The capital asset pricing model (CAPM)

BKM Chapter 9

Outline

- 9.1 The development and implications of the CAPM
- *Skip the remaining sections*
- Problems: Concept check problems; Problem sets: 1-3, 6, 7, 10-16, 17; CFA problems 3-9
The individual forms a portfolio

- Estimate $E_r, \sigma$, $\rho$ for all risky securities.
- Construct set of efficient portfolios
- Look up $r_f$
- Identify the tangency portfolio $P$
- Pick a point on the resulting Capital Allocation Line

$P = \{SEP500\}$

100% in SEP500
The CAPM framework

- The perspective shifts from one investor to a market that includes all investors.
- If all investors agree on expected returns, standard deviations and correlations, then they agree on:
  - The composition of portfolio $P$.
  - The position of the capital allocation line.
- Note: they can still disagree about how much of their net worth will be in $P$ (relative to $r_f$).

The CAPM’s two main results are

1. $P$ is the “market portfolio” denoted “$M$”
2. The expected return on any stock (or portfolio) depends only on its $\beta$ (from the index model)
Caution

- The assumptions and results are linked by a formal mathematical proof.
- However
  - The assumptions are violated (significantly).
  - The model is impossible to verify empirically.
- Therefore, the model is ...
  - An approximation.
  - Viewed with skepticism.

CAPM: The first main result

- "P" is the market portfolio $M$
- $M$ is formed by taking all stocks in proportion to their market capitalization.
The market portfolio of large U.S. stocks

- The market portfolio (in theory) covers all securities
  - The market portfolio of large US common stocks is often the starting point.
- A firm’s weight in the market portfolio is its market capitalization relative to the capitalization of the entire market
  - The market value of firm’s stock (“equity capitalization,” “market capitalization”) = number of shares × price per share
- The S&P 500 index comprises the 500 largest US firms and uses market value weights.
- Next overhead from S&P Fact Sheet (February, 2015)

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>NUMBER OF CONSTITUENTS</th>
<th>TOTAL MARKET CAP [USD MILLION]</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>502</td>
<td>19,760,271.61</td>
</tr>
</tbody>
</table>

Top 10 Constituents By Index Weight

<table>
<thead>
<tr>
<th>CONSTITUENT</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apple Inc.</td>
<td>AAPL</td>
</tr>
<tr>
<td>Exxon Mobil Corp</td>
<td>XOM</td>
</tr>
<tr>
<td>Microsoft Corp</td>
<td>MSFT</td>
</tr>
<tr>
<td>Johnson &amp; Johnson</td>
<td>JNJ</td>
</tr>
<tr>
<td>Berkshire Hathaway B</td>
<td>BRK.B</td>
</tr>
<tr>
<td>General Electric Co</td>
<td>GE</td>
</tr>
<tr>
<td>Wells Fargo &amp; Co</td>
<td>WFC</td>
</tr>
<tr>
<td>Procter &amp; Gamble</td>
<td>PG</td>
</tr>
<tr>
<td>JP Morgan Chase &amp; Co</td>
<td>JPM</td>
</tr>
<tr>
<td>Pfizer Inc</td>
<td>PFE</td>
</tr>
</tbody>
</table>
Suppose that the market consists of three stocks. What are their weights in the market portfolio?

<table>
<thead>
<tr>
<th>Stock</th>
<th>Price per share</th>
<th>Shares (MM)</th>
<th>Market cap.</th>
<th>Market portfolio weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>80</td>
<td>400</td>
<td>0.4</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>10</td>
<td>100</td>
<td>0.1</td>
</tr>
<tr>
<td>C</td>
<td>25</td>
<td>20</td>
<td>500</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Total: 1,000</td>
<td></td>
</tr>
</tbody>
</table>
If everyone (collectively) holds the market portfolio and $P$ is the same for everyone, then $P$ is the market portfolio ($M$).

\[
Er_c = r_f + \frac{Er_M - r_f}{\sigma_M} \sigma_C
\]

A practical implication: “Indexing”

- Matching your portfolio (of risky assets) to a popular index.
- CAPM provides a rationale for indexing based on a market-value-weighted index.
- Indexing is a passive strategy.
  - No security analysis; no “market timing”
What is the real “market portfolio”?

- What assets/securities do we want to consider?
- Theory says we should invest in everything.
  - But the theory ignores transaction costs.
  - As we move away from large domestic stocks and domestic government bonds, trading costs get much higher.

Should we always buy the market portfolio?

- Theory assumes everyone has the same beliefs.
  - We may feel especially positive or negative on a sector.
- Theory assumes that everyone is exposed to the same risks.
  - By virtue of where we live, work, etc., we’re exposed to different risks.
    - Employees of a firm are already highly exposed to the firm without taking on additional stock ownership.
  - Any or all of these imply we might want to diverge (“tilt”) from the market portfolio.
- Still, a well-diversified portfolio of stocks is a good starting point.
CAPM: The second main result

- The Security Market Line
  - A line (like the Capital Market Line) that gives the risk-return trade-off for individual stocks.

Why isn’t the Capital Market Line enough?

- The CML specifies the tradeoff between risk and return in complete portfolios (“C”).
- An individual stock will not lie on the CML.
How should we measure the risk of Home Depot (HD)?

- An investor cares only about risk in complete portfolio ($\sigma_C$).
- The risk in $C$ comes from the risk in the market portfolio $M$.
  - $\sigma_C = y \sigma_P = y \sigma_M$.
- What does HD contribute to $\sigma_M$?
- Answer based on:
  - Diversifiable vs. nondiversifiable risk.
  - The (market) index model.

Recall: Index (Market) Model

- $R_{HD}(t) = \alpha_{HD} + \beta_{HD} \times R_M(t) + e_{HD}(t)$
- $\beta_{HD}$ measures how much HD is driven by the market return.
- It measures the market portion of HD’s risk.
- Diversifiable risk will be eliminated in a large portfolio.
- So $E r_{HD}$ should depend only on market risk ($\beta_{HD}$).
Consider adding a new stock to your portfolio

- If the stock has a high positive $\beta$ ...  
  - It will have large price swings driven by the market  
  - It will increase the risk of my portfolios.  
  - I’ll demand a high $Er$ in compensation.
- If the stock has a negative $\beta$ ...  
  - It will move “against” the market.  
  - It will decrease the risk of my portfolio  
  - I’ll accept a lower $Er$ (in exchange for the risk reduction).

Security Market Line (SML)

- SML gives the relationship between $\beta$ and $Er$.

![SML diagram](image)

Note that $\beta$ can be negative.
Can we pin down the SML?

- In order to fix the position of the SML, we need to know two points.
- For the risk-free security, $\beta = 0$. (The risk-free return is constant; it isn’t “driven” by the market.)
- For the market itself, $\beta = 1$.
  - If we run a regression of $R_M$ vs $R_M$, the slope is 1. (Think about the scattergram.)

$Er = r_f + \beta_i(\hat{Er}_M - r_f)$

where

$Er_i$ is the expected return on stock (or security) $i$.
$\beta_i$ is security $i$’s beta.
Most stocks have $\beta$s between 0 and 3.

- In principle, a stock with $\beta < 0$ will have $E_r < r_f$
  - If $\beta \ll 0$, could have $E_r < 0$

- Like the CML, the SML is a statement about expected returns.
  - In any given year, a low-$\beta$ stock could have a high return and a high-$\beta$ stock could have a low return.

Assume $r_f = 1\%$ and $E_{r_m} - r_f = 9\%$

\[ E_{r_HD} = 1\% + 1.09 \times 9\% = 10.8\% \]
What are the expected returns?

- Assume \( r_f = 1\% \) and \( E_r_m - r_f = 9\% \)
- For E*Trade, \( \beta_{ETFC} = 1.7 \)
  - \( E_r_{ETFC} = r_f + \beta_{ETFC} \times (E_r_m - r_f) \)
  - \( 1\% + 1.7 \times 9\% = 16.3\% \)
- What is the expected return for Canyon Resources (\( \beta_{CAU} = -0.4 \))?
  - \( E_r_{CAU} = 1\% - 0.4 \times 9\% = -2.6\% \).
  - In principle, we should be willing to accept a low expected return on Canyon because it is such a great hedge against market risk. (Doubtful)
When a stock or fund is “off” the SML ...

\[ r_f + \beta_{XYZ} (E_r - r_f) \]

\[ \alpha_{XYZ} \]

The hunt for “positive alpha”

- \( \alpha \) is a measure of performance
  - if \( \alpha > 0 \), then the stock gives a higher return than its (market) risk would justify. **It is underpriced.**
  - if \( \alpha < 0 \), the stock is an underperformer. **It is overpriced.**

- Recall
  - If the CAPM is 100\% correct, all securities should have \( \alpha = 0 \)
  - To use the SML when we admit the possibility of \( \alpha \neq 0 \), we assume the CAPM is approximately correct or applies to “most” securities in the market.
How does the market react to a non-zero $\alpha$?

- If $\alpha > 0$, the stock is desirable (it is undervalued)
- Buying drives up the price ...
- ... and drives down the $Er$.
- If $\alpha < 0$, the stock is overvalued.
- Selling (and short-selling) drives the price down ...
- ... and drives the $Er$ up.

Finding alphas and betas

- $\beta$s are available on financial information sites
  - finance.yahoo.com: enter a ticker symbol (like “MSFT”),
    go to “key statistics”, look under “stock price history”.
- $\alpha$s are not often reported for individual stocks, but are used as a performance measure for mutual funds.
- Next: SPY and XLK from the New York Times
SPDR® S&P 500 ETF (SPY)

RISK METRICS
- Beta: 1
- Alpha: -0.1
- R-squared: 100
- Standard deviation: 9.48
- Sharpe ratio: 1.78

Technology Select Sector SPDR® Fund (XLK)

RISK METRICS
- Beta: 0.8
- Alpha: 6.49
- R-squared: 60.79
- Standard deviation: 10.73
- Sharpe ratio: 1.45

Source: New York Times, 10 March 2015
Estimates of mutual fund alphas, 1972-1991


$$\alpha$$ in the index model and the SML

- The single-index model is $R_i = \alpha_i + \beta_i R_M + e_i$
  - $R_i = r_i - r_f$; $R_M = r_M - r_f$
- The expected excess return is: $ER_i = \alpha_i + \beta_i ER_M + \overline{ee_i}$
- Expected errors average out to zero
- $ER_i - r_f = \alpha_i + \beta_i (ER_M - r_f)$
- Rearrange: $ER_i = r_f + \beta_i (ER_M - r_f) + \alpha_i$
- $\alpha_i$ measures performance not explained by the SML
The CML, SCL and SML: A summary

- Capital Market Line and Security Market Line
  - Both specify a relation between risk and \( E_r \)
  - Applicability.
    - CML is applicable only to an investor’s Complete portfolio
    - SML is applicable to any security, asset or portfolio.
  - Measure of risk.
    - In the CML, risk is measured by \( \sigma \).
    - In the SML, risk is measured by \( \beta \).
- The Security Characteristic Line is a regression ("best fit") line based on historical data (not expected returns).

\[
E_r = r_f + \frac{(E_r - r_f)}{\sigma_M} \sigma_c
\]

\[
E_r = r_f + \beta_i (E_r - r_f)
\]

Slope = estimate of \( \beta_i \)