1. For the following problems, assume that:

| Asset | Er (*not* ER=Er–$r_f$) | $\sigma$ |
|---|---|---|
| Stock Fund | 12% | 20% |
| Gold | 5% | 25% |
| T-Bills ($r_f$) | 4% | |

The correlation between the stock fund and gold is $\rho$ = –0.2. Midas advisors' recommended portfolio ("P") is 50% in stock and 50% gold. What are the expected return and standard deviation of P?

$Er_P$ = 0.5 x 5 + 0.5 x 12 = 8.5%

$\sigma_P = [(0.5 \times 20)^2 + (0.5 \times 25)^2 + 2 \times 0.5 \times 0.5 \times 20 \times 25 \times (-0.2)]^{1/2}$

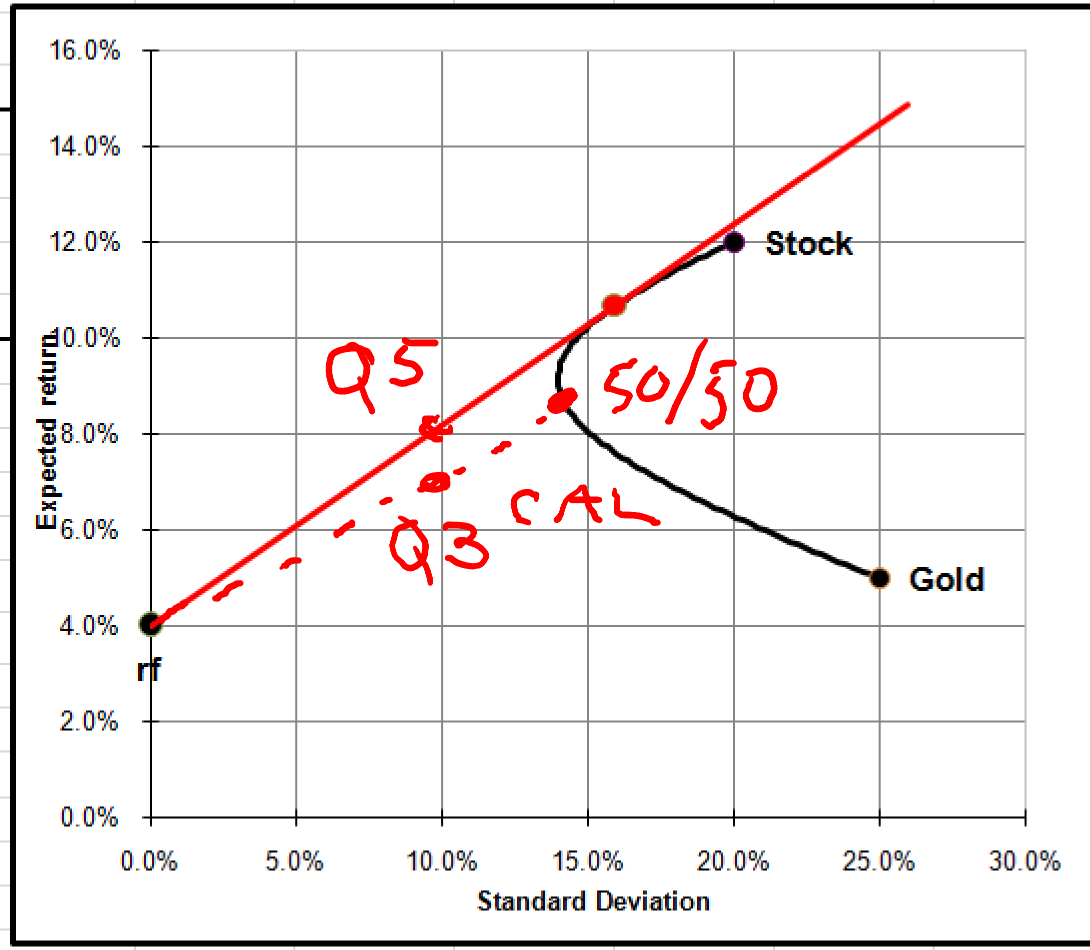
$= [100 + 156.25 - 50]^{1/2} = 14.36\%$

**Two-Security Portfolios: +RiskFree Worksheet**

Asset Allocation Analysis: Risk and Return

| | Er | σ | Corr (ρ) |
|---|---|---|---|
| **Stock** | **12.00%** | **20.00%** | **-0.2** |
| **Gold** | **5.00%** | **25.00%** | -0.010000 |
| $r_f$ | **4.00%** | 0 | |

| Optimal Risky Port | | **Short Sales Allowed** | **No Short Sales** |
|---|---|---|---|
| | w(Stock) | 0.80952 | 0.80952 |
| | w(Gold) | 0.19048 | 0.19048 |
| | $ER_P$ | 10.6667% | 10.6667% |
| | $\sigma_P$ | 15.9364% | 15.9364% |
| | | | |
| Reward to variability | | 0.41833 | 0.41833 |



2. Beth forms a portfolio (“C”) consisting of 25% in $r_f$ and 75% in “P” (as defined above). What are the risk and return of her portfolio?

$y=0.75$, so $Er_C = 0.25 \times 4 + .75 \times 8.5 = 7.375\%$; $\sigma_C = 0.75 \times 14.36 =$ 10.77

$r_f$ $Er_p$

3. Andrei has determined that the maximum risk he can accept in his combined portfolio ("C") is $\sigma_C$ = 10%. What is the expected return on this portfolio?

CAL $Er_C = r_f + [(Er_P - r_f)/\sigma_P] \times \sigma_C = 4 + 0.313 \times 10 = 7.13\%$ NOTE: Andrei's portfolio has lower expected return than Beth's, ~~but higher risk. Although Andrei~~ *could* invest in this ~~portfolio, he'd be better off in Beth's~~.

4. With the given parameters, Midas' recommendation is not the optimal tangency portfolio. What are the weights (proportions of gold and stock) for the optimal tangency portfolio? What are the expected return and standard deviation of this portfolio? (For this question, you will need the equations in section 7.3, or you can use the spreadsheet Two_security_portfolio.xlsx posted to the web site.)

Using equation (7.13) on p. 207 ("D"="Gold", "E"="Stock"],

$w_D$ = [(5–4)x400 – (12-4)x(20 x 25 x –0.2)] /

[(5–4)x400 + (12–4)x625 – [(5–4)+(12–4)](20 x 25 x –0.2)]

= [400 + 800] / [400 + 5,000 + 900] = 1,200 / 6,300 = 0.190 → wE = 0.810

$Er_P$ = 0.19 x 5 + 0.81 x 12 = 10.67%;

$\sigma_P = (.19^2 25^2 + .81^2 20^2 + 2x.19x.81x20x25x(-.2))^{1/2}$

$= (22.5625 + 262.4400 - 30.7800)^{1/2} = 254.2225^{1/2} = 15.94\%$

5. Now suppose that Andrei uses the optimal portfolio to construct his combined portfolio. If the maximum risk that he can tolerate is still 10%, what is the expected return on his combined portfolio?

Using the new values for $Er_P$ and $\sigma_P$,

$Er_C = r_f + [(Er_P - r_f)/\sigma_P] \times \sigma_C = 4 + 0.4184 \times 10 = 8.184\%$, i.e. an improvement of about one percent relative to the solution to Q3.