of the coupon rule that makes the inverse floater worth par?

$$Price\ of\ Inverse\ Floater(k) = 2 \times Price\ of\ Fixed(k/2) - 100$$

$\Rightarrow 100 = 2 \times Price\ of\ Fixed(k/2) - 100$

$\Rightarrow Price\ of\ Fixed(k/2) = 100$

$\Rightarrow k/2 = \text{par rate, or } k = 2 \times \text{par rate.}$

- For example, suppose the 30-year par rate is 7% (i.e., a 30-year 7% fixed rate bond is worth par).
- Then a 30-year inverse floater with coupon equal to
  
  $2 \times 7\% - \text{floating} = 14\% - \text{floating}$

  will be worth par.

- If the 0.5-year rate is 6%, the first inverse floater coupon will be 14%-6% = 8%.