New York University
Stern School of Business

Debt Instruments and Markets
Professor Carpenter

Practice Midterm

Name:

Section:

Social Security Number:

This exam has a total of 8 pages, including this one. Please make sure you have them all. Please do all work on these pages. You may not submit additional pages with your exam.

All rates are annualized with semi-annual compounding. There are no bid-ask spreads and there is no default risk. All coupon-bearing instruments make payments semi-annually.
1. Suppose the following two coupon bonds are trading at the yields indicated in the following table:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Coupon rate</th>
<th>Maturity</th>
<th>Yield</th>
<th>Price per $100 par</th>
</tr>
</thead>
<tbody>
<tr>
<td># 1</td>
<td>3.00%</td>
<td>0.5 years</td>
<td>2.00%</td>
<td></td>
</tr>
<tr>
<td># 2</td>
<td>4.00%</td>
<td>1.0 years</td>
<td>4.00%</td>
<td></td>
</tr>
</tbody>
</table>

(a) (6 points) Fill in the bond prices in the last column of the table.

(b) (8 points) Construct a portfolio of these two coupon bonds that synthesizes $100 par of a zero maturing at time 1, i.e., determine the par amounts $N_{0.5}$ and $N_1$ of the 0.5- and 1-year coupon bonds in the replicating portfolio.
(c) (8 points) What is the no-arbitrage price of $100 par of the zero maturing at time 1?

(d) (7 points) What is the implied 1-year zero rate?

(e) (10 points) What is the no-arbitrage forward rate from time 0.5 to time 1?
(f) Consider $100 notional par amount of an old plain vanilla interest rate swap with one year left to maturity. Counterparty A receives 5% and pays the floating 0.5-year rate.

i. (9 points) Describe a portfolio (i.e., par amounts) of bonds and plain vanilla floating-rate notes that generates the same cash flows as A’s position.

ii. (8 points) What is the time 0 value of A’s position in the swap (excluding the time 0 swap payment)?
2. A bank’s assets and liabilities have value, duration, and convexity given in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>$100M</td>
<td>$50M</td>
</tr>
<tr>
<td>Duration</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Convexity</td>
<td>40</td>
<td>5</td>
</tr>
</tbody>
</table>

(a) (2 points) What is the value of the bank’s net equity?

(b) (5 points) What is the duration of the bank’s net equity?

(c) (5 points) What is the convexity of the bank’s net equity?
(d) (5 points) Using duration and/or dollar duration alone (i.e., not convexity), approximate the change in the value of the bank’s net equity if all zero rates rise by 50 basis points.

(e) (5 points) Using both duration and convexity (and/or dollar duration and dollar convexity), approximate the change in the value of the bank’s net equity if all zero rates rise by 50 basis points.
3. The diagram below shows the time 0 and time 0.5 prices and possible payoffs of $1 par of the zero maturing at time 0.5 and $1 par of the zero maturing at time 30. The diagram also shows the possible time 0.5 payoffs of a call on $100 par of the zero maturing at time 30. (The call expires at time 0.5 and its strike price is $50).

<table>
<thead>
<tr>
<th>Time 0</th>
<th>Time 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{0.5} = 0.99$</td>
<td>$d_{0.5}^{u} = 1 $</td>
</tr>
<tr>
<td>$d_{30} = 0.50$</td>
<td>$d_{30}^{u} = 0.49$</td>
</tr>
<tr>
<td>$C^u = 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$d_{0.5}^{d} = 1 $</td>
</tr>
<tr>
<td></td>
<td>$d_{30}^{d} = 0.51$</td>
</tr>
<tr>
<td></td>
<td>$C^d = 1$</td>
</tr>
</tbody>
</table>

(a) (8 points) Construct a portfolio of the two zeroes that has the same time 0.5 payoff as the call, i.e., determine the par amounts $N_{0.5}$ and $N_{30}$ of the zeroes maturing at time 0.5 and 30, respectively, in the replicating portfolio.
(b) (5 points) What is the time 0 value of this call-replicating portfolio?

(c) (5 points) What are the “risk-neutral” probabilities $p$ and $1 - p$ of the up state and the down state, respectively, that make the expected return from time 0 to time 0.5 on the zero maturing at time 30 the same as that on the zero maturing at time 0.5?

(d) (4 points) Price the call using these risk-neutral probabilities.
Solutions to Practice Midterm

1. Suppose the following two coupon bonds are trading at the yields indicated in the following table:

<table>
<thead>
<tr>
<th>Bond</th>
<th>Coupon rate</th>
<th>Maturity</th>
<th>Yield</th>
<th>Price per $100 par</th>
</tr>
</thead>
<tbody>
<tr>
<td># 1</td>
<td>3.00%</td>
<td>0.5 years</td>
<td>2.00%</td>
<td>101.5/1.01 = 100.495</td>
</tr>
<tr>
<td># 2</td>
<td>4.00%</td>
<td>1.0 years</td>
<td>4.00%</td>
<td>100</td>
</tr>
</tbody>
</table>

(a) (6 points) Fill in the bond prices in the last column of the table.
(b) (8 points) Construct a portfolio of these two coupon bonds that synthesizes $100 par of a zero maturing at time 1, i.e., determine the par amounts $N_{0.5}$ and $N_1$ of the 0.5- and 1-year coupon bonds in the replicating portfolio.

\[
1.015 \times N_{0.5} + 0.02 \times N_1 = 0 \\
0 \times N_{0.5} + 1.02 \times N_1 = 100 \\
\Rightarrow N_{0.5} = -1.932 \\
N_1 = 98.039
\]

(c) (8 points) What is the no-arbitrage price of $100 par of the zero maturing at time 1?

\[
N_{0.5} \times 1.00495 + N_1 \times 1 = 96.10
\]

(d) (7 points) What is the implied 1-year zero rate?

\[
r_1 = 2\left(\frac{1}{0.9610}\right)^{0.5} - 1 = 4.018%
\]

(e) (10 points) What is the no-arbitrage forward rate from time 0.5 to time 1?

\[
2\left[\frac{1/0.9610}{1.01} - 1\right] = 6.06%
\]

(f) Consider $100 notional par amount of an old plain vanilla interest rate swap with one year left to maturity. Counterparty A receives 5% and pays the floating 0.5-year rate.

i. (9 points) Describe a portfolio (i.e., par amounts) of bonds and plain vanilla floating-rate notes that generates the same cash flows as A’s position.

Long $100 par of one-year 5%-coupon bonds, short $100 par one-year floating rate notes.

ii. (8 points) What is the time 0 value of A’s position in the swap (excluding the time 0 swap payment)?

\[
(2.5/1.01 + 102.5 \times 0.9610) - 100 = 0.975
\]
2. A bank’s assets and liabilities have value, duration, and convexity given in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>$100M</td>
<td>$50M</td>
</tr>
<tr>
<td>Duration</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Convexity</td>
<td>40</td>
<td>5</td>
</tr>
</tbody>
</table>

(a) (2 points) What is the value of the bank’s net equity?

\[ 100 - 50 = 50 \]

(b) (5 points) What is the duration of the bank’s net equity?

\[ \frac{6 \times 100M - 2 \times 50M}{50M} = 10 \]

(c) (5 points) What is the convexity of the bank’s net equity?

\[ \frac{40 \times 100M - 5 \times 50M}{50M} = 75 \]

(d) (5 points) Using duration and/or dollar duration alone (i.e., not convexity), approximate the change in the value of the bank’s net equity if all zero rates rise by 50 basis points.

\[ \Delta V = -dur \times \Delta r = -50M \times 10 \times 0.005 = -2.5M \]

(e) (5 points) Using both duration and convexity (and/or dollar duration and dollar convexity), approximate the change in the value of the bank’s net equity if all zero rates rise by 50 basis points.

\[ \Delta V = -dur \times \Delta r + 0.5 \times con \times (\Delta r)^2 = -2.5M + 0.5 \times 50M \times 75 \times 0.005^2 = -2.45M \]
3. The diagram below shows the time 0 and time 0.5 prices and possible payoffs of $1 par of the zero maturing at time 0.5 and $1 par of the zero maturing at time 30. The diagram also shows the possible time 0.5 payoffs of a call on $100 par of the zero maturing at time 30. (The call expires at time 0.5 and its strike price is $50).

<table>
<thead>
<tr>
<th>Time 0</th>
<th>Time 0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5(d_{0.5}^u = 1) (0.5d_{30}^u = 0.49) (C^u = 0)</td>
</tr>
<tr>
<td>(0d_{0.5} = 0.99)</td>
<td>0.5(d_{0.5}^d = 1) (0.5d_{30}^d = 0.51) (C^d = 1)</td>
</tr>
<tr>
<td>(0d_{30} = 0.50)</td>
<td></td>
</tr>
<tr>
<td>(C = ?)</td>
<td></td>
</tr>
</tbody>
</table>

(a) (8 points) Construct a portfolio of the two zeroes that has the same time 0.5 payoff as the call, i.e., determine the par amounts \(N_{0.5}\) and \(N_{30}\) of the zeroes maturing at time 0.5 and 30, respectively, in the replicating portfolio.

\[
\begin{align*}
1 \times N_{0.5} + 0.49 \times N_{30} &= 0 \\
1 \times N_{0.5} + 0.51 \times N_{30} &= 1
\end{align*}
\Rightarrow N_{0.5} = -24.5 \quad N_{30} = 50
\]

(b) (5 points) What is the time 0 value of this call-replicating portfolio?

\[-24.5 \times 0.99 + 0.5 \times 50 = 0.745\]

(c) (5 points) What are the “risk-neutral” probabilities \(p\) and \(1 - p\) of the up state and the down state, respectively, that make the expected return from time 0 to time 0.5 on the zero maturing at time 30 the same as that on the zero maturing at time 0.5?

\[
[p \times 0.49 + (1 - p) \times 0.51]/0.50 = 1/0.99 \Rightarrow p = 0.24747, (1 - p) = 0.75252
\]

(d) (4 points) Price the call using these risk-neutral probabilities.

\[(0 \times 0.24747 + 1 \times 0.75252) \times 0.99 = 0.745\]