Asset Pricing with Status Risk*

Job Market Paper

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Abstract

I examine the impact of status-seeking considerations on investors' portfolio choices and asset prices in a general equilibrium setting. The economy I study consists of traditional ("Markowitz") investors as well as status-seekers who are concerned about relative wealth. The model highlights the strategic and interdependent nature of portfolio selection in such a setting: low-status investors look for portfolio choices that maximize their chances of moving up the ladder while high-status investors look to maintain the status quo and hedge against these choices of the low-status investors. In equilibrium, asset returns obey a novel two-factor model in which one factor is the traditional market factor and the other is a particular "high volatility factor" that does not appear to have been identified so far in the theoretical or empirical literature. I test this two-factor model using stock market data and find significant support for it. Of particular interest, the model and the empirical results attribute the low returns on idiosyncratic volatility stocks documented by Ang, Hodrick, Xing and Zhang (2006) to their covariance with the portfolio of highly volatile stocks held by investors with relatively low status.

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1 Introduction

The empirical literature in finance has provided many challenges to traditional asset pricing theories. While diversification is a fundamental tenet of modern portfolio theory, in reality investors tend to hold underdiversified portfolios. Many households that hold individual stocks directly hold only a single stock, and the median number of stocks held is only about three (Liu 2008). In addition, less wealthy investors are even less diversified than wealthier investors. A recent study by Kumar (2009) finds that investors who earn less than their neighbors (within 25-mile radius) invest more in stocks with lottery-like payoffs, and experience greater underperformance. The latter finding agrees with a recent empirical asset pricing study by Bali, Cakici, and Whitelaw (2009) showing that stocks with lottery-like payoffs have significantly low returns. This work coincides with the puzzling empirical finding documented by Ang, Hodrick, Xing, and Zhang (2006); that stocks with high idiosyncratic volatility have exceptionally low average returns. These findings stand in stark contrast to asset pricing theories that imply that idiosyncratic risk should not be priced, or that in a market in which investors cannot properly diversify, one would expect idiosyncratic risk to be positively related to expected returns (Merton, 1987).

In this paper I argue that taking status concerns into account sheds some light on these puzzles. I develop a model that introduces status-conscious investors, whose status is defined as their wealth-based rank in their reference group, into an economy, populated with traditional "Markowitz" investors (mean-variance optimizers). Because some of these investors have low status and others have high status, each group employs a different investment strategy. I obtain closed-form solutions for portfolio choice in equilibrium and show that the low status investors hold portfolios concentrated in high volatility assets in order to leapfrog the high status investors. High status investors strategically hedge against these attempts by investing in a portfolio with high exposure to high volatility assets. This gives rise to a 2-factor asset pricing model in a general equilibrium. I obtain exact solutions for prices and show that expected returns are a sum of a positive return premium on the market beta and a negative return premium on the beta with a high volatility factor, a portfolio of the assets with the highest volatility in the economy.

I test the asset pricing implications of the model using stock market data and find significant economic and statistical support. The 2-factor model proves to price assets with high exposure to high volatility stocks significantly better than the CAPM, the Fama-French (1993) 3-factor model, and the Carhart (1997) 4-factor model. The model sheds light on the idiosyncratic volatility puzzle of Ang et al. (2006), as assets with high idiosyncratic risk are positively correlated with assets with high exposure to the high volatility factor. Nevertheless, the 2-factor model has a cross-sectional pricing ability above and beyond that of idiosyncratic risk.

The idea that individuals are often motivated in their behavior by a quest for social status is not new. It has been acknowledged by economists such as Smith (1776), Marx (1849), Veblen (1899), and Duesenberry (1949). The importance of social status has long been recognized in psychology, sociology and more recently in economics. These studies highlight the effects that status concerns have on individuals' decisions. I ask how such concerns affect agents' financial decisions - portfolio choices - and how these decisions affect aggregate outcomes, namely asset prices. In the context of financial markets, status concerns affect not only individual investors, but also mutual fund managers. The mutual fund tournament literature, spawned by Brown, Harlow and Starks (1996), hypothesizes that portfolio choices of fund managers reflect their concern about their position relative to other managers, as their compensation is linked to their performance relative to their peers.

I devise a model that captures several salient characteristics of the preferences for status. Most importantly, status is inherently positional in the sense of Heffez and Frank (2008). I capture the positionality of status by modeling status as the ordinal wealth-based rank of investors in their reference group. Modeling status as indicated by the ordinal rank in the distribution of wealth was pioneered by Frank (1985) in a study of the demand for positional and non-positional goods. Later, Robson (1992) and Becker, Murphy and Werning (2005) considered preferences over absolute wealth as well as ordinal rank in wealth.

One consequence of the positionality of status is that an increase in one agent's relative status automatically translates to a decrease in the relative status of others in the relevant reference group. This zero-sum feature of status gives rise to a strategic interaction among agents in portfolio choice. Anecdotal evidence of this sort is stated by Harrison Hong (2008) during an interview given to the Wall Street Journal regarding the High-Tech bubble: "My sister's getting rich. My friends are getting rich... I think this is all crazy, but I feel so horrible about missing out, about being left out of the party. I finally caved in, I put in some money just as a hedge against other people getting richer than me and feeling better than me." To capture the strategic interaction among investors, status conscious investors in my model strategically choose portfolios in a Nash Equilibrium.

My work is consistent with the assumption that status concerns are primarily local. Sociological studies consistently confirm that the comparisons that really matter are highly local in character (Frank 1985). As Bertrand Russell once remarked, "Beggars don't envy millionaires; they envy other beggars who earn more than they do" (Frank 2009). Recent empirical work indeed confirms the importance of local comparisons. Luttmer (2005) shows that higher earnings of neighbors are associated with lower levels of self-reported happiness, and Clark and Oswald (1996) regress job satisfaction on personal income and on predicted income of a comparison group and find coefficients of equal magnitude but opposite sign. Similarly, in the mutual funds context, local competitions emerge among funds with the same investment style. Within each such competition, there could be a nested local competition among the best performing funds that compete for the top positions in the tournament, and another local competition among funds that compete to avoid the bottom positions. There could also be a competition among funds of the same family, as suggested by Kempf and Ruenzi (2008).

Just as in every competition there are winners and losers, in every local reference group there must be high status investors that are ahead in the competition ("leaders"), and low status investors that fall behind ("laggards")[1]. If status incentives are strong enough, then the leaders are satisfied with their position, and are interested in maintaining the status quo. The laggards, on the other hand, are dissatisfied with their low rank and are interested in moving up the status ladder. I model each competition within a certain reference group as a two-player game, where the leader is the richer investor, and the laggard is the poorer investor. The players compete over wealth-based rank. I obtain the portfolio choices of the leader and the laggard in a Nash Equilibrium, and then examine asset prices in an economy with many pairs of status investors, representing many reference groups, as well as many traditional Markowitz investors who care only about maximizing mean and minimizing variance.

At the heart of my model is a game between two players, a laggard and a leader, who compete against each other for the top (wealth-based) rank. The leader is defined as the wealthier player at the onset of the game. The attitudes of investors toward the moments of the return on their portfolios are determined endogenously as a function of this initial status. The laggard pursues a volatile portfolio, as he has 'nothing to lose.' In addition, he seeks minimal correlation with the portfolio of the leader, as differentiating himself is necessary to overtake her. On the other hand, the leader faces "status risk," the risk of losing her leadership. This status risk has two components. The first is that she will earn a low return and fall behind the laggard, and the second is that the laggard will earn a high return and overtake her. To manage the first risk, the leader tries to minimize the variance of her portfolio. Practically, this risk is the same as the risk faced by a traditional "Markowitz" investor. To manage the second risk, the leader is interested in increasing the covariance of her portfolio with that of the laggard. In what follows, I show that this two-fold concern of the leader gives rise to a 2-factor asset pricing model.

In order to study portfolio choices and the cross-section of expected returns in this status-conscious economy, I introduce an economy with many groups of similar assets.

[1] I use the terms "leader" and "laggard" following Cabral (2002, 2003) and Anderson and Cabral (2007), who study the strategy choices of players who compete in a dynamic race over positional rewards.

There are many pairs of laggard and leader investors (status-conscious investors) and many Markowitz investors, who care only about maximizing expected return and minimizing volatility. I find a Nash Equilibrium, in which each laggard uses a mixed strategy to invest in a single asset from a specific group ("group V"), which is characterized by the high volatility of its assets. I obtain a closed-form solution for the leader's response. The leader's portfolio reflects her two-fold concern for reducing her variance and increasing her covariance with the laggard as she invests in a linear combination of the tangency portfolio and group V. The trade-off of the leader between the tangency portfolio and group V depends on her "hedging demand," which captures the extent to which the leader can hedge against the laggard. As the correlation within group V increases, the covariance between the return of group V and the return of the laggard's portfolio increases. Therefore, the leader can better hedge against the laggard using group V, and she increases her invested portion in group V accordingly.

I obtain exact solutions for asset prices and show that they follow a 2-factor beta pricing model where one factor is the market excess return and the other is the excess return on group V over the market return (VMM). Assets with high exposure to VMM obtain lower expected returns as they provide a hedge against the status risk of the leader. The negative premium for assets with high exposure to VMM depends on the hedging demand, derived from the correlation within group V, and on the variance of VMM. When the variance of VMM is lower, the leader can use other assets in the economy to hedge against the laggard, the overall demand for group V assets decreases, and exposure to VMM is rewarded less in equilibrium.

The model provides both portfolio choice and asset pricing implications. It explains why some investors (the laggards in the model) hold undiversified portfolios of stocks with lottery-like payoffs and experience underperformance relative to traditional asset pricing models. The model generates a cross-sectional 2-factor beta pricing equation. It suggests that assets with higher exposure to the most volatile group of stocks in the market should have a negative return premium, as they provide a hedge against status risk. In addition, the model provides a closed-form expression for the premium on this portfolio in terms of the correlation within that portfolio and its variance.

I test the asset pricing implications of the model using stock market data. Following the model predictions, I construct 25 portfolios designed to maximize the cross-sectional variation in expected returns. I sort stocks first by their exposure to the market and then by their exposure to the high volatility factor (VMM). The empirical counterpart of group V is the portfolio of stocks with the highest total volatility. I examine the monthly returns on these portfolios and show that portfolios with higher exposure to VMM earn significantly lower risk-adjusted returns. For example, the portfolio with the highest exposure to VMM has monthly alphas of -1.08%, -0.84%, and -0.75% using, respectively, the CAPM, the Fama-French (1993) 3-factor model and the Carhart (1997) 4-factor model during the period 1945-2008.

I test the cross-sectional implications of the 2-factor model by time-series regressions, the Fama-MacBeth method and GMM-SDF tests. All tests agree that the unconditional version of my 2-factor model proves to price the 25 test assets significantly better than the CAPM, the Fama-French 3-factor model, and the Carhart 4-factor model. Using time-series regressions, I show that a joint test that all pricing errors are zero rejects the CAPM, the Fama-French 3-factor model, and the Carhart 4-factor model at the 1% significance level. However, the same test cannot reject my 2-factor model even at the 5% significance level. The GMM tests reinforce this finding, as the Hansen's (1982) test of overidentifying restrictions rejects the CAPM, the Fama-French 3-factor model, and the Carhart 4-factor model at the 5% significance level, while it cannot reject the unconditional 2-factor model at the 10% significance level. Using the Fama-MacBeth method and GMM-SDF tests, I also find that the exposure to the VMM factor remains statistically and economically significant in explaining the cross-section of expected returns after controlling for the Fama-French 3-factor model, and the Carhart 4-factor model. Using GMM tests I also show that the conditional version of the two factor asset pricing model, which incorporates variations over time in the risk premium of the VMM factor, performs better than the unconditional version.

Finally, I test the performance of this 2-factor model versus that of idiosyncratic volatility, as measured by Ang, Hodrick, Xing, and Zhang (2006), using Fama-MacBeth regressions and GMM-SDF tests. The results show that the 2-factor model performs better in the cross-section. Nevertheless, I find high positive correlation in the cross-section between idiosyncratic volatility and exposure to high-volatility stocks. The model and this empirical finding lend an explanation for the idiosyncratic volatility puzzle: the low returns that have been assigned to idiosyncratic risk are actually a result of the negative premium for covariance with the portfolio of highly volatile stocks. In this spirit, I show that the two factor pricing model reduces the monthly alpha of the High/Low idiosyncratic risk portfolio of Ang, Hodrick, Xing, and Zhang (2006) from -1.23%, using the Fama-French 3-factor model, to -0.32%, in the period of 1963 to 2008.

The rest of this paper proceeds as follows. In the next section I review the literature related to this work. In section 3, I present the model. In section 4, I test the asset pricing implications of the model. Section 5 concludes.

2 Related Literature

Several papers in the economics literature study the effect of status concerns on risk preferences. In fact, Friedman and Savage (1948) use status concerns to interpret the concave-convex-concave shape of their renown utility function. They interpret the convex segment in their utility function as a transition between two socioeconomic levels and suggest that people are willing to take great risks to obtain a chance to move to a higher social class. Robson (1992) and Becker, Murphy and Werning (2005) illustrate that including status, modeled as ordinal rank in the distribution of wealth in the utility function, leads some agents to take on fair lotteries.

The behavior of the laggards in my model is consistent with the agents that are induced to take on fair lotteries in the above studies. However, an important distinguishing feature of my model is that the investment opportunity set is common to all investors and assets are not independent of one another. Thus, the preference of the laggards for high-volatility assets cascades in my model to other investors, the leaders, as they can create a hedge against the laggards. Becker, Murphy and Werning (2005) stress that fair lotteries may take the form of gambles through occupational choices and entrepreneurial activities and not necessarily actual lotteries. The appropriate interpretation of assets in my model is public equity. Unlike occupational choices and entrepreneurial activities, investment in public equity is quite accessible and does not require drastic changes in the daily life of an individual.

In the context of portfolio choice and asset prices, Cole, Mailath, and Postlewaite (2001) study the effects of relative wealth concerns in a general framework and show that these concerns can have two opposite effects: investors can bias their portfolios either toward or away from the portfolios held by other investors. Abel (1990) and other papers studying interdependent preferences in finance model relative wealth using utility functions that exhibit the first effect, which is commonly termed "keeping up with the Joneses." Since investors in these models tend to bias their portfolio toward the portfolio held by the reference group, such models yield herding behavior. For example, DeMarzo, Kaniel, and Kremer (2004) show that preference for a local good can give rise to relative wealth concerns, leading to undiversified portfolios, with households in each community tilting their portfolios toward community-specific assets. Others, such as Lauterbach and Reisman (2004) and Cole, Mailath, and Postlewaite (2001) have used such preferences to explain the home bias. Roussanov (2009) is the first paper, to my knowledge, in the finance literature that specifies a utility function that leads investors to "get ahead of the Joneses" and seek portfolios that are biased away from the aggregate portfolio.

My model takes the view that merging these two effects into a single framework is essential to understanding the effects of status on portfolio choice and asset pricing.

Investors who are satisfied with their position relative to others will purse the "keeping up with the Joneses" approach and will bias their portfolios towards the portfolio of others, whereas other investors who are dissatisfied with their position relative to others will pursue the "getting ahead of the Joneses" approach of Roussanov (2009) and will bias their portfolio away from the portfolios of others. Juxtaposing these two approaches gives rise to a strategic game as the decisions of both types of investors are interdependent. This paper is a first step in studying such models.

This paper also adds to earlier work that shows that relative wealth concerns might lead to a preference for lottery-like securities. Robson (1996) uses a biologically motivated model to show that agents who are fundamentally risk neutral are induced to take fair bets involving small losses and large gains in an environment in which the rewards are a function of relative wealth. Barberis and Huang (2008) study the implications of prospect theory (Tversky and Kahneman, 1979) on asset prices. They show that errors in the probability weighting of investors cause agents to over-value stocks that have a small probability of a large positive return. The optimal beliefs framework of Brunnermeier, Gollier and Parker (2007) also predicts a preference for lottery-like securities. In their economy, agents optimally choose to distort their beliefs about future probabilities in order to maximize their current utility. In both papers, assets with positively skewed payoffs are overpriced. The asset pricing implications of my model are different in two dimensions from the above-mentioned works. First, in my economy, high volatility securities obtain high prices even if they are not positively skewed. Second, in my economy, agents seek *covariance* with high volatility assets, giving rise to a 2-factor beta pricing model, in which covariance with high volatility assets commands a negative premium.

Other studies that use relative wealth concerns in order to explain under-diversification are Roussanov (2009) and DeMarzo, Kaniel, and Kremer (2004). Unlike these studies, this paper identifies the holders of under-diversified portfolios as investors who fall behind in the competition over status and identifies the assets held in these portfolios as the most volatile assets. A model that associates assets held in under-diversified portfolios with high risk assets is provided by Van-Nieuwerburgh and Veldkamp (2010) who argue that information acquisition can rationalize investing in a concentrated set of assets. In particular, they formalize the conditions under which the informed investor would hold an under-diversified portfolio of the riskiest assets. Liu (2008) argues that portfolio insurance leads the poorest investors to hold under-diversified portfolios with assets that have the highest expected return and highest risk. Unlike these works, my model predicts that the underdiversified portfolios held by poorer investors contain assets with low expected returns. Thus, it is better aligned with the findings of Kumar (2009) who shows that investors who invest disproportionately more in lottery-type stocks experience greater underperformance.

Another potential consequence of relative wealth concerns is the creation of financial bubbles. Demarzo, Kaniel and Kremer (2007, 2008) have already pointed out that herding behavior due to relative wealth concerns can play a role in explaining financial bubbles, and in particular, the high-tech bubble. This paper adds to this literature by identifying that the source of asset pricing bubbles is likely to be high volatility stocks, especially in times when they co-vary more and have high variance relative to the market. During the high-tech bubble, sophisticated investors, such as hedge fund managers, were heavily invested in technology stocks (Brunnermeier and Nagel, 2004). From the vantage point of the paper, these managers were using technology stocks as a hedge against their status risk. Corresponding to the characteristics that lead to high prices in my model, at that time high-tech stocks were the most volatile stocks in the market, had high covariance with one another, and the high-tech industry as a whole had high variance relative to the market.

Another related strand of literature is the mutual fund tournament literature, which examines whether underperforming mutual funds (the laggards in my model) increase risk in the latter part of the year. This phenomenon has been studied using return data, with different studies reaching different conclusions (see a review by Elton, Gruber, Blake, Krasny, and Ozelge, 2009). There are at least two issues with this empirical approach that might hinder reaching a decisive conclusion about mutual fund tournament behavior. First, while most studies examine the risk taken by the leaders versus the laggards as measured by volatility or beta, Chen and Pennacchi (2009) show that laggard funds should increase the fund's "tracking error" volatility and not necessarily its return volatility. My paper provides support for both of these approaches, as the laggards in my model seek higher variance and lower covariance with the leaders.

The second issue with this literature is the identity of the leaders and the laggards. While the common and intuitive approach is to view the under-performing funds as the laggards and the top-performers as the leaders, this might not necessarily be the case. Chevalier and Ellison (1997) study the relationship between new cash flows and returns, and find that it is nonlinear, with an extreme payoff from winning the tournament. Moreover, Chevalier and Ellison (1998) identify implicit incentives created by the relationship between job termination and performance. Thus, career concerns of mutual fund managers may provide an incentive to avoid ending up as the worst performers. These studies suggest that the competition structure might be more complex and localized, where the best performers compete amongst themselves, as do the worst performers. Furthermore, it could be the case that funds compete within families as suggested by Kempf and Ruenzi (2008). My model lends an additional test to this literature, as it predicts that both the leaders and the laggards should increase holdings of highly volatile assets as the tournament-based incentives intensify (i.e., towards the end of the calendar year).

Finally, my paper relates to the race literature that studies the choices of efforts and strategies of agents in a setting where rewards are positional by nature. In particular, Cabral (2003) and Anderson and Cabral (2007) provide conditions under which the laggard chooses a risky strategy, while the leader chooses a safe strategy. Cabral (2002) shows that when the competitors choose covariance, the laggard is willing to trade off lower expected value for lower correlation with respect to the leader. Both effects are consistent with my model.

3 The Model

In this section, I first define and examine the status game between the leader and the laggard in a general setting where assets are multivariate normal. Next, I define an economy with additional structure imposed on the distribution of assets and derive the Nash Equilibrium in the two-player game. Finally, I add many pairs of leaders and laggards (status-conscious investors) and many traditional Markowitz investors and examine asset prices in this economy.

3.1 The Two Player Status Game

The model has two players: the laggard ("he") with an initial wealth normalized to 1, and the leader ("she") with an initial wealth normalized to $k > 1$. There are finitely many risky assets with returns from a multivariate normal distribution as well as a risk-free asset with a gross return of 1 available for investment. Short sales are not allowed [2]. Time is discrete and runs for one period.

The gross return of the leader's portfolio is given by r_d and the gross return of the laggard's portfolio is given by r_g. Therefore, at the end of the period, the wealth of the leader is kr_d, and the wealth of the laggard is r_g. The utility of the players is given by their wealth based rank - the initial utility of the leader is 1, and the initial utility of the laggard is 0. At the beginning of the period, the players choose a portfolio to maximize their expected utility, which is equivalent to their probability to be the leader at the end of the period. I denote the difference in the end-of-period wealth between the leader and the laggard by $D(r_d, r_g)$:

$$D(r_d, r_g) = kr_d - r_g. \tag{1}$$

This is a zero-sum game, in which the leader (laggard) tries to maximize (minimize) the expression:

$$Pr(D(r_d, r_g) > 0). \tag{2}$$

[2] The results of this section can be derived under the assumption of limited short sales as well.

Since D is normal, the objective function of the leader can be written as:

$$Max\ \Phi\left(\frac{kE(r_d) - E(r_g)}{\sqrt{k^2Var(r_d) - 2kCov(r_d, r_g) + Var(r_g)}}\right). \tag{3}$$

In the above, $\Phi(.)$ is the CDF of a standard normal distribution. If short sales are not allowed, the expected return on any portfolio must be finite. Therefore, the leader can guarantee an expected return on her portfolio at least as high as that of the laggard, and the following proposition holds:

Proposition 1. *In a Nash Equilibrium, the choices of the players must satisfy:*

$$kE(r_d) - E(r_g) > 0, \tag{4}$$

where r_d is the gross return of the leader, r_g is the gross return of the laggard, and $k > 1$ is the wealth ratio between the players.

Proof. The short sales constraint guarantees that any strategy η of the laggard yields a finite expected return. For any such strategy, the leader can use an imitation strategy with a portion 1 of her wealth invested with η, and the rest (k-1) invested in the risk-free asset. This guarantees a probability of winning the game (i.e., remaining the leader) of more than 0.5. In a Nash Equilibrium the leader uses a strategy that is at least as good as the imitation strategy and therefore yields a probability of more than 0.5 to win the game. Since assets are multivariate normal, equation (3) implies that a probability of more than 0.5 to win the game is obtained if and only if the numerator of the CDF's argument is positive. Thus, it must be that $kE(r_d) - E(r_g) > 0$. □

In other words, in equilibrium, the laggard cannot choose a portfolio such that his end-of-period wealth, on average, will exceed or be equal to that of the leader. Otherwise, the leader will imitate his strategy.

The leader (laggard) chooses her (his) portfolio in order to maximize (minimize) the probability of the leader maintaining her first place rank at the end of the game. Using proposition 1 and equation (3), we can gain some insight into the endogenous translation of the players' initial status to their portfolio choice preferences. Both players prefer higher expected returns on their portfolios. In addition, since the numerator in equation (3) is positive, the laggard (leader) prefers higher (lower) variance of his (her) portfolio. Finally, the laggard (leader) prefers lower (higher) covariance between the wealth of the players.

3.2 The Economy

To further examine portfolio choices and asset prices, I impose additional structure on the distribution of assets, and introduce G groups of assets. To simplify notation, I will use capital letters to denote variables that relate to groups, and small letters to denote variables that relate to individual assets.

All assets are multivariate normal. Each group $I \in \{1, ..., G\}$ has N_I similar assets with the same distribution. The return of asset $k \in \{1, ..., N_I\}$ in group I is denoted by r_i^k. The return of the equally-weighted portfolio over all assets in group I is denoted by r_I. I refer to the "equally weighted portfolio over all assets in group I" as "group I" for brevity.

The expected return of every individual asset in the same group I is identical and denoted by μ_i. Since the return of group I is the average of the returns of individual assets, the expected return of group I, μ_I, is equal to the expected return of individual securities:

$$E(r_i^k) = \mu_i = E(r_I) = \mu_I,$$
$$\forall k \in \{1, ..., N_I\}, I \in \{1, ..., G\}.$$

The variance of every individual security in group I is the same and denoted by σ_i^2. The correlation of every pair of assets in group I is the same and denoted by ρ_I. Hence, the N_I by N_I covariance matrix of group I follows:

$$Cov(r_i^k, r_i^l) = \begin{cases} \sigma_i^2 \text{ if } k = l \\ \rho_I \sigma_i^2 \text{ if } k \neq l \end{cases}.$$

I assume that the correlation of every pair of assets within any group is positive, but less than one:

$$0 < \rho_I < 1, \ \ \forall I \in \{1, ..., G\}. \tag{5}$$

The correlation between two assets of different groups $I, J \in \{1, ..., G\}$ is denoted by $\rho_{i,j}$. Hence, the G by G covariance matrix for individual assets across different groups follows:

$$Cov(r_i^k, r_j^k) = \sigma_{ij} = \begin{cases} \sigma_i^2 \text{ if } i = j \\ \rho_{i,j} \sigma_i \sigma_j \text{ if } i \neq j \end{cases}.$$

The G by G covariance matrix for groups is denoted by Σ and follows:

$$Cov(r_I, r_J) = \sigma_{I,J} = \begin{cases} \sigma_I^2 = \sigma_i^2(\rho_I + \frac{1-\rho_I}{N_I}) \text{ if } I = J \\ \rho_{i,j}\sigma_i\sigma_j \text{ if } I \neq J \end{cases}.$$

Note that the covariance of two individual assets in different groups is equal to the covariance of these two groups.

3.3 Portfolio Choice in a Nash Equilibrium

First, I characterize the laggard's response to the leader's strategy. Proposition 1 suggests that the strategy of the leader in a Nash Equilibrium guarantees that her expected wealth in the next period is higher than that of the laggard, regardless of the response of the laggard. Therefore, I examine the response of the laggard to such strategies. The following proposition characterizes the best response of the laggard (all remaining proofs are delegated to the appendix):

Proposition 2. *Given a strategy of the leader that satisfies condition (4) for all possible strategies of the laggard, the best response of the laggard is to use a mixed strategy whose support consists exclusively of pure strategies that employ a single asset.*

In other words, the laggard either chooses a single asset or mixes over which single asset to employ. That asset can be either the risk-free asset or a risky asset. Since the laggard cannot choose a strategy such that his end-of-period wealth, on average, will exceed or be equal to that of the leader, he prefers higher volatility and lower covariance with the leader. Investing in a single risky asset serves both objectives. The laggard might choose to invest in the risk-free asset if the leader is invested in risky assets and either the expected returns on these risky assets are too low, or the correlation between the risky assets is too high. In either of these cases, the laggard is better off waiting on the sideline, hoping that the leader obtains a low return using the risky assets.

The symmetry of assets within each group makes it natural to focus the Nash Equilibrium analysis on strategies that are symmetric within each group. Such strategies invest, on average, the same amount in each of the assets in the same group. Therefore, proposition 2 implies that in a Nash Equilibrium with symmetric strategies within groups, if the laggard invests in some group I, then he must choose a single risky asset using a uniform mixed strategy over all assets in group I. If he were to invest exclusively in the risk-free asset, then the leader would imitate his investment to guarantee her first rank.

We now turn to the leader's response to the laggard's strategy of choosing a single risky asset using a uniform mixing strategy over a specific group that I denote group V. The following proposition suggests that if the number of assets in group V is large enough, the leader's problem is reduced to choosing a portfolio over groups.

Proposition 3. *For an N_V large enough, and given that the laggard invests his wealth in a single asset chosen uniformly by a mixed strategy over group V, within each group the leader invests an equal amount in each of the assets.*

The reason that a large enough number of assets in group V is required can be illustrated by the following example. Suppose there is only one group in the economy, and there are only two assets in that group. In addition, the wealth ratio, k, is very close to 1. In this case, if the leader holds the equally-weighted portfolio, the probability she remains the leader is a little more than 0.5, since the result of the game depends primarily on whether the laggard's chosen asset performs better than the other asset. However, in the event that she invests all in one asset, she will obtain a probability of a little more than 0.75 to win the game, because if she 'catches' the laggard, and invests in the same asset, she will undoubtedly remain the leader. Having a large enough number of assets is important to discourage the leader from pursuing such strategies.

With N_V large enough, since the laggard invests in an asset v, and within each group the leader invests an equal amount in each of the assets, we can treat the leader's problem as choosing a vector θ of length G over groups. Therefore, the problem of the leader can be represented in the following way:

$$Max_{\theta} \ \frac{k\left(\theta'\tilde{\mu}+1\right)-\left(\tilde{\mu}_v+1\right)}{\sqrt{k^2\theta'\Sigma\theta-2k\theta'\Sigma E_v+\sigma_v^2}}, \tag{6}$$

where $\tilde{\mu}$ is the vector of expected excess returns over the G groups, $\tilde{\mu}_v$ is the expected excess return of a group V asset, Σ is the covariance matrix over groups, and E_v is a vector of zeros except for entry v, which is 1.

The leader prefers higher expected return, lower variance and higher covariance with group V. Therefore, the leader's problem is a special case of the problem of an ICAPM investor who cares about her covariance with state variables. In my model, the only "state" variable is the return on group V, and naturally the mimicking portfolio for group V is simply the return on group V. Following Fama (1996), I conclude that the leader chooses a Multifactor-Minimum-Variance portfolio, in the sense of Fama (1996). This portfolio is a combination of the risk-free asset, the tangency portfolio and group V. Hence, the leader's risky portfolio can be written in the following way:

$$\theta = x\Sigma^{-1}\tilde{\mu} + yE_v, \tag{7}$$

where x and y are scalars. Using this insight, I can now solve for a unique Nash Equilibrium:

Theorem 4. *Under the conditions described in Appendix 6.3, there exists a unique Nash Equilibrium in which the laggard chooses a single asset using a uniform mixed strategy over group V assets, and the leader invests in a risky portfolio θ over groups:*

$$\theta = \frac{\sigma_v^2 - \sigma_V^2}{k(k-1)}\Sigma^{-1}\tilde{\mu} + \frac{1}{k}E_v, \tag{8}$$

where $k > 1$ is the wealth ratio between the players, σ_v is the volatility of an individual asset of group V, σ_V is the volatility of group V, Σ is the covariance matrix for groups, $\tilde{\mu}$ is the expected excess return over the groups, and E_v is a vector of zeros except entry v, which is 1.

The investment of the leader in the risk-free asset, the tangency portfolio, and group V reflects her preferences for lower variance, higher expected return and higher covariance with the laggard. As the wealth ratio between the players, k, increases, the leader becomes less threatened by the laggard. Her status risk, the risk of losing her first rank, becomes primarily the risk of obtaining too low of a return, and in some sense she becomes her 'own worst enemy'. Hence, as k increases, the leader increases the portion invested in the risk-free asset.

The leader's two-fold concern for low variance and high covariance with the laggard is reflected in her risky portfolio represented in equation (8), which is a linear combination of the tangency portfolio and group V. To obtain high covariance with the laggard she matches his investment in group V in the second term $\frac{1}{k}E_v$. To obtain an efficient mean-variance trade-off she invests in the tangency portfolio. The balance between the two terms is a function of the following term:

$$\psi = \frac{k-1}{\sigma_v^2 - \sigma_V^2}, \tag{9}$$

ψ reflects the hedging demands of the leader. When the volatility of an individual asset of group V (asset v) relative to the volatility of group V increases, the correlation within group V decreases, and it is harder for the leader to obtain a high covariance with the laggard. Therefore, her hedging demands, reflected in ψ, decrease.

When hedging demands are high, the leader concentrates her efforts on the covariance with the laggard and decreases her investment in the tangency portfolio. When hedging demands are low, she finds it difficult to obtain a high covariance with the laggard, and she channels her concerns to obtaining a more efficient mean-variance portfolio by investing more in the tangency portfolio.

The first condition for the equilibrium described in Theorem 4 is that the number of assets in group V is large enough in the sense of Proposition 3. The second set of conditions is sufficient to have the laggard not deviating from investing in asset v. Condition (2.a), $\sigma_v > \sqrt{2}\sigma_j \ \forall j \in \{1, ..., g\}$, says that the variance of asset v should be high enough relative to assets of other groups in order to encourage the laggard to invest in it. This condition identifies the attribute that makes group V the laggard's choice. It is the high volatility of group V that distinguishes it from other groups in the economy. Condition (2.b), $\sigma_{v,j} > 0 \ \forall j$, tells us that there is no group with negative covariance with group V. If there were one, the laggard might have been enticed to invest in it in order to obtain a negative correlation with the leader who tilts her portfolio towards group V. Condition (2.c), $\sigma_v > 2\sigma_V$, says that the volatility of asset v should be high enough relative to the volatility of group V. In other words, the correlation within group V should be low enough, otherwise, the leader can easily hedge against the laggard. To have the laggard not deviating to the risk-free asset, a sufficient condition is $\sigma_v > \sqrt{2}\sigma_V$, which is included in the above conditions. The third set of conditions is necessary to ensure that the leader refrains from taking a short position in the risk-free asset or in any of the risky assets.

3.4 Asset Prices in a General Equilibrium

To examine asset prices in this economy, I add many leader-laggard pairs (status investors) and many Markowitz investors. Let us examine the risky portfolio held by each type of investor. The risky portfolio of every Markowitz investor is simply the tangency portfolio. There are many laggards in the economy, and each laggard chooses a single asset out of group V using a uniform mixed strategy. Therefore, the law of large numbers suggests that the aggregation of the portfolios of the laggards is simply group V. As we have seen, the portfolio of the leader is a linear combination of the tangency portfolio and group V. Hence, the market portfolio, which is a linear combination of the portfolios held by all investors, is also a linear combination of the tangency portfolio and group V. This guarantees the existence of a 2-factor beta pricing model. To obtain a closed form solution for prices, I rearrange equation (8):

$$\tilde{\mu} = \psi \left(k\Sigma\theta - \Sigma E_v \right). \tag{10}$$

The expected return of an asset positively relates to its covariance with the portfolio of the leader and negatively relates to its covariance with group V. Since the leader's portfolio is a combination of group V and the tangency portfolio, it is also a combination of the market and group V. Hence, there exists a 2-factor beta pricing model, where one of the factors is the market and the other is group V. To derive the closed form of this 2-factor beta pricing model, further assumptions about the proportion of the status-conscious investors versus the Markowitz investors are required.

3.4.1 An Economy with Only Status Investors

Suppose that there are only status investors in the economy. In this case, the market portfolio can be written as a combination of the leader's portfolio and the laggard's portfolio:

$$\theta_M = \frac{k\theta + E_v}{k\theta'\iota + 1}. \tag{11}$$

Using the expression for the leader's portfolio I obtained in (8) and the pricing equation (10), the expected excess return for an individual asset can be represented as:

$$\tilde{\mu_i} = \left[\iota'\Sigma^{-1}\tilde{\mu} + 2\psi\right] Cov(r_i, r_M) - 2\psi Cov(r_i, r_V). \tag{12}$$

An exposure of an asset to the market positively contributes to its expected return. The term $\iota'\Sigma^{-1}\tilde{\mu}$ is the expected excess return divided by the variance of the tangency portfolio. It reflects the implicit risk aversion of the leader. The term ψ reflects the hedging demands of the leader, as defined in equation (9). If the hedging demands are high, the leader is inclined to invest more in group V at the expense of other groups, leading to a higher price for group V and a lower price for the market.

A compact way to express the asset pricing relation as a 2-factor beta pricing model is to have the first factor as the excess return of the market portfolio, and the second factor as the excess return on group V over the market return. I denote the second factor by VMM. This form is not only algebraically simpler, but also leads to sharp empirical predictions, as we will see shortly. From equation (12), the 2-factor beta pricing model for this economy can be derived:

Theorem 5. *In an economy with many pairs of leaders and laggards, where the Nash Equilibrium described in Theorem 4 holds, the expected excess return of asset i ($\tilde{\mu}_i$) is:*

$$\tilde{\mu_i} = \beta^I_{i,MKT}\left[\iota'\Sigma^{-1}\tilde{\mu}\right] Var\left(r_{MKT}\right) + \beta^I_{i,VMM}\left[-2\psi\right] Var\left(r_{VMM}\right), \tag{13}$$

where $\beta^I_{i,MKT}$ is the slope from a univariate regression of the asset excess return on the excess market return ($r_{MKT} - r_f$), $\beta^I_{i,VMM}$ is the slope from a univariate regression *of the asset excess return on the return of VMM. The return of VMM equals to the return of group V minus the return on the market* $(r_V - r_{MKT})$*,* ι *is a vector of ones,* Σ *is the covariance matrix of groups, and* $\tilde{\mu}$ *is a the expected excess returns of groups.*

Equation (13) illustrates how exposure to the high volatility factor, VMM, translates into prices. Fixing the univariate beta of an asset on the market ($\beta^I_{i,MKT}$), a higher univariate beta on VMM ($\beta^I_{i,VMM}$) leads to lower expected return.

The negative premium on VMM beta has two determinants. First, the hedging demands ψ - when there are higher hedging demands, the premium becomes more negative as the demand for exposure to group V increases. The second determinant is the variance of the VMM factor. When $Var\left(r_{VMM}\right)$ increases, it is harder to hedge against group V using other groups in the market, group V becomes more special in its effectiveness as a hedge, and its price increases.

To derive the Stochastic Discount Factor in this economy, equation (12) can be manipulated to obtain a 2-factor SDF:

$$M = 1 + \tilde{\mu}'\Sigma^{-1}\tilde{\mu} - \iota'\Sigma^{-1}\tilde{\mu}\left(r_{MKT} - r_f\right) + 2\psi\left(r_V - r_{MKT}\right). \tag{14}$$

The SDF expression is useful if we want to think about the model in conditional terms. So far, our analysis focused on a one-period game. To extend the asset pricing implications to a multi period setting, I use an non-overlapping generations approach. In this case, every time period t I can write the SDF as:

$$M_{t+1} = 1 + \left(\tilde{\mu}'\Sigma^{-1}\tilde{\mu}\right)_t - \left(\iota'\Sigma^{-1}\tilde{\mu}\right)_t\left(r_{MKT,t+1} - r_{f,t+1}\right) + 2\psi_t(r_{V,t+1} - r_{MKT,t+1}). \tag{15}$$

This form of the SDF stresses the role of the hedging demands term in the conditional asset pricing model. The return on VMM is scaled by the hedging demands term. It implies that the importance of covariance of a return with the VMM factor at time $t+1$ depends on the hedging demands at time t. At times when the correlation within the most volatile group of stocks increases, the hedging demands term increases, and covariance with VMM leads to higher prices.

3.4.2 Status Investors and Markowitz Investors

Let us suppose that along with the status investors, there are Markowitz investors who invest solely in the tangency portfolio. In particular, suppose that for every pair of leader/laggard investors with wealth of $(k+1)$, there is one Markowitz investor that invests ϕ in the tangency portfolio. ϕ is positively related to the wealth of the

Markowitz investors in the economy, and negatively related to their risk aversion. Now the market portfolio is:

$$\theta_M = \frac{k\theta + E_v + \frac{\phi\Sigma^{-1}\tilde{\mu}}{\iota'\Sigma^{-1}\tilde{\mu}}}{k\theta'\iota + 1 + \phi}. \tag{16}$$

I can use the same derivation used in the previous section to obtain the 2-factor beta pricing model as a function of ϕ:

$$\tilde{\mu}_i = \beta^I_{i,MKT}\left[\frac{\iota'\Sigma^{-1}\tilde{\mu} + \phi\psi}{1 + \frac{\psi\phi}{\iota'\Sigma^{-1}\tilde{\mu}}}\right]Var\left(r_{MKT}\right) + \beta^I_{i,VMM}\left[\frac{-2\psi}{1 + \frac{\psi\phi}{\iota'\Sigma^{-1}\tilde{\mu}}}\right]Var\left(r_{VMM}\right). \tag{17}$$

As ϕ increases, the effect of the status investors on prices in this economy decreases. In particular, the negative premium on VMM becomes less negative as ϕ increases. Nevertheless, the Markowitz investors do not reverse the effect of the status investors. In fact, in the presence of status investors, the Markowitz investors require a lower expected return on assets with high VMM beta than the expected return they require in a CAPM world. The Markowitz investors care about the beta of an asset with the tangency portfolio. The tangency portfolio, however, has a short position in group V, and therefore a higher beta with group V leads to a lower beta with the tangency portfolio.

Not surprisingly, as ϕ goes to infinity, the model converges to the CAPM world:

$$\lim_{\phi\to\infty} \tilde{\mu}(\phi) = \iota'\Sigma^{-1}\tilde{\mu}\Sigma\theta_M. \tag{18}$$

Finally, the Stochastic Discount Factor can be obtained as a function of ϕ:

$$M = 1 + \tilde{\mu}'\Sigma^{-1}\tilde{\mu} - \left[\frac{\iota'\Sigma^{-1}\tilde{\mu} + \phi\psi}{1 + \frac{\psi\phi}{\iota'\Sigma^{-1}\tilde{\mu}}}\right](r_{MKT} - r_f) - \left[\frac{-2\psi}{1 + \frac{\psi\phi}{\iota'\Sigma^{-1}\tilde{\mu}}}\right](r_{VMM}). \tag{19}$$

4 Empirical Tests

The model provides two sets of testable implications. The first set relates to the portfolio choice of investors as a function of their status in their reference group. According to the model, low-status investors hold under-diversified portfolios which are concentrated in highly volatile securities, while high-status investors weight these assets more than traditional Markowitz investors. Kumar (2009) provides empirical support for the portfolio choice implication as he shows that investors who earn less than their neighbors hold under-diversified portfolios, concentrated in lottery like stocks. The mutual fund tournament literature provides indecisive empirical findings regarding the tournament behavior of mutual funds. As discussed in the related literature section, it is perhaps difficult to identify the "laggards" and the "leaders" in such a tournament. The second set of implications relate to the asset pricing results of the model. In this section, I concentrate on testing these implications.

The model provides a linear 2-factor beta pricing model, in which the factors are excess returns. The first factor is the return on the market minus the risk free rate, and the second factor is the return of the most volatile group of stocks minus the market (henceforth VMM). Since the model is a member in the family of linear factor models, we can use an array of statistical tests provided by the empirical asset pricing literature to evaluate it. In addition, the model suggests that assets with higher exposure to the portfolio of the most volatile stocks (henceforth portfolio V) should obtain lower returns. If the model is true, then assets with high exposure to portfolio V (i.e., assets with high VMM beta) are over-priced relative to asset pricing models, such as the CAPM, the Fama-French 3-factor model (1993), and the Carhart 4-factor model (1997), that do not take this negative premium into account. In particular, the model predicts that the CAPM alpha follows:

$$\alpha_{i,capm} = \beta^I_{i,MKT} C + \beta^I_{i,VMM} \lambda^I_{VMM}, \tag{20}$$

where $\beta^I_{i,MKT}$, and $\beta^I_{i,VMM}$ are the univariate slopes of the return of asset i on the market and VMM respectively, C is a constant, and λ^I_{VMM} is the premium on the VMM factor. For a given $\beta^I_{i,MKT}$, a higher $\beta^I_{i,VMM}$ of an asset leads to lower CAPM alpha, since λ^I_{VMM} is a negative number.

I will start from examining, using time series regressions, whether stocks with high exposure to the portfolio of high volatility stocks (i.e., stocks with high VMM beta) obtain low returns relatively to other asset pricing models. Then, I examine the unconditional version of the 2-factor model directly using time series regressions, Fama-MacBeth (1973) regressions, and GMM-Stochastic Discount Factor tests. I find that VMM beta is positively correlated with idiosyncratic risk across our test asset, and therefore I examine the explanatory power of VMM beta versus idiosyncratic risk using Fama-MacBeth regressions and GMM-SDF tests. In addition, I form double-sorted portfolios - I sort first by idiosyncratic risk and then by VMM beta and examine the returns obtained by these portfolios. Finally, I examine the conditional version of the model using GMM.

In testing the asset pricing model, I first need to create test assets that have dispersion in their exposure to the market and to VMM. To successfully create such assets, I need to take into account time variation not only in the volatility of stock returns, but also in the cross-section of stocks volatility. In particular, the composition of the most volatile portfolio of stocks may frequently change and therefore the sensitivity of an individual stock to VMM can dramatically change in a short period of time. For example, in the second half of 1978, stocks of the petroleum industry (SIC 1311) constituted only 2.7% of portfolio V, while in the second half of 1979, stocks of the petroleum industry constituted 28.1% of portfolio V. The dramatic change was caused by the oil crisis of 1979. Hence, to form the V portfolio using the most volatile stocks and to obtain up-to-date VMM beta estimators, short windows with daily data are preferable. However, to obtain more accurate estimators, longer windows are better. Most studies that estimated betas use a formation period of more than a year, on the other hand, Ang, Hodrick, Xing and Zhang (2006), use a formation period of one month to estimate idiosyncratic volatility. I choose a formation period of six month to balance this tradeoff[3].

My sample includes all daily and monthly data of AMEX, NASDAQ, and the NYSE stocks. For the time-series regressions, I use the sample period of January 1945 to December 2008. I do not use earlier data, as during and prior to world war II, there were several periods in which less than five stocks satisfied the conditions necessary to be included in portfolio V. For the Fama-MacBeth regressions and for the GMM-SDF tests, I use the period of July 1963 to December 2008. These tests are used to examine the explanatory power of the model versus idiosyncratic risk, which was found to have explanatory power in the cross-section of expected return in a study by Ang, Hodrick, Xing and Zhang (2006). To comply with this study, I start the sample period in July 1963. In my tests, I use the Fama-French (1993) factors, MKTRF, SMB, and HML, and the momentum factor, UMD, constructed by Kenneth French[4].

4.1 Forming the Test Assets

My model has a sharp prediction regarding the relationship between univariate market betas, univariate VMM betas, and stock expected returns. Higher univariate VMM beta leads to lower expected return, while higher univariate market beta leads to higher expected return. Following the spirit of the model, I construct strategies that select stocks based on their univariate slopes to the market and to VMM. I first form portfolio V as the value-weighted top decile of stocks sorted by total volatility. I estimate total volatility using daily returns of the past six months. I then sort stocks into $5 \times 5 = 25$ portfolios. First, I sort stocks into quintiles according to their univariate market beta, estimated using daily returns of the past six months. Next, for every quintile, I sort stocks into sub-quintiles based on their univariate VMM beta.

[3]The results are robust to a formation period of three months.

[4]The data source for these four factors is Kenneth French's web site at Dartmouth.

Since a sound estimation of VMM beta is crucial for my tests, I follow Pastor and Stambaugh (2003) and forecast the next period VMM beta using a cross-sectional predictive model that relates the VMM beta of a specific stock in a certain month to the lagged VMM beta and other predictive variables detailed below.

4.1.1 Choosing Portfolio V

I use the following procedure to form portfolio V every month:

1. I include only stocks that have daily returns for all trading days in the past six months.

2. I exclude the lowest decile of stocks in terms of dollar volume.

3. I rank stocks according to their total volatility estimated in the previous six months, according to equation (21) below, and pick the value-weighted top decile as portfolio V.

I use the liquidity-based filtration, because I am interested in capturing the co-movements across volatile stocks and in measuring sensitivities to VMM. Hence, I refrain from using noisy stocks with low-quality daily return data that suffer from micro-structure issues and from the problem of zero returns that might obscure my estimations.

I estimate $Var(r_i)$ using daily returns of the past six months. Since non-synchronous trading of securities causes daily portfolio returns to be autocorrelated, I follow French, Schwert, and Stambaugh (1987), and estimate $Var(r_i)$ as the sum of the squared daily return plus twice the sum of the products of adjacent returns:

$$\hat{\sigma}^2_{i,t} = \sum_{\tau=1}^{N_t} (r_{i,\tau,t})^2 + 2 \sum_{\tau=1}^{N_t-1} (r_{i,\tau,t})(r_{i,\tau+1,t}), \tag{21}$$

where there are N_t daily stock returns, $r_{i,\tau,t}$, in formation period t. After obtaining the variance for the entire six months, I divide by six to obtain an estimator for the monthly variance.

Table 1 provides some statistics about VMM and its relation with other well known factors. VMM has an average monthly return of -0.86% and its monthly standard deviation 8.22% is, not surprisingly, the highest among the factors. VMM has a correlation of 0.69 with SMB, and a correlation of -0.53% with HML, suggesting that small growth stocks set the tone within portfolo V. VMM has a low correlation with UMD, suggesting that its return is not driven by momentum effects. Finally, the correlation of VMM with MKT is 0.52. Although the VMM factor has a short position in the market portfolio, the market beta of portfolio V is high enough to make the correlation between VMM and MKT positive.

4.1.2 Forecasting VMM beta

To forecast the next period β_{VMM} for a specific stock, I start by regressing the daily return of the stock on the daily returns of VMM in the formation period. To account for nonsynchronous price movements in returns, I follow Lewellen and Nagel (2006), who include four lags of factor returns, imposing the constraint that lags 2, 3, and 4 have the same slope to reduce the number of parameters. The Lewellen and Nagel method is an extension of Dimson (1979), who included current and lagged factor returns in the regression, and addressed the finding that small stocks tend to react with a week or more delay to common news (Lo and MacKinlay, 1990), so a daily beta will miss much of the small-stock covariance with market returns. Specifically, I estimate β^I_X, where X is either the excess return on the market or VMM, using the following regression, in which the dependent variable is an excess return of a stock:

$$r_{i,t} = \alpha_i + \beta_{i,0} r_{X,t} + \beta_{i,1} r_{X,t-1} + \beta_{i,2}\left[r_{X,t-2} + r_{X,t-3} + r_{X,t-4}\right] + \epsilon_{i,t}. \tag{22}$$

The estimated beta is then:

$$\beta_{i,X} = \beta_{i,0} + \beta_{i,1} + \beta_{i,2}.$$

The empirical literature has shown that stock level beta estimators are quite noisy and not persistent (e.g. Blume, 1971). In our case, the problem is exacerbated, since I am using a short period to estimate VMM beta, and the VMM factor is very volatile. Table 2 presents cross-sectional predictive regression results of next month's VMM beta ($\beta^I_{VMM,t+1}$) on various variables estimated in the previous six months. Every month t, I measure the next month's VMM beta using the daily returns of month $t+1$. On average, a cross-sectional regression of $\beta^I_{VMM,t+1}$ on $\beta^I_{VMM,t}$ yields an R^2 of just 0.02.

To improve the predicting ability of VMM beta, I start by adding volatility - a stock cannot have high exposure to portfolio V without being volatile itself. The reverse argument, however, is not necessarily true. A stock with high volatility can have low exposure to volatile stocks, for example, in the event that its volatility is purely idiosyncratic and is driven by factors that are not common with any other stock. Nevertheless, the formation period volatility has a significant predictive power for the next period VMM beta. In fact, as a stand-alone predictor, it is not inferior to the lagged VMM beta, judging by the average R^2 which is 0.02 in both cases. Table 2 also shows that the measure of idiosyncratic risk using the last month daily returns has significant predicting ability for the next period VMM beta.

If a certain industry is extremely volatile at a certain point in time, then portfolio V is likely to contain a high proportion of stocks associated with that industry and the factor of high volatility stocks will be dominated by this industry. In such an event, a stock belonging to that industry is likely to have high exposure to portfolio V. To quantify this intuition, I measure the percentage proportion of every industry i (4 digit SIC code[5]) in the market (denoted by m_i) and the percentage proportion of every industry i (4 digit SIC code) in the V portfolio (denoted by v_i). I construct a measure of industry affiliation to portfolio V by:

$$\phi_i = v_i - m_i. \tag{23}$$

A stock of industry i that has the same proportion of market cap in portfolio V and in the market portfolio will have a neutral affiliation of $\phi_i = 0$. For example, in the period of October 1999 to March 2000, the most volatile industry was SIC 7372, "Prepackaged software," with a market proportion of 7.69%, a portfolio V porportion of 33.19%, and $\phi_i = 25.50$. Table 2 shows that higher ϕ_i is indeed positively related with higer VMM beta in the following period.

Table 2 shows that all three variables are jointly significant in forecasting the next period VMM beta, and so I use all three variables to forecast the next period VMM beta:

$$\beta^I_{VMM,i,t+1} = C_0 + C_1\beta^I_{VMM,i,t} + C_2\sigma_{i,t} + C_3\phi_{i,t}. \tag{24}$$

Each month, I estimate the coefficients in (24) in the following way. I run 240 monthly cross sectional regressions over the previous 20 years and estimate the coefficients in (24) as the average of values obtained in the 240 regressions. It is important to update the predictive regression as the relationship between the variables can change over time. For example, SIC codes have become more accurate and informative with time and indeed untabulated results show that C_3 is increasing with time.

4.1.3 The 25 Test Assets

Stocks are sorted first into quintiles based on market beta, and then in each quintile, they are sorted again into sub-quintiles according to the forecasted VMM beta. Table 3 depicts statistics for the 25 portfolios. For each market beta quintile, the raw returns on the highest VMM beta sub-quintile (the right most column) are generally lower than the returns of other portfolios. This result is most pronounced for the (5,5) portfolio - the portfolio with the highest market beta and the highest VMM beta.

[5] I use the four digit SIC code in order to obtain the most informative partition of industries. The results are robust to a specification of 3 digit SIC code.

This portfolio earns an average return of 0.37% per month, which is by far the lowest return among all portfolios. Portfolios with higher VMM beta are more volatile - the standard deviation of portfolio monthly returns are increasing with VMM beta. In addition, the value-weighted average volatility and idiosyncratic volatility across stocks in each portfolio are increasing with VMM beta. Idiosyncratic volaility is measured following Ang et al. (2006), with respect to the Fama-French (1993) 3-factor model, using the daily returns of the prior month. Intuitively, a portfolio with higher VMM beta contains stocks with higher sensitivities to portfolio V, which contains the most volatile stocks, and therefore we expect it to be more volatile as well.

To test a factor model, previous studies form portfolios using pre-formation criteria, but examine post-ranking factor loadings that are computed over the full sample. To provide a convincing factor risk explanation, I need to show that the portfolios also exhibit persistent loadings on VMM over the same period used to compute the alphas. The pair of panels in the first row of Table 4 depict the post-formation VMM betas of the 25 portfolios and their t-statistics. Indeed, in each and every row the post-formation coefficients on VMM follow the pre-formation coefficients in terms of ranking. Moreover, the dispersion in VMM beta among portfolios is quite high. I form a high VMM beta minus low VMM beta portfolio for each market beta quintile, and show in Table 4 that the VMM beta on these five portfolios are $0.6, 0.46, 0.52, 0.44,$ and 0.66.

4.2 Time Series Analysis

To examine whether other asset pricing models overprice portfolios with high VMM beta, I examine the Jensen alphas obtained for the 25 portfolios. The pair of panels in the second row of Table 4 show the CAPM alphas and their t-statistics for each of the 25 portfolios. Indeed, for each market beta quintile, we see that alphas are generally decreasing with VMM beta. The portfolio with the highest VMM beta has the lowest CAPM alpha for each and every market beta quintile. Moreover, for each market beta quintile, the alphas of the high VMM beta minus low VMM beta portfolio are -0.35%, -0.48%, -0.62%, -0.39%, and -1.01% (from the quintile with the lowest market beta to the quintile with the highest market beta). The t-statistics values for these alphas are -1.66, -2.69, -3.31, -1.90, and -4.19 respectively. These findings indeed support the prediction of the model - stocks with higher exposure to portfolio V are overpriced by the CAPM.

The results are qualitatively the same using the Fama-French 3-factor model and the Carhart 4-factor model (the panels in the third and fourth rows of Table 4). A joint test for the 25 alphas equal to zero rejects the CAPM, the Fama-French 3-factor model, and the Carhart 4-factor model with p-values of 0.0014, 0.0001, and 0.0001 respectively. On the other hand, the 2-factor beta pricing model, advocated by this paper, does a significantly better job in pricing these portfolios. A joint test for the 25 alphas equal to zero cannot reject the model at the 5% significance level (the p-value is 0.095).

An important empirical issue is whether our results coincide with those of Ang et al. (2006), who found that stocks with high idiosyncratic volatility (IVOL) relative to the Fama and French (1993) model have abysmally low average returns. The cross-sectional correlation between VMM beta and idiosyncratic volatility across the test assets is 0.84, where VMM beta is obtained for each portfolio using a post-formation regression over the sample of July 1963 to December 2008, and idiosyncratic volatility is calculated for each portfolio every month, following Ang et al. (2006), and averaged across the sample of July 1963 to December 2008. This high cross-sectional correlation makes it difficult to disentangle these two effects.

I start addressing this by sorting stocks into IVOL quintiles, and then for each IVOL quintile, I sort them again into two portfolios based on VMM beta. Tables 5 and 6 show the results of this analysis. For the quintile with the highest idiosyncratic risk, the portfolio with lower VMM beta has a simple return of 0.31% per month, while the portfolio with higher VMM beta has a simple return of −0.39% (Table 5). For the quintile with the highest idiosyncratic risk, the risk-adjusted returns on the high VMM beta portfolio are significantly lower than those of the low VMM portfolio (Table 6). For example, the CAPM alpha for the portfolio of high VMM beta minus low VMM beta in the quintile with the highest diosyncratic risk is −0.87% per month, statistically significant at the 1% level. The results suggest the VMM beta has an explanatory power beyond that of idiosyncratic volatility. A caveat is in order - the two variables are positively correlated and therefore sorting by VMM beta creates a distinguishable difference in IVOL (Table 5). I will revisit this issue using Fama-MacBeth analysis and GMM-SDF tests.

Finally, Table 6 shows that the 2-factor model proves to better price the quintile portfolios sorted by idiosyncratic volatility, following Ang et al. (2006). While the monthly alphas of the high idiosyncratic risk minus low idiosyncratic risk portfolio are −1.20%, −1.23%, and −0.96% using the CAPM, the Fama-French 3-factor model, and the Carhart 4-factor model, the monthly alpha of that portfolio using the 2-factor model is −0.32%.

4.3 Fama-MacBeth Analysis

The 2-factor asset pricing model suggests that expected returns follow:

$$E(r_i - r_f) = \beta^I_{i,MKT}\lambda^I_{MKT} + \beta^I_{i,VMM}\lambda^I_{VMM}, \quad (25)$$

where $\beta^I_{i,MKT}$, and $\beta^I_{i,VMM}$ are the univariate slopes of the return of asset i on the market and VMM respectively, and λ^I_{VMM} and λ^I_{MKT} are the premiums on the VMM factor and the market factor respectively. We can also write the model using the bivariate slopes from a multiple regression of stock return on both the excess return of the market and VMM:

$$E(r_i - r_f) = \beta^{II}_{i,mf}\lambda^{II}_{MKT} + \beta^{II}_{i,VMM}\lambda^{II}_{VMM}. \tag{26}$$

With this form, λ^{II}_{MKT} is the expected excess market return and λ^{II}_{VMM} is the expected return on VMM. Since VMM and the market are positively correlated with a correlation of 0.52, and since the model suggests that $\lambda^I_{VMM} < 0$, and $\lambda^I_{MKT} > 0$, the model implies the following relationship between the premiums:

- $\lambda^I_{VMM} < 0$.
- $\lambda^I_{MKT} > 0$.
- $\lambda^{II}_{VMM} > \lambda^I_{VMM}$.
- $\lambda^{II}_{MKT} < \lambda^I_{MKT}$.

Naturally, the model suggests that the bivariate premium on each factor equals its expected return. The Fama-MacBeth method is a convenient framework to evaluate the model and its predictions. In addition, it allows us to confront the factors suggested by the model with various factors that have proven to have an explanatory power for the cross section of expected returns such as the Fama-French SMB and HML factors and the momentum factor UMD. Finally, the Fama-MacBeth procedure provides a closer look into the pricing relationship between idiosyncratic volatility and VMM beta.

Following Fama-MacBeth (1973), I first perform time series regressions where I regress the excess portfolio returns on a constant and on various factors - MKT, SMB, HML, UMD, and VMM. In the second step, the excess portfolio returns are regressed on the estimated factor loadings in each month in the sample. Then, a time series average of the estimated coefficients is taken to arrive at point-estimates and statistical significance of the factor premia. To examine the role of idiosyncratic risk (IVOL), I calculate the IVOL of each portfolio in each month following Ang et al. (2006), and use the averaged IVOL value for each portfolio for the entire sample.

Table 7 depicts the results of the Fama-MacBeth regressions. The first row shows that the CAPM can account for 16% of the cross-sectional variation in the returns on the 25 portfolios. In contradiction to the theoretical expectation of the CAPM, the market factor (MKT) shows up with an insignificantly negative factor premium.

My 2-factor model suggests that the CAPM is misspecified. Since the univariate market beta and VMM beta are positively correlated, the CAPM attributes high market beta to portfolios with high VMM beta that obtain low returns. The second row shows that VMM beta on its own also shows up with an insignificantly negative factor premium. Now, the cross-sectional R^2 is 0.28.

Rows 3 and 4 show the Fama-MacBeth results for my 2-factor model. Row 3 depicts the results with the univariate betas, while row 4 depicts the results with the bivariate betas. In both cases, the R^2 with the 2-factor model jumps to 0.61. The predictions of the model regarding the risk premiums are supported. The estimated univariate market premium is 1.67% and the estimated VMM premium is -2.47%. Both premiums are significant at the 1% level. The bivariate market premium is 0.97%, significant at the 1% level, and the bivariate VMM premium is -0.86%, significant at the 5% level. The bivariate premium of VMM, -0.86%, is exactly the mean monthly return of VMM for the sample period.

Row 5 shows the results for the Fama-French 3-factor model, and row 6 shows the results for the Fama-French 3-factor model augmented with VMM. Adding VMM to the Fama-French 3-factor model increases the R^2 from 0.47 to 0.73 and VMM shows up with a factor premium of -1.02%, significant at the 5% level. In fact, row 6 shows that VMM is the only factor that is significant at the 5% level among the four factors. In the absence of VMM, SMB and HML observe the effect of VMM. The SMB factor, which is positively correlated with VMM, shows up with a negative premium of -0.39%. The HML factor, which is negatively correlated with VMM, shows up with a positive premium of 0.36%. However, adding VMM changes the coefficient on SMB to -0.1% and renders the HML negative with a premium of -0.48%. The results are qualatitively the same with the Carhart 4-factor model (rows 7 and 8).

Finally, I examine the role of idiosyncratic risk. Row 10 shows that a model with the market and idiosyncratic volatility has an R^2 of 0.39, which is significantly lower than the R^2 of the 2-factor model, which is 0.61. Nevertheless, since IVOL is positively correlated with VMM beta, the premium on IVOL is negative, -0.28%, and significant at the 5% level. Row 11 shows that when IVOL is added to univariate market beta and the univariate VMM beta, it becomes economically and statistically insignificant with a premium of -0.02%. However, VMM is still significant at the 1% level. Hence, the results suggest that VMM drives out idiosyncratic volatility, and not the other way around.

4.4 GMM - SDF Tests

My model provides an explicit Stochastic Discount Factor, which is linear in the market return and the return on the high volatility portfolio:

$$M_{t+1} = 1 + \left(\tilde{\mu}'\Sigma^{-1}\tilde{\mu}\right)_t - \left(\iota'\Sigma^{-1}\tilde{\mu}\right)_t (r_{MKT,t+1} - r_{f,t+1}) + 2\psi_t(r_{V,t+1} - r_{MKT,t+1}).$$

The model suggests that the coefficient on the excess market return in the SDF is negative, while the coefficient on the return of high volatility assets minus the market in the SDF is positive. I examine the predictions of the model by estimating the model $E[MR] = 1$ using the GMM of Hansen (1982). In the analysis below, I choose the weighting matrix W to be the asymptotically optimal one, given by the inverse of the covariance matrix of the moment conditions. I use the same 25 portfolios used in the previous sections. As Cochrane (2005) notes, when the factors are correlated, one should test whether the SDF-parameter coefficients equals zero to see whether a certain factor helps to price the assets rather than to test whether the factor premium obtained from the Fama-MacBeth method equals zero. In our case, the excess market return and the VMM factor indeed have a positive correlation of 0.52.

I start by examining the pricing ability of the unconditional 2-factor model versus the CAPM, the Fama-French 3-factor model and the Carhart 4-factor model. In particular, I estimate the following moment conditions:

$$1 = E\left[(B_0 - B'F_{t+1}) \cdot R_{t+1}\right], \tag{27}$$

where B_0 is a constant, B is the vector of coefficients to be estimated, F_{t+1} is a vector of factors included in the specification of the SDF, and R_{t+1} is the vector of the returns on the 25 portfolios. Using this specification, a factor that commands a positive risk premium should have a positive coefficient.

Consistent with the Fama-MacBeth regressions, row 1 of Table 8 documents the failure of the CAPM in pricing the 25 portfolios. The CAPM is formally rejected by Hansen's (1982) test of overidentifying restrictions with a p-value of 1%, and the coefficient on the market is negative and statistically insignificant. Row 2 shows the results with the unconditional version of the Stochastic Discount Factor advocated by the model. Adding VMM along with the market excess return in the specification of the stochastic discount factor significantly improves the pricing ability of the SDF. Now, the test of overidentifying restrictions cannot reject the 2-factor SDF at the 10% significance level (the p-value is 13%). Both the excess market return and the return of the VMM portfolio show up with the expected signs, and both are statistically significant at the 1% level.

The 2-factor SDF performs better than the Fama-French 3-factor model (row 3) and the Carhart 4-factor model (row 5), as both are rejected by the test of overidentifying restrictions at the 5% significance level. Adding VMM to the Fama-French 3-factor model renders both the SMB and the HML statistically insignificant, while VMM remains significant at the 1% level. Moreover, after adding VMM to the Fama-French 3-factor SDF, the test of overidentifying restrictions cannot reject the SDF at the 5% significance level. The results for the Carhart 4-factor models are qualitatively the same (rows 5 and 6).

The conditional SDF model obtained by the model implies that the effect of the covariance between an asset's return and the VMM factor depends on the hedging demands term - ψ. When the hedging demands term is high, covariance with VMM leads to lower expected returns. To estimate the conditional model, I first need to estimate the hedging demands term ψ. The expression for ψ in the model, when the number of assets in group V goes to infinity, converges to the following term:

$$\psi_t = \frac{\rho_{v,t}(k-1)}{\sigma_{V,t}^2(1-\rho_{v,t})}, \tag{28}$$

where $\rho_{v,t}$ is the correlation of each pair of assets in portfolio V at time t, and $\sigma_{V,t}$ is the volatility of portfolio V at time t. I estimate ψ_t using daily returns in the six months of the formation period. I estimate $\rho_{v,t}$ as the average correlation between all pairs in portfolio V in the six months formation period, and I estimate the monthly total volatility of portfolio V, $\sigma_{V,t}$, using daily returns in the formation period, following equation (21) above. Following Ferson and Harvey (1999), I demean the scaling variable ψ_t. After obtaining an estimate for ψ_t for every month, I estimate the following moment conditions:

$$1 = E\left[\left(B_0 - B_1\left(r_{MKT,t+1} - r_{f,t+1}\right) - B_2(1 + B_3\psi_t)(r_{V,t+1} - r_{MKT,t+1})\right) \cdot R_{t+1}\right]. \tag{29}$$

The model suggests that the coefficient B_3 should be positive - in times where ψ_t is high, covariance with VMM leads to lower returns. Row 7 of Table 8 reports the results of this estimation. The conditional model performs better than any other model, including the unconditional one, according to the p-value of the test of overidentifying restrictions, which is 0.21. The coefficients on the market excess return and on VMM remain significant and with the expected sign. The coefficient on ψ_t has the expected positive sign, however, it is statistically insignificant.

To examine the role of idiosyncratic volatility, I follow Nyberg (2008), and examine whether idiosyncratic volatility has pricing power beyond that of the stochastic discount factors specified above. I estimate the parameters of the stochastic discount factors using GMM with the following moment conditions:

$$1 = E\left[(B_0 - B'F_{t+1}) \cdot R_{t+1} - \gamma_{IVOL} IVOL_t\right]. \tag{30}$$

The idiosyncratic volatility for every portfolio is estimated using last month's daily returns, following Ang et al. (2006). If the stochastic discount factors specified above cannot capture the negative relation between idiosyncratic volatility and returns, then γ_{IVOL} should have a negative coefficient. Rows 8, 10, and 11 show that indeed idiosyncratic volatility has a pricing ability above that of the SDF, using CAPM, Fama-French 3-factor model, and the Carhart 4-factor model. In all cases, IVOL is significant at the 1% level. In fact, the test cannot reject both the Fama-French 3-factor model and the Carhart 4-factor model when IVOL is added to the specification. However, when idiosyncratic volatility is added to the 2-factor model with the market return and VMM, IVOL becomes statistically insignificant. Moreover, the p-value of the test of overidentifying restrictions decreases from 0.13, without IVOL, to 0.10 with IVOL.

The GMM-SDF tests agree with the Fama-MacBeth analysis on several results. First, the 2-factor model advocated by the model proves to better price the test assets than the CAPM, Fama-French 3-factor model, and the Carhart 4-factor model. Controlling for SMB, HML, UMD and idiosyncratic volatility, VMM beta has a significant pricing ability for the cross section of the expected returns of the test assets. Nevertheless, idiosyncratic risk and VMM beta are positively correlated, and in the absence of VMM beta, idiosyncratic risk can help in pricing the test assets. Moreover the GMM-SDF tests provides support to the conditional version of the 2-factor model, as it does a better job in pricing the test assets than the unconditional version.

5 Conclusion

A fundamental assumption that underlies the lion's share of portfolio choice and asset pricing theories is that investors care only about their absolute consumption or wealth. However, both in the social context and in the mutual fund arena, a growing literature suggests that investors are concerned about their status relative to a reference group of other investors. In this paper, I examine the implications of such concerns on portfolio choice and asset pricing in an economy with status conscious investors and traditional Markowitz investors. I devise a model that captures the fundamental features of the concern for status. Status is inherently positional and therefore gives rise to a strategic interaction among investors. In addition, different investors define their status with respect to different local reference groups. Hence, in each reference group, there must be investors with high relative status and investors with low relative status. My model introduces many pairs of low-status and high-status investors into an economy populated with traditional Markowitz investors. These status-conscious investors strategically choose their investment in order to maximize their expected status.

The model provides a rich set of implications. In a general equilibrium, low-status investors hold a single high volatility asset in order to move up the status ladder. Since high-status investors are concerned about the risk of losing their status, they demand assets that co-vary with high volatility assets, as a hedge against low-status investors. In equilibrium, the demand for exposure to high-volatility leads to a 2-factor model, in which the first factor is the market and commands a positive premium, while the second factor is a portfolio of high volatility stocks and commands a negative premium. The model also has dynamic implications. At times when the returns of high-volatility assets co-vary more, and the variance of the high-volatility factor is higher, high-status investors are induced to invest more in assets with high exposure to the high-volatility factor, and its premium becomes more negative.

The general equilibrium asset pricing model derived in the paper is novel in at least two important dimensions. First, it is not driven by the preferences of a single representative agent, but rather it is a result of strategic interactions among heterogeneous investors. Second, the general theme of asset pricing models is that factor risk premiums arise due to the fact that risk-averse investors seek to limit their exposure to systematic risk factors. In my model, the high-volatility factor premium arises due to the fact that status-conscious investors seek exposure to this factor in order to hedge against their status risk.

I test the asset pricing implications of the model using stock market data and find significant economic and statistical support. The two factor model, advocated by this paper, does a better job than the CAPM, the Fama-French 3-factor model (1993), and the Carhart 4-factor model (1997) in pricing assets with dispersion in their exposure to the market and exposure to high-volatility stocks. In particular, the model provides an explanation to the idiosyncratic volatility puzzle of Ang, Hodrick, Xing and Zhang (2006) as there is a positive cross-sectional correlation between high exposure to the high-volatility factor and idiosyncratic risk. Nevertheless, I show that my 2 factor model has a cross-sectional pricing ability above and beyond that of idiosyncratic risk, suggesting that the empirical results of this paper cannot be solely explained by the negative discount on idiosyncratic risk, documented by Ang et al. (2006).

6 Appendix

6.1 Proof Of Proposition 2

I will prove the proposition in several steps. First, I examine the best risky strategy of the laggard, given that he invests some wealth in the risky assets. The following lemma argues that within any group I the laggard will not invest in more than one asset. In particular, the risky asset chosen by the laggard out of group I is one in which the leader invests the lowest amount out of all assets in group I.

Lemma 6. *Given a strategy of the leader that satisfies condition (4) for all possible strategies of the laggard, and given that the laggard invests some wealth in the risky assets, within each group I the laggard holds at most one risky asset. If the laggard invests in one asset out of group I, then that asset belongs to the set* $S_I(\theta_{I,d})$ *where*

$$S_I(\theta_{I,d}) = \underset{j\in\{1,\ldots,N_I\}}{\text{argmin}}\ \theta_{I,d,j},$$

and where $\theta_{I,d}$ *is the length* N_I *leader's group I portfolio, and* $\theta_{I,d,j}$ *is the amount invested by the leader in asset* j *of group* I*.*

Proof. Since the leader uses a strategy that satisfies condition (4) for all possible strategies of the laggard, equation (3) implies that the laggard prefers portfolios with higher expected return, higher variance and lower covariance with the leader. Let us fix the portfolios that the laggard holds in the non-I groups, fix the amount the laggard invests in group I, and examine the optimal portfolio within group I. First, note that since all assets are similar within group I, any portfolio yields the same expected return. Therefore, we can focus on the way choosing a portfolio within group I affects the laggard's variance and covariance with the leader.

To maximize the variance of his portfolio, in what follows I show that the laggard is strictly better off investing in a single asset of group I. The contribution of the group I portfolio to the total variance of the laggard is via the covariance terms of the laggard's group I portfolio with the laggard's other group portfolios, and via the variance of the group I portfolio. Fixing the wealth the laggard invests in group I, it is not difficult to show that the covariance terms of the group I portfolio with his non-I group portfolios do not depend on the way he distributes his wealth across the assets in group I. However, the variance of the group I portfolio is maximized by investing in a single asset of group I, since the correlation of any pair of assets in group I is less than one according to equation (5). The similarity of assets across group I suggests that the laggard is indifferent with respect to which asset to hold.

To minimize the covariance of the laggard with the leader, in what follows I show that the laggard is strictly better off investing in assets that belong to $S_I(\theta_{I,d})$, the set of assets in which the leader invests the lowest amount out of group I. The covariance of the laggard's group I portfolio with the leader's portfolio depends on the covariance of the laggard's group I portfolio with the leader's non-I group portfolios and the covariance of the laggard's group I portfolio with the leader's group I portfolio. Again, fixing the amount the laggard invests in group I, the covariance of his group I portfolio with the leader's non-I group portfolios is constant. However, the covariance between the laggard's group I portfolio and the leader's group I portfolio can be written in the following way:

$$Cov(r_{I,d}, r_{I,g}) = \theta_{I,d} \Sigma_I \theta_{I,g},$$

where $r_{I,d}$ is the return on the leader's group I portfolio, $r_{I,g}$ is the return on the laggard's group I portfolio, $\theta_{I,d}$ is the leader's group I portfolio, $\theta_{I,g}$ is the laggard's group I portfolio, and Σ_I is the covariance matrix of group I. The similarity of assets within group I suggests that Σ_I can be written in the following way:

$$\Sigma_I = \sigma_i^2 \left(\rho_I \Omega + (1 - \rho_I) I\right),$$

where Ω is an N_I by N_I all-ones matrix, and I is the N_I by N_I identity matrix. Now, I can write the covariance between the laggard's group I portfolio and the leader's group I portfolio as:

$$Cov(r_{I,d}, r_{I,g}) = \sigma_i^2 \left(\rho_I + (1 - \rho_I) \theta'_{I,d} \theta_{I,g}\right).$$

Therefore, the laggard can minimize this covariance by investing in group I assets that belong to $S_I(\theta_{I,d})$. From covariance point of view, it does not matter how the laggard distributes his wealth within the assets in $S_I(\theta_{I,d})$. However, taking into account the laggard's variance considerations, he is strictly better off investing in a single asset out of $S_I(\theta_{I,d})$. □

I further examine the best risky strategy of the laggard, given that he invests some wealth in the risky assets. The following lemma argues that the laggard will invest only in a single risky asset out of all risky assets in the economy.

Lemma 7. *Given a strategy of the leader that satisfies condition (4) for all possible strategies of the laggard, and given that the laggard invests some wealth in the risky assets, the laggard invests in a single risky asset.*

Proof. Using lemma 6, the risky strategy of the laggard can be characterized by a length G vector θ_g, $\theta'_g \iota = 1$, that reflects his investment in a single asset of each group.

Let θ_d represent the portion the leader invests in every group, such that $\theta_d'\iota = 1$. Let w_d be the portion the leader invests in the risky assets, and w_g be the portion the laggard invests in the risky assets. Let μ be the length G vector of expected returns for groups. Let $\tilde{\Sigma}$ be the G by G covariance matrix for individual assets across different groups, and let $\hat{\Sigma}$ be the G by G covariance matrix that reflects the covariance of the G assets chosen by the laggard with the G portfolios chosen by the leader out of each group. Finally, let Var_d be the variance of the risky portfolio of the leader. We can write the laggard's problem as:

$$Min_{\theta_g}\ U(\theta_g) = \frac{k\left(w_d\theta_d'\mu + 1 - w_d\right) - \left(w_g\theta_g'\mu + 1 - w_g\right)}{\sqrt{k^2{w_d}^2 Var_d - 2kw_g w_d \theta_d'\hat{\Sigma}\theta_g + {w_g}^2\theta_g'\tilde{\Sigma}\theta_g}}. \tag{31}$$

Now, I will show that the best response of the laggard is not only to invest in a single asset out of every group, but also to invest in a specific group out of all groups. That is, $\theta_g = E_i$, where E_i is a vector of zeros except for entry i, which is one. By showing this, I will conclude that given an investment in the risky assets, the best response of the laggard is to invest in a single risky asset out of all risky assets in the economy.

Assume by contradiction that the best response of the laggard is not to invest in a single asset. In this case, there must be a pair of groups I and J where he invests a portion of his risky portfolio w_i^* and w_j^*. Due to the short sales constraint, it must be that $0 < w_i^*, w_j^* < 1$. We can examine strategies in which the laggard transfers wealth from i to j and invests $w_i = w_i^* + z$ and $w_i = w_j^* - z$ in a single asset of group i and a single asset of group j. Now, we can write the objective function of the laggard as $U(z)$ instead of $U(\theta_g)$.

Consider the unconstrained problem of the laggard, when short sales are allowed, solving for z. First, note that $U(z)$ is continuous for all z, since its denominator represents the variance of the wealth difference between the players. This variance cannot be zero, because if the leader chooses a single asset out of every group, lemma 6 suggests that the laggard will choose a different asset than the leader out of every group. Therefore, it must be that the portfolios of the two players are different. Now, the optimality of w_i^* and w_j^*, and the fact that $0 < w_i^*, w_j^* < 1$, guarantees that $z = 0$ is a local minimum for this problem. At this point, $U(z = 0)$, the value of the objective function of the laggard, is positive, as the leader has a higher expected wealth than the laggard. Note that in the unconstrained problem of the laggard, he can increase the weight on the security (either i or j) that has higher expected return and by doing so he can reach negative values for $U(z)$. Alternatively, if both assets have the same expected return, he can increase the weight on one of them, taking the denominator to infinity and the value of the objective function $U(z)$ to zero. Therefore, since $U(z = 0) > 0$, and $z = 0$ is a local minimum, the function $U(z)$ must have at least one maximum point. Hence, $U(z)$ has at least two extremum points. However, it is not difficult to show that the function $U(z)$ is of the form:

$$U(z) = \frac{a + bz}{\sqrt{Az^2 + Bz + C}}. \tag{32}$$

By taking the first derivative of U(z), it is easy to show that each member in this family of functions has at most one extremum point. Contradiction! Hence, the laggard will invest only in one risky asset. □

Next, I will introduce the risk-free asset into the analysis. I will show that if the laggard invests in a risky asset, he will not invest in the risk-free asset as well:

Lemma 8. *Given a strategy of the leader that satisfies condition (4) for all possible strategies of the laggard, the laggard will not invest both in a risky asset and in the risk-free asset.*

Proof. This proof is similar to the previous one. We assume by contradiction that the best response of the laggard is to invest both in a risky asset and in the risk-free asset. We let z be the amount invested in the risky portfolio. Now, we observe that $U(z)$ still follows equation (32), and thus we can use the same line of reasoning to show that the laggard must invest all in the risk-free asset or all in the risky portfolio. □

I use lemma 7 and lemma 8 to conclude that the best response of the laggard is to invest in a single asset - either a risky asset or the risk-free asset. In the event that the laggard invests in a risky asset of some group I, he is indifferent among the assets in group I that belongs to $S_I(\theta_{I,d})$, thus, any mixed strategy across $S_I(\theta_{I,d})$ is a best response as well.

6.2 Proof Of Proposition 3

In this section, I provide a sketch of the proof for proposition 3. If the laggard invests in a single asset, chosen uniformly by a mixed strategy over group V, in what follows I will show that in the non-V groups the leader will invest the same amount in each of the assets. Choosing a portfolio that is different from the equally-weighted portfolio for some group $I \neq V$ will not change the leader's expected return and the covariance with the laggard. However, it will increase the variance of her portfolio, because the covariance between groups will remain the same, whereas the variance of the group V portfolio will increase. Thus, not investing in an equally-weighted portfolio within a non-V group decreases the value of the objective function of the leader.

What left to show is that for N_v large enough, the leader will invest the same amount in each of the assets in group V. The leader faces the following trade-off when she chooses a portfolio within group V. While any portfolio within group V yields the same expected return on the leader's portfolio, choosing the equally-weighted portfolio is the best choice in terms of the overall variance of the leader's portfolio. However, the leader might be induced to invest in other portfolios within group V in order to increase her covariance with the laggard. For example, she can choose only a single asset in group V, hoping that the laggard chooses the same asset. I will show that when N_v is large enough though, the leader is better off concentrating on decreasing her variance, as the large number of assets makes it impossible for the leader to find a portfolio that yields higher covariance with the laggard in a way that offsets the variance inefficiency resulted from not investing in the equally-weighted portfolio. This proof uses only one group in the economy, group V, and it can be generalized to many groups. The presence of more groups in the economy does not change the nature of the covariance-variance trade-off faced by the leader.

Let w_d be the portion invested in the risky assets by the leader. Let μ be the expected return of asset v. Let θ be the risky portfolio of the leader within group V. Let ρ be the correlation of any pair in group V. Let n be the number of assets in group V. Given the mixed strategy of the laggard, the leader seeks a risky portfolio θ to maximize:

$$U_n(\theta) = \frac{1}{n}\sum_{j=1}^{n} \Phi\left(\frac{k\left(\mu w_d + 1 - w_d\right) - \mu}{\sigma\sqrt{1 - 2kw_d\left(\rho + (1-\rho)\theta_j\right) + k^2 {w_d}^2\left(\rho + (1-\rho)\theta'\theta\right)}}\right). \tag{33}$$

First, I will characterize the general form of the leader's solution to this problem.

Lemma 9. *The portfolio that maximizes the function $U_n(\theta)$ takes the following form:*

$$\theta = z\frac{1}{n}\iota + (1-z)E_j;\ j \in \{1, ..., n\}. \tag{34}$$

That is, it is a linear combination of the equally-weighted portfolio and a single asset j, where z is the amount invested in the equally-weighted portfolio

The leader's response reflects the trade-off between variance and covariance with the laggard. If the leader wants to minimize variance, she should invest in the equally-weighted portfolio. However, if she wants to maximize the covariance with the laggard, then the prospects of a successful bet on the laggard's asset might increas the covariance with the laggard. The solution reflects this trade-off as it is a linear combination between the equally-weighted portfolio and a single asset j.

Lemma 10. *There exists an n_0 such that for every $n > n_0$, the equally-weighted portfolio is a local maximum.*

Proof. Now the leader solves for z to find the weight invested in the equally-weighted portfolio. WLOG, the leader invests $(1-z)$ in asset 1. We can write the leader's problem as a function of n, the number of assets in group V in the following way:

$$U_n(z) = \frac{n-1}{n}\Phi\left(\frac{A}{\sqrt{f_n(z)}}\right) + \frac{1}{n}\Phi\left(\frac{A}{\sqrt{g_n(z)}}\right). \tag{35}$$

where the expected value of the wealth difference between the players is A. In addition, the variance of the wealth difference between the players in the event that the laggard chooses an asset different than 1 is expressed as a function of z, the weight invested in the equally-weighted portfolio:

$$f_n(z) = 1 - 2kw_d\left(\rho + (1-\rho)\left[\frac{z}{n}\right]\right) + k^2 {w_d}^2\left(\rho + (1-\rho)\left[\frac{z(2-z)}{n} + (1-z)^2\right]\right), \tag{36}$$

and the variance of the wealth difference between the players in the event that the laggard chooses asset 1:

$$g_n(z) = f_n(z) - 2kw_d(1-\rho)(1-z). \tag{37}$$

Some algebra shows that the derivative of the objective function at $z = 1$ is zero; $U_n'(z = 1) = 0$. Hence, we conclude that the equally-weighted portfolio is an extremum point. Moreover, the second derivative of the objective function at $z = 1$ converges to a negative number as the number of assets in V goes to infinity:

$$\lim_{n\to\infty} U_n''(z = 1) < 0. \tag{38}$$

We conclude that there is an n_0 such that for all $n > n_0$, the equally-weighted portfolio is a local maximum.

□

Lemma 11. *There exists an n_0 such that for every $n > n_0$, the equally-weighted portfolio is a* ***global*** *maximum*

Proof. Given that the solution for this problem has the form of (34), some algebra shows that any extremum point must be a local maximum point for n large enough. Since the equally-weighted portfolio is a local maximum, for large enough n, there cannot be any other local maxima. If there were, then the continuity of $U_n(z)$ implies that there should also be a local minimum point between these maxima. This contradicts our previous observation that for n large enough, any extremum point must be a local maximum.

□

6.3 Conditions for Theorem 4

1. N_V should be large enough in the sense of Proposition 3
2. Conditions to have the laggard not deviating from investing in asset v:
   (a) $\sigma_v > \sqrt{2}\sigma_j \ \forall j \in \{1, ..., g\}$.
   (b) $\sigma_{v,j} > 0 \ \forall j$.
   (c) $\sigma_v > 2\sigma_V$.
   (d) $k(\theta'\tilde{\mu} + 1) > (\tilde{\mu}_j + 1) \ \forall j \in \{1, ..., g\}$.
3. Conditions to have the leader refraining from taking a short position in the risk-free asset or in any of the risky assets
   (a) $0 < \psi^{-1}\iota'\Sigma^{-1}\tilde{\mu} + 1 < k$.
   (b) $E_j\Sigma^{-1}\tilde{\mu} > 0 \ \forall j$.
   (c) $E_v\Sigma^{-1}\tilde{\mu} > -\psi$.

where $\sigma_{v,j}$ is the covariance between an asset in group V and an asset in group J, σ_j is the volatility of an individual asset of group J, σ_V is the volatility of group V, Σ is the covariance matrix for groups, $\tilde{\mu}$ is the expected excess return over the groups, E_v is a vector of zeros except entry v, which is 1, N_V is the number of assets in group V, θ is the optimal portfolio of the leader, and $\psi = \frac{k-1}{\sigma_v^2 - \sigma_V^2}$.

6.4 Proof for Theorem 4

Denoting $A = \tilde{\mu}'\Sigma^{-1}\tilde{\mu}$, we can simplify the following terms:

$$\theta'\tilde{\mu} = xA + y\tilde{\mu}_v,$$

$$\theta'\Sigma\theta = x^2A + 2xy\tilde{\mu}_v + y^2\sigma_V^2,$$

$$\theta'\Sigma E_v = x\tilde{\mu}_v + y\sigma_V^2.$$

Now, we can write the leader's problem as:

$$Max_{(x,y)}\ \frac{k\left([xA+y\tilde{\mu}_v]+1\right)-(\tilde{\mu}_v+1)}{\sqrt{k^2[x^2A+2xy\tilde{\mu}_v+y^2\sigma_V^2]-2k\left[x\tilde{\mu}_v+y\sigma_V^2\right]+\sigma_v^2}}. \tag{39}$$

Taking the first order conditions for x and y and equating both to zero lead to the solution:

$$x=\frac{\sigma_v^2-\sigma_V^2}{k(k-1)},$$
$$y=\frac{1}{k}.$$

So the leader's risky portfolio over groups is:

$$\theta=\frac{\sigma_v^2-\sigma_V^2}{k(k-1)}\Sigma^{-1}\tilde{\mu}+\frac{1}{k}E_v. \tag{40}$$

Given the leader's strategy, we revisit the problem of the laggard and find the conditions required to keep the laggard investing in group V. Since the laggard invests only in a single risky asset or in the risk-free asset, we examine his utility from investing in asset j (note that the laggard is interested in minimizing U_j):

$$U_j=\frac{k\left(\theta'\tilde{\mu}+1\right)-(\tilde{\mu}_j+1)}{\sqrt{k^2\theta'\Sigma\theta-2k\theta'\Sigma E_j+\sigma_j^2}}. \tag{41}$$

Plugging the investment of the leader, we obtain:

$$U_j=\frac{k\left(xA+\frac{\tilde{\mu}_v}{k}+1\right)-(\tilde{\mu}_j+1)}{\sqrt{k^2\left[x^2A+2x\frac{\tilde{\mu}_v}{k}+\frac{\sigma_V^2}{k^2}\right]-2k\left[x\tilde{\mu}_j+\frac{\sigma_{v,j}}{k}\right]+\sigma_j^2}}. \tag{42}$$

We examine the conditions to guarantee that $U_v<U_j$ for all j. The conditions are algebraically involved, but we can find simple sufficient conditions to satisfy this inequality:

1. $\sigma_{v,j}>0\ \forall j$.
2. $\sigma_v>\sqrt{2}\sigma_j\ \forall j$.
3. $\sigma_v>2\sigma_V$.

Restricting the analysis to strategies that are symmetric within each group, proposition 2 implies that in a Nash Equilibrium, the laggard must use a mixed strategy, in which he invests in a single risky asset chosen uniformly over a specific group or over several groups. The conditions above guarantee that the laggard prefers an asset of group V to an asset of any other group. Therefore he will use a mixed strategy only over group V. Since the unique best response of the leader is determined by solving her maximization problem, this equilibrium is a unique one among strategies that are symmetric within each group.

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Table 1: **Factors Statistics** The table reports the means, standard deviations and correlations of VMM and various factors. The factors MKTRF, SMB, HML are the Fama and French (1993) factors, the momentum factor UMD is constructed by Kenneth French. The sample period is July 1963 to December 2008, and the estimated values relate to monthly returns. VMM is constructed every month using the following procedure. Only stocks with daily returns for every trading day in the previous six months are considered as candidates for portfolio V. Next, stocks in the lowest decile in terms of dollar volume (volume times price) are eliminated. Then, the monthly return volatility of stocks is estimated, using the six months daily returns corrected for one-lag autocorrelation as in French, Schwert, and Stambaugh (1987) and in equation (21). A value-weighted portfolio is formed out of the highest decile of stocks sorted by total volatility. The monthly return of this portfolio is obtained for the consecutive month. Finally, the Fama and French MKT return is subtracted to obtain the monthly return of VMM.

Factor	Mean	StdDev	ρ(i,MKTRF)	ρ(i,SMB)	ρ(i,HML)	ρ(i,UMD)	ρ(i,VMM)
MKTRF	0.38	4.45	1.00	0.30	-0.38	-0.08	0.52
SMB	0.24	3.19	0.30	1.00	-0.26	0.01	0.69
HML	0.43	2.89	-0.38	-0.26	1.00	-0.13	-0.53
UMD	0.86	4.03	-0.08	0.01	-0.13	1.00	-0.06
VMM	-0.86	8.22	0.52	0.69	-0.53	-0.06	1.00

Table 2: **Predictive Regressions of VMM beta** - The table summarizes the results of stock-level cross-sectional predictive regressions of VMM beta ($\beta^I_{VMM,i,t+1}$) on various lagged variables in the period of July 1963 to December 2008. The dependent variable, $\beta^I_{VMM,i,t+1}$, is estimated using a univariate regression of daily stock returns on VMM in the holding month $(t+1)$, accounting for nonsynchronous price movements in returns, as in Lewellen and Nagel (2006) and in equation (22). The independent variables, $\beta^I_{VMM,i,t}$, $\sigma_{i,t}$, and $\phi_{i,t}$ are estimated using the six month formation period prior to month $(t+1)$. For each stock, $\beta^I_{VMM,i,t}$ is estimated by a univariate regression of daily stock returns on VMM in the six month formation period, accounting for nonsynchronous price movements in returns, as in Lewellen and Nagel (2006) and in equation (22). The monthly volatility of a stock, $\sigma_{i,t}$, is estimated using the prior six month daily returns, corrected for one-lag autocorrelation, as in French, Schwert, and Stambaugh (1987) and in equation (21). $\phi_{i,t}$ measures the affiliation of the 4 digit SIC industry of stock i with portfolio V in the six months formation period and is constructed for each 4 digit SIC industry in the following way. The percentage market-cap proportion of every industry i (4 digit SIC code) in the market (denote by m_i), and the percentage market-cap proportion of every industry i (4 digit SIC code) in portfolio V (denote by v_i) are measured as of the end of the six month formation period. A measure of industry affiliation to portfolio V is constructed by $\phi_i = v_i - m_i$. IVOL is idiosyncratic risk, and it is estimated following Ang et al. (2006), relative to the Fama and French (1993) 3-factor model, using the daily returns of the one month prior to $(t+1)$. The cross sectional regressions are run over all stocks, each month, from July 1963 to December 2008. A time series average of the estimated coefficients is taken to arrive at point estimates. T-stats are reported in parenthesis. The t-stats are obtained from the time series of estimated coefficients and include a GMM correction for heteroskedasticity and serial correlation. The column $\bar{R}^2$ represents the average R^2 across time.

const	$\beta^I_{VMM,i,t}$	$\sigma_{i,t}$	$\phi_{i,t}$	$IVOL_{i,t}$	$\bar{R}^2$
0.35	0.38				0.02
(9.15)	(11.31)				
0.11		0.04			0.02
(2.77)		(7.09)			
0.56			0.05		0.01
(8.79)			(14.71)		
0.34				0.09	0.01
(7.05)				(5.96)	
0.16	0.22	0.02	0.03		0.03
(4.32)	(11.78)	(6.23)	(9.30)		

Table 3: **Statistics for 5x5 portfolios** - 25 value-weighted portfolios are constructed, sorted first by univariate market beta, β^I_{mkt}, and then sorted by univariate VMM beta, β^I_{vmm}. Portfolio VMM is formed as described in Table 1. Only stocks with more than 12 days and more than 75% of trading days in each month in the past six months are considered. β^I_{mkt} is estimated by a univariate regression of stock daily returns on the market in the six month formation period, accounting for nonsynchronous price movements in returns, as in Lewellen and Nagel (2006) and in equation (22). β^I_{vmm} is forecasted using a cross-sectional predictive model, in which the independent variables are lagged β^I_{vmm}, estimated in the six month formation period, stock volatility, and stock's industry affiliation to portfolio V, as described in Table 2. The predictive model is estimated using 240 cross-sectional regressions for each month in the 20 years prior to the holding period. Stocks are then sorted into quintiles according to β^I_{mkt}, and within each equintile, they are sorted into 5 sub quintiles according to β^I_{vmm}. The statistics in the pair of panels in the first row labeled Raw Returns Mean and Std.Dev. are measured in monthly percentage terms and apply to total simple returns. The panel Market Share is in percentage points and represents the average market share of each portfolio, measured as of the end of the six month formation period. The values in the panels Book To Market, IVOL, and Volatility are calculated in each formation period for each portfolio using a value-weighted average across stocks, and then averaged across time. The panel Book To Market represents the book-to-market ratio within each portfolio, calculated following Fama-French. IVOL is idiosyncratic risk, measured following Ang et al. (2006), relative to the Fama and French (1993) 3-factor model, using the daily returns of the last month in the formation period. Volatility is estimated using the prior six month daily returns, corrected for one-lag autocorrelation, as in French, Schwert, and Stambaugh (1987) and in equation (21). The period is from January 1945 to December 2008.

	Univariate β^I_{VMM} Quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	Mean Return					Std.dev Return				
Low β^I_{MKT}	0.89	0.79	1.00	0.96	0.95	3.28	3.62	4.50	5.38	7.31
2	0.94	1.05	1.09	0.95	0.81	3.63	4.22	4.54	5.20	6.66
3	1.08	1.06	1.09	0.95	0.78	4.21	4.79	5.41	6.17	7.54
4	1.01	0.96	1.01	1.07	0.90	5.29	5.81	6.52	7.29	8.35
High β^I_{MKT}	1.09	1.10	0.89	0.66	0.37	6.67	8.06	8.44	9.20	10.58
	Market Share					Book To Market				
Low β^I_{MKT}	0.09	0.05	0.03	0.02	0.01	0.66	0.57	0.59	0.63	0.63
2	0.12	0.06	0.04	0.02	0.01	0.55	0.54	0.58	0.59	0.59
3	0.13	0.05	0.03	0.02	0.01	0.51	0.55	0.57	0.57	0.54
4	0.10	0.04	0.03	0.02	0.01	0.53	0.54	0.56	0.55	0.52
High β^I_{MKT}	0.05	0.02	0.02	0.01	0.01	0.54	0.55	0.54	0.54	0.54
	IVOL (daily)					Volatility (Monthly)				
Low β^I_{MKT}	0.88	1.17	1.48	1.91	2.85	4.88	6.26	7.71	9.59	13.01
2	0.95	1.13	1.33	1.62	2.24	5.70	6.65	7.60	8.92	11.62
3	1.04	1.26	1.46	1.74	2.24	6.54	7.53	8.57	9.87	11.99
4	1.21	1.47	1.70	1.97	2.37	7.65	8.88	9.94	11.24	12.78
High β^I_{MKT}	1.59	1.95	2.21	2.53	3.18	9.69	11.46	12.66	14.06	16.21

Table 4: **Post-Formation Regressions** - The table depicts results of various post-formation monthly regressions for the 25 portfolios described in Table 3. There are 10 panels in the table. The left panels depict point estimations and the right panels report robust NeweyWest (1987) t-statistics. Each panel contains 25 values corresponding to the 25 portfolios, and 5 values corresponding to 5 portfolios of high VMM beta minus low VMM beta for each market beta quintile. The first pair of panels depicts the post-formation VMM beta estimated using a regression of portfolio monthly returns on VMM monthly returns for each portfolio. The next four pairs of panels depict the Jensens alphas with respect to the CAPM, the FamaFrench (1993) 3-factor model, the Carhart 4-factor model, and the MKT+VMM 2-factor model advocated by this paper. The factors MKTRF, SMB, HML are the Fama and French (1993) factors, the momentum factor UMD is constructed by Kenneth French. For the panels that report point estimates of Jensen's alpha, the last row depicts the p-value obtained for a joint test for the 25 alphas equal to zero. The test is conducted by first estimating all 25 regressions simultaneously using GMM with robust heteroskedasticity and autocorrelation consistent covariance matrix, and then using a Wald test. The sample period is from January 1945 to December 2008

	Univariate β^I_{VMM} Quintiles											
	Low	2	3	4	High	H-L	Low	2	3	4	High	H-L
	Post-Form β^I_{VMM}						$t(\beta^I_{VMM})$					
Low β^I_{MKT}	0.04	0.11	0.21	0.32	0.64	0.60	1.15	2.37	4.06	5.67	9.45	10.00
2	0.08	0.19	0.22	0.30	0.54	0.46	1.40	3.76	3.48	4.98	8.21	10.72
3	0.18	0.25	0.36	0.47	0.70	0.52	3.51	3.90	5.71	6.98	11.51	15.07
4	0.34	0.42	0.53	0.70	0.77	0.44	6.24	6.73	8.97	17.90	8.11	7.37
High β^I_{MKT}	0.55	0.76	0.85	1.01	1.21	0.66	10.04	16.71	18.22	15.99	14.53	11.16
	Post-Form α_{CAPM}						$t(\alpha_{CAPM})$					
Low β^I_{MKT}	0.22	0.05	0.14	0.03	-0.13	-0.35	2.46	0.47	1.24	0.20	-0.70	-1.66
2	0.19	0.19	0.19	-0.03	-0.28	-0.48	2.48	2.28	2.32	-0.30	-2.02	-2.69
3	0.21	0.11	0.06	-0.14	-0.41	-0.62	3.17	1.56	0.78	-1.44	-2.53	-3.31
4	0.00	-0.12	-0.14	-0.12	-0.39	-0.39	-0.03	-1.15	-1.46	-0.79	-2.15	-1.90
High β^I_{MKT}	-0.07	-0.18	-0.42	-0.69	-1.08	-1.01	-0.57	-1.02	-2.40	-3.18	-4.11	-4.19
$P(\alpha = 0)$	0.0014											
	Post-Form α_{FF3}						$t(\alpha_{FF3})$					
Low β^I_{MKT}	0.07	-0.09	-0.01	-0.14	-0.17	-0.25	0.92	-0.95	-0.14	-1.12	-1.08	-1.41
2	0.09	0.09	0.08	-0.15	-0.30	-0.38	1.25	1.19	1.13	-1.85	-2.43	-2.79
3	0.16	0.06	0.04	-0.14	-0.36	-0.52	2.54	0.77	0.53	-1.55	-2.52	-3.38
4	0.05	-0.04	-0.05	-0.01	-0.32	-0.37	0.68	-0.45	-0.49	-0.06	-1.92	-2.05
High β^I_{MKT}	0.05	0.02	-0.24	-0.50	-0.84	-0.90	0.47	0.13	-1.76	-2.79	-3.58	-3.60
$P(\alpha = 0)$	0.0001											
	Post-Form α_{CAR4}						$t(\alpha_{CAR4})$					
Low β^I_{MKT}	0.14	-0.04	0.00	-0.15	-0.20	-0.34	1.45	-0.41	0.02	-1.08	-1.11	-1.64
2	0.14	0.14	0.13	-0.15	-0.25	-0.39	1.71	1.80	1.59	-1.66	-1.99	-2.62
3	0.17	0.13	0.13	-0.09	-0.23	-0.40	2.81	1.87	1.44	-0.98	-1.32	-2.23
4	0.09	0.01	0.05	0.05	-0.24	-0.33	0.98	0.07	0.44	0.38	-1.43	-1.83
High β^I_{MKT}	0.09	0.04	-0.19	-0.38	-0.75	-0.84	0.70	0.30	-1.28	-2.17	-2.81	-3.04
$P(\alpha = 0)$	0.0001											
	Post-Form $\alpha_{MKT+VMM}$						$t(\alpha_{MKT+VMM})$					
Low β^I_{MKT}	0.06	-0.08	0.08	0.08	0.25	0.19	0.69	-0.89	0.78	0.53	1.45	1.03
2	0.02	0.09	0.11	-0.05	-0.06	-0.08	0.24	1.15	1.40	-0.54	-0.48	-0.53
3	0.10	0.04	0.09	-0.01	-0.03	-0.12	1.48	0.53	1.08	-0.09	-0.17	-0.73
4	0.00	-0.04	0.03	0.26	0.03	0.03	0.00	-0.37	0.36	2.08	0.17	0.16
High β^I_{MKT}	0.13	0.23	0.08	0.00	-0.16	-0.29	1.13	1.68	0.58	0.01	-1.01	-1.51
$P(\alpha = 0)$	0.0953											

Table 5: **Statistics for Double-sorted IVOL and VMM beta portfolios** - Each month, stocks are sorted into quintiles based on idiosyncratic volatility, measured following Ang et al. (2006), relative to the Fama and French (1993) 3-factor model, using the daily returns of the prior month. Then, each quintile is sorted into two deciles by VMM beta, calculated as in Table 3. There are six panels in the table. The first row in each panel relates to the 5 idiosyncratic risk quintile base portfolios. The second row relates to the 5 low VMM beta portfolios, and the third row relates to the 5 high VMM beta portfolios. The statistics correspond to the ones in Table 3. The sample period is from July 1963 to December 2008.

	Idiosyncratic Risk Quintiles									
	beta	2	3	4	High	Low	2	3	4	High
	Mean Return					Std.dev Return				
Base	0.89	0.93	0.95	0.81	-0.01	3.83	4.88	6.02	7.57	8.71
Low β^I_{VMM}	0.85	0.94	0.90	0.86	0.31	3.55	4.40	5.21	6.29	7.47
High β^I_{VMM}	0.96	0.98	1.06	0.69	-0.39	4.91	6.26	7.59	9.43	10.67
	Market Share					Book To Market				
Base	0.57	0.24	0.11	0.05	0.02	0.52	0.54	0.56	0.58	0.68
Low β^I_{VMM}	0.38	0.15	0.07	0.03	0.01	0.53	0.56	0.58	0.62	0.72
High β^I_{VMM}	0.19	0.09	0.05	0.02	0.01	0.50	0.52	0.53	0.55	0.64
	IVOL (daily)					Volatility (Monthly)				
Base	0.92	1.49	2.06	2.84	4.66	6.68	8.76	10.86	13.30	16.28
Low β^I_{VMM}	0.88	1.47	2.04	2.80	4.39	6.07	7.84	9.51	11.52	14.23
High β^I_{VMM}	1.01	1.52	2.09	2.88	4.98	7.95	10.26	12.74	15.49	18.79

Table 6: **Double-sorted IVOL and VMM beta portfolios - Post Formation Regressions** - Each month, stocks are sorted into quintiles based on idiosyncratic volatility, measured following Ang et al. (2006), relative to the Fama and French (1993) 3-factor model, using the daily returns of the prior month. Then, each quintile is sorted into two deciles by VMM beta, calculated as in Table 3. There are 10 panels in the table. The left panels depict point estimations and the right panels report robust NeweyWest (1987) t-statistics. In each panel, the first row relates to the 5 idiosyncratic risk quintile base portfolios. The second row relates to the 5 low VMM beta portfolios, and the third row relates to the 5 high VMM beta portfolios. The fourth and last row relates to high VMM beta minus low VMM beta portfolio for each idiosyncratic risk quintile. The sixth and last column in each row represents high idiosyncratic risk minus low idiosyncratic risk portfolio for each row. The first pair of panels depicts the post-formation VMM beta estimated using a regression of portfolio monthly returns on VMM monthly returns for each portfolio. The next four pairs of panels depict the Jensen's alphas with respect to the CAPM, the Fama-French (1993) 3-factor model, the Carhart 4-factor model, and the MKT+VMM 2-factor model advocated by this paper. The factors MKTRF, SMB, HML are the Fama and French (1993) factors, the momentum factor UMD is constructed by Kenneth French. The sample period is from July 1963 to December 2008.

	Idiosyncratic Risk Quintiles											
	Low	2	3	4	High	H-L	Low	2	3	4	High	H-L
	Post-Form β_{VMM}						$t(\beta_{VMM})$					
Base	0.14	0.29	0.48	0.71	0.89	0.75	3.43	5.23	8.94	16.99	19.19	23.34
Low β_{VMM}	0.08	0.17	0.33	0.49	0.63	0.55	1.94	3.02	6.56	10.81	11.62	12.30
High β_{VMM}	0.26	0.47	0.68	0.95	1.16	0.90	5.58	8.35	11.76	21.12	23.07	30.49
H-L β_{VMM}	0.19	0.30	0.34	0.46	0.53	0.35	13.85	11.90	12.48	23.03	13.56	8.87
	Post-Form α_{CAPM}						α_{CAPM}					
Base	0.12	0.06	0.01	-0.22	-1.07	-1.20	2.14	1.32	0.09	-1.35	-5.19	-4.78
Low β_{VMM}	0.13	0.13	0.02	-0.08	-0.67	-0.80	1.69	1.86	0.25	-0.60	-3.77	-3.55
High β_{VMM}	0.11	0.02	0.02	-0.44	-1.55	-1.65	1.61	0.21	0.14	-2.01	-5.37	-5.36
H-L β_{VMM}	-0.02	-0.11	0.00	-0.36	-0.87	-0.86	-0.21	-0.79	0.01	-2.01	-3.66	-3.78
	Post-Form α_{FF3}						α_{FF3}					
Base	0.10	0.06	0.03	-0.16	-1.14	-1.23	2.18	1.15	0.49	-1.27	-7.60	-6.95
Low β_{VMM}	0.06	0.04	-0.05	-0.17	-0.86	-0.92	1.03	0.71	-0.62	-1.62	-6.34	-5.46
High β_{VMM}	0.13	0.13	0.12	-0.26	-1.52	-1.65	2.25	1.38	1.15	-1.50	-6.95	-7.22
H-L β_{VMM}	0.07	0.08	0.17	-0.10	-0.66	-0.73	0.77	0.76	1.29	-0.56	-2.89	-3.15
	Post-Form α_{CAR4}						α_{CAR4}					
Base	0.08	0.12	0.10	-0.07	-0.88	-0.96	1.54	2.14	1.54	-0.60	-5.45	-4.88
Low β_{VMM}	0.05	0.06	-0.02	-0.07	-0.67	-0.72	0.77	0.98	-0.29	-0.66	-4.41	-3.83
High β_{VMM}	0.10	0.20	0.18	-0.16	-1.21	-1.31	1.51	2.08	1.71	-0.90	-4.74	-4.80
H-L β_{VMM}	0.05	0.14	0.20	-0.09	-0.54	-0.59	0.62	1.12	1.56	-0.45	-1.97	-2.17
	Post-Form $\alpha_{MKT+VMM}$						$\alpha_{MKT+VMM}$					
Base	-0.03	0.04	0.21	0.26	-0.34	-0.32	-0.63	0.69	3.51	2.44	-2.79	-2.21
Low β_{VMM}	-0.07	-0.02	0.05	0.14	-0.28	-0.21	-1.37	-0.24	0.70	1.20	-1.61	-1.04
High β_{VMM}	0.05	0.19	0.42	0.32	-0.47	-0.52	0.82	2.06	3.57	2.71	-3.75	-3.66
H-L β_{VMM}	0.12	0.21	0.37	0.17	-0.19	-0.32	1.41	1.86	2.41	1.20	-1.09	-1.61

Table 7: **Fama-MacBeth Analysis** This table shows the estimated Fama-MacBeth (1973) factor premiums on 25 portfolios sorted first by market beta and then by VMM beta, as described in Table 3. Following Fama-MacBeth, in the first step, time series regressions, in which excess portfolio returns are regressed on a constant and on a set of factors, corresponding to each row in the table, are run for the entire sample. In the second step, the excess portfolio returns are regressed on the estimated factor loadings in each month in the sample. Then, a time series average of the estimated coefficients is taken to arrive at point estimates and statistical significance of the factor premia. MKT is the market factor, SMB and HML are the Fama-French (1993) factors and UMD is a momentum factor, constructed by Kenneth French. VMM is constructed as explained in Table 1. IVOL is the value weighted average of idiosyncratic risk for each portfolio, as calculated by Ang et al (2006), measured each month using the prior month daily returns and then averaged across the entire sample period. Fama-MacBeth (1973) t-values are shown in brackets. A notation ^{I} means that I used the beta obtained from a univariate time series regression. R^2 and adjusted R^2 are obtained from a single cross-section regression of the average excess returns for each portfolio on the factor loadings. The sample period is from July 1963 to December 2008.

	c	λ_{MKT}	λ_{SMB}	λ_{HML}	λ_{UMD}	λ_{VMM}	$IVOL$	R^2	$Adj.R^2$
1	0.68	-0.24						0.16	0.13
	(2.76)	(-0.80)							
2	0.56					-0.41		0.28	0.25
	(3.51)					(-1.05)			
3	-0.54	1.67^I				-2.47^I		0.61	0.58
	(-1.67)	(3.31)				(-3.64)			
4	-0.54	0.97				-0.86		0.61	0.58
	(-1.67)	(2.68)				(-2.22)			
5	-0.33	0.75	-0.39	0.36				0.37	0.28
	(-0.99)	(2.01)	(-1.86)	(1.26)					
6	-0.02	0.49	-0.10	-0.48		-1.02		0.69	0.63
	(-0.06)	(1.28)	(-0.45)	(-1.42)		(-2.57)			
7	-0.20	0.75	-0.40	0.14	1.85			0.47	0.36
	(-0.59)	(1.98)	(-1.92)	(0.47)	(2.15)				
8	0.05	0.50	-0.13	-0.57	1.62	-1.11		0.73	0.66
	(0.13)	(1.32)	(-0.58)	(-1.66)	(1.90)	(-2.75)			
9	0.77						-0.20	0.37	0.34
	(3.86)						(-1.34)		
10	0.70	0.18					-0.28	0.39	0.34
	(2.84)	(0.54)					(-2.14)		
11	-0.50	1.63^I				-2.39^I	-0.02	0.61	0.56
	(-1.48)	(3.51)				(-2.99)	(-0.11)		

Table 8: **SDF-GMM Tests** This table shows results from estimating the stochastic discount factor M, using the moment conditions E[MR] = 1. MKT is the market factor, SMB and HML are the Fama-French (1993) factors and UMD is a momentum factor, constructed by Kenneth French. IVOL is the value weighted average of idiosyncratic risk for each portfolio, as calculated by Ang et al. (2006), measured each month using the prior month daily returns. The hedging demans term ψ is estimated each month using daily returns in the six months of the formation period, following equation (29). The moment conditions for rows 1 to 6 are $1 = E\left[(B_0 - B'F_{t+1}) \cdot R_{t+1}\right]$. The moment conditions for row 7 are $1 = E\left[(B_0 - B_1\left(r_{mkt,t+1} - r_{f,t+1}\right) - B_2(1 + B_3\psi_t)(r_{V,t+1} - r_{mkt,t+1})\right) \cdot R_{t+1}]$. The moment conditions for rows 8 to 11 are $1 = E\left[(B_0 - B'F_{t+1}) \cdot R_{t+1} - \gamma_{IVOL}IVOL_t\right]$. The parameter estimates are obtained from minimizing the GMM-criterion function where the weighting matrix of moment conditions is the asymptotically optimal one. TJ is Hansen's (1982) test of overidentifying restrictions, and p-val is the corresponding p-value. T-statistics are reported in brackets, and include a GMM correction for heteroskedasticity and serial correlation. The sample period is from July 1963 to December 2008.

	C	b_{MKT}	b_{SMB}	b_{HML}	b_{UMD}	b_{VMM}	γ_{IVOL}	ψ	TJ	p-val
1	0.99	-0.16							41.82	0.01
	(195.93)	(-0.13)								
2	1.10	10.64				-5.34			29.47	0.13
	(38.93)	(4.91)				(-4.86)				
3	1.04	8.35	-8.62	6.77					35.47	0.03
	(29.27)	(2.92)	(-3.23)	(1.46)						
4	1.10	9.63	3.83	-2.50		-6.15			29.79	0.07
	(26.86)	(3.49)	(0.99)	(-0.48)		(-3.59)				
5	1.12	6.71	-8.55	2.16	12.19				31.85	0.04
	(20.02)	(2.51)	(-3.22)	(0.49)	(2.68)					
6	1.09	4.44	4.24	-16.28	7.47	-7.14			29.18	0.06
	(19.21)	(1.62)	(0.97)	(-2.86)	(1.64)	(-3.23)				
7	1.10	11.20				-6.19		127.62	25.88	0.21
	(36.63)	(4.84)				(-3.66)		(1.13)		
8	1.00	3.25					-0.005		36.05	0.03
	(140.32)	(2.39)					(-4.55)			
9	1.08	9.59				-4.21	-0.002		29.41	0.10
	(36.99)	(4.27)				(-3.26)	(-1.54)			
10	1.09	8.84	3.48	13.08			-0.007		28.83	0.09
	(23.48)	(3.33)	(0.92)	(2.71)			(-3.62)			
11	1.18	7.97	3.55	10.08	11.55		-0.007		27.76	0.09
	(16.95)	(2.38)	(0.84)	(1.63)	(2.07)		(-3.07)			