Foundations of Finance, Fall, 2008

Prof. Joel Hasbrouck

Problem set 2 M DUE IN CLASS Monday OCTOBER 27

Although you may discuss these problems with others, you should calculate everything yourself (just to make sure that you can get your calculator to give the same answers that everyone else is getting). Your solutions must be written up by yourself: you must turn in the original. If you won't be in class on the day the problem set is due, you can fax me a pdf, and turn in the original at the next meeting.

For the following problems, assume that the risk and return on two hedge funds (Semper and Apex) are given by:

Asset	Er (*not* ER=Er–r_f)	σ
Semper Fund	8%	20%
Apex Fund	12%	18%

The correlation between Semper and Apex is $\rho = -0.2$.

1. Brianne and Arkady are both risk-averse, but Arkady is more risk-averse. If we restrict both of them to choosing between putting 100% of their wealth in Semper or 100% of their wealth in Apex, can we determine which choices they will make?

Apex has higher expected return *and* lower risk than Semper: They would both choose Apex.

We now allow them to hold portfolio combinations of Semper and Apex.

2. If Arkady wants an expected return of $Er_P = 9\%$, what are his portfolio weights in Semper and Apex? What is the risk (σ_P) of the portfolio?

   Let "S" denote Semper; "A" denotes Apex.

   Then $Er_P = w_S \times Er_S + (1 - w_S) \times Er_{Apex}$:

   $9 = w_S \times 8 + (1 - w_S) \times 12 \rightarrow w_S = 0.75$ (and $w_A = 0.25$).

   For the risk, $\sigma_P = [w_S^2\sigma_S^2 + w_A^2\sigma_A^2 + 2w_S w_A \sigma_S \sigma_A \rho]^{1/2} = 14.8\%$

3. If Brianne wants an expected return of $Er_P = 11\%$, what are her portfolio weights in Semper and Apex? What is the risk (σ_P) of the portfolio?

Using the same computations as in the last question, we get $w_S=0.25$ (and $w_A=0.75$) and $\sigma_P = 13.4\%$.

Note that Brianne's portfolio has lower risk and higher expected return than Arkady's. Although Arkady set out looking for a portfolio with $Er_P=9\%$, he would prefer Brianne's portfolio
(if he were aware of it)

We now allow them to hold portfolio combinations involving Semper, Apex and a T-Bill (a risk free security) paying $r_f = 5\%$.

4. What are the weights of the optimal (tangency) portfolio? (For this question, you will need the equations in section 7.3, or you can use the "+Riskfree" worksheet in Two_security_portfolio.xlsx.)

Using the equation in the text for the weights on the optimal tangency portfolio,

w_A = [(12–5)x400 – (8-5)x20x18x(–0.2)] /
[(12–5)x400 + (8–5)x324 – (12–5 + 8–5]x20x18x(–0.2)] = 3,016 / 4,492 = 0.67

and $w_S = 0.33$. Then, $Er_P = w_S \times Er_S + (1–w_S) \times Er_{Apex}) = 10.7\%$,
and $\sigma_P = [w_S^2\sigma_S^2 + w_A^2\sigma_A^2 + 2w_Sw_A\sigma_S\sigma_A\rho]^{1/2} = 12.5\%$

5. Arkady still wants an expected return for his combined portfolio of $Er_C = 9\%$. What is the risk of this portfolio (σ_C)? If he has $100,000 to invest, how much goes into T-bills, Apex and Semper?

The CAL is $Er_C = r_f + [(Er_P – r_f)/\sigma_P] \times \sigma_C$.

So for Arkady, $9 = 5 + [(10.7–5)/12.5 \times \sigma_C \rightarrow 4 = 0.46\ \sigma_C \rightarrow \sigma_C = 8.7\%$. We get y from $Er_C = r_f + y(Er_P – r_f) \rightarrow 9 = 5 + y(10.7–5) \rightarrow y=0.70$.

With 100,000, the allocations are 30,000 into r_f; 70,000 into P, of which 0.67 x 70,000 = 46,900 goes into Apex and 23,100 goes into Semper.

	A	B	C	D
1	**Two-Security Portfolios: +RiskFree Worksheet**			
2				
3	Asset Allocation Analysis: Risk and Return			
4		Er	σ	Corr (ρ)
5	**Semper**	**8.00%**	**20.00%**	**-0.2**
6	**Apex**	**12.00%**	**18.00%**	-0.007200
7	r_f	**5.00%**	0	
8				
9				
10	Optimal Risky Port		**Short Sales**	**No Short**
11			**Allowed**	**Sales**
12		w(Semper)	0.32858	0.32858
13		w(Apex)	0.67142	0.67142
14		ER_P	10.6857%	10.6857%
15		σ_P	12.5490%	12.5490%
16				
17	Reward to variability		0.45308	0.45308
18				
19				
20				
21				
22