Problem Set 1: Zeroes, Coupon Bonds, Yield, Forwards

1) $100 par of a 0.5-year 8%-coupon bond has a price of $101.
   $100 par of a 1-year 10%-coupon bond has a price of $104.
   a) What is the price of $1 par of a 0.5-year zero?
   b) What is the price of $1 par of a 1-year zero?
   c) Suppose $100 of a 1-year 6%-coupon bond has a price of $99. Is there an arbitrage opportunity? If so, how?
   d) What is the 1-year par rate, i.e., what coupon rate would make the price of a 1-year coupon bond equal to par?
   e) What is the 0.5-year zero rate?
   f) What is the 1-year zero rate?
   g) Considering the shape of the yield curve, should the yield on the 1-year 10%-coupon bond be higher or lower than the 1-year par rate?

2) Suppose the yield curve is downward-sloping and there is no arbitrage. Two ordinary fixed coupon bonds, bond A and bond B, have the same maturity, but bond A has a higher yield. Which bond has the higher coupon? Just give your answer, do not give an explanation.

3) Suppose that at time 0 you buy a 6%-coupon 30-year bond priced at par, and at time 0.5 you sell this bond at a yield of 8%.
   a) What is your time 0.5 payoff per $1 of initial investment?
   b) What is the rate of return on your investment (annualized, with semi-annual compounding)?

4) Consider the following information about three fixed income securities. Two, security A and security B, are ordinary fixed coupon bonds. The third, security C, is a forward contract.

<table>
<thead>
<tr>
<th>Bonds</th>
<th>Maturity (yrs)</th>
<th>Coupon rate (%)</th>
<th>Price (per $100 par)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.5</td>
<td>6</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td>1.0</td>
<td>8</td>
<td>101</td>
</tr>
<tr>
<td>C: Forward contract: The underlying asset is $100 par of a zero maturing at time 1. The forward price is $97, to be paid at time 0.5. The contract costs zero today.</td>
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Do these three securities present an arbitrage opportunity? If so, describe a trade that makes arbitrage profits.
5) The current price of $1 par of a zero maturing at time 2 is $0.90. In addition, you can contract today to purchase, at time 2, $1 par of a zero maturing at time 3. The forward price to pay at time 2 for this zero maturing at time 3 is $0.94. It costs nothing today to enter into this forward contract.

a) What is the forward rate from time 2 to time 3?

b) Describe transactions in the 2-year zero and the forward contract that together synthesize a spot purchase of $1 par of the zero maturing at time 3. (With the spot purchase, you pay for the zero today, rather than in 2 years.)

c) Assuming there are no arbitrage opportunities, what is the current spot price (the price to pay today) for $1 par of the zero maturing at time 3?

6) a) At time 0, Investor A enters into a forward contract, at no cost, to buy, at time 2, $100,000 par of a zero maturing at time 3. The forward price this investor locks in to pay at time 2 is $92,000. What forward rate does this investor lock in at time 0, through this forward contract, for lending from time 2 to time 3?

b) At time 1, the spot price of $1 par of a zero maturing at time 2 is 0.96 and the spot price of $1 par of a zero maturing at time 3 is 0.93.

   i) At time 1, what is the forward price an investor could lock in to pay, at time 2, for $100,000 par of a zero maturing at time 3?

   ii) What is the value, at time 1, of Investor A’s position in the forward contract from part (a)?