This is true even though we don't know what the best bidding strategy is!

2.7 The first-price auction with uniform private values: a very special case.

Recall that:

$$E\pi = \left[E(V \mid W) - B\right] \times \Pr(W)$$

$$\underbrace{=}_{\text{because private values}} [V - B] \times \Pr(W)$$

$$\text{As } B\uparrow,\ \underbrace{[V-B]}_{\downarrow} \times \underbrace{\Pr(W)}_{\uparrow}$$

Suppose that for each of *n* bidders, *V* is uniformly random between 0 and some upper limit (that is the same for all bidders). Then the bid *B* that yields the highest $E\pi$ for a bidder with signal *V* is:

$$B = \frac{n-1}{n} \times V$$

Correction

Note that as $n\uparrow$, $B\uparrow$.

Intuition: with more bidders, we need to bid higher to win.

(See appendix to Bergstrom and Miller, or Klemperer (1999), appendix D)

2.8 (Expected) revenue equivalence

The seller will choose the format of the auction based on the revenue (sale price). What format should she choose?

$$\text{Dutch} \overset{\text{Strategic}}{\Leftrightarrow} \text{First price}$$

This implies that they'll always generate the same revenue.

With private values:

$$\text{English} \overset{\text{Strategic}}{\Leftrightarrow} \text{Second Price}$$

This pair will generate the same revenue.

But even with private values, how do we rank English/2nd vs. Dutch/1st ?

The Theorem (Klemperer (1999), p. 232)

Table 7.1. Price Differences: Dutch versus First-Price Auctions

n	First Price	Dutch Price	Difference
3	2.36	1.98	.38
	2.60	2.57	.03
4	5.42	4.98	.44
	5.86	5.68	.18
5	9.15	8.72	.43
	9.13	8.84	.29
6	13.35	13.25	.10
	13.09	12.89	.20
9	31.02	30.32	.70

Source: Cox, Roberson, and Smith 1982, table 7.
Note: Means for paired-comparison auction sequences. Two entries indicate two paired-comparison sequences for that value of *n*.

(cited in Research in Experimental Economics, V. L. Smith, ed., JAI Press)

2.10 Common value auctions: the bidder's view

Suppose each bidder gets a random signal of what the object is worth.

It is called a "signal" because it is a measurement (with error) of the object's value.

Example:

The object is an oil field with unknown production potential. The signal is a geologist's estimate of the oil reserves.

Extended example

There are four bidders. Bidder 1 gets a signal s_1, a random number between 0 and 100.

The other bidders receive signals s_2, s_3 and s_4.

Suppose that the true value of the object is the average of the signals:

$$V = (s_1 + s_2 + s_3 + s_4)/4$$

Without knowing anything about any of the signals, my estimate of V is EV=50.

Suppose that I know my signal is s_1=90. Now what is $E(V \mid s_1 = 90)$?

$$E\left(V \mid s_1 = 90\right) = \left(90 + 50 + 50 + 50\right)/4 = 60$$

Should I bid 60?

What if I win?

Suppose that we (all the bidders) are identical. If we get a higher signal, then we make a higher bid.

If I win the auction, I must have made the best bid ⇒ I must have had the highest signal.

$$E\left(V \mid s_1 = 90 \text{ and } W\right)$$
$$= E\left(V \mid s_1 = 90 \text{ and } s_2, s_3, s_4 < 90\right)$$

Result: Suppose that *s* is uniformly random between 0 and 100. Conditional on knowing that $s<90$, *s* is uniformly random between 0 and 90.

This implies that $E\left(s \mid s < 90\right) = 45$, and

$$E\left(V \mid s_1 = 90 \text{ and } s_2, s_3, s_4 < 90\right)$$
$$= \left(90 + 45 + 45 + 45\right)/4 = 56.25$$

Should I bid 56.25?

In a first-price or Dutch auction: No. (Suppose the price drops to 56.25 and I call "Mine!." What have I won?)

In a second-price auction: Yes.

In an English auction, should I bid up to 56.25? No. (By the time we get up to 56.25, I'll find out something about the other bids.)

The effect of increasing the number of bidders

Now suppose that there were five bidders (including myself).

Then

$$E\left(V \mid s_1 = 90 \text{ and } W\right)$$
$$= \left(90 + 45 + 45 + 45 + 45\right)/5 = 54$$

So in going from 4 to 5 bidders, I'll drop my bid.

(If my estimate is highest in a field of 100, my overestimate of the true value is worse than if my estimate were highest in a field of 3.)

New material

How should you bid with a low signal/estimate?

To this point, we've seen that if my signal is 90, I should shade my bid way down, taking into account the possibility that my signal is in error.

Is my signal an overestimate simply because it happened to be above the overall expectation of 50?

Let's see what would happen if my signal is $s_1 = 10$.

With three other bidders,

$$E(V \mid s_1 = 10) = (10 + 50 + 50 + 50)/4 = 40$$

So all else equal, my signal is likely to be an underestimate.

But if I'm bidding, all else is not equal. I need to focus on what happens if I win. Specifically:

$$E(V \mid s_1 = 10 \text{ and I win})$$
$$= E(V \mid s_1 = 10 \text{ and } s_2, s_3, s_4 < 10)$$

Conditional on knowing that $s<10$, s is uniformly random between 0 and 10.

This implies that $E(s \mid s < 10) = 5$, and

$$E(V \mid s_1 = 10 \text{ and } s_2, s_3, s_4 < 10)$$
$$= (10 + 5 + 5 + 5)/4 = 6.25$$

Intuitively, if I win, my signal must be higher than all of the others, *even if my signal is extremely low relative to my prior expectations*.

As always, you need ask yourself the question: "Will I be happy if I win?"

2.11 Common value auctions: the seller's view

The winner's surplus

Suppose:

- We have two bidders (1 and 2) who get signals s_1 and s_2
- The true value of the object/mine/oil field is the average $V = (s_1 + s_2)/2$.

