Review

- An American call gives the holder the right to buy the underlying stock ($S$) at exercise/strike price $X$ up to and including maturity date $T$.
- An American put gives the holder the right to sell the underlying ...
- A European option can only be exercised at maturity.
- An option comes into existence when it is traded.
  - The person who sells a call has written the call, and is short the call

Payoff graphs for long positions in puts and calls.

- The payoff to the call at maturity is $\text{Max}(S_T - X, 0)$
- The payoff to the put at maturity is $\text{Max}(X - S_T, 0)$
Payoffs to long positions in the underlying and bond.

- The payoff to the stock.
- The payoff to a zero-coupon bond with par value 30

The payoff graphs for short positions (write a put, write a call, short the stock, borrow money) are vertically-flipped images of the payoff graphs to long positions.

- European put-call parity:
  - $Put + Stock = Bond + Call$
  - The put and call have the same strike price $X$ and the bond has a par value of $X$.
  - “=” means “has the same payoff”
    - This often implies “have the same market value”
And for practice ...

- Draw the payoff graph for a straddle (long one call with \( X = 40 \) and long one put with \( X = 40 \)).

- Draw the payoff graph for a combo (long one call with \( X = 40 \) and short one call with \( X = 60 \)).
Option valuation, hedging and replication
Outline

- Readings
  - 21.1 The time value of a call.
  - 21.2 Bounds on option values
  - 21.3 The binomial valuation model. Skip.
  - 21.4 The Black-Scholes valuation model.
  - 21.5, up to (but not including) “Hedging bets on mispriced options”
  - Also: BKM 20.5

- Problems
  - All concept checks
  - Practice problems 1-6, 8, 14,16-21, 32, CFA 5

A call with exercise price $X$.

- If the current stock price is $S$, the gain from immediate exercise is
- \textit{Intrinsic value} = \textit{Max}(S - X, 0)

\[ Call \ with \ X: \ 40 \]

\[ S \]

\[ \text{Max} \ S \ - \ X \ , \ 0 \]
The call value

- The call value is computed from the Black-Scholes equation, which will be covered in a while. For now, take the red line as the observed market value.
- The vertical difference between the two lines is the time value of the call.
- Example: If $X = 40$, $S = 48$ and $C = 15$, then the intrinsic value is $Max[48 - 40, 0] = 8$. The time value is $15 - 8 = 7$.

Problem

- Estimate the time value of the call at $S = 28$ and $S = 36$. 
The time value of a call is (somewhat, loosely) related to the time value of money.

- Suppose that the call is deep in the money: \( S \gg X \).
  - “The stock price is so high that exercise is certain.”
- If we want the stock at maturity, we can either
  - Buy the stock today and hold it. \( Cost = S \).
  - Buy the call today and spend \( X \) at maturity.
    - We can’t say the cost today is \( C + X \) because we don’t actually spend \( X \) until the option matures.
    - We can reason as: “To have \( X \) at maturity, we must invest \( PV(X) \) today.” \( Cost = C + PV(X) \)

Time value of call / money (an interpretation)

- Since both alternatives lead to the ownership of the stock, they should have the same outlay:
  - \( C + PV(X) = S \) or \( C = S - PV(X) \)
- The difference between \( S - PV(X) \) and the intrinsic value is
  - \( S - PV(X) - [S - X] = X - PV(X) \)
- \( X - PV(X) \approx \)
  - The accrued interest (at maturity) on an initial investment of \( PV(X) \).
- Suppose that we want \$11 in one year, and \( r = 10\% \).
  - The PV of the \$11 is \$10.
  - \$11 - \$10 = \$1 = “10% in the initial \$10 investment”
$C$, the value of a call depends on ...

- $S$ (if $S \uparrow$, then $C \uparrow$)
- $X$ (if $X$ could be retroactively increased after the call was issued, $X \uparrow$, then $C \downarrow$)
  - In fact, $X$ can’t be reset in this way. Think about comparing two calls with $X_1 > X_2$. $C_1 < C_2$.
- $r$, the interest rate (if $r \uparrow$, then $C \uparrow$).
- $T$, the maturity (if $T \uparrow$, then $C \uparrow$).
- For $r$ and $T$, recall that if the call is deep in the money, $C \approx S - PV(X)$.
  - If $r \uparrow$ or $T \uparrow$, then $PV(X) \downarrow$, and $- PV(X) \uparrow$. \[ PV(X) = \frac{X}{(1 + r)^T} \]

... and $C$ depends on the volatility of the underlying.

- If $\sigma$, the volatility, $\uparrow$, then $C \uparrow$.
- A volatility increase is symmetric
  - Good and bad outcomes become more extreme in equal measure.
- Call payoffs are asymmetric.
  - When you own a call, you benefit from the “upside” more than you’re hurt by the “downside”.

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A Low \( \sigma \) vs. a High \( \sigma \) return distribution.
- Relative to Low, High is spread-out symmetrically.
- But call payoffs are asymmetric.

... and \( C \) depends on the expected dividends paid on the underlying.

- Anything that causes the stock price to drop prior to maturity will hurt the value of the call.
- After a firm pays a dividend, its stock price drops by an equal amount.
  - The ex-dividend day drop.
- Exercise prices of options are not “adjusted for dividends”
Does $C$ depend on the stock’s expected return? Two views

- “Yes. High expected return means high stock price growth, so the call is more likely to have high value at maturity.”
  - To insiders with advance knowledge of good news announcements, calls are especially valuable.
- “No. The stock’s expected return does not appear in the Black-Scholes equation for $C$.”

Should we exercise an American call early?

- Generally “No”
  - If the stocks pays dividends, “maybe”.
- Recall that if $S \gg X$, $C = S - PV(X) > S - X$
  - If we exercise immediately we get $S - X$.
  - If we sell the call, we get $C > S - X$.
- For a stock that doesn’t pay dividends, don’t exercise early. Sell the call instead.
- This rule applies more generally, not just in cases where $S \gg X$. 
Suppose we own a call and \( S > X \)

- If we exercise immediately, our net inflow is \( S - X > 0 \).
- The alternative (deferred exercise)
  - Today
    - Keep the call unexercised.
    - Short the stock.
    - From the short sale proceeds, invest an amount equal to \( PV(X) \).
  - Our net inflow today is \( +S - PV(X) \).
  - This is more than we’d realize today if we exercise immediately.
  - What happens with this alternative at maturity?

The alternative strategy at maturity

- If \( S_T \geq X \) ...
  - Investment of \( PV(X) \) matures, \( +X \)
  - We exercise the call, paying \( X \) for the stock, \(-X\).
  - We deliver the stock to settle our short sale.
  - Net cash flow is \( +X - X = 0 \)

- If \( S_T < X \) ...
  - Investment of \( PV(X) \) matures, \( +X \)
  - We let call expire unexercised.
  - We buy the stock in the market, \(-S_T\) and deliver it.
  - Net cash flow is \( +X - S_T > 0 \)

The deferred-exercise alternative at maturity is at least as good as (and might be better than) the immediate exercise result.
Summary

- The deferred-exercise alternative at maturity is at least as good as (and might be better than) the immediate exercise result.
- We didn’t have to assume that the call could be sold for its “correct” price (whatever that might be).
- Our own expectations or beliefs didn’t enter into the argument.
- Early exercise is a poor choice even if
  - We can’t sell the call for some reason.
  - We believe that the stock is certain to fall below $X$ in the near future.

What if the market price of the call is $C < S - PV(X)$?

- Even if we don’t have a prior position in any security. A strategy similar to the deferred exercise alternative generates riskless profits.
- Today
  - Buy the call, $-C$
  - Short the stock, $+S$
  - Invest to realize $X$ at maturity, $-PV(X)$
  - Net cash flow today: $S - PV(X) - C > 0$.
- The cash flows at maturity are identical to those in the deferred exercise alternative.
- Arbitrageurs will buy the call, driving its price up, until $C \geq S - PV(X)$.
- A call (on a non-dividend-paying stock) must sell for at least $S - PV(X)$.
Why might you want to exercise early if the stock pays a dividend?

- Suppose that $S = $50 and after the close of trading, the firm “goes ex” a $10 dividend.
  - If you buy the stock today, you get $10.
  - If you buy tomorrow, you don’t get the $10.
- Suppose that you hold a call with $X = $45 and a maturity of two days.
  - The call can be exercised today for a gain of $S - X = 50 - 45 = 5$.
  - Tomorrow the stock will be trading around $40, and exercise is worthless.

We might want to exercise an American put early.

- Suppose that
  - We have a put option with $X = 100$ and a one-year maturity.
  - The current stock price is $S = $0.01.
- If we exercise the put immediately (force the put writer to buy at $X$) the gain is
  $$-0.01 + 100 = 99.99$$
- Why wait? The most we can gain is another $0.01.
Making option valuation more exact

- Two ways of getting a number for $C$, the value of a call.
  - Binomial option valuation (BKM 21.3, skip)
    - Strong assumptions
    - relatively simple math
    - not used in practice.
  - Black-Scholes option valuation (BKM 21.4)
    - Assumptions are strong, but more realistic than those for the Binomial.
    - Complex math.
    - Widely used in practice
  - The link: the Black-Scholes equation can be derived as a limiting case of the Binomial model.
Black-Scholes: the assumptions

- The stock (and risk-free bonds) may be bought and sold at any time without cost.
  - This allows us to construct and maintain perfectly hedged (risk-free) portfolios of stocks, bonds and calls.
  - In fact, real-world options traders use approximate (low risk) hedges.
- The stock price moves as the accumulation of small (infinitesimal) random changes that have constant volatility.
  - There are no sudden announcements or surprises.
  - In fact, the possibility of “jumps” can lead to discrepancies between Black-Scholes valuations and actual market prices.

The Black-Scholes equation for \( C \), the value of a European call on a non-dividend paying stock.

\[
C = S_0 \times N(d_1) - X \times e^{-rT} \times N(d_2)
\]

- \( r \) is the (risk-free) interest rate for borrowing and lending.
- \( T \) is the time remaining to maturity.
- \( d_1, d_2 \), and \( N(\cdot) \) will be discussed later.
Continuous compounding

- At annual interest rate \( r \), for \( T \) years, the future value of $1 is ...
  - \((1 + r)^T\) when \( r \) is compounded annually.
  - \(\left(1 + \frac{r}{m}\right)^{T \times m}\) when \( r \) is compounded \( m \) times per year.
- In the limit, as \( m \to \infty \),
  the future value converges to \( e^{rT} \).
  - \( e \approx 2.71828 \) is the base of the natural logarithms
- The present value of $1 with continuous compounding is \( PV = e^{-rT} \)

Working with continuous compounding

- Calculators
  - HP-10BII: orange \( e^x \) function (on the “1” key)
  - HP-12C: blue \( e^x \) function (on the “1/x” key)
- Examples
  - The FV of 100 invested for 2 years at 10% cont. comp. is \( 100 \times e^{2 \times 0.10} = 100 \times e^{0.2} = $122.14 \)
  - The PV of 90 to be received in 5 years at 10% cont. comp. is \( 90 \times e^{-5 \times 0.10} = 90 \times e^{-0.5} = 54.59 \)
For a call deep in the money \((S \gg X)\), \(N(d_1) \approx N(d_2) \approx 1\)

- \(C = S_0 \times N(d_1) - X \times e^{-rT} \times N(d_2)\)
- \(\approx S_0 - X \times \frac{e^{-rT}}{PV(X)}\)
- The formula is consistent with the \(C \geq S - PV(X)\) result developed earlier.

More generally (whether or not \(S_0 \gg X\))

- \(d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma \sqrt{T}}\) and \(d_2 = d_1 - \sigma \sqrt{T}\)
- \(N(d)\) is the cumulative distribution for the standard normal density evaluated at \(d\).
Example

- The parameters used to generate the call values for the graph on overhead 4 are
  - \( X = 40 \)
  - \( T = 1 \text{ year} \)
  - \( r = 0.10 \) (10\% per year)
  - \( \sigma = 0.50 \) (50\% per year)

- At \( S_0 = 48 \)
  - \( d_1 = 0.815, d_2 = 0.315, \)
  - \( N(d_1) = 0.792, N(d_2) = 0.623 \)
  - \( C = 15.47 \)

Why doesn’t expected return matter?

- The value of the call is determined in the hedge portfolio. In the hedge portfolio,
  - The stock is never held for its “long term investment” properties.
  - It is held only to offset the risk in the call.
- The amount we have to hold doesn’t depend on \( Er \).
- The amount \( does \) depend on \( \sigma \).
European vs. American calls

- $C$ is technically the value of a European call.
  - If we had an American call, though, we’d never exercise it early.
  - The early-exercise feature of an American call has no value.
  - $C$ is also the value of an American call.
- This is only true if the stock doesn’t pay dividends.

European calls and European puts

- Put call parity for European options
- A stock + a put has the same payoff as a bond + a call
  - where the put and call have the same exercise price $X$ and the bond has par value $X$. 

![Graph 1](image1.png)

![Graph 2](image2.png)
Law of one price (portfolios that deliver identical payoffs must sell in the market for the same price)

\[ S + P = Bond + C \]

- The value of the bond is \( PV(X) = Xe^{-rT} \)

- The Black-Scholes value of a European put is

\[ P = Xe^{-rT} + \underbrace{C}_{\text{from Black-Scholes}} - \underbrace{S}_{\text{market}} \]

- Recall that an American put might be exercised early, so \( P \) is not also the value of an American put.

- With \( S = 48, X = 40, r = 0.10, \sigma = 0.5, T = 1, \)

\[
\begin{align*}
P &= 40 \times e^{-0.1 \times 1} + 15.47 - 48 \\
&= 36.20 + 15.47 - 48 \\
&= 3.67
\end{align*}
\]

The Black-Scholes value of a European put option

- Note: the Black-Scholes value for the European put is sometimes below the intrinsic value of an American put.

  "We’d like to exercise it (but we can’t)."
The online.cboe/ivolatility options calculator

- The calculator uses a very general algorithm to deal with dividends, American vs. European options, and so on.
- The calculator can automatically link to real-world parameters for traded options.
- Caution: the calculator only gives approximate answers.
  - For $C(X = 40, S = 48, r = 10\%, T = 1, \sigma = 50\%)$:
    - A Black-Scholes calculation on Excel gives $C = 15.47$
    - The online calculator gives $C = 15.39$. 
Options and volatility

- Value of a call option depends on $S, X, T, r$ and $\sigma$.
- $\sigma$ = the standard deviation of the stock's annual return.
- We can't observe this directly.
- A common approach is to estimate from past data.
  - In portfolio theory applications, often estimate over sample of two prior years.
  - In option applications, use shorter samples, e.g., 10, 20 or 30 days.
- Next pages from ivolatility.com

Amazon (AMZN) volatility (from ivolatility.com)

$\sigma_m \approx 20\%$

<table>
<thead>
<tr>
<th></th>
<th>Current</th>
<th>1 WK AGO</th>
<th>1 MO AGO</th>
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<tbody>
<tr>
<td>HISTORICAL VOLATILITY</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 days</td>
<td>26.52%</td>
<td>13.96%</td>
<td>21.07%</td>
</tr>
<tr>
<td>20 days</td>
<td>20.88%</td>
<td>13.82%</td>
<td>19.83%</td>
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<tr>
<td>30 days</td>
<td>20.62%</td>
<td>17.52%</td>
<td>18.68%</td>
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<tr>
<td>IMPLIED VOLATILITY</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>IV Index call</td>
<td>41.87%</td>
<td>39.56%</td>
<td>28.08%</td>
</tr>
<tr>
<td>IV Index put</td>
<td>41.08%</td>
<td>39.04%</td>
<td>27.18%</td>
</tr>
<tr>
<td>IV Index mean</td>
<td>41.47%</td>
<td>39.30%</td>
<td>27.63%</td>
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</tbody>
</table>
Implied volatilities

Using the Black-Scholes formula to find $C$:

$$ C = C(S, X, r, T, \sigma) $$

What we're known or easily estimated from observed historical data

Using the Black-Scholes formula to find implied volatility:

$$ C = C(S, X, r, T, \sigma) $$

Set equal to the current market observed price of call

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AMZN: DAILY 6 MONTHS VOLATILITY CHART (3 months 6 months 1 year)

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The VIX

- VIX is the implied volatility on the S&P 500 computed by the Chicago Board Options Exchange.
  - Recall: the historical average standard deviation on “large US stocks” is $\sigma \approx 20\% \text{ per year}$.  
- The VIX itself is used as an underlying.
  - Options on the S&P 500 $\rightarrow$ VIX
  - Options and futures on VIX
  - These allow traders to speculate/hedge on movements in “risk”.
- As of Monday, April 20, 2015 ... (cboe.com)...
Hedging

- The stock and the call move in the same direction.
  - They are very highly correlated.
  - In the near term (over a day or so) $\rho \approx 1$.
- A short position in the call and a long position in the stock move in opposite directions.
  - They are very negatively correlated.
  - In the near term (over a day or so) $\rho \approx -1$.
- Can we find a risk-free portfolio combination?
- Why would we want to?
A call is sold in the institutional market

- With the SPY trading at 180, hedge fund A calls bank B:
  - “We’d like to trade a ten-year call on the SPY for one million shares at an exercise price of 190.”
- Bank B computes that the Black Scholes value of a call with \( S = 180, X = 190, r = 2\% , T = 10 \text{ years}, \sigma = 12\% \) is \( C = 39.29. \)
  - “Okay, we’re bidding \$38.90, asking \$39.70.”
- Hedge fund: “We’ll buy at \$39.70.”
- Now Bank B has written a (large) call option.
  - It has made a profit of \$0.41 relative to the “fair” (Black Scholes) value ...
  - ... but it is exposed to a lot of risk if the S&P index goes up.
At bank B: the trader and the risk manager.

- Trader: “I’ve just made a good sale of SPY calls.”
- Risk-manager: “You’re short a call. You need to buy some SPY shares to offset those short calls.”
- Trader: “How much should I buy?”

The Black-Scholes value of a 10-year call on SPY with $X = $190.

The slope of the red curve at 180 gives the change in the call value for every $1 change in the stock price. This slope is 0.72: “For every $1 the stock goes up, the call goes up by $0.72.”
Trader “How much SPY should I buy?”

- Risk manager: If we buy 0.72 shares of SPY for each call we’ve written, then if the SPY goes up by $1 ...
  - We’ll have a $0.72 gain on the stock ...
  - Offsetting the $0.72 loss on our short call position.
- 0.72 is the *hedge ratio* of the call.

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The hedge ratio, \( \Delta \) (“delta”)

- The \( \Delta \) for a call is the number of shares you should buy for each call that you’ve written.
- The Black-Scholes \( \Delta \) for a call is \( \Delta = N(d_1) \).
- The hedge ratio for a put is \( N(d_1) - 1 = -0.28 \).
Another trade on bank B’s equity desk.

- Trader: “I’ve just sold some 10-year SPY put options with $X = 190.$”
- Risk manager: “For every put that you’ve sold, go short 0.28 shares of SPY.”
- If SPY drops by $1, the $0.28 loss on each put the bank has written will be offset by a gain of $0.28 on the short stock position.

Back to the 10-year call on SPY with $X = 190.$

- The trader sold the calls when the SPY was 180.
  - It’s been a rough day. The SPY is now at 150.
- Trader: “Good thing I bought 0.72 shares. We’re hedged. No worries.”
- Risk manager (recalculating): “At $S = 150, \Delta = N(d_1) = 0.54$ shares. To stay hedged, you need to sell 0.18 shares to get from 0.72 to 0.54.”
- Trader: “You want me to sell more SPY into a falling market?!”
Dynamic hedging ("All is flux.")

- Hedge ratios change when anything in the model changes.
  - This includes $r$ and $\sigma$, but most importantly, $S$, the price of the underlying.
- To maintain the hedge, you have to assume that you’ll have ongoing access to a liquid market in the underlying.
- Even then:
  - ... on a written call, you must sell in a falling market and buy in a rising market.
  - ... on a written put, you must sell short more stock in a falling market and buy back (your short stock) in a rising market.
  - These trades aggravate price trends.

Hedge ratio’s ($\Delta$s) are additive

- With $S = 41, T = 1, \sigma = .40, r = .05$, the prices and delta’s for puts and calls are:

<table>
<thead>
<tr>
<th>Calls</th>
<th>Price</th>
<th>$\Delta$</th>
<th>Puts</th>
<th>Price</th>
<th>$\Delta$</th>
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<tbody>
<tr>
<td>X=40</td>
<td>7.84</td>
<td>0.65</td>
<td>X=40</td>
<td>4.91</td>
<td>-0.35</td>
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<tr>
<td>X=50</td>
<td>4.20</td>
<td>0.43</td>
<td>X=50</td>
<td>10.79</td>
<td>-0.57</td>
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</tbody>
</table>

- The hedge ratio for a combo that is long $C(X = 40)$ and short $C(X = 50)$ is $0.65 - 0.43 = 0.22$.
- What is the hedge ratio for a straddle that is long $C(X = 40)$ and long $P(X = 40)$? $+0.65 - 0.35 = +0.30$
Replication

- Using a dynamic trading strategy to create an option.
  - In contrast, hedging uses a dynamic trading strategy to lay off or defuse the risk in an already-existing option.
- Useful when the desired option can’t be purchased directly.

Replication of a protective put

- A hedge fund manager wants downside protection on an existing stock portfolio (“portfolio insurance”)
  - She wants a protective put = stock + put.
- Buying a put option might not be possible because
  - Puts aren’t traded on this portfolio.
  - A bank won’t provide quotes on over-the-counter puts or calls unless the manager tells the bank exactly what’s in the portfolio.
- Alternative method
  - Reallocate the portfolio as a (dynamically changing) mix of stock + bonds that replicates the protective put payoff.
Example

- We’ll treat the portfolio as if it were a single stock with price \( S = 100 \) per share.
- The manager wants a floor of \( X = 90 \) one year from now \( (T = 1) \); the volatility is \( \sigma = 50\% \), \( r = 10\% \).
- Using the options calculator:

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<td>Strike:</td>
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<td>Expiration Date:</td>
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<td>Days to Expiration:</td>
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<td>Volatility %:</td>
<td>50.</td>
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<td>Interest Rate %:</td>
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<table>
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<tr>
<td>Delta:</td>
<td>0.7434</td>
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The two alternative approaches

- Buy the put for $10.20
  - If we do this, the delta of the overall portfolio (stock + put) is
    \[
    \Delta = 1 - 0.26 = 0.74
    \]
- Dynamic replication
  - Sell 0.26 shares of stock (putting the proceeds in bonds)
  - The delta of the remaining shares is 0.74.
The dynamic replication over time

- If the stock price falls:
  - The delta of the portfolio falls.
  - We'll shift more out of stock and into bonds.
- If the stock price rises:
  - The delta of the portfolio will rise (up to a maximum of $\Delta = 1$)
  - We'll be mostly in stock.

Options on portfolios

- Stock A is trading at $50; a call with $X = 50$ is priced at $6.$
- Stock B is trading at $60; a call with $X = 60$ is priced at $7.$
- An exchange-traded fund $BA$ consists of one share of A and one share of B.
- $BA$ is priced at $110; a call with $X = 110$ is priced at $5.$
- How is this possible?
Options are everywhere

- Securities with option-like features
  - Warrants
  - Callable bonds
  - Convertible bonds
- Embedded options
  - Hedge fund management contracts.
- See BKM 20.5.

Warrants

- A warrant is a call option issued by the corporation.
- At exercise, the corporation
  - receives the exercise price
  - issues new shares
- Exercise may dilute the pre-existing stock.
  - The Black-Scholes formula must be adjusted for this dilution.
- Warrants are often issued in conjunction with other securities (as a “sweetener” to the sale)
- Detachable warrants may be sold off separately from the other securities (and may trade separately).
- Executive and employee stock options are similar to warrants.
Callable bonds

- A callable bond can be repurchased by the issuer.
- ABC issues a $1,000 bond, callable in 5 years at $1,050.
  - After the issue, ABC holds a call option on the bond.
- The buyer of the bond is (implicitly) short the call.
- If I purchase ABC’s callable bond, I hold a portfolio consisting of
  - A $1,000 non-callable bond.
  - A short position in a call with an exercise price of $1,050.
  - This is a covered call.
- These two components do not normally trade separately.
  - The call might be exercised against me if the market price of the bond exceeds $1,050.
- A callable bond will sell for less than a non-callable bond.

**Figure 20.11** Values of callable bonds compared with straight bonds
Convertible bonds

- A convertible bond is a bond that may be converted into stock (at the holders option).
  - Conversion ratio is # shares per bond.
  - Conversion price is (par value of bond)/(# shares)
- This option is neither a put nor a call: it is an *exchange* option.
- Convertible bonds are generally callable.
- Like warrants, conversion generally dilutes existing equity.
- Next slide: Figure 20.12
Hedge fund management contracts

- The standard “two and twenty”
  - Manager collects an annual fee of 2% of the assets under management, plus 20% of the cumulative profits.
- If the fund is started with outside investment $X$, management gets 20% of any increase above $X$.
- Management holds 20% of a call option on the value of the fund with an exercise price of $X$.
- What can the management do to increase the value of their “call option”?

Collateralized (non-recourse) loans

- Amount borrowed is principal amount $L$.
- Borrower posts collateral.
- At maturity, lender gets $\min(L, S_T)$
  - where $S_T$ is the value of the collateral at “expiration”.
- “Non-recourse”
  - If there is a shortfall ($S_T < L$), lender does not have a claim on borrower’s other assets.
- Option structure
  - Lender’s position is a covered call (on the collateral assets)
  - Borrower’s is short a covered call
- Next slide: Figure 20.13
6. In each of the following questions, you are asked to compare two options with parameters as given. The risk-free interest rate for all cases should be assumed to be 6%. Assume the stocks on which these options are written pay no dividends.

<table>
<thead>
<tr>
<th>Put</th>
<th>T</th>
<th>X</th>
<th>σ</th>
<th>Price of Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>.5</td>
<td>50</td>
<td>.20</td>
<td>$10</td>
</tr>
<tr>
<td>B</td>
<td>.5</td>
<td>50</td>
<td>.25</td>
<td>$10</td>
</tr>
</tbody>
</table>

Which put option is written on the stock with the lower price?

i. A.
ii. B.
iii. Not enough information.