2. AUCTIONS

2.1 Situations

Recall that bargaining is: “One buyer; one seller; one deal.”

Auctions are used when we have: “One seller; many (potential) buyers; one deal.”

> We’ll be using this perspective, but auctions also used when we have: “One buyer, many (potential) sellers; one deal.”

The “one” (buyer or seller) usually gets to set the rules.

The economic theory of auctions is more well-developed than that of bargaining.

We also have more field observations.

2.2 Auction formats (types of seller’s auctions)

► English (“Ascending”)

Open outcry by bidders until only one bidder is left.

Examples: Wine, art, most internet auctions

► Dutch

Auctioneer calls out a high price and methodically drops it until someone says “mine”

Examples: Dutch flower market; thrift markets

► First price, sealed bid auctions (“first price”)

Bidders submit sealed bids

Winner is the highest bid

Amount paid is the highest bid

Examples: Sealed bid, first price format is used mostly in buyers’ auctions, esp. government procurement.

The format also arises “by accident”, e.g., an English auction in which bidder’s won’t get a chance to revise their bids.

- Second price, sealed bid auctions (“Vickrey”, “second price”)

  Bidders submit sealed bids

  Winner is the highest bid

  Amount paid is the *second* highest bid

  Example: If the top two bids are Smith bidding 50 and Brown bidding 45, Smith gets the object and pays 45.

  Examples: stamp auctions, some internet auctions.

“Dutch” auctions: A note on terminology

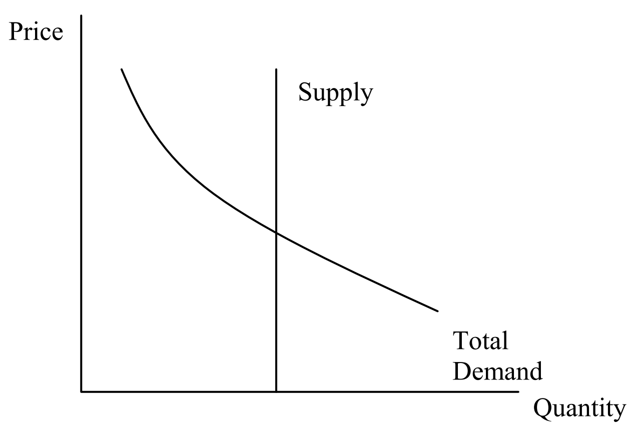
In securities markets, Dutch auction used to refer to situations with multiple units and sealed bids or offers.

Example: Dutch IPO auction (W.R. Hambrecht).

An issuer has one million shares of a new issue to sell. Bidders name prices and quantities:

“I’ll buy 2,000 shares at $20; I’ll buy a total of 3,000 shares at $19.00.” These bids are not visible.

“Auctioneer” marks down the price until the quantity sought matches the quantity available.



The “mark down” does not occur in real time; it is purely notional.

2.3 Some key concepts in the economic analysis of auctions.

- Type of “value”

What is the object worth to me?

Do I know this before the bidding starts?

Is this value dependent on/related to the values of others?

Will I learn anything about this value in the course of bidding?

- Risk-neutrality

A risk-neutral person cares only about the expected ("average") outcome. He is indifferent between

- receiving \$100 cash
- receiving a lottery ticket that pays \$200 or \$0 (with 50% probabilities))
- taking a gamble that will pay \$9,100 if he wins (a 10% chance) or *costs* him \$900 if he loses (a 90% chance)

> The expected outcome is $0.5 \times \$200 + 0.5 \times \$0 = \$100$.

> The expected outcome is $0.1 \times \$9{,}100 - 0.9 \times \$900 = \$100$.

- Nash equilibrium

A state of balance in a game:
All of the bidders' strategies and conjectures about other bidders' strategies are mutually consistent.

Auctions are played against others. Our choice strategy depends on what we think others will do.

Suppose that my conjectures of others strategies are correct.

> This does not mean that I know exactly what they're going to do.
>
> I may not know all of the information available to them, so I may not know exactly what they're going to do. But I know how they use information: if I knew everything they new, I could predict exactly what they'd do.

Knowing this I formulate my own strategy.

Now consider anyone else, if they correctly conjecture my strategy, will they change their strategy?

If the answer is "no", and if this holds for all bidders, we have a Nash equilibrium.

Nobody has an incentive to deviate from their planned strategies.

The combination of risk neutrality and Nash equilibrium is invoked so often that it's commonly abbreviated "RNNE".

2.4 Value

The value is what the object is worth to a bidder.

Value is often at least partially specific to the bidder.

I might value a particular painting more than someone else.

I probably don't know other bidders' values.

I might not even know everything about my own value.

If I plan to resell the painting, what will its resale value be?

Value is generally random variable

The collection of all bidders' values is a set of random variables *that might be dependent*.

Usually we know something about the *distribution* of the values.

Knowledge of the distribution of others' values ⇒ conjectures about others' bidding strategies.

Sources and types of value

- ► Independent private values

  I know my valuation and it's not related to the valuation of any other bidders.

  Example: At the end of the day, a produce market auctions off the unsold strawberries, which must be consumed that day.

  Other peoples bids and the final price tell me nothing that influences my personal value.

- ► Affiliated values

  I know my valuation and it is positively related to the private values of the other bidders.

  E.g., (in the strawberry example), there's something in the weather that's just making people feel like eating strawberry pie.

  If my valuation is "high", I know that other valuations are likely to be high. This may affect my bidding strategy.

- ► Common values

The true value to me is unknown, but it will be the same across all bidders.

Example: drilling rights to an off-shore oil tract.

We have different guesses ("signals") as to what it's worth.

Other bids and the final price may be informative about value.

Values in securities markets

In security markets, components of private or affiliated values can arise from idiosyncratic tax situations, investment horizons, control rights.

BUT securities markets are dominated by common values. This is due to

Likelihood of resale

Future investment opportunities

2.5 Auction outcomes

If I win, my profits are

$$\underbrace{\pi}_{\text{Profit}} = \underbrace{V}_{\text{Value}} - \underbrace{P}_{\text{Price paid}}$$

"If I win" ⇔ "Conditional on winning"

We write "conditional on x" as "$|x$" as in:

$\pi|winning$ ("profits conditional on winning")

E denotes "expected" (in the statistical sense)

Example of conditioning:

z is the outcome on a throw of a fair die.

$Ez = (1+2+3+4+5+6)/6 = 3.5$

Note that a throw of "3.5" never occurs.

One interpretation: "I'll pay you $3.50 if you pay me the number of dollars on the throw of a die" is a fair game.

Conditional expectations:

$$E(z \mid z \text{ is even}) = (2+4+6)/3 = 4$$
$$E(z \mid z \text{ is odd}) = (1+3+5)/3 = 3$$

We can write *Ez* as:

$$Ez = E(z \mid even) \times \Pr(even) + E(z \mid odd) \times \Pr(odd) = 4 \times 0.5 + 3 \times 0.5 = 3.5$$

In an auction, the expected profits are

$$E\pi = E(\pi \mid winning) \times \Pr(winning) + E(\pi \mid losing) \times \Pr(losing)$$

Usually, we assume $E(\pi \mid losing) = 0$

We didn't have to bet in the auction at all; If we lose, we're no worse off than if we'd never heard about the auction in the first place. (BUT: Costs of entry? Psychic costs of trying for something and *then* losing?)

So:

$$E\pi = E(\pi \mid W) \times \Pr(W) = \left[E(V \mid W) - E(P \mid W)\right] \times \Pr(W)$$

Let *B* be my bid. (Why might $B \neq P$?)

As $B\uparrow$: $\Pr(W)\uparrow$, $E(P|W)\uparrow$, but $E(V|W)$?

If *V* is my own private value (e.g., my enjoyment from the painting): $E(V|W)$ doesn't change.

But if *V* is driven in whole or part by the resale value.

W (I win) $\Rightarrow$ My estimate of the resale value is higher than anyone else's.
$\Rightarrow$ I probably made a mistake.
$\Rightarrow E(V|W)\downarrow$

2.6 Substantive results for private value auctions

Strategic equivalence of English (open outcry) and second-price sealed bid formats

Example: assume private value V=25

- English auction

  Suppose the current bid is 19. Should I bid 20? Will I be happy if I win?

Suppose the current bid is 24. Should I bid 25?

Suppose the current bid is 25. Should I bid 26?

If my V is the highest, and the next highest V is 22, how will the bidding go?

- Second price, sealed bid; My private value V=25

  Bid less than V, e.g. 23.

    If my bid is one of the two highest bids, and the highest bid not mine (H, "the other bid") is 20, I win & pay 20, payoff = 25–20 = 5

    If H=26, they win and pay 23 (my bid); my payoff=0.

    If H=24, they win and pay 23 (my bid); my payoff=0.

  Bid more than V, e.g., 27.

    If H=20, I win and pay 20; payoff=5

    If H=26, I win and pay 26; payoff=-1

    If H=24, I win and pay 24; payoff=1

  Bid V=25

    If H=20, I win and pay 20; payoff=5

    If H=26, they win and pay 25; my payoff=0

    If H=24, I win and pay 24; my payoff=1;

In open outcry English, bidding beyond your private value will incur a loss (if you win)

Also (sometimes) true in second-price, sealed bid auctions.

Let H=highest bid not mine. My payoffs are

	My Bid:	V	V+x where x>0
H:			
H<V		V–H>0	V–H >0
H>V+x		0	0
V<H<V+x		0	V–H <0

With private values, English and second-price auctions are strategically equivalent: bid your value (no higher).

Two games are strategically equivalent if players use the same strategies in each (which means the outcomes will be identical).

The strategic equivalence of English and second-price auctions holds with private values. Does it hold with common values? (Hint: does hearing the bids of others tell you anything about value?)

Strategic equivalence of Dutch and first-price sealed bid formats

Maintain assumption that my private value is V=25

- Dutch auction

  Suppose the auctioneer is calling out 30. Should I say “mine”?

  If the auctioneer is calling out 25?

  If my V is the highest, and the next highest V is 22, when will the next bidder jump in and take the object from me?

- The first-price sealed-bid auction

  Suppose I’m contemplating a bid of 23. I’ll get the item at 23 if nobody else bids higher.

  Alternatively, in an open outcry Dutch auction, suppose I precommit myself to crying “mine” if the bidding gets down to 23.

The Dutch auction can be played as a first-price sealed-bid auction:

> Write down my first-price bid on a piece of paper.
>
> When the Dutch auctioneer calls out that price: “mine”.
>
> In both formats, we make a decision before knowing any other players bids.
>
> Dutch and first-price auctions are strategically equivalent. (This holds for common values as well as private values.)

This is true even though we don't know what the best bidding strategy is!

2.7 The first-price auction with uniform private values: a very special case.

Recall that:

$$E\pi = \left[E(V \mid W) - B\right] \times \Pr(W)$$
$$\underbrace{=}_{\text{because private values}} \left[EV - B\right] \times \Pr(W)$$

$$\text{As } B\uparrow,\ \underbrace{\left[EV - B\right]}_{\downarrow} \times \underbrace{\Pr(W)}_{\uparrow}$$

Suppose that for each of n bidders, V is uniformly random between 0 and 100. Then the B that yields that highest $E\pi$ is:

$$B = 100 \times \frac{n-1}{n+1}$$

Note that as $n\uparrow$, $B\uparrow$.

Intuition: with more bidders, we need to bid higher to win.

(See appendix to Bergstrom and Miller, or Klemperer (1999), appendix D)

2.8 (Expected) revenue equivalence

The seller will choose the format of the auction based on the revenue (sale price). What format should she choose?

$$\text{Dutch} \overset{\textit{Strategic}}{\Leftrightarrow} \text{First price}$$

This implies that they'll always generate the same revenue.

With private values:

$$\text{English} \overset{\textit{Strategic}}{\Leftrightarrow} \text{Second Price}$$

This pair will generate the same revenue.

But even with private values, how do we rank English/2nd vs. Dutch/1st ?

The Theorem (Klemperer (1999), p. 232)

Assume all bidders are risk-neutral and symmetric. Each bidder's value is independent and randomly drawn from a common distribution. Then all auctions yield the same expected revenue.

"It doesn't matter" (an irrelevancy proposition).

A strong result, but note the (implausibly) strong assumptions.

2.9 Experimental evidence

Experimental evidence on revenue equivalence: English vs. second-price (sealed bid) auctions

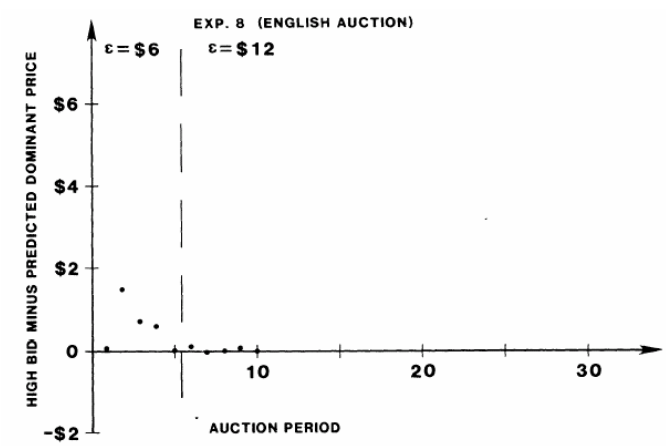
English and second-price auctions are not equivalent.

There is more overbidding in second-price auctions.

On average, second price auctions yield higher revenues than English auctions.

Source: Kagel, Harstad, and Levin (1987)

► Overbidding in English auctions:



► Overbidding in second-price auctions:

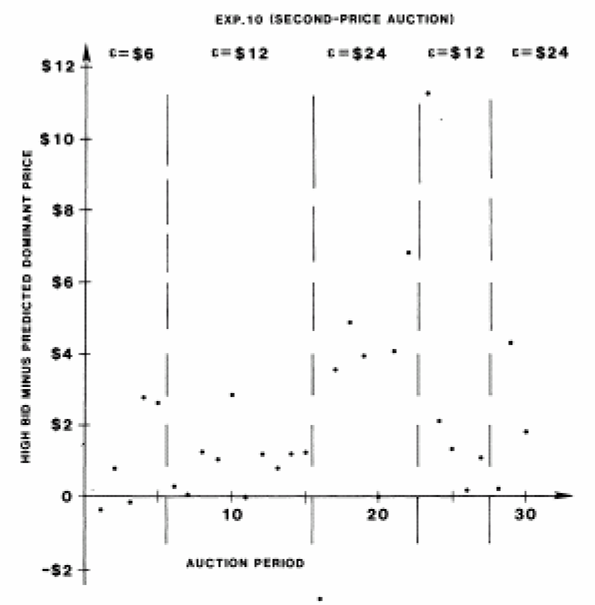


FIGURE 8

Revenue equivalence: Dutch and first-price (sealed bid) auctions

Bids in both Dutch and first-price auctions tend to exceed what would be predicted in the RNNE (risk-neutral Nash equilibrium)

First-price bids are generally higher than Dutch auction bids.

This implies that first-price auctions raise more revenue for the seller:

Table 7.1. Price Differences: Dutch versus First-Price Auctions

n	First Price	Dutch Price	Difference
3	2.36	1.98	.38
	2.60	2.57	.03
4	5.42	4.98	.44
	5.86	5.68	.18
5	9.15	8.72	.43
	9.13	8.84	.29
6	13.35	13.25	.10
	13.09	12.89	.20
9	31.02	30.32	.70

Source: Cox, Roberson, and Smith 1982, table 7.
Note: Means for paired-comparison auction sequences. Two entries indicate two paired-comparison sequences for that value of n.

(cited in Research in Experimental Economics, V. L. Smith, ed., JAI Press)

2.10 Common values auctions

Suppose each bidder gets a random signal of what the object is worth.

It is called a "signal" because it is a measurement (with error) of the object's value.

Example:

The object is an oil field with unknown production potential. The signal is a geologist's estimate of the oil reserves.

Example:

There are four bidders. Bidder 1 gets a signal s_1, a random number between 0 and 100.

The other bidders receive signals s_2, s_3 and s_4.

Suppose that the true value of the object is the average of the signals:

$$V = (s_1 + s_2 + s_3 + s_4)/4$$

Without knowing anything about any of the signals, my estimate of V is EV=50.

Suppose that I know my signal is s_1=90. Now what is $E(V \mid s_1 = 90)$?

$$E(V \mid s_1 = 90) = (90 + 50 + 50 + 50)/4 = 60$$

Should I bid 60?

What if I win?

Suppose that we (all the bidders) are identical. If we get a higher signal, then we make a higher bid.

If I win the auction, I must have made the best bid ⇒ I must have had the highest signal.

$$E(V \mid s_1 = 90 \text{ and } W)$$
$$= E(V \mid s_1 = 90 \text{ and } s_2, s_3, s_4 < 90)$$

Result: Suppose that s is uniformly random between 0 and 100. Conditional on knowing that s<90, s is uniformly random between 0 and 90.

This implies that $E(s \mid s < 90) = 45$, and

$$E(V \mid s_1 = 90 \text{ and } s_2, s_3, s_4 < 90)$$
$$= (90 + 45 + 45 + 45)/4 = 56.25$$

Should I bid 56.25?

In a first-price or Dutch auction: No.

Suppose the price drops to 56.25 and I call "Mine!." What have I won?

In a second-price auction: Yes.

In an English auction, should I bid up to 56.25?

No. By the time we get up to 56.25, I'll find out something about the other bids.

What is the best strategy?

Now suppose that there were five bidders (including myself).

Then

$$E(V \mid s_1 = 90 \text{ and } W)$$
$$= (90 + 45 + 45 + 45 + 45)/5 = 54$$

So in going from 4 to 5 bidders, I'll drop my bid.

(If my estimate is highest in a field of 100, my overestimate of the true value is worse than if my estimate were highest in a field of 3.)

2.11 Reserve prices (To be added)

2.12 Manipulations (To be added)

2.13 Multiple unit auctions (To be added)

2.14 Security auctions (To be added)

2.15 Auction problems

1. I'm selling my Dali lithograph by an open-outcry English auction and I'm trying to set a reserve price R. Consider the highest bid H and the second highest bid SH. Ignoring equalities, there are three possibilities: (1) R<SH<H; (2) SH<R<H; and (3) SH<H<R. In each of these ranges, how does my reserve price affect the auction outcome? If I know H and SH, where would I set R?

2. Assume that the setup is an independent private-values descending-price Dutch auction with identical bidders. My aunt Florence can't be at present, so I'll be representing her in the bidding for a Tiffany lamp. I ask:

   (a) "If this were an English auction, what's the highest you'd bid?"
   Aunt Florence: "$800"

   (b) "If this were a second-price sealed-bid auction, what would you bid?"
   Aunt Florence: "$850"

   (c) "If this were a first-price sealed-bid auction, what would you bid?"
   Aunt Florence: "$750"

   Are Aunt Florence's response consistent? Which answer should come closest to AF's true value for the lamp? Which answer should come closest to the price at which (in the actual auction) I should claim the lamp?

3. Buyers on eBay can use a proxy bidder. To do so, they set a maximum price. The proxy bidder keeps bidding just high enough to be the best bid (up to where the maximum price is hit). It has been stated that the proxy bidder transforms the format of eBay's English auction. In what sense is this true?

4. On the eBay web site it is stated, "We will thoroughly investigate bid retractions, and abuse of this feature may result in the suspension of your eBay account." Why does eBay take such a hard line on retractions?

5. On the eBay web site, it is stated, "eBay originally allowed sellers to bid on their own auctions as a way to close their auction without selling to the highest bidder." This is no longer allowed. Why not?

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GameID: 1 ValueType: Fixed

Ultimatum, I & II

ValueCode: ValueA: ValueB:

Bid range ($)	Accept	Reject	Total	% Accept
1 to 5	4	7	11	36%
6 to 10	1	1	2	50%
11 to 15		2	2	0%
16 to 20	3	1	4	75%
21 to 25	1	2	3	33%
26 to 30	4	1	5	80%
31 to 35	2	1	3	67%
36 to 40	7	1	8	88%
41 to 45	5	1	6	83%
46 to 50	6		6	100%
56 to 60	4		4	100%
61 to 65	3		3	100%
66 to 70	3		3	100%
91 to 95		1	1	0%
	43	18	61	70.5%

GameID: 2 ValueType: Fixed

Ultimatum, session I

ValueCode: ValueA: 0 ValueB: 0

Bid range ($)	Accept	Reject	Total	% Accept
1 to 5		3	3	0%
6 to 10	1		1	100%
11 to 15		1	1	0%
21 to 25	1	1	2	50%
26 to 30		1	1	0%
36 to 40	6	3	9	67%
41 to 45	4	3	7	57%
46 to 50	8		8	100%
51 to 55	3		3	100%
71 to 75	1		1	100%
	24	12	36	66.7%

GameID: 3 ValueType: Random; buyer knows

Asy info, session I

ValueCode: A ValueA: 150 ValueB: 50

Bid range ($)	Accept	Reject	Total	% Accept
1 to 5		1	1	0%
21 to 25	3		3	100%
26 to 30	2		2	100%
36 to 40	2	1	3	67%
46 to 50	3		3	100%
96 to 100	1		1	100%
	11	2	13	84.6%

ValueCode: B ValueA: 150 ValueB: 50

Bid range ($)	Accept	Reject	Total	% Accept
16 to 20		2	2	0%
21 to 25	4	5	9	44%
26 to 30	1		1	100%
36 to 40	5	1	6	83%
41 to 45	2		2	100%
46 to 50	1	1	2	50%
	24	11	35	59.1%

GameID: 4 ValueType: Fixed

Ultimatum, session II

ValueCode: ValueA: 0 ValueB: 0

Bid range ($)	Accept	Reject	Total	% Accept
11 to 15	1		1	100%
26 to 30		1	1	0%
31 to 35	1	1	2	50%
36 to 40	3	3	6	50%
41 to 45	9	1	10	90%
46 to 50	9		9	100%
51 to 55	1		1	100%
66 to 70	1		1	100%
81 to 85	1		1	100%
	26	6	32	81.3%

GameID: 5 ValueType: Random; seller knows

Asy info, session II

ValueCode: A ValueA: 50 ValueB: 150

Bid range ($)	Accept	Reject	Total	% Accept
21 to 25	2		2	100%
31 to 35	2		2	100%
41 to 45	2	1	3	67%
46 to 50	3		3	100%
66 to 70	1		1	100%
71 to 75	3		3	100%
76 to 80	1		1	100%
81 to 85	1		1	100%
96 to 100	2	1	3	67%
	17	2	19	89.5%

ValueCode: B ValueA: 50 ValueB: 150

Bid range ($)	Accept	Reject	Total	% Accept
21 to 25		1	1	0%
26 to 30		1	1	0%
36 to 40	1		1	100%
41 to 45	2	1	3	67%
46 to 50	3	1	4	75%
66 to 70	2		2	100%
86 to 90	1		1	100%
	26	6	32	69.2%