Return and risk, then and now

BKM Chapter 5

Readings and problems

- Readings
  - 5.1 Interest rates, inflation and taxes
  - 5.2 EAR and APR (review from first class session)
  - 5.3 skip
  - 5.4 Measuring returns to risky securities, scenario analysis
  - 5.5 Statistical measures that summarize historical data
  - 5.6 The normal distribution
  - 5.7-5.9 skip
  - Note “Five important numbers” (the last slide in the Ch. 5 deck)
- Practice problems: 1, 2, 6, 7, CFA problems 3, 4, 5.
Interest rates and inflation

- The interest rate is sometimes called “the price of money”
- But the value of money changes over time.
- How do markets recognize and adjust for this?

Measuring the value of money

- The US Consumer Price Index (CPI)
  - Compiled by the US Bureau of Labor Statistics
  - A survey measure of the cost of things purchased by a representative consumer
  - Indexed to 100 as of 1982-84 (“base period”)
  - Widely used to adjust (“index”) labor contracts, social security payments, etc.
Inflation, $i$

- Inflation is the rate of change in a price index.
  - Over 2012, the CPI rose by 1.7%
  - Over 1990, +6%
- The inflation rate can be negative.
- Some components of the CPI are volatile.
  - In April, 2013, the CPI energy index changed by $-4.3\%$
- A condition of pervasive negative inflation is called deflation.
  - In April, 2013, the overall CPI changed by $-0.4\%$. 
The nominal interest rate, $R$

- The interest rate measured in currency ($$)
- The usual, conventionally quoted rate.
- Suppose a loan is negotiated at a given $R$.
- Inflation over the life of the loan hurts the lender:
  - The $ received in payment have declined in value.
- ... and helps the borrower
  - The loan is repaid with “devalued” $ $

The real interest rate, $r$

- The interest rate measured in purchasing power.
- Suppose that an investment pays $R = 5\%$.
- If inflation is $i = 2\%$, the increase in the investors purchasing power is approximately $3\%$. 
Approximately?

- Correct relation:
  - \(1 + R = (1 + r)(1 + i)\)
  - \(R, r\) and \(i\) are in decimals (“0.04, not 4%”)
- Rearranging: \(R = r + i + r \times i\)
- Example: If \(r = 2\%\), \(i = 3\%\) then
  - \(R = 0.02 + 0.03 + 0.0006 = 0.0506\) (“5.06%”)
- Approximation \(R \approx r + i\)
  - works with decimals or percentages.

Problems

- Robert seeks a real return of \(r = 1\%\).
  - If he expects inflation to be \(i = 4\%\), what nominal rate does he need (approx.)? \(R = 1\% + 4\% = 5\%\)
- Rachel also seeks \(r = 1\%\), but expects inflation to be 6%.
  - What nominal rate does she need? \(R = \nosearch\)
The (Irving) Fisher Hypothesis

- Investors demand nominal rates that include expected inflation.
  - \[ R \approx r + E_i \]
  - This is a statement that involves future beliefs and behavior.
- “\[ R \approx r + i \]” is usually stated as the definition of the real rate.
  - “If the nominal rate over last year was 6% and inflation was 4%, what was the real return?”

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Historical interest rates and inflation

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![Graph showing historical interest rates and inflation](image)
Nominal returns and inflation: the history

Taxes and inflation

- Taxes are based on nominal income (interest and capital gains).
- A representative US personal tax rate is \( t = 30\% \).
- After-tax nominal return = \( R(1 - t) \)  \( \frac{1\% (1 - 0.3)}{1\%} = 0.7\% \).
- After-tax real return = \( R(1 - t) - i \)  \( 0.7\% - 3\% = -2.3\% \).
- Suppose:
  - A money-market account pays 1%.
  - The tax rate is 30%.
  - The inflation rate is 3%.
- What are the after-tax nominal and real rates of return?
Measuring (nominal) returns on securities

- Most investments give us a return from
  - Cash payments, and
  - Price appreciation (or depreciation).
- The holding period return (HPR) is a summary estimate of the overall return.

The HPR on a stock investment

- \[ HPR = \frac{\text{Price}_{\text{Ending}} + \text{Dividend} - \text{Price}_{\text{Beginning}}}{\text{Price}_{\text{Beginning}}} \]
- “Period:” a day, month, quarter or year.
- Dividends assumed to be at end of the period.
- This is a “mark to market” return.
- The HPR is intended to represent the return an investor could actually have obtained.
- Also note the discussion of EAR and APR, BKM pp. 122-123.
HPR Example

- On Jan 31, XYZ stock closes at $31.00
- On Feb 28, the closing price is $30.00
- On Feb 20, XYZ pays a dividend of $0.43.
- The HPR over February is computed as
  \[ HPR = \frac{30.00 - 31.00 + 0.43}{31.00} = -0.0184 = -1.84\% \]
- Sometimes decomposed as
  \[ HPR = \frac{30 - 31}{31} + \frac{0.43}{31} = -3.23\% + 1.39\% = -1.84\% \]

HPR Embedded problem

- On June 30, ABC closed at $43.20
- On July 31, ABC closed at $44.92
- The HPR over July was 4.84%.
- What was the dividend yield?
Embedded problem (Answer)

- On June 30, ABC closed at $43.20
- On July 31, ABC closed at $44.92
- The HPR over July was 4.84%.
- What was the dividend yield?
- \[
    \frac{44.92 - 43.20}{43.20} + \text{div yield} = 3.98\% + \text{div yield} = 4.84\% \\
    \Rightarrow \text{div yield} = 4.84\% - 3.98\% = 0.86\%
    \]

The HPR and risk

- \( \text{HPR} = \frac{\text{Price}_{\text{Ending}} + \text{Dividend} - \text{Price}_{\text{Beginning}}}{\text{Price}_{\text{Beginning}}} \)
- In forecasting the HPR, the final price of the stock is a very large source of uncertainty.
  - The current (beginning) price is known; dividends are relatively stable.
- We try to summarize the uncertainty by viewing the ending price (or the HPR) as a *random variable* and computing its *statistics*. 
Describing random returns: “scenario analysis”

- In “scenario analysis” we summarize our beliefs in a table of states, probabilities, and returns.
- Example:

<table>
<thead>
<tr>
<th>State (of the economy)</th>
<th>Probability</th>
<th>Return on ABC stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>0.30</td>
<td>0.15</td>
</tr>
<tr>
<td>Normal</td>
<td>0.50</td>
<td>0.09</td>
</tr>
<tr>
<td>Bad</td>
<td>0.20</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

- From this table, we can compute expected returns, return standard deviations, and so forth (see BKM 5.4)
- In practice: How many states? Where do the probabilities come from? What’s the correspondence between states and returns?
  - These are difficult things to determine.

Describing random returns: “time series analysis”

- We collect a sample of past returns.
- Plot the returns to see how they are distributed.
  - These plots are called histograms.
- Compute sample statistics
  - The average return, the sample variance and standard deviation, and so forth.
- The sample statistics are estimates.
  - The average return over a sample is an estimate of the “true” (unknown) mean return.
  - Sample variances and standard deviations are also estimates of the corresponding “true” (but unknown) values.
The US experience

- The S&P 500 index is an average of the prices of the 500 largest US-listed corporations.
  - You can “buy” the S&P 500.
  - An investment in ticker symbol SPY will have a return equal to the return on the S&P 500 index.
- Spreadsheet `histrepSP.xls` contains annual returns on the S&P 500 index and T-bills from 1928 to the present.
  - [http://www.stern.nyu.edu/~adamodar/pc/datasets/histretSP.xls](http://www.stern.nyu.edu/~adamodar/pc/datasets/histretSP.xls)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Year</td>
<td>Annualized</td>
<td>Stocks (S&amp;P 500)</td>
<td>3-month T-Bill</td>
<td>10-year T-Bond</td>
<td>Stocks – T-Bills</td>
</tr>
<tr>
<td>2</td>
<td>1928</td>
<td></td>
<td>43.81%</td>
<td>3.08%</td>
<td>0.84%</td>
<td>40.73%</td>
</tr>
<tr>
<td>3</td>
<td>1929</td>
<td></td>
<td>-8.30%</td>
<td>3.16%</td>
<td>4.20%</td>
<td>-11.46%</td>
</tr>
<tr>
<td>4</td>
<td>1930</td>
<td></td>
<td>-25.12%</td>
<td>4.55%</td>
<td>4.54%</td>
<td>-29.67%</td>
</tr>
<tr>
<td>5</td>
<td>1931</td>
<td></td>
<td>-43.84%</td>
<td>2.31%</td>
<td>-2.56%</td>
<td>-46.15%</td>
</tr>
<tr>
<td>6</td>
<td>1932</td>
<td></td>
<td>-8.64%</td>
<td>1.07%</td>
<td>8.79%</td>
<td>-9.71%</td>
</tr>
</tbody>
</table>

...  

| 85  | 2011       | 2.10%      | 0.03%        | 16.04%       | 2.07%        | -13.94%      |
| 86  | 2012       | 15.89%     | 0.05%        | 2.97%        | 15.84%       | 12.92%       |
| 87  | 2013       | 32.15%     | 0.07%        | -9.10%       | 32.08%       | 41.25%       |
| 88  | 2014       | 13.48%     | 0.05%        | 10.75%       | 13.42%       | 2.73%        |

Note on data sources: BKM report statistics on a portfolio of all US stocks (not just the S&P 500), so their estimates differ slightly from what you might compute in this spreadsheet.
Useful statistics

- Average returns
  - Arithmetic mean
  - Geometric mean
- Average excess returns
  - In excess of the risk-free return.
- Standard deviation of returns and excess returns
- Sharpe ratio
  - Reward-to-variability ratio

Measuring average returns

- Suppose a risky investment had a return \( r \) of 15% in the first year and 5% in the second year.
- The arithmetic mean return is \( r_a = \frac{15 + 5}{2} = 10\% \)
  - This is the best estimate of the return we should expect over any one year
- What if the investment horizon is two years?
  - If we put in $1 and compound \( r_a = 10\% \) over two years, we'd expect to have $1.21
  - If we actually invest $1 in for two years, we'd have \( 1.15 \times 1.05 = 1.2075 \) (NOT 1.21)
  - What is the actual compound return? \(-1 \text{ PV}; 1.2075 \text{ FV}; 2 \text{ N} ; 1/YR \rightarrow 9.89\% \)
- 9.89% is the geometric mean return, \( r_g \).
  - The compound return over the whole sample.
An investor looks back ...

- “I had a bad year in 2008. My portfolio was down 50%. But in 2009 it was up 50%, so I got everything back.”

- Arithmetic mean return? \[ \frac{50\% - 50\%}{2} = 0 \]

- Geometric mean return? \[ \prod \left( 1 - 0.50 \right) \times \left( 1 + 0.50 \right) = \$ 0.75 \]

- Do these numbers depend on the order of the outcomes?

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### Historical arithmetic and geometric means

**1929 - 2014**

<table>
<thead>
<tr>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annualized returns on:</td>
<td>Stocks (S&amp;P 500)</td>
<td>3-month T-Bill</td>
<td>10-year T-Bond</td>
</tr>
<tr>
<td>N</td>
<td>87</td>
<td>87</td>
<td>87</td>
</tr>
<tr>
<td>Mean</td>
<td>11.53%</td>
<td>3.53%</td>
<td>5.28%</td>
</tr>
<tr>
<td>Variance</td>
<td>396 ( %^2 )</td>
<td>9 ( %^2 )</td>
<td>61 ( %^2 )</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>19.90%</td>
<td>3.06%</td>
<td>7.83%</td>
</tr>
<tr>
<td>Std. Err. of Mean</td>
<td>2.13%</td>
<td>0.33%</td>
<td>0.84%</td>
</tr>
<tr>
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<td>9.97%</td>
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Assessing risk: the variance and the standard deviation

- Recall the sample estimate of the variance is
  \[ \hat{\sigma}^2 = \frac{1}{n} \sum_{s=1}^{n} [r(s) - \bar{r}]^2 \]

- The standard deviation is the square root of the variance: \( \hat{\sigma} = \sqrt{\hat{\sigma}^2} \)
  - In finance, the standard deviation is sometimes called "volatility".
- Both measure the sample dispersion (spread) about the mean.
  - But \( \hat{\sigma} \) has the same units as the original data, so it is often easier to understand.

\( \sigma \) and the Normal Distribution

“three-sigma events”

**FIGURE 5.4** The normal distribution

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### Variance and standard deviation in the sample

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### The risk-return trade-off

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Uncertainty about the (arithmetic) mean

- The arithmetic mean is an estimate of the true ("population") mean.
- The standard deviation of the estimation error in the mean is called the *standard error of the mean*.
- The estimate \( \pm 2 \times \text{standard error} \) defines a (somewhat) customary ("95\%") *confidence interval*.

<table>
<thead>
<tr>
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<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{r} )</td>
<td>Annualized returns on:</td>
<td>Stocks (S&amp;P 500)</td>
<td>3-month T-Bill</td>
<td>10-year T-Bond</td>
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<td>0.33%</td>
<td>0.84%</td>
<td></td>
</tr>
<tr>
<td>Confidence interval</td>
<td>((7.26% \text{ to } 15.79%))</td>
<td>((2.87% \text{ to } 4.19%))</td>
<td>((3.60% \text{ to } 6.95%))</td>
<td></td>
</tr>
<tr>
<td>Geometric mean</td>
<td>9.97%</td>
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</table>

- "We are 95\% sure that the true mean return on the S&P 500 lies between 7.26\% and 15.79\%"
- This is an imprecise statement, and it is contingent on a number of assumptions that aren’t spelled out here.
The excess returns

- We expect that a risky investment (like stock) should outperform a risk-free investment (like T-bills).
  - Investors demand extra compensation for bearing risk.
- \( \text{Excess Return} = \text{HPR}_{stock} - r_f \)
  - where \( r_f \) is the HPR on T-bills (or similar)

<table>
<thead>
<tr>
<th>Year</th>
<th>Annualized returns on:</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
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</tr>
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<td></td>
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<td>Stocks (S&amp;P 500)</td>
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</tr>
<tr>
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<td></td>
<td>-8.30%</td>
<td>3.16%</td>
<td>4.20%</td>
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<tr>
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<td></td>
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<td>4.55%</td>
<td>4.54%</td>
<td>-29.67%</td>
<td>-29.66%</td>
</tr>
</tbody>
</table>

...
The average excess return

- Recall that for a single period $Excess\ Return = HPR_{risky\ investment} - r_f$
  - where $r_f$ is the HPR on T-bills (or similar)
- The (arithmetic) average excess return is often used as an estimate of the expected risk premium
  - Expected return on the risky security over and above the risk-free return.

Variation in outcomes

- How representative is the mean?
  - Is the mean a good approximation to the return we’d have obtained in a randomly chosen year?
- How much variation is there (above and below the mean)?
- Are there “extreme” returns?
- Do these tend to be positive or negative?
The Sharpe (Reward to volatility) ratio

- Measures “return relative to risk”
- \[ \text{Sharpe} = \frac{(\text{Arithmetic}) \text{ average excess return}}{\text{Standard deviation of excess return}} \]
- For S&P 500:
  \[ \text{Sharpe} \approx \frac{8}{20} = 0.4 \]
- “For every extra 1% of (standard deviation) risk we take on, we earn an additional 0.4% in average return.”
Five Important Numbers

- S&P 500 stocks 1928-2014
- Mean returns
  - Arithmetic mean return: 11.5%
  - Geometric mean return: 10.0%
- Excess return
  - Arithmetic mean: 8.0%
  - Standard deviation: 19.9%
- Sharpe ratio 0.4