Problem Set 5: Dynamic Trading Strategies and Interest Rate Modeling

1) a) Construct an interest rate tree according to the method presented in class with step size $h = 0.5$, volatility $\sigma = 0.2$, drift $m = 0.1$, and initial 0.5-year rate $r = 8\%$. The tree should give short rates out to time 0.5.
   b) What is the time 0 price of $1$ par of a zero maturing at time 0.5?
   c) What is the time 0 price of $1$ par of a zero maturing at time 1?

2) Show that a term structure model with flat yield curves and parallel yield curve shifts contains arbitrage opportunities as follows. Consider the following model: At time 0 the yield curve is flat at 8%. At time 0.5 there are two possible states, the yield curve is either flat at 10% or flat at 6%. This is depicted below:

   Time 0
   Yield curve flat at 8%

   Time 0.5
   Yield curve flat at 10%
   Yield curve flat at 6%

   a) Draw a tree containing the time 0 price and the two possible time 0.5 payoffs (or prices) for three different assets: a zero maturing at time 0.5, a zero maturing at time 10, and a zero maturing at time 30, $100$ par value each.
   b) Consider a portfolio of 0.5- and 30-year zeroes that replicates $100$ par of the 10-year zero, or in other words, that has the same value as $100$ par of the 10-year zero in both states at time 0.5.
      (i) Determine the par amounts of the 0.5- and 30-year zeroes in this replicating portfolio.
      (ii) What is the cost of this replicating portfolio at time 0?
   c) Describe an arbitrage opportunity in this market and indicate the arbitrage profit.
   d) Suppose we change the yield on the 10-year zero to eliminate the arbitrage opportunity. What would the yield have to be?

3) Prices of zeroes at time 0 and time 0.5 are listed in order of maturity in the tree below:

   Time 0
   0.9713
   0.9433
   0.9146

   Time 0.5
   0.9678
   0.9745
   0.9483
Consider a forward contract to buy, at time 1, $1 par of the zero maturing at time 1.5.
  a) What is the forward price, agreed upon at time 0, to pay at time 1, for $1 par of the zero maturing at time 1.5, that makes this contract worth zero at time 0?
  b) What is the interest rate delta of this forward contract?