CHAPTER 8
NET PRESENT VALUE AND OTHER INVESTMENT CRITERIA

Answers to Concepts Review and Critical Thinking Questions

1. A payback period less than the project's life means that the NPV is positive for a zero discount rate, but nothing more definitive can be said. For discount rates greater than zero, the payback period will still be less than the project's life, but the NPV may be positive, zero, or negative, depending on whether the discount rate is less than, equal to, or greater than the IRR.

2. If a project has a positive NPV for a certain discount rate, then it will also have a positive NPV for a zero discount rate; thus the payback period must be less than the project life. If NPV is positive, then the present value of future cash inflows is greater than the initial investment cost; thus PI must be greater than 1. If NPV is positive for a certain discount rate R, then it will be zero for some larger discount rate R*; thus the IRR must be greater than the required return.

3. *a.* Payback period is simply the break-even point of a series of cash flows. To actually compute the payback period, it is assumed that any cash flow occurring during a given period is realized continuously throughout the period, and not at a single point in time. The payback is then the point in time for the series of cash flows when the initial cash outlays are fully recovered. Given some predetermined cutoff for the payback period, the decision rule is to accept projects that payback before this cutoff, and reject projects that take longer to payback.

b. The worst problem associated with payback period is that it ignores the time value of money. In addition, the selection of a hurdle point for payback period is an arbitrary exercise that lacks any steadfast rule or method. The payback period is biased towards short-term projects; it fully ignores any cash flows that occur after the cutoff point.

c. Despite its shortcomings, payback is often used because (1) the analysis is straightforward and simple and (2) accounting numbers and estimates are readily available. Materiality consider-ations often warrant a payback analysis as sufficient; maintenance projects are another example where the detailed analysis of other methods is often not needed. Since payback is biased towards liquidity, it may be a useful and appropriate analysis method for short-term projects where cash management is most important.

4. *a.* The average accounting return is interpreted as an average measure of the accounting perfor-mance of a project over time, computed as some average profit measure due to the project divided by some average balance sheet value for the project. This text computes AAR as average net income with respect to average (total) book value. Given some predetermined cutoff for AAR, the decision rule is to accept projects with an AAR in excess of the target measure, and reject all other projects.

b. AAR is not a measure of cash flows and market value, but a measure of financial statement accounts that often bear little semblance to the relevant value of a project. In addition, the selection of a cutoff is arbitrary, and the time value of money is ignored. For a financial manager, both the reliance on accounting numbers rather than relevant market data and the exclusion of time value of money considerations are troubling. Despite these problems, AAR continues to be used in practice because (1) the accounting information is usually available, (2) analysts often use accounting ratios to analyze firm performance, and (3) managerial compensation is often tied to the attainment of certain target accounting ratio goals.

5. *a.* NPV is simply the sum of the present values of a project's cash flows. NPV specifically measures, after considering the time value of money, the net increase or decrease in firm wealth due to the project. The decision rule is to accept projects that have a positive NPV, and reject projects with a negative NPV.

b. NPV is superior to the other methods of analysis presented in the text because it has no serious flaws. The method unambiguously ranks mutually exclusive projects, and can differentiate between projects of different scale and time horizon. The only drawback to NPV is that it relies on cash flow and discount rate values that are often estimates and not certain, but this is a problem shared by the other performance criteria as well. A project with NPV = $2,500 implies that the total shareholder wealth of the firm will increase by $2,500 if the project is accepted.

6. *a.* The IRR is the discount rate that causes the NPV of a series of cash flows to be equal to zero. IRR can thus be interpreted as a financial break-even rate of return; at the IRR discount rate, the net value of the project is zero. The IRR decision rule is to accept projects with IRRs greater than the discount rate, and to reject projects with IRRs less than the discount rate.

b. IRR is the interest rate that causes NPV for a series of cash flows to be zero. NPV is preferred in all situations to IRR; IRR can lead to ambiguous results if there are non-conventional cash flows, and also ambiguously ranks some mutually exclusive projects. However, for stand-alone projects with conventional cash flows, IRR and NPV are interchangeable techniques.

c. IRR is frequently used because it is easier for many financial managers and analysts to rate performance in relative terms, such as “12%”, than in absolute terms, such as “$46,000.” IRR may be a preferred method to NPV in situations where an appropriate discount rate is unknown or uncertain; in this situation, IRR would provide more information about the project than would NPV.

7. *a.* The profitability index is the present value of cash inflows relative to the project cost. As such, it is a benefit/cost ratio, providing a measure of the relative profitability of a project. The profitability index decision rule is to accept projects with a PI greater than one, and to reject projects with a PI less than one.

b. PI = (NPV + cost) / cost = 1 + (NPV / cost). If a firm has a basket of positive NPV projects and is subject to capital rationing, PI may provide a good ranking measure of the projects, indicating the “bang for the buck” of each particular project.

8. For a project with future cash flows that are an annuity:

Payback = I / C

And the IRR is:

0 = – I + C / IRR

Solving the IRR equation for IRR, we get:

IRR = C / I

Notice this is just the reciprocal of the payback. So:

IRR = 1 / Payback

For long-lived projects with relatively constant cash flows, the sooner the project pays back, the greater is the IRR.

9. There are a number of reasons. Two of the most important have to do with transportation costs and exchange rates. Manufacturing in the U.S. places the finished product much closer to the point of sale, resulting in significant savings in transportation costs. It also reduces inventories because goods spend less time in transit. Higher labor costs tend to offset these savings to some degree, at least compared to other possible manufacturing locations. Of great importance is the fact that manufacturing in the U.S. means that a much higher proportion of the costs are paid in dollars. Since sales are in dollars, the net effect is to immunize profits to a large extent against fluctuations in exchange rates. This issue is discussed in greater detail in the chapter on international finance.

10. The single biggest difficulty, by far, is coming up with reliable cash flow estimates. Determining an appropriate discount rate is also not a simple task. These issues are discussed in greater depth in the next several chapters. The payback approach is probably the simplest, followed by the AAR, but even these require revenue and cost projections. The discounted cash flow measures (NPV, IRR, and profitability index) are really only slightly more difficult in practice.

11. Yes, they are. Such entities generally need to allocate available capital efficiently, just as for-profits do. However, it is frequently the case that the "revenues" from not-for-profit ventures are not tangible. For example, charitable giving has real opportunity costs, but the benefits are generally hard to measure. To the extent that benefits are measurable, the question of an appropriate required return remains. Payback rules are commonly used in such cases. Finally, realistic cost/benefit analysis along the lines indicated should definitely be used by the U.S. government and would go a long way toward balancing the budget!

12. The yield to maturity is the internal rate of return on a bond. The two concepts are identical with the exception that YTM is applied to bonds and IRR is applied to capital budgeting.

13. The MIRR is calculated by finding the present value of all cash outflows, the future value of all cash inflows to the end of the project, and then calculating the IRR of the two cash flows. As a result, the cash flows have been discounted or compounded by one interest rate (the required return), and then the interest rate between the two remaining cash flows is calculated. As such, the MIRR is not a true interest rate. In contrast, consider the IRR. If you take the initial investment, and calculate the future value at the IRR, you can replicate the future cash flows of the project exactly.

14. The statement is incorrect. It is true that if you calculate the future value of all intermediate cash flows to the end of the project at the required return, then calculate the NPV of this future value and the initial investment, you will get the same NPV. However, NPV says nothing about reinvestment of intermediate cash flows. The NPV is the present value of the project cash flows. What is actually done with those cash flows once they are generated is not relevant. Put differently, the value of a project depends on the cash flows generated by the project, not on the future value of those cash flows. The fact that the reinvestment "works" only if you use the required return as the reinvestment rate is also irrelevant simply because reinvestment is not relevant in the first place to the value of the project.

One caveat: Our discussion here assumes that the cash flows are truly available once they are generated, meaning that it is up to firm management to decide what to do with the cash flows. In certain cases, there may be a requirement that the cash flows be reinvested. For example, in international investing, a company may be required to reinvest the cash flows in the country in which they are generated and not "repatriate" the money. Such funds are said to be "blocked" and reinvestment becomes relevant because the cash flows are not truly available.

15. The statement is incorrect. It is true that if you calculate the future value of all intermediate cash flows to the end of the project at the IRR, then calculate the IRR of this future value and the initial investment, you will get the same IRR. However, as in the previous question, what is done with the cash flows once they are generated does not affect the IRR. Consider the following example:

	C_0	C_1	C_2	IRR
Project A	–$100	$10	$110	10%

Suppose this $100 is a deposit into a bank account. The IRR of the cash flows is 10 percent. Does the IRR change if the Year 1 cash flow is reinvested in the account, or if it is withdrawn and spent on pizza? No. Finally, consider the yield to maturity calculation on a bond. If you think about it, the YTM is the IRR on the bond, but no mention of a reinvestment assumption for the bond coupons is suggested. The reason is that reinvestment is irrelevant to the YTM calculation; in the same way, reinvestment is irrelevant in the IRR calculation. Our caveat about blocked funds applies here as well.

Solutions to Questions and Problems

Basic

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

1. To calculate the payback period, we need to find the time that the project has recovered its initial investment. After two years, the project has created:

$1,200 + 1,500 = $2,700

in cash flows. The project still needs to create another:

$3,400 – 2,700 = $700

in cash flows. During the third year, the cash flows from the project will be $900. So, the payback period will be 2 years, plus what we still need to make divided by what we will make during the third year. The payback period is:

Payback = 2 + ($700 / $900)
Payback = 2.78 years

2. To calculate the payback period, we need to find the time that the project has recovered its initial investment. The cash flows in this problem are an annuity, so the calculation is simpler. If the initial cost is $3,400, the payback period is:

Payback = 4 + $360 / $760
Payback = 4.47 years

There is a shortcut to calculate payback period when the future cash flows are an annuity. Just divide the initial cost by the annual cash flow. For the $3,400 cost, the payback period is:

Payback = $3,400 / $760
Payback = 4.47 years

For an initial cost of $4,450, the payback period is:

Payback = $4,450 / $760
Payback = 5.86 years

The payback period for an initial cost of $6,800 is a little trickier. Notice that the total cash inflows after eight years will be:

Total cash inflows = 8($760)
Total cash inflows = $6,080

If the initial cost is $6,800, the project never pays back. Notice that if you use the shortcut for annuity cash flows, you get:

Payback = $6,800 / $760
Payback = 8.95 years

This answer does not make sense since the cash flows stop after eight years, so again, we must conclude the payback period is never

3. Project A has cash flows of:

Cash flows = $13,000 + 19,000
Cash flows = $32,000

during the first two years. The cash flows are still short by $3,000 of recapturing the initial investment, so the payback for Project A is:

Payback = 2 + ($3,000 / $14,000)
Payback = 2.21 years

Project B has cash flows of:

Cash flows = $18,000 + 27,000 + 38,000
Cash flows = $83,000

during the first three years. The cash flows are still short by $12,000 of recapturing the initial investment, so the payback for Project B is:

Payback = 3 + ($12,000 / $225,000)
Payback = 3.05 years

Using the payback criterion and a cutoff of 3 years, accept project A and reject project B.

4. Our definition of AAR is the average net income divided by the average book value. The average net income for this project is:

Average net income = ($1,643,000 + 1,987,000 + 1,523,000 + 1,308,000) / 4
Average net income = $1,615,250

And the average book value is:

Average book value = ($16,000,000 + 0) / 2
Average book value = $8,000,000

So, the AAR for this project is:

AAR = Average net income / Average book value
AAR = $1,615,250 / $8,000,000
AAR = .2019 or 20.19%

5. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:

$0 = – \$130,000 + \$68,000/(1+IRR) + \$71,000/(1+IRR)^2 + \$54,000/(1+IRR)^3$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 23.65%

Since the cash flows are conventional and the IRR is greater than the required return, we would accept the project.

6. The NPV of a project is the PV of the outflows minus by the PV of the inflows. The equation for the NPV of this project at an 11 percent required return is:

$NPV = – \$130,000 + \$68,000/1.11 + \$71,000/1.11^2 + \$54,000/1.11^3$
NPV = $28,730.79

At an 11 percent required return, the NPV is positive, so we would accept the project.

The equation for the NPV of the project at a 27 percent required return is:

$NPV = – \$130,000 + \$68,000/1.27 + \$71,000/1.27^2 + \$54,000/1.27^3$
NPV = – $6,074.35

At a 27 percent required return, the NPV is negative, so we would reject the project.

7. The NPV of a project is the PV of the outflows minus by the PV of the inflows. Since the cash inflows are an annuity, the equation for the NPV of this project at an 8 percent required return is:

$NPV = – \$7,200 + \$1,700(PVIFA_{8\%, 9})$
NPV = $3,419.71

At an 8 percent required return, the NPV is positive, so we would accept the project.

The equation for the NPV of the project at a 24 percent required return is:

NPV = – \$7,200 + \$1,700($PVIFA_{24\%, 9}$)
NPV = –\$1,138.65

At a 24 percent required return, the NPV is negative, so we would reject the project.

We would be indifferent to the project if the required return was equal to the IRR of the project, since at that required return the NPV is zero. The IRR of the project is:

0 = – \$7,200 + \$1,700($PVIFA_{IRR, 9}$)
IRR = .1848 or 18.48%

8. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:

$0 = – \$36,000 + \$14,700/(1+IRR) + \$19,600/(1+IRR)^2 + \$13,100/(1+IRR)^3$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 15.37%

9. The NPV of a project is the PV of the outflows minus by the PV of the inflows. At a zero discount rate (and only at a zero discount rate), the cash flows can be added together across time. So, the NPV of the project at a zero percent required return is:

NPV = – \$36,000 + 14,700 + 19,600 + 13,100
NPV = \$11,400

The NPV at a 10 percent required return is:

$NPV = – \$36,000 + \$14,700/1.10 + \$19,600/1.10^2 + \$13,100/1.10^3$
NPV = \$3,404.21

The NPV at a 20 percent required return is:

$NPV = – \$36,000 + \$14,700/1.20 + \$19,600/1.20^2 + \$13,100/1.20^3$
NPV = –\$2,557.87

And the NPV at a 30 percent required return is:

$NPV = – \$36,000 + \$14,700/1.30 + \$19,600/1.30^2 + \$13,100/1.30^3$
NPV = – \$7,132.00

Notice that as the required return increases, the NPV of the project decreases. This will always be true for projects with conventional cash flows. Conventional cash flows are negative at the beginning of the project and positive throughout the rest of the project.

10. *a.* The IRR is the interest rate that makes the NPV of the project equal to zero. The equation for the IRR of Project A is:

$0 = –\$40,000 + \$24,000/(1+IRR) + \$20,000/(1+IRR)^2 + \$16,000/(1+IRR)^3 + \$12,000/(1+IRR)^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 32.98%

The equation for the IRR of Project B is:

$0 = –\$40,000 + \$14,000/(1+IRR) + \$18,000/(1+IRR)^2 + \$22,000/(1+IRR)^3 + \$26,000/(1+IRR)^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 30.72%

Examining the IRRs of the projects, we see that the IRR_A is greater than the IRR_B, so IRR decision rule implies accepting project A. This may not be a correct decision; however, because the IRR criterion has a ranking problem for mutually exclusive projects. To see if the IRR decision rule is correct or not, we need to evaluate the project NPVs.

b. The NPV of Project A is:

$NPV_A = –\$40,000 + \$24,000/1.11+ \$20,000/1.11^2 + \$16,000/1.11^3 + \$12,000/1.11^4$
$NPV_A = \$17,457.90$

And the NPV of Project B is:

$NPV_B = –\$40,000 + \$14,000/1.11 + \$18,000/1.11^2 + \$22,000/1.11^3 + \$26,000/1.11^4$
$NPV_B = \$20,435.03$

The NPV_B is greater than the NPV_A, so we should accept Project B.

c. To find the crossover rate, we subtract the cash flows from one project from the cash flows of the other project. Here, we will subtract the cash flows for Project

B from the cash flows of Project A. Once we find these differential cash flows, we find the IRR. The equation for the crossover rate is:

Crossover rate: $0 = \$10,000/(1+R) + \$2,000/(1+R)^2 – \$6,000/(1+R)^3 – \$14,000/(1+R)^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

R = 22.42%

At discount rates above 22.42% choose project A; for discount rates below 22.42% choose project B; indifferent between A and B at a discount rate of 22.42%.

11. The IRR is the interest rate that makes the NPV of the project equal to zero. The equation to calculate the IRR of Project X is:

$0 = –\$8,000 + \$4,300/(1+IRR) + \$2,700/(1+IRR)^2 + \$3,800/(1+IRR)^3$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 17.16%

For Project Y, the equation to find the IRR is:

$0 = –\$8,000 + \$4,100/(1+IRR) + \$2,775/(1+IRR)^2 + \$3,950/(1+IRR)^3$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 16.98%

To find the crossover rate, we subtract the cash flows from one project from the cash flows of the other project, and find the IRR of the differential cash flows. We will subtract the cash flows from Project Y from the cash flows from Project X. It is irrelevant which cash flows we subtract from the other. Subtracting the cash flows, the equation to calculate the IRR for these differential cash flows is:

Crossover rate: $0 = \$200/(1+R) – \$75/(1+R)^2 – \$150/(1+R)^3$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

R = 7.36%

The table below shows the NPV of each project for different required returns. Notice that Project Y always has a higher NPV for discount rates below 7.36 percent, and always has a lower NPV for discount rates above 7.36 percent.

R	NPV_X	NPV_Y
0%	2,800.00	2,825.00
5%	1,826.80	1,833.93
10%	995.49	988.35
15%	279.28	260.71
20%	–342.59	–370.37
25%	–886.40	–921.60

12. *a.* The equation for the NPV of the project is:

$NPV = – \$25,000,000 + \$51,000,000/1.10 – \$9,000,000/1.10^2 = \$13,925,619.83$

The NPV is greater than 0, so we would accept the project.

b. The equation for the IRR of the project is:

$0 = –\$28M + \$53M/(1+IRR) – \$8M/(1+IRR)^2$

From Descartes' rule of signs, we know there are two IRRs since the cash flows change signs twice. From trial and error, the two IRRs are:

IRR = 84.49%, –80.49%

When there are multiple IRRs, the IRR decision rule is ambiguous. Both IRRs are correct, that is, both interest rates make the NPV of the project equal to zero. If we are evaluating whether or not to accept this project, we would not want to use the IRR to make our decision.

13. The profitability index is defined as the PV of the cash inflows divided by the PV of the cash outflows. The equation for the profitability index at a required return of 10 percent is:

$PI = (\$13,000/1.10 + \$8,500/1.10^2 + \$5,500/1.10^3) / \$20,000$
PI = 1.149

The equation for the profitability index at a required return of 15 percent is:

$PI = (\$13,000/1.15 + \$8,500/1.15^2 + \$5,500/1.15^3) / \$20,000$
PI = 1.067

The equation for the profitability index at a required return of 22 percent is:

$PI = (\$13,000/1.22 + \$8,500/1.22^2 + \$5,500/1.22^3) / \$20,000$
$PI = 0.970$

We would accept the project if the required return were 10 percent or 15 percent since the PI is greater than one. We would reject the project if the required return were 22 percent since the PI is less than one.

14. *a.* The profitability index is defined as the PV of the cash inflows divided by the PV of the cash outflows. The equation for the profitability index for each project is:

$PI_I = (\$17,000/1.11 + \$20,000/1.11^2 + \$24,000/1.11^3) / \$45,000$
$PI_I = 1.091$

$PI_{II} = (\$6,000/1.11 + \$13,000/1.11^2 + \$9,000/1.11^3) / \$20,000$
$PI_{II} = 1.127$

The profitability index decision rule implies that we accept project II, since PI_{II} is greater than the PI_I.

b. The NPV of each project is:

$NPV_I = – \$45,000 + \$17,000/1.11 + \$20,000/1.11^2 + \$24,000/1.11^3$
$NPV_I = \$4,096.36$

$NPV_{II} = – \$20,000 + \$6,000/1.11 + \$13,000/1.11^2 + \$9,000/1.11^3$
$NPV_{II} = \$2,537.22$

The NPV decision rule implies accepting Project I, since the NPV_I is greater than the NPV_{II}.

c. Using the profitability index to compare mutually exclusive projects can be ambiguous when the magnitude of the cash flows for the two projects are of different scale. In this problem, project I is roughly 2 times as large as project II and produces a larger NPV, yet the profitability index criterion implies that project II is more acceptable.

15. *a.* The payback period for each project is:

A: 3 + ($162,000/$480,000) = 3.34 years

B: 21 + ($1,100/$12,700) = 2.09 years

The payback criterion implies accepting project B, because it pays back sooner than project A.

b. The NPV for each project is:

A: NPV = – \$325,000 + \$43,000/1.15 + \$51,000/1.15^2 + \$69,000/1.15^3 + \$480,000/1.15^4
NPV = \$70,764.81

B: NPV = – \$30,000 + \$15,800/1.15 + \$13,100/1.15^2 + \$12,700/1.15^3 + \$9,700/1.15^4
NPV = \$7,541.08

NPV criterion implies we accept project A because project A has a higher NPV than project B.

c. The IRR for each project is:

A: \$325,000 = \$43,000/(1+IRR) + \$51,000/$(1+IRR)^2$ + \$69,000/$(1+IRR)^3$ + \$480,000/$(1+IRR)^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 21.91%

B: \$30,000 = \$15,800/(1+IRR) + \$13,100/$(1+IRR)^2$ + \$12,700/$(1+IRR)^3$ + \$9,700/$(1+IRR)^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 28.02%

IRR decision rule implies we accept project B because IRR for B is greater than IRR for A.

d. The profitability index for each project is:

A: PI = (\$43,000/1.15 + \$51,000/1.15^2 + \$69,000/1.15^3 + \$480,000/1.15^4) / \$325,000
PI = 1.218

B: PI = (\$15,800/1.15 + \$13,100/1.15^2 + \$12,700/1.15^3 + \$9,700/1.15^4) / \$30,000
PI = 1.251

Profitability index criterion implies accept project B because its PI is greater than project A's.

e. In this instance, the NPV criterion implies that you should accept project A, while payback period, IRR, and the profitability index imply that you should accept project B. The final decision should be based on the NPV since it does not have the ranking problem associated with the other capital budgeting techniques. Therefore, you should accept project A.

16. *a.* The IRR for each project is:

M: $\$190{,}000 = \$75{,}000/(1+IRR) + \$90{,}000/(1+IRR)^2 + \$85{,}000/(1+IRR)^3 + \$70{,}000/(1+IRR)^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 24.94%

N: $\$300{,}000 = \$110{,}000/(1+IRR) + \$145{,}000/(1+IRR)^2 + \$130{,}000/(1+IRR)^3 + 95{,}000/(1+IRR)^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 22.38%

IRR decision rule implies we accept project M because IRR for M is greater than IRR for N.

b. The NPV for each project is:

M: $NPV = -\$190{,}000 + \$75{,}000/1.15 + \$90{,}000/1.15^2 + \$85{,}000/1.15^3 + \$70{,}000/1.15^4$
NPV = $39,181.93

N: $NPV = -\$300{,}000 + \$110{,}000/1.15 + \$145{,}000/1.15^2 + \$130{,}000/1.15^3 + \$95{,}000/1.15^4$
NPV = $45,086.67

NPV criterion implies we accept project N because project N has a higher NPV than project M.

c. Accept project N since the NPV is higher. IRR cannot be used to rank mutually exclusive projects.

17. *a.* The profitability index for each project is:

Y: PI = ($16,000/1.12 + $13,000/$1.12^2$ + $15,000/$1.12^3$ + $11,000/$1.12^4$) / $35,000
PI = 1.209

Z: PI = ($25,000/1.12 + $24,000/$1.12^2$ + $22,000/$1.12^3$ + $21,000/$1.12^4$) / $60,000
PI = 1.174

Profitability index criterion implies accept project Y because its PI is greater than project Z's.

b. The NPV for each project is:

Y: NPV = – $35,000 + $16,000/1.12 + $13,000/$1.12^2$ + $15,000/$1.12^3$ + $11,000/$1.12^4$
NPV = $7,316.64

Z: NPV = – $60,000 + $25,000/1.12 + $24,000/$1.12^2$ + $22,000/$1.12^3$ + $21,000/$1.12^4$
NPV = $10,459.13

NPV criterion implies we accept project Z because project Z has a higher NPV than project Y.

c. Accept project Z since the NPV is higher. The profitability index cannot be used to rank mutually exclusive projects.

18. To find the crossover rate, we subtract the cash flows from one project from the cash flows of the other project, and find the IRR of the differential cash flows. We will subtract the cash flows from Project J from the cash flows from Project I. It is irrelevant which cash flows we subtract from the other. Subtracting the cash flows, the equation to calculate the IRR for these differential cash flows is:

Crossover rate: 0 = $24,000/(1+R) + $4,000/$(1+R)^2$ – $15,000/$(1+R)^3$ – $20,000/$(1+R)^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

R = 9.65%

At a lower interest rate, project J is more valuable because of the higher total cash flows. At a higher interest rate, project I becomes more valuable since the differential cash flows received in the first two years are larger than the cash flows for project J.

19. If the payback period is exactly equal to the project's life then the IRR must be equal to zero since the project pays back exactly the initial investment. If the project never pays back its initial investment, then the IRR of the project must negative.

20. At a zero discount rate (and only at a zero discount rate), the cash flows can be added together across time. So, the NPV of the project at a zero percent required return is:

NPV = – $487,160 + 170,605 + 189,895 + 150,387 + 135,867
NPV = $159,594

If the required return is infinite, future cash flows have no value. Even if the cash flow in one year is $1 trillion, at an infinite rate of interest, the value of this cash flow today is zero. So, if the future cash flows have no value today, the NPV of the project is simply the cash flow today. So at an infinite interest rate:

NPV = – $487,160

The interest rate that makes the NPV of a project equal to zero is the IRR. The equation for the IRR of this project is:

$$0 = -\$487{,}160 + 170{,}605/(1 + \text{IRR}) + 189{,}895/(1 + \text{IRR})^2 + 150{,}387/(1 + \text{IRR})^3 + 135{,}867/(1 + \text{IRR})^4$$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 13.01%

21. *a.* The payback period for each project is:

F: 2 + ($25,000/$80,000) = 2.31 years

G: 3 + ($40,000/$190,000) = 3.21 years

The payback criterion implies accepting project F because it pays back sooner than project G. Project G does not meet the minimum payback of three years.

b. The NPV for each project is:

F: $\text{NPV} = -\$175{,}000 + \$85{,}000/1.10 + \$65{,}000/1.10^2 + \$80{,}000/1.10^3 + \$70{,}000/1.10^4 + \$55{,}000/1.10^5$
NPV = $98,058.53

G: NPV = – $275,000 + $55,000/1.10 + $70,000/$1.10^2$ + $110,000/$1.10^3$
+ $190,000/$1.10^4$ + $135,000/$1.10^5$
NPV = $129,092.80

NPV criterion implies we should accept project G because project G has a higher NPV than project H.

c. Even though project G does not meet the payback period of three years, it does provide the largest increase in shareholder wealth, therefore, choose project G. Payback period should generally be ignored in this situation.

22. The MIRR for the project with all three approaches is:

Discounting approach:

In the discounting approach, we find the value of all cash outflows to time 0, while any cash inflows remain at the time at which they occur. So, the discounting the cash outflows to time 0, we find:

Time 0 cash flow = –$12,000 – $4,300 / 1.10^5
Time 0 cash flow = –$14,669.96

So, the MIRR using the discounting approach is:

0 = –$14,669.96 + $5,800/(1 + MIRR) + $6,500/$(1 + \text{MIRR})^2$ + $6,200/$(1 + \text{MIRR})^3$
+ 5,100/$(1 + \text{MIRR})^4$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

MIRR = 22.63%

Reinvestment approach:

In the reinvestment approach, we find the future value of all cash except the initial cash flow at the end of the project. So, the reinvesting the cash flows to time 5, we find:

Time 5 cash flow = $5,800($1.10^4$) + $6,500($1.10^3$) + $6,200($1.10^2$) + $5,100(1.10) – $4,300
Time 5 cash flow = $25,955.28

So, the MIRR using the discounting approach is:

0 = –$12,000 + $25,955.28/$(1+\text{MIRR})^5$
$25,955.28 / $12,000 = $(1+\text{MIRR})^5$

MIRR = ($25,955.28 / $12,000)$^{1/5}$ – 1
MIRR = .1668 or 16.68%

Combination approach:

In the combination approach, we find the value of all cash outflows at time 0, and the value of all cash inflows at the end of the project. So, the value of the cash flows is:

Time 0 cash flow = –$12,000 – $4,300 / 1.10^5
Time 0 cash flow = –$14,669.96

Time 5 cash flow = $5,800(1.10^4) + $6,500(1.10^3) + $6,200(1.10^2) + $5,100(1.10)
Time 5 cash flow = $30,255.28

So, the MIRR using the discounting approach is:

0 = –$14,669.96 + $30,255.28/(1 + MIRR)^5
$30,255.28 / $14,669.96 = (1 + MIRR)^5
MIRR = ($30,255.28 / $14,669.96)$^{1/5}$ – 1
MIRR = .1558 or 15.58%

Intermediate

23. With different discounting and reinvestment rates, we need to make sure to use the appropriate interest rate. The MIRR for the project with all three approaches is:

Discounting approach:

In the discounting approach, we find the value of all cash outflows to time 0 at the discount rate, while any cash inflows remain at the time at which they occur. So, the discounting the cash outflows to time 0, we find:

Time 0 cash flow = –$12,000 – $4,300 / 1.11^5
Time 0 cash flow = –$14,551.84

So, the MIRR using the discounting approach is:

0 = –$14,551.84 + $5,800/(1+MIRR) + $6,500/(1+MIRR)^2 + $6,200/(1+MIRR)^3
+ 5,100/(1+MIRR)^4

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

MIRR = 23.08%

Reinvestment approach:

In the reinvestment approach, we find the future value of all cash except the initial cash flow at the end of the project using the reinvestment rate. So, the reinvesting the cash flows to time 5, we find:

Time 5 cash flow = $\$5,800(1.08^4) + \$6,500(1.08^3) + \$6,200(1.08^2) + \$5,100(1.08) – \$4,300$
Time 5 cash flow = $24,518.64

So, the MIRR using the discounting approach is:

$0 = –\$12,000 + \$24,518.64/(1+\text{MIRR})^5$
$\$24,518.64 / \$12,000 = (1+\text{MIRR})^5$
$\text{MIRR} = (\$24,518.64 / \$12,000)^{1/5} – 1$
MIRR = .1536 or 15.36%

Combination approach:

In the combination approach, we find the value of all cash outflows at time 0 using the discount rate, and the value of all cash inflows at the end of the project using the reinvestment rate. So, the value of the cash flows is:

Time 0 cash flow = $–\$12,000 – \$4,300 / 1.11^5$
Time 0 cash flow = –$14,551.84

Time 5 cash flow = $\$5,800(1.08^4) + \$6,500(1.08^3) + \$6,200(1.08^2) + \$5,100(1.08)$
Time 5 cash flow = $28,818.64

So, the MIRR using the discounting approach is:

$0 = –\$14,551.84 + \$28,818.64/(1+\text{MIRR})^5$
$\$28,818.64 / \$14,551.84 = (1+\text{MIRR})^5$
$\text{MIRR} = (\$28,818.64 / \$14,551.84)^{1/5} – 1$
MIRR = .1464 or 14.64%

24. To find the crossover rate, we subtract the cash flows from one project from the cash flows of the other project, and find the IRR of the differential cash flows. We will subtract the cash flows from Project S from the cash flows from Project R. It is irrelevant which cash flows we subtract from the other. Subtracting the cash flows, the equation to calculate the IRR for these differential cash flows is:

$0 = \$25,000 + \$2,000/(1+R) + \$3,000/(1+R)^2 – \$13,000/(1+R)^3 – \$26,000/(1+R)^4 – \$4,000/(1 + R)^5$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

R = 10.75%

The NPV of the projects at the crossover rate must be equal, The NPV of each project at the crossover rate is:

R: NPV = – \$60,000 + \$21,000/1.1075 + \$24,000/1.1075^2 + \$17,000/1.1075^3 + \$11,000/1.1075^4
+ \$9,000/1.1075^5
NPV = \$3,754.83

S: NPV = – \$85,000 + \$19,000/1.1075 + \$21,000/1.1075^2 + \$30,000/1.1075^3 + \$37,000/1.1075^4
+ \$13,000/1.1075^5
NPV = \$3,754.83

25. The IRR of the project is:

\$48,000 = \$26,000/(1+IRR) + \$38,000/$(1+IRR)^2$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

R = 20.09%

At an interest rate of 12 percent, the NPV is:

NPV = \$48,000 – \$26,000/1.12^2 – \$38,000/1.12^2
NPV = –\$5,507.65

At an interest rate of zero percent, we can add cash flows, so the NPV is:

NPV = \$48,000 – \$26,000 – \$38,000
NPV = –\$16,000.00

And at an interest rate of 24 percent, the NPV is:

NPV = \$48,000 – \$26,000/1.24^2 – \$38,000/1.24^2
NPV = +\$2,318.42

The cash flows for the project are unconventional. Since the initial cash flow is positive and the remaining cash flows are negative, the decision rule for IRR in invalid in this case. The NPV profile is upward sloping, indicating that the project is more valuable when the interest rate increases.

26. By definition, the profitability index is:

PI = Discounted Value of Future Cash Flows / Initial Cost

But note that the discounted value of future cash flows is just the NPV overstated by the neglected initial costs, so:.

NPV = Discounted Value of Future Cash Flows – Initial Cost
Discounted Value of Future Cash Flows = NPV + Initial Cost

Substituting, we get:

PI = (NPV + Initial Cost) / Initial Cost
PI = NPV / Initial Cost + 1
PI = NPV Index + 1
NPV Index = PI – 1

27. *a.* To have a payback equal to the project's life, given C is a constant cash flow for N years:

C = I/N

b. To have a positive NPV, I < C (PVIFA$_{R\%,\,N}$). Thus, C > I / (PVIFA$_{R\%,\,N}$).

c. Benefits = C (PVIFA$_{R\%,\,N}$) = 2 × costs = 2I
C = 2I / (PVIFA$_{R\%,\,N}$)

Challenge

28. *a.* Here the cash inflows of the project go on forever, which is a perpetuity. Unlike ordinary perpetuity cash flows, the cash flows here grow at a constant rate forever, which is a growing perpetuity. If you remember back to the chapter on stock valuation, we presented a formula for valuing a stock with constant growth in dividends. This formula is actually the formula for a growing perpetuity, so we can use it here. The PV of the future cash flows from the project is:

PV of cash inflows = $C_1/(R – g)$
PV of cash inflows = $60,000/(.13 – .06) = $857,142.86

NPV is the PV of the outflows minus by the PV of the inflows, so the NPV is:

NPV of the project = –$925,000 + 857,142.86 = –$67,857.14

The NPV is negative, so we would reject the project.

b. Here we want to know the minimum growth rate in cash flows necessary to accept the project. The minimum growth rate is the growth rate at which we would have a zero NPV. The equation for a zero NPV, using the equation for the PV of a growing perpetuity is:

$0 = –\$925,000 + \$60,000/(.13 – g)$

Solving for g, we get:

$g = .0651$ or 6.51%

29. The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the project is:

$0 = \$20,000 – \$26,000 / (1 + \text{IRR}) + \$13,000 / (1 + \text{IRR})^2$

Even though it appears there are two IRRs, a spreadsheet, financial calculator, or trial and error will not give an answer. The reason is that there is no real IRR for this set of cash flows. If you examine the IRR equation, what we are really doing is solving for the roots of the equation. Going back to high school algebra, in this problem we are solving a quadratic equation. In case you don't remember, the quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case, the equation is:

$$x = \frac{-(-26{,}000) \pm \sqrt{(-26{,}000)^2 - 4(20{,}000)(13{,}000)}}{2(26{,}000)}$$

The square root term works out to be:

676,000,000 – 1,040,000,000 = –364,000,000

The square root of a negative number is a complex number, so there is no real number solution, meaning the project has no real IRR.

30. First, we need to find the future value of the cash flows for the one year in which they are blocked by the government. So, reinvesting each cash inflow for one year, we find:

Year 2 cash flow = \$165,000(1.04) = \$171,600
Year 3 cash flow = \$190,000(1.04) = \$197,600
Year 4 cash flow = \$205,000(1.04) = \$213,200
Year 5 cash flow = \$183,000(1.04) = \$190,320

So, the NPV of the project is:

NPV = –\$450,000 + \$171,600/1.11^2 + \$197,600/1.11^3 + \$213,200/1.11^4 + \$190,320/1.11^5
NPV = \$87,144.93

And the IRR of the project is:

0 = –\$450,000 + \$171,600/(1 + IRR)^2 – \$197,600/(1 + IRR)^3 + \$213,200/(1 + IRR)^4 + \$190,320/(1 + IRR)^5

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:
IRR = 16.95%

While this may look like a MIRR calculation, it is not an MIRR, rather it is a standard IRR calculation. Since the cash inflows are blocked by the government, they are not available to the company for a period of one year. Thus, all we are doing is calculating the IRR based on when the cash flows actually occur for the company.