Index models

BKM Ch. 8
Readings and problems

- 8.1 (skip) “It is difficult to compute the optimal portfolio when we have many stocks.”
- **8.2 The single index model.**
  (skip: “the set of estimates needed for the single-index model”)
- **8.3 Estimating the single-index model.**
- 8.4 skip.
- 8.5 Read “Alpha Betting” on p. 283, otherwise skip.
- Practice problems 4, 6-8; CFA problems 2-5
Factor Models

- A factor model is a statistical model of stock returns.
- A stock return is assumed to be driven by one or more factors, like...
  - Exchange rates, commodity prices, interest rates, etc.
- A factor model
  - Helps us to understand the sources of risk for a stock or portfolio.
  - Makes possible the determination of the optimal portfolio $P$ when we have many risky assets.
An index model is a kind of factor model in which the factors are returns on one or more stock indexes.

- For example, the S&P 500, industry indexes, etc.

A single-index model is based on one index/factor.

- Usually a market-wide index (like S&P 500)
- In this context, “M” is the market-wide index.
Example: A single-index model for HomeDepot (HD)

- Collect historical data on the returns to HD and M (the S&P 500): \( r_{HD} \) and \( r_M \)
- Compute excess returns (above the risk-free return)
  \[ R_{HD} = r_{HD} - r_f \] and \( R_M = r_M - r_f \)
- Run a simple linear regression of \( R_{HD} \) against \( R_M \)
  \[ R_{HD}(t) = \alpha_{HD} + \beta_{HD} \times R_M(t) + e_{HD}(t) \]
  - \( R_{HD}(t) \) is the excess return in a particular time period, \( t \).
  - The regression line (the best-fit line) is also called the Security Characteristic Line (SCL)
Security Characteristic Line for HD from Bloomberg

- Notes
  - Bloomberg actually uses returns (instead of excess returns), but the interpretation is similar.
  - The data are weekly returns.
  - The sample is two years (September 2005 to September 2007).
For explanation.

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Raw BETA</td>
<td>1.091</td>
</tr>
<tr>
<td>Adj BETA</td>
<td>1.001</td>
</tr>
<tr>
<td>ALPHA(Intercept)</td>
<td>-0.255</td>
</tr>
<tr>
<td>R2(Correlation)</td>
<td>0.282</td>
</tr>
<tr>
<td>Std Dev Of Error</td>
<td>2.669</td>
</tr>
<tr>
<td>Std Error Of ALPHA</td>
<td>0.266</td>
</tr>
<tr>
<td>Std Error Of BETA</td>
<td>0.173</td>
</tr>
<tr>
<td>Number Of Points</td>
<td>103</td>
</tr>
</tbody>
</table>

$Y = HOME\ DEPOT\ INC$

$X = S&P\ 500\ INDEX$

$Y = 1.091X - 0.255$

$R^2 = 0.282$

Equity BETAS

$\beta_{HD} = \frac{\text{S&P 500 Index}}{\text{Home Depot Inc}}$

$\beta_{RM} = \frac{\text{S&P 500 Index}}{\text{Market Index}}$

$R^2_{HD} = 11.7\%$

$R^2_{RM} = 6.2\%$

Last Observation

*Last Observation*
Interpretation

- \( R_{HD}(t) = \alpha_{HD} + \beta_{HD} \times R_M(t) + e_{HD}(t) \)
  - \( \beta_{HD} \) is like a multiplier.
    - If the market is up 1%, HD should be up 1.091%.
  - \( \alpha_{HD} \) measures the average excess return on HD net of market movements.
    - Over this sample, HD had an average excess return of \(-0.255\%\) per week, after removing what can be explained by market movements.
- The standard deviation of the error is
  - \( \sigma(e_{HD}) = 2.669 \)
The regression correlation

- The regression correlation ("R2") is
  - $R^2 = 0.282$
    - "The regression explains 28.2% of the variation in $R_{HD}$.”
  - $R^2 = \rho^2 = 0.282 \Rightarrow \rho = \sqrt{0.282} = 0.531$
  - We know that $\rho > 0$ because the regression line slopes upwards.
Interpretation of a single observation

Actual \( R_{HD} = 15\% \)

\( e_{HD} = 4.345\% \)

Predicted \( R_{HD} = -0.255 + 1.091 \times 10\% \)

\[ = 10.655 \]

\( \alpha_{HD} = -0.255 \)

\( R_{HD}(t) = \alpha_{HD} + \beta_{HD} \times R_M(t) + e_{HD}(t) \)

\( R_M = 10\% \)

\( e_{HD} = 4.345\% \)
Interpreting SCLs

- The following graphs present data for two stocks, A and B.
- Draw where you think the best fit lines should go.
- Based on these lines, what can we say about the relative $\alpha$s, $\beta$s and $\sigma_e$s?
Embedded problem: Compare the $\alpha$, $\beta$, and $\sigma_e$ for A and B.

Higher?  
Lower?  
About the same?

$\alpha_B > 0$  $\alpha_A < 0$  
$\beta_A \sim \beta_B$  
$\sigma_{e,A} \sim \sigma_{e,B}$
Answer: Compare the $\alpha$, $\beta$, and $\sigma_e$ for A and B.

Higher?  
Lower?  
About the same?

$\alpha_A < \alpha_B$  
$\beta_A \approx \beta_B$  
$\sigma_{e,A} \approx \sigma_{e,B}$

The best fit lines have about the same slope (same $\beta$), but the intercepts differ: $\alpha_B > \alpha_A$. 
Embedded problem: Compare the $\alpha$, $\beta$, and $\sigma_e$ for A and B.

Higher?
Lower?
About the same?

$\alpha$
$\beta$ $\beta_B > \beta_A$
$\sigma_e$ $\sigma_{e,A} \approx \sigma_{e,B}$
Answer: Compare the $\alpha$, $\beta$, and $\sigma_e$ for A and B.

The best fit lines have about the same intercept (same $\alpha$), but the slopes differ: $\beta_B > \beta_A$. 

$\alpha_A \approx \alpha_B$

$\beta_B > \beta_A$

$\sigma_{e,A} \approx \sigma_{e,B}$
Embedded problem: Compare the $\alpha$, $\beta$, and $\sigma_e$ for A and B.

Higher?  
Lower?  
About the same?

$\alpha$ same  
$\beta$ same  
$\sigma_e$ $\sigma_{e,B} > \sigma_{e,A}$
Answer: Compare the $\alpha$, $\beta$, and $\sigma_e$ for A and B.

Higher?  
Lower?  
About the same?

$\alpha_A \approx \alpha_B$  
$\beta_A \approx \beta_B$  
$\sigma_{e,A} < \sigma_{e,B}$

The best fit lines have about the same intercept (same $\alpha$) and about the same slope (same $b$). However, the points for A lie almost exactly on the line, while B’s points are farther from the line. This means that B’s errors are larger: $\sigma_{e}(B) > \sigma_{e}(A)$. 

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Raw returns or excess returns?

- Bloomberg (and many financial sites) estimate $\beta$ based on raw returns: $r_{HD}$ and $r_M$ are the “percent” returns on HD and the market.

- Bodie, Kane and Marcus (and many practitioners) estimate $\beta$ based on excess returns, the $Rs$. 
Interpreting risk

- $R_{HD}(t) = \alpha_{HD} + \beta_{HD} \times R_M(t) + e_{HD}(t)$
  - $\alpha_{HD}$ constant
  - $\beta_{HD}$ market-driven, systematic
  - $e_{HD}(t)$ non-market, idiosyncratic, firm-specific

- $\text{Variance}(R_{HD}) = \sigma_{HD}^2 = \beta_{HD}^2 \times \sigma_M^2 + \sigma^2(e_{HD})$
  - $\sigma_{HD}^2$ total risk
  - $\beta_{HD}^2 \times \sigma_M^2$ market or systematic risk
  - $\sigma^2(e_{HD})$ non-market or idiosyncratic risk

- For this period, $\sigma_M \approx 1.153\%$ per week, so
  - $\sigma_{HD}^2 = 1.091^2 \times 1.153^2 + 2.669^2 \approx 2.8 + 7.1 = 9.9$

- Note: $\frac{2.8}{9.1} = 0.283 \approx R^2$
- Note: $\beta$s are often used to rank stocks’ market risks.
SCL Example (E*Trade is a brokerage)

\[ Y = \text{E*TRADE FINANCIAL CORP} \]
\[ X = \text{S&P 500 INDEX} \]

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
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<tbody>
<tr>
<td>Raw BETA</td>
<td>1.726</td>
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<tr>
<td>Adj BETA</td>
<td>1.486</td>
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<tr>
<td>ALPHA(Intercept)</td>
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<td>R2(Correlation)</td>
<td>0.306</td>
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<tr>
<td>Std Dev Of Error</td>
<td>4.761</td>
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<tr>
<td>Std Error Of ALPHA</td>
<td>0.778</td>
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<tr>
<td>Std Error Of BETA</td>
<td>0.433</td>
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<td>Number Of Points</td>
<td>38</td>
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\[ \beta_{HP} \approx 1.09 \]
\[ \beta_{ETR0} = 1.7 \]

* Last Observation
SCL Example (Canyon is a mining stock.)

$$\beta_{CAU} = -0.43$$

**CONF**
GLD (the gold ETF, www.nytimes.com)
A long-short portfolio

- Hedge funds often pursue “long-short” strategies.
- The fund believes that the stocks held long will appreciate, and that the stocks held short will depreciate.
- The proceeds from the short sales partly finance the long positions.
- The amounts are set so that there is no net exposure to the market.
- The fund is “market-neutral.”
Example

- Suppose that
  - $R_{MSFT} = \beta_{MSFT} R_M + e_{MSFT}$ where $\beta_{MSFT} = \frac{1}{3}$
  - $R_{GE} = \beta_{GE} R_M + e_{GE}$ where $\beta_{GE} = 1$
- We set up the following:

<table>
<thead>
<tr>
<th></th>
<th>Assets</th>
<th>Liabilities+NW</th>
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</thead>
<tbody>
<tr>
<td>MSFT</td>
<td>15,000</td>
<td>5,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GE sold short</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Net Worth</td>
</tr>
</tbody>
</table>
If $R_M = 10\%$, we expect: $R_{GE} = 10\%$ and $R_{MSFT} = 3.33\%$

<table>
<thead>
<tr>
<th></th>
<th>Assets</th>
<th>Liabilities+NW</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSFT +3.33%</td>
<td>15,500</td>
<td>5,500 + 1D 9%</td>
</tr>
<tr>
<td>GE sold short</td>
<td></td>
<td>GE sold short</td>
</tr>
<tr>
<td></td>
<td>10,000</td>
<td>Net Worth</td>
</tr>
</tbody>
</table>

- The strategy has removed market risk
  - “Market neutral” / “Delta neutral”
  - This is what makes a hedge fund “hedged”

- We’re betting that MSFT’s idiosyncratic return components will exceed GE’s: $e_{MSFT} > e_{GE}$

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Does diversification really work when we need it?

- Some recent situations where we thought the correlation was low (but it really turned out to be high)
  - International diversification
  - Default risk of mortgages
  - Funds of [hedge] funds
International diversification

- Securities: stock markets in US, Europe and Asia
- Historical correlations were positive, but small.
  - There appeared to be a benefit to a US investor in diversifying across countries.
- During the financial crisis there were widespread and pervasive market declines in all countries.
Default risks

- Securities: commercial and residential mortgages.
- Historical correlations on defaults were small.
  - We thought that we’d get diversification by pooling mortgages and mortgage-backed securities.
- During the financial crisis, it turned out that:
  - Defaults were highly correlated.
  - Benefits of diversification were small.
  - Credit ratings overstated.
Funds of Funds

- Hedge funds attempt to achieve high returns (at the price of higher risk)
  - Could we keep the high returns (on average) while reducing the risks by diversification?
- A fund of funds attempts to hold a portfolio of hedge funds that are not highly correlated.
  - Historical evidence suggested that this was possible.
- Recent experience suggests:
  - There has been great convergence in funds’ trading strategies.
  - Hedge fund returns are positively correlated.
same P for everybody

P is market portfolio

easy way to apply portfolio

Risk-return tradeoff for indiv. stocks.