Trading Costs and Returns for US Equities: Estimating Effective Costs from Daily Data

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This paper is about …

- Trading cost estimates for US stocks
  - formed from daily CRSP data.

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- Trading cost estimates for US stocks
  - formed from daily CRSP data.
- Expected return specifications with the cost estimates
  - Characteristic
  - Risk factor
  - 1926 to present

Asset pricing / liquidity: Where does the paper fit?

- A. Studies using microstructure measures of trading cost.
  - Stoll and Whalley (1983)
    Amihud and Mendelson (1986)
    Brennan and Subrahanyam (1986)
    Korajczk and Sadka (2006)
  - Trading cost estimation is direct, but time span of sample limited.
- B. Studies using liquidity measures based on daily returns and/or volumes
  - Amihud (2002)
    Pastor and Stambaugh (2003)
    Acharya and Pedersen (2005)
  - Long time series, but liquidity measures are indirect.
- This paper
  - long time series, daily data (like B)
  - cost measure is validated against microstructure data (like A)

Key results: Estimation and Validation of cost measure

- Effective cost (of trading) U.S. equity
  - Approximately one-half the bid-ask spread.
  - Usually estimated with TAQ (trade and quote) data.
  - Here, estimated from daily CRSP data.
  - The CRSP/Gibbs estimates are very close to TAQ estimates
  - → The daily (Gibbs) estimates have strong validity.
- CRSP/Gibbs methodology extended to allow for common variation in effective costs.
  - CRSP-based factor is highly correlated with the TAQ-based factor.

Key results: The historical perspective

- Extend the Gibbs estimates back to 1926 (for NYSE stocks).
  - Trading costs have sharp peak in the early 1930’s.
  - Cross-sectional variation in cost larger than time-series variation.
  - Effective costs (levels and variation) much larger in the lower capitalization groups

Key results: Asset pricing

- The common effective cost *factor* adds little explanatory power to return regressions.
- The expected return is not affected by:
  - A stock's return beta on the common factor of effective cost.
  - The sensitivity of a stock's effective cost to the common factor of effective cost.
- The level of effective cost *as a characteristic* is a positive determinant of expected returns.
  - But this effect is strongly concentrated in January.

Organization of talk

- The effective cost and its estimation (no time variation)
  - Performance of the CRSP/Gibbs level estimates
- CRSP/Gibbs analysis of variation in effective cost.
  - Performance of CRSP/Gibbs variation estimates
- Historical perspective on CRSP/Gibbs estimates
- Asset pricing tests
- Discussion

Trading/liquidity costs: working definitions

- Trading cost
  - Agent's perceived cost of executing an order (completing an intended trade)
- Liquidity cost
  - Economic distortions resulting from trading activity.

Effective cost of trading

- m_k = log bid/ask midpoint prior to the kth trade
  p_k = log trade price
- (Log) effective cost:

$$c_k = \begin{cases} p_k - m_k, & \text{for a buy order} \\ m_k - p_k, & \text{for a sell order} \end{cases}$$

- SEC Rule 605: markets report their effective costs.
- When we don't know order direction $c_k \approx |p_k - m_k|$
- Shortcomings
  - No dependence on order size
  - No differentiation between permanent (informational) and transient (non-informational) effects of order.

The effective cost in the Roll (1984) model

- m_k is also the "efficient price" prior to the trade.

$$m_k = m_{k-1} + u_k$$

- Trade price is

$$p_k = m_k + c\, q_k$$

- q_k is the trade direction indicator (+1 = "buy"; –1 = "sell")
- Implied price changes: $\Delta p_k = c\, \Delta q_k + u_k$
- Roll (1984) and most subsequent implementations apply the model to *daily* data:

$$\Delta p_t = c\, \Delta q_t + u_t$$

where p_t is the closing price for day t.

The moment estimate of c

$$\Delta p_t = c\Delta q_t + u_t \quad \Rightarrow \quad Cov\left(\Delta p_t, \Delta p_{t-1}\right) = -c^2$$

$$c^{Moment} = \begin{cases} \sqrt{-Cov\left(\Delta p_t, \Delta p_{t-1}\right)}, \text{ if } Cov\left(\Delta p_t, \Delta p_{t-1}\right) < 0 \\ 0, \text{ otherwise} \end{cases}$$

- Often satisfactory, but:
  - Infeasible when return autocovariance is positive
  - Many sample paths include quote midpoints interspersed with trade prices.
- Alternative: Bayesian estimation via Gibbs sampler.

Estimating the effective cost with the Gibbs sampler

- Model: $\Delta p_t = c\,\Delta q_t + u_t$
- Bayesian approach
  - Observed data: $p_1,\ldots, p_T$
  - Unobserved data:
    - Parameters, c and σ_u
    - Latent data $q = \{q_1,\ldots,q_T\}$ and $m = \{m_1,\ldots,m_T\}$
- To complete the framework, need:
  - Distributional assumptions on u_t: $u_t \sim N\left(0,\sigma_u^2\right)$
  - Priors (half-normal for c; inverted gamma for σ_u)
- Posterior is $f\,(c,\ \sigma_u,\ q,\ m|\ p_1,\ldots,\ p_T)$

Components of the Gibbs sampler

- Basic specification is:

$$\Delta p_t = c\,\Delta q_t + u_t$$

- Given the q_t this is a normal Bayesian regression model
  - Apply standard results
- Nonstandard part of this model:
  - Given c and σ_u, construct posterior for $q_1, \ldots, q_T$.
- Gibbs sampler constructs full posterior by iteratively simulating from full conditional distributions for c, σ_u, and the q_t.

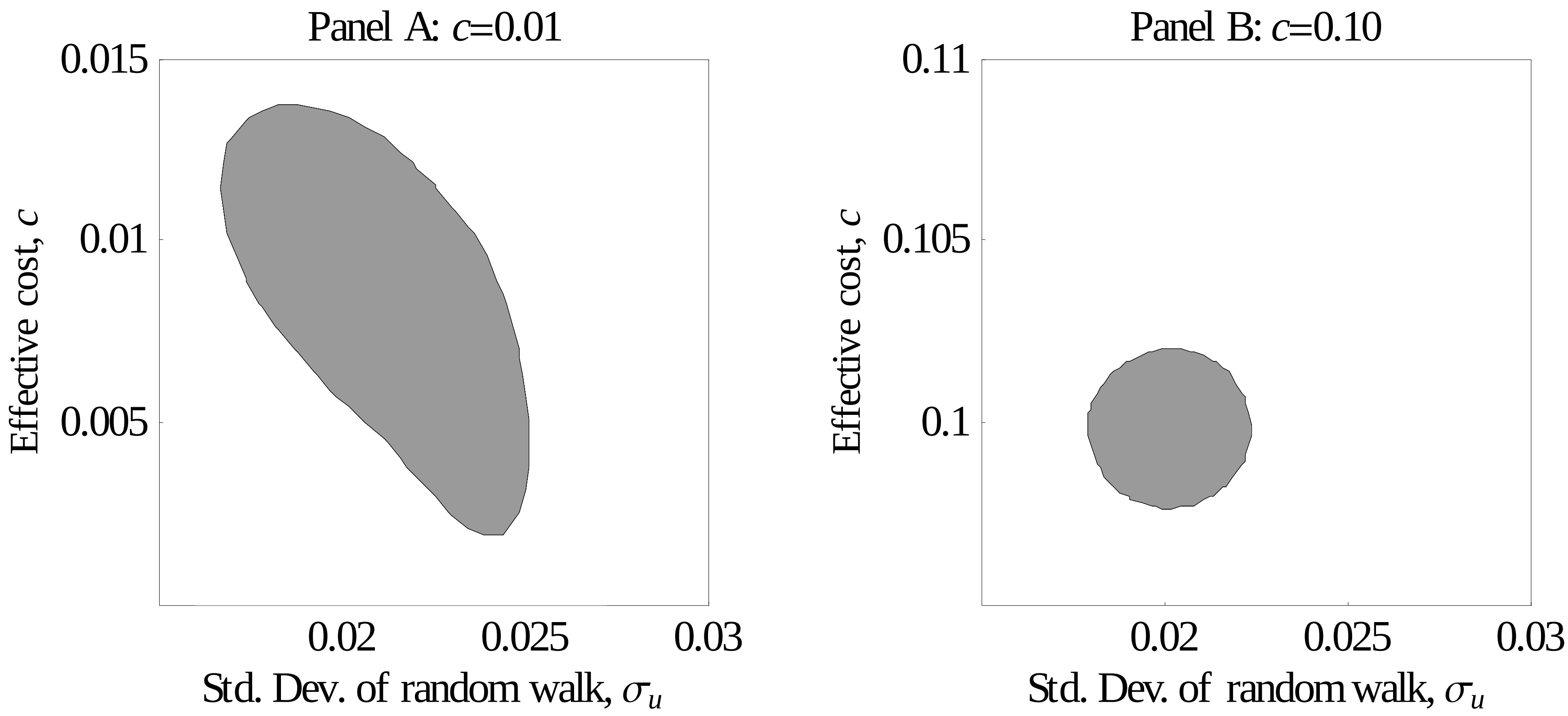
Specification issues

- Normality of u_t?
- Serial independence of q_t?
- Discreteness?
- …
- The method will be validated by correlating estimates with those independently derived.

The intuition: What the estimation procedure does (and when it will encounter difficulties)

- A sample price path is composed of:
  - Permanent (random-walk) innovations
  - Transient *c*-related components (reversals, bid-ask bounce)
- When we look at a price path, we try to resolve the two.
- Resolution will be …
  - clean when reversals are distinct: $c >> \sigma_u$
  - Murky when reversals are lost in the random walk innovations: $c << \sigma_u$
- Next page: simulated posteriors

Figure 1. Posteriors for simulated price paths with σ_u = 0.02



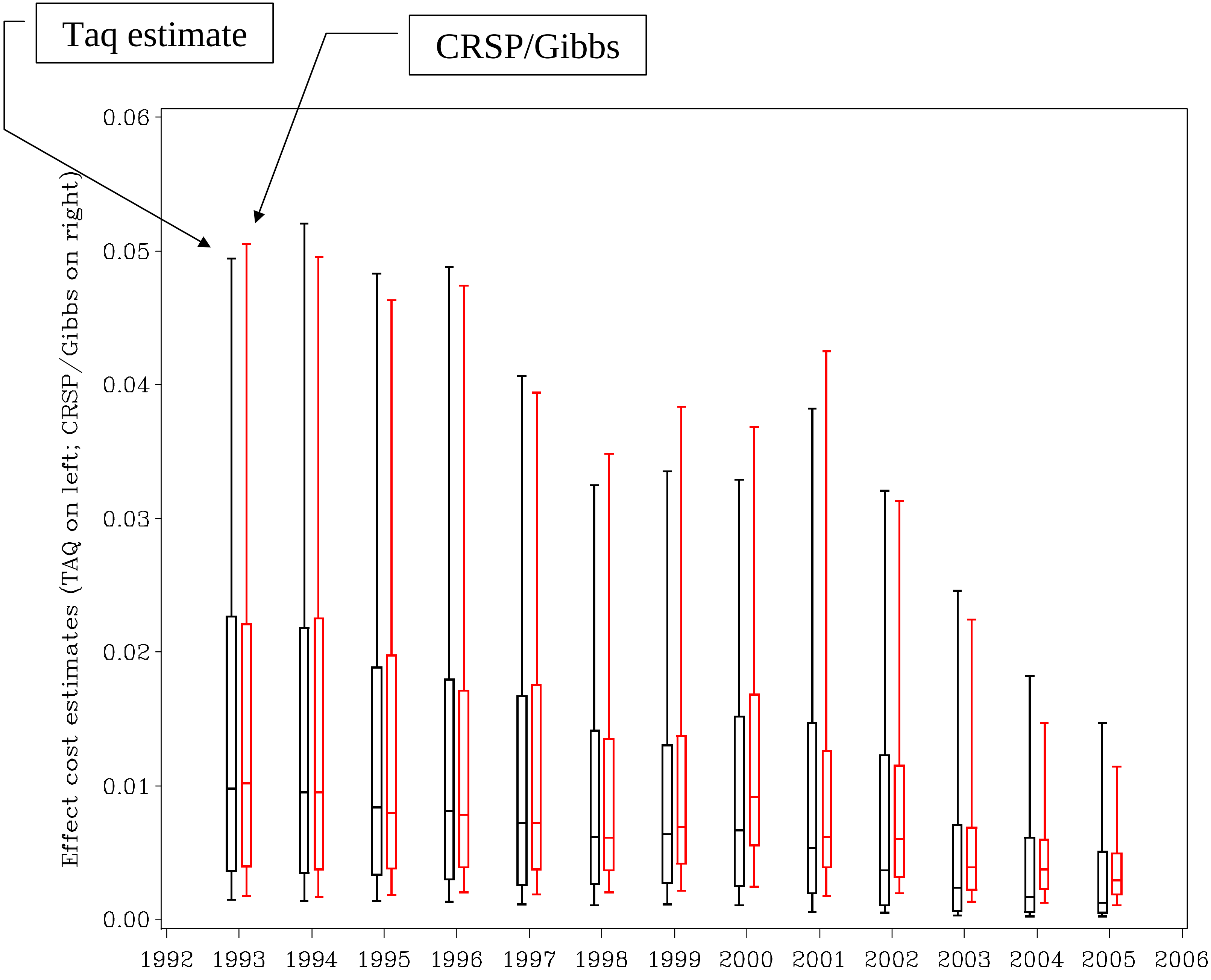
Practical implications: Two situations

- Ticker symbol NEWE, January, 1990.
  - Bid=3.625 Ask=4.125
    $c \approx$ (Relative spread)/2 = 0.064
  - Annual log volatility = 0.40 (?) → daily $\sigma_u = 0.025$
  - Clean resolution
- Ticker symbol MSFT, January, 2005
  - Closing price ≈ \$28, spread ≈ \$0.01 → $c \approx 0.0002$
  - Poor resolution

Validation in the comparison sample

- Comparison sample: In each year from 1993 to 2005
  - Randomly select 150 Nasdaq firms + 150 NYSE/Amex firms.
  - Approx 300 x 13 = 3,900 firm-years
- Compute TAQ estimates of effective cost
- Compute CRSP/Gibbs estimate using:
  - Basic market-adjusted model: $\Delta p_t = c\,\Delta q_t + \beta\, r_{mt} + u_t$

- Next: Figure 2. Effective costs in comparison sample


Taq estimate
CRSP/Gibbs
Effect cost estimates (TAQ on left; CRSP/Gibbs on right)
0.00
0.01
0.02
0.03
0.04
0.05
0.06
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006

Other liquidity measures

- TAQ-based measure of price impact
- Daily
  - (Amihud) Illiquidity ratio I = avg of $|r_t|$/Vol
  - Proportion of zero returns (Lesmond et al.)

Table 2. Correlations [Extract] ***(Estimates formed annually)***

	Eff. cost (TAQ)	*Eff. cost, Gibbs*	*Eff. cost, Moment*	*Prop. zero returns*	*Price Impact (TAQ)*	*Illiquidity*
Pearson correlation						
Eff. cost (TAQ)	1.000	0.965	0.878	0.611	0.513	0.612
Eff. cost, Gibbs	0.965	1.000	0.917	0.579	0.450	0.589
Eff. cost, Moment	0.878	0.917	1.000	0.451	0.378	0.504
Prop. zero returns	0.611	0.579	0.451	1.000	0.311	0.252
Price impact (TAQ)	0.513	0.450	0.378	0.311	1.000	0.668
Illiquidity	0.612	0.589	0.504	0.252	0.668	1.000

- Liquidity measures are generally positively correlated (Korajczyk and Sadka)

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- Liquidity measures are generally positively correlated (Korajczyk and Sadka)
- Illiquidity good proxy for price impact
- Gibbs estimate dominates as proxy for effective cost

Figure 3. Effective cost, NYSE/Amex 1926-2005

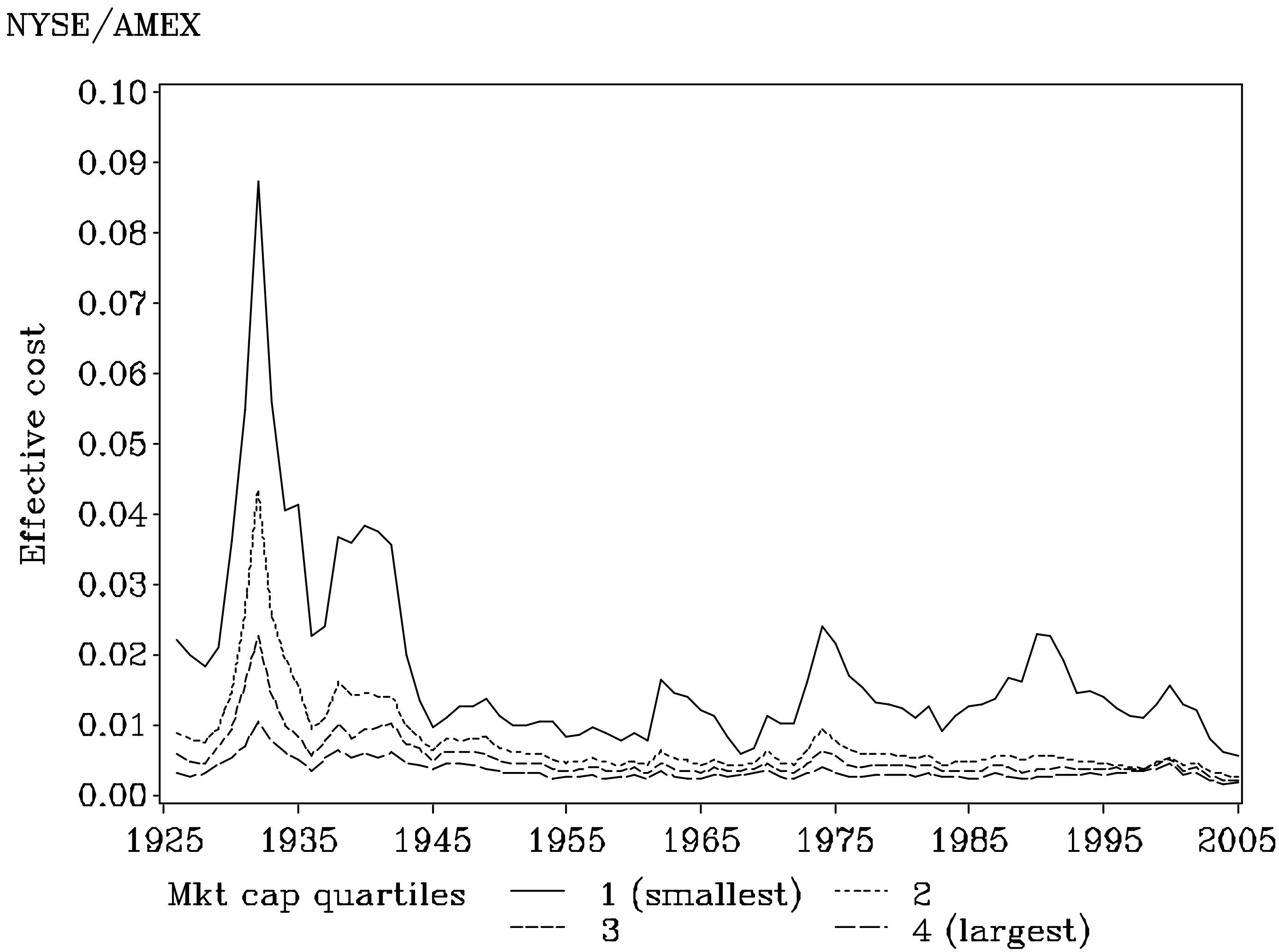
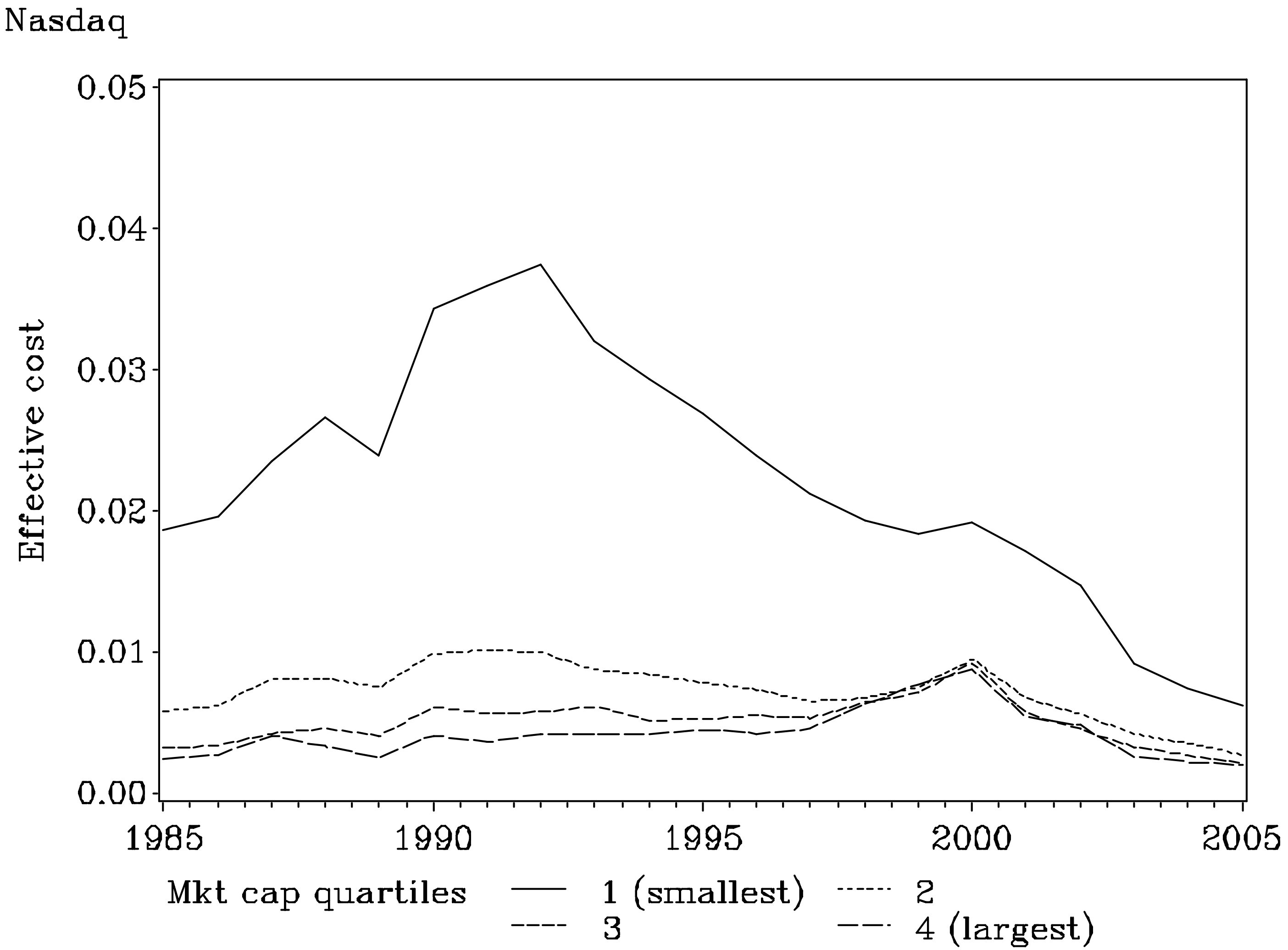


Figure 3. Effective cost, Nasdaq, 1985-2005



Variation in liquidity

- Is there cross-sectional variation (across firms, securities)
  - Drivers: firm size, industry, regulation, trading mechanism
- Is there time-series variation (for individual firms)
  - Drivers: corporate-specific news announcements and other value-relevant events (splits, earnings surprises, etc.)
- Is there (cross-firm) commonality in liquidity variation?
  - Drivers: common-factor macroeconomic value determinants.

Usual approach

- ❑ λ_t = unobserved liquidity measure on day t
- ❑ Model: $\lambda_t = \gamma x_t$ (conditioning variables x_t)
- ❑ Estimate a monthly proxy based on daily data.

Illiquidity measure (Amihud) $$I_m = \overline{\left(\frac{|r_d|}{Vol_d} \right)}$$

- ❑ Average over all days d in month m
- ❑ Relate to conditioning variables at monthly frequency

$$I_m = \gamma \overline{\left(x_t \right)}_m + \varepsilon_m$$

- ❑ Estimate in a two-step procedure

One-step Bayesian estimation of model with variation in *c*

- For stock i:
  - Efficient price: $m_{i,t} = m_{i,t-1} + u_{i,t}$
  - Trade price: $p_{i,t} = m_{i,t} + c_{i,t}\, q_{i,t}$
- Price changes: $\Delta p_{i,t} = c_{i,t}\, q_{i,t} - c_{i,t-1}\, q_{i,t-1} + u_{i,t}$
- Model for variation in costs: $c_{i,t} = \gamma_i\, x_{i,t}$
- New model:

$$\Delta p_{i,t} = \gamma_i (x_{i,t}\, q_{i,t} - x_{i,t-1}\, q_{i,t-1}) + u_{i,t}$$

- The γ parameters are regression coefficients (easy)
- Limitation: *All* variation in c is attributed to x.

Latent Common Factor Model

- Model: $c_{i,t} = \gamma_{i,0} + \gamma_{i,1} z_t$
  - Where z_t is an unobserved factor common to all $i=1,\ldots n$ securities.
- Price change equation becomes:
  - $\Delta p_{i,t} = (\gamma_{i,0} + \gamma_{i,1} z_t)\, q_{i,t} - (\gamma_{i,0} + \gamma_{i,1} z_{t-1})\, q_{i,t-1} + u_{i,t}$
- Rearrange as:
  - $\Delta p_{i,t} - \gamma_{i,0}(q_{i,t} - q_{i,t-1}) = z_t\,(\gamma_{i,1}\, q_{i,t}) - z_{t-1}(\gamma_{i,1}\, q_{i,t-1}) + u_{i,t}$
- The z_t are coefficients in a panel regression (easy)

The Latent Common Factor model in the comparison sample

- Estimation output:
  - z_t common effective cost factor (daily)
  - For each firm i: γ_{0i} (intercept), γ_{1i} (loading)

Analysis of estimated common factor

- Compare z_t to $\overline{c}_t^{TAQ}$ (= daily TAQ-based effective cost, averaged across firms)
- Correlation is 0.447
- Using monthly averages, correlation is 0.670

Analysis of factor coefficients (loadings)

- Estimate TAQ common factor coefficient as:

$$c_{it} = \gamma_{0i}^{TAQ} + \gamma_{1i}^{TAQ}\,\overline{c}_t^{TAQ} + e_{it}$$

$$Corr\left(\underbrace{\gamma_{i1}}_{CRSP/Gibbs},\underbrace{\overline{\gamma}_{i1}^{TAQ}}_{TAQ}\right) = 0.328$$

Asset Pricing

- CRSP monthly returns file 1926-2005
- Fama-French factors
- CRSP/Gibbs liquidity estimates

Estimation

- Form portfolios
- Conceptually a two-step procedure
  - Estimate factor loadings: $R_t = a + b\, f_t + u_t$
    - Factors: FF + innovation in liquidity factor z_t
  - Estimate expected returns: $ER_t = \beta\,\lambda + Z_t\,\delta$.
    - Z_t are characteristics: c, γ_{0i}, γ_{1i} , seasonals
- Estimation approach GMM (Cochrane) is one-step
  - Point estimates of β, λ and δ = OLS values
  - SE's corrected for estimation error in β's.

Portfolio construction

- Effective cost / beta
  - 5 x 5 independent rankings on c and β
  - Gibbs estimates, basic market adjusted model
- Intercept / loading
  - 5 x 5 independent rankings on γ_{0i}, γ_{1i}
  - Gibbs estimates, latent common factor model

Results: cost as a risk factor

- Estimates of return-generating process: $R_t = a + b\, f_t + u_t$
  - f_t: *FF* factors + z_t (liquidity factor innovation)
- z_t generally insignificant
  - cost variation doesn't help explain returns
- Estimates of expected returns: $ER_t = \beta\,\lambda + Z_t\,\delta$.
- λ coefficient corresponding to liqudity factor insignificant
  - Findings do not support other studies.

Results: cost as a characteristic

- Next slides report δs from:

$$ER_t = \beta \lambda + Z_t \delta$$

- (Extracts from Table 8.)

Table 8, Panel A (c/β portfolios, NYSE, 1927-2005) *Extract*

Characteristic	Coefficient			
c_{it}	1.18642			
	(1.42)			
γ_{0i}		1.21408	1.21443	
		(1.46)	(1.46)	
γ_{1i}		0.57275	0.57190	
		(0.63)	(0.62)	
d_t^{Jan}				0.01279
				(1.27)
$d_t^{Jan} c_{it}$				3.94894
				(2.67)
$\left(1 - d_t^{Jan}\right) c_{it}$				0.93529
				(1.05)

Table 8, Panel B (*c*/β portfolios, Amex, 1962-2005) *Extract*

Characteristic	Coefficient			
c_{it}	1.56484			
	(2.87)			
γ_{0i}		1.33776	1.36922	
		(2.76)	(2.82)	
γ_{1i}		1.08496	1.01960	
		(1.73)	(1.59)	
d_t^{Jan}				0.01279
				(1.19)
$d_t^{Jan} c_{it}$				6.20826
				(6.59)
$\left(1 - d_t^{Jan}\right) c_{it}$				1.14271
				(2.66)

Table 8, Panel B (c/β portfolios, Nasdaq, 1985-2005) *Extract*

Characteristic	Coefficient			
c_{it}	0.29523			
	(1.10)			
γ_{0i}		0.19558	0.19568	
		(0.73)	(0.73)	
γ_{1i}		1.18753	1.18551	
		(2.10)	(1.96)	
d_t^{Jan}				0.01699
				(0.95)
$d_t^{Jan} c_{it}$				2.39502
				(5.20)
$\left(1 - d_t^{Jan}\right) c_{it}$				0.10434
				(0.43)

Seasonality in trading cost effects
Coefficients in expected return regressions
(Extract, Table 8)

	NYSE	Amex	Nasdaq
	1927-2005	1962-2005	1985-2005
$d_t^{Jan} c_{it}$	3.94894	6.20826	2.39502
	(2.67)	(6.59)	(5.20)
$\left(1 - d_t^{Jan}\right) c_{it}$	0.93529	1.14271	0.10434
	(1.05)	(2.66)	(0.43)

- Trading cost effects concentrated in January
- The effects appear too large.

Other studies of trading cost as a characteristic

- Positive
  - Stoll & Whalley (1983)
  - Amihud & Mendelson (1986)
  - Eleswarapu (1997)
  - Brennan and Subrahmanyam (1996)
- Negative
  - Chen and Kan (1989)
  - Chalmers and Kadlec (1998)
- Seasonality: Eleswarapu & Reinganum (1993)

Studies that have found liquidity risk *does* matter

- Pastor and Stambaugh (2003)
  - Reversal measure of liquidity, 1966-1999
- Acharya and Pedersen (2005),
  - Amihud illiquidity measure, 1964-1999
- Korajczk and Sadka (2006)
  - Principal component of a set of TAQ based measures, 1983-2000

Why the present lack of support for liquidity risk?

- Quality of proxy
- Sample (time span)
- Methodology
- A fundamental difference in liquidity measures.
  - Other studies: Measures based on prices/returns and trading volume
  - This study: The effective cost uses prices *only*.
- Present negative results may be supporting the importance of volume.

What do we know about trading volume?

- Statistically: Poorly behaved
  - Highly nonstationary
  - Many extreme values
  - Strong commonality
- Economically: open to variety of interpretations
- Most directly a measure of trading activity (not cost)
- Relation to trading cost ambiguous
  - Holding constant the demand for trading …
    - High volume → lower trading cost
  - Holding constant supply of market-making services …
    - High volume → higher trading cost

Conclusion

- For expected returns …
  - Weak evidence of trading cost as a characteristic
  - No evidence for trading cost variation as a risk factor.
- Strengths of present study
  - Long-term
  - Trading cost estimates subjected to extensive validity checks.
- Negative results indirectly suggest the importance of volume