Interest Rate Swaps

Key Concepts and Buzzwords

- Swaps
- Swap Spreads
- Credit Risk of Swaps
- Uses of Swaps

Readings
- Tuckman, Chapter 18.
Description of a Swap

- An interest rate swap is a contract which commits two counterparties to exchange, over an agreed period, two streams of interest payments, each calculated using a different interest rate index, but applied to a common notional principal amount.

The Plain Vanilla Swap

- A plain vanilla or generic swap is a fixed-for-floating swap with:
  - constant notional principal
  - constant fixed interest rate
  - floating interest rate such as 6-month LIBOR (London Interbank Offer Rate), a Treasury bill rate, Prime rate, Fed Funds
  - semi-annual payments of fixed and floating.
- The swap rate quoted is the fixed rate.
Which Side is Which?

- The party who is long the swap is paying floating and receiving fixed.
- The party who is short the swap is paying fixed and receiving floating.
- Long the bond market.

Cash Flow Rule

- Every six months until maturity, the party who is long the swap receives a fixed rate $k$, and pays the 6-month rate set 6-months earlier and.
- If the notional amount of the swap is $N$ and the maturity is $T$, the time $t$ cash flow to this party is
  \[ N(k - r_{t-0.5})/2 \text{ for } t = 0.5, 1, 1.5, ..., T. \]
- * Note that no principal is exchanged.
**Example: Cash Flows to Long Position in 5.5% 2-Year Swap with $100 Notional Amount**

Possible path of future 0.5-year rates:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.54%</td>
<td>6.00%</td>
<td>5.44%</td>
<td>6.18%</td>
<td></td>
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</tbody>
</table>

Cash flows to long swap position:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.75</td>
<td>2.75</td>
<td>2.75</td>
<td>2.75</td>
<td></td>
</tr>
<tr>
<td>-2.77</td>
<td>-3.00</td>
<td>-2.72</td>
<td>-3.09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>= -0.02</td>
<td>= -0.25</td>
<td>= 0.03</td>
<td>= -0.34</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Decomposition of a Swap**

Consider again the cash flows of the plain vanilla swap with fixed rate \( k \), notional amount \( N \) and maturity \( T \):

\[ N(k_{t,x}^2)/2 \text{ for } t=0.5, 1, 1.5, \ldots, T. \]

These are the same as the cash flows from a portfolio consisting of:

- A long position in a \( T \)-year fixed rate note with par amount \( N \) and coupon rate \( k \), and
- A short position in a \( T \)-year floating rate note with par amount \( N \).

*In particular, the difference between the coupons of the two notes equals the swap payment, and the difference between their principal payments is zero.*

\[ \text{Swap}(k) = \text{Fixed Rate Note}(k) - \text{Floating Rate Note} \]
Swap = Long a fixed rate note, short a floater

\[ \text{Swap value} = \text{value of fixed rate note} - \text{value of floater} = \text{value of fixed rate note} - 100 \]

\[ \text{Swap dollar duration} = \text{dollar duration of fixed rate note} - \text{dollar duration of floater} \]

\[ \text{Swap dollar convexity} = \text{dollar convexity of fixed rate note} - \text{dollar convexity of floater} \]

### Example

Interest rate sensitivity of 2-year 5.5% swap, $100 notional amount.

<table>
<thead>
<tr>
<th>Security</th>
<th>Market Value</th>
<th>Dollar Duration</th>
<th>Dollar Convexity</th>
<th>Duration</th>
<th>Convexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-Year 5.5%</td>
<td>100.0019</td>
<td>187</td>
<td>449</td>
<td>1.87</td>
<td>4.49</td>
</tr>
<tr>
<td>Coupon Bond</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two Year Floater</td>
<td>100</td>
<td>49</td>
<td>47</td>
<td>0.49</td>
<td>0.47</td>
</tr>
<tr>
<td>Swap</td>
<td>0.0019</td>
<td>134</td>
<td>401</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The Swap Rate

- The fixed rate in the swap is called the swap rate.
- The swap rate in a newly negotiated swap is set to make the contract \textit{worth zero at inception}.

\[ \text{Swap}(k, T) = \text{Fixed Rate Bond}(k, T) - \text{Floater} \]

- The value of the floater is par
- To make the swap worth zero, the swap rate must make the fixed rate bond worth par as well
  - The swap rate must be the par rate for maturity \( T \)
  - New Swap\((k^*, T) = \text{Par bond}(k^*, T) - \text{Floater} \)
    - Because only then you get \( 0 = 100 - 100 \)

Example

- The 2-year swap with fixed rate 5.5% is worth 0.0019 per $100 notional amount:
  - The 2-yr 5.5% bond is worth 100.0019
  - The floater is worth 100
  - swap value = 100.0019 - 100 = 0.0019
- To make the swap worth exactly zero, the swap rate must be set equal to the par rate for 2 year maturity:
  - 2-year par rate =
  \[ \frac{2(1-0.897166)}{(0.973047+0.947649+0.922242+0.897166)} \]
  \[ = 5.499\% \]
Swap-Based Products

- **Swaption**: An option on a swap (usually with strike price 0), i.e., the right to enter into a swap with specified terms at some future date (premium paid up front)
- **Putable Swap**: The fixed interest receiver (long the swap) has the right to cancel the swap before maturity (the premium for the cancelation option is paid up front)
- **Callable Swap**: The fixed interest payer (short the swap) has right to cancel the swap before maturity (premium paid up front)
- **Forward Swap**: The swap begins at some specified future date with the terms set in advance (mutually binding)
- **Amortizing Swap**: Notional amount of swap, and thus, the size of the coupon payments, changes over time according to a schedule.
- **Zero Swap**: There is no exchange of payments until maturity. Then a fixed amount of accumulated interest is exchanged for interest that has accumulated at the floating interest rate.
Swap Spreads and the Swap Curve

- For various maturities, dealers quote the swap rate for a generic swap of fixed for 6-month LIBOR.
- For such swaps, called “plain vanilla,” there is typically a bid-ask spread of 3 basis points.
- Recall that the swap rate represents a par rate.
- The rate on current Treasury bond with the same maturity also represents a par rate since the current Treasury is generally trading near par.
- The swap rate is typically higher, and the difference is called the swap spread.
- The plot of swap rates for different maturities is called the swap curve.

Swap and Treasury Curves 9/28/06

- The graph shows the swap rates and Treasury rates for different maturities.
Why Is There a Swap Spread?

In the previous graph, the swap spread ranges from 30 to 50 bps. Why is there a spread?

– LIBOR represents the average rate for dollar deposits in London at five major banks (A/AA rating), and thus offers a spread over default-free Treasuries.

– Thus, if the swap is to be priced at zero, swaps offering fixed-against-LIBOR must offer a spread over Treasuries.

Why Does the Swap Spread Vary with the Swap Maturity?

• Think of the counterparties as not only exchanging fixed for floating bond coupons but also fixed for floating spreads.

• The long side receives a fixed swap spread in exchange for the floating spread in the LIBOR rate over the life of the swap.

• The fair fixed spread to receive depends on the swap maturity for the essentially same reason that the par rate varies with maturity—differing expectations and uncertainties about the path of the floating spread over different time horizons.
Credit Risk of Swaps

- The spread in the swap rate comes from the fact that the floating index used in practice (LIBOR) has a spread in it. It is not because of a risk of default by one of the swap counterparties.
- Note that the swap dealer could be on either side of the swap, so he could be paying the spread just as well as receiving it.
- Counterparty credit risk is controlled in other ways:
  - Dealers won’t transact with really poor credits.
  - Credit triggers are provided which force a downgraded counterparty to settle his side of the swap at market rates.
  - In any case, the amount at risk is much smaller than the swap notional amount since the swap is a combined long-short position.

Uses of Swaps

- A swap is essentially a fully levered position in a bond, providing an easy way to take a position without capital.
- Swaps are often used for hedging bond positions.
- Another traditional use of swaps is to convert floating rate debt to fixed. A conventional wisdom in corporate borrowing has been that poorer credits have a comparative advantage issuing shorter term debt and rolling it over because they pay a lower credit spread than on longer term debt. Some firms like to issue short term debt and roll it over and then convert their debt service to fixed by entering into a swap in which they pay fixed and receive floating. The idea is to synthesize long term fixed rate debt with a lower spread.
- The risk is that their credit quality worsens and the spread on their short term debt widens, which would not happen if they issue long debt directly.