Swaptions

Key Concepts and Buzzwords
- Swaps
- Swaptions
- Connection to Callable Bonds
- Putable Swap, Callable Swap
- Cancelable Swaps

Reading
- Tuckman, chapter 19.
**Review of Interest Rate Swaps**

- A plain vanilla semi-annual swap is a contract to receive a fixed interest rate and pay a floating interest rate on a given notional par amount every 6 months until maturity.
- A T-year swap with notional par amount $N$ and fixed rate $k$ is the same as the portfolio:
  - long $N$ par of T-year fixed rate bonds with coupon rate $k$ and
  - short $N$ par of T-year floaters.
- The value of a swap is the value of the fixed rate bond minus par.

**Example: Value of Long Position in 5.5% 2-Year Swap with $100 Notional Amount**

Each node lists the value of a 5.5% fixed rate note and the value of the swap.

<table>
<thead>
<tr>
<th>Time</th>
<th>Value of Fixed Rate Note</th>
<th>Value of Swap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time 0</td>
<td>100.0019</td>
<td>-0.9110</td>
</tr>
<tr>
<td>Time 0.5</td>
<td>99.0890</td>
<td>100.0207</td>
</tr>
<tr>
<td>Time 1</td>
<td>98.6063</td>
<td>100.3113</td>
</tr>
<tr>
<td>Time 1.5</td>
<td>98.8626</td>
<td>100.8227</td>
</tr>
</tbody>
</table>

Swap = Long Fixed Rate Note, Short Floater
Swap Value = Value of Fixed Rate Note - 100
Swaptions

Swap-Based Products

- **Swaption (option on a swap)**
  - The right to enter into (buy or sell) a swap with pre-specified fixed rate for given strike price (usually zero) at some future date.
  - Receiver swaption: the right to enter into a swap as the fixed rate receiver (a call on a swap)
  - Payer swaption: the right to enter into a swap as the fixed rate payer (a put on a swap)

- **Cancelable swap (swap with embedded swaption)**
  - Putable Swap: The fixed interest receiver (the long side) has the right to cancel the swap before maturity.
  - Callable Swap: The fixed interest payer (the short side) has the right to cancel the swap before maturity.

Example

- Consider the following American receiver swaption:
  - The right to enter into a swap to receive a fixed rate of 5.5% and pay floating, until time 2;
  - i.e., an American option to buy the 2-year, 5.5% swap with $100 notional amount for a strike price of zero.
  - ...the right to be long the swap
Swaptions

Modeling the Receiver Swaption

The swaption gives the owner the right to buy the swap for zero, i.e., to enter into a long position in the swap at no cost.

Modeling the Receiver Swaption...
Modeling the Receiver Swaption...

\[
\text{Swaption value} = 0.5003
\]

Modeling the Receiver Swaption

\[
\text{Swaption value} = 0.5003
\]
Swaptions and Callable Bonds

- Notice that this swaption is the same as the option embedded in the 2-year 5.5%-coupon bond callable at par.
- In general, in the absence of default risk:
  - an option to buy a swap with maturity T, and swap rate C for a strike price of zero
  - = an option to buy a T-year fixed rate bond with coupon rate C at a strike price of par
  - = the option embedded in a T-year bond with coupon C that is currently callable at par.
Swaptions and Embedded Options

In order to prove that:

\[ \text{an option to buy a swap with maturity } T, \text{ and swap rate } C \text{ for a strike price of zero} \]
\[ = \text{an option to buy a } T\text{-year fixed rate bond with coupon rate } C \text{ at a strike price of par} \]

we show they have the same exercise value at all states and times:

Receiver swaption exercise value
\[ = \text{swap value - 0} \]
\[ = \text{bond value - floater value - 0} \]
\[ = \text{bond value} - \text{par} \]
\[ = \text{exercise value of call on bond with par strike} \]

Ways A Callable Bond Issuer Can Get Value from An Embedded Call Option

- Get current exercise value by doing a refunding now
  - Issue new noncallable bonds with same coupon and maturity.
  - Call old bonds.
  - Profit = value of noncallable - strike price.
- Wait for payoff from future exercise (future refunding)—uncertain payoff.
- Try to extract value of future refunding today by selling option:
  - Sell embedded option to option dealer.
    - Difficulty: must find buyer—generally someone with expertise in hedging.
    - Especially hard when credit quality is an important factor driving bond value.
  - Synthetically sell option
    - Do-it-yourself hedge with dynamic trading strategy using liquid bonds (feasible?)
  - Sell option back to investors.
    - Issue new noncallable bonds with same coupon and maturity.
    - Tender for old bonds in open market.
    - Profit = noncallable value - callable value = option value.
    - Problem: if bondholders collude they can drive up price of callable.
  - Sell receiver swaption to swap dealer.
    - Problem: Subject to risk of own credit quality decreasing. If own debt is
down-graded but riskless bonds rally, call option may not be in the money when
swaption may is. (Swaption is generally worth more than embedded option.)
Swaptions and Cancelable Swaps

- **Cancelable swap** = swap +/- option to cancel
  
  = swap +/- option to enter offsetting swap at no cost
  
  = long swap +/- swaption with zero strike price.

- **Putable swap** = swap + payer swaption
  
  - Fixed receiver has the option to cancel.
  
  - He is long the swap and long the option to cancel.
  
  - The option to cancel is a payer swaption.

- **Callable swap** = swap - receiver swaption
  
  - Fixed payer has the option to cancel.
  
  - Fixed payer’s option to cancel is a receiver swaption.

---

Example: Callable Swap

- Consider a 5.5% 2-year callable swap, i.e. the swap is cancelable at the fixed payer's option.
  
  - The fixed payer is short the swap and long a call swaption.
  
  - The time 0 value of the fixed payer’s position is
    
    \[ \text{- swap + swaption} = -0.0019 + 0.5003 = 0.4984. \]

  - The time 0 value of the party who is long the swap, the fixed receiver, is
    
    \[ \text{- swap - swaption} = 0.0019 - 0.5003 = -0.4984 \]
Par Rate for Callables?
Bond and Option Prices for Different Coupon Rates:

<table>
<thead>
<tr>
<th>Coupon</th>
<th>Bond</th>
<th>Call option</th>
<th>Callable</th>
<th>Swap</th>
<th>Callable swap</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.500</td>
<td>$100.00</td>
<td>$ 0.50</td>
<td>$99.50</td>
<td>$ 0.00</td>
<td>$-0.50</td>
</tr>
<tr>
<td>5.600</td>
<td>$100.19</td>
<td>$ 0.58</td>
<td>$99.61</td>
<td>$ 0.19</td>
<td>$-0.39</td>
</tr>
<tr>
<td>5.700</td>
<td>$100.38</td>
<td>$ 0.65</td>
<td>$99.72</td>
<td>$ 0.38</td>
<td>$-0.28</td>
</tr>
<tr>
<td>5.800</td>
<td>$100.56</td>
<td>$ 0.75</td>
<td>$99.82</td>
<td>$ 0.56</td>
<td>$-0.18</td>
</tr>
<tr>
<td>5.900</td>
<td>$100.75</td>
<td>$ 0.84</td>
<td>$99.91</td>
<td>$ 0.75</td>
<td>$-0.09</td>
</tr>
<tr>
<td>6.000</td>
<td>$100.93</td>
<td>$ 0.93</td>
<td>$100.00</td>
<td>$ 0.93</td>
<td>$ 0.00</td>
</tr>
</tbody>
</table>

Par Coupon Rate for Callable Bond or Swap

- A 6% 2-year swap is worth $0.928
- A call option on the 6% swap is worth $0.928

⇒ The callable 6% swap is worth $0.928-0.928=0

⇒ The par callable swap rate is 6%, 50bp over the rate for a plain vanilla swap

⇒ Similarly, a 6% 2-year bond is that is callable at par is worth par.