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On the Rationality of the Post-Announcement Drift

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Abstract. This paper demonstrates that a post-announcement earnings drift, which is often advanced as an example of market irrationality, can arise even if traders act rationally on their information. Specifically, we show that in the presence of share supply variations which are unrelated to information, there is a positive correlation between the unexpected component of current public signals and future price changes. Such a correlation arises from the fact that while prices reveal private information that cannot be found in public signals, non-information based trading distorts the information content of prices relative to the implications of both private and public information. Under these circumstances, markets may appear semi-strong inefficient and slow to respond to earnings announcements even though information is processed in a timely and efficient manner. Our findings correspond well with previously documented empirical evidence and suggest that the robustness of earnings-based "anomalies" may be rational outcomes of varying uncertain share supply.

Keywords: post-announcement drift, earnings announcements, noisy rational expectations equilibrium, non-information based trading

JEL Classification: M41-Accounting

A number of empirical studies show that risk and/or size-adjusted abnormal returns in the period following an earnings announcement tend to be positively correlated with the "earnings surprise" associated with the announcement (see Bernard, 1992a for a survey). Usually referred to as a "post-announcement drift," this empirical phenomenon has been viewed as a market anomaly—i.e., a violation of semi-strong efficiency. Previous attempts to explain this anomaly have invoked inefficient processing of public information, or some form of investor myopia.[1] In contrast, we demonstrate that a post-announcement drift can arise naturally within a "multi-period noisy rational expectations" framework. Consequently, the existence of a post-announcement drift may not imply investor "irrationality" with respect to the processing or the usage of information.

For reasons that will be made clear below, the key to understanding post-announcement drift lies in the fact that equilibrium prices are affected by economic factors other than information. "Non-information based trading" (NIB) is a fundamental observed feature of

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securities markets which Grossman (1995) characterized as follows: “In general, there may be many reasons for trade other than information. After all, the traditional view of the market is of a location where resources are reallocated. Reasons for these non-informational trades include cross-sectional changes in wealth, risk-preferences, liquidity needs, unanticipated investment opportunities and all other factors that do not directly relate to the payoffs of traded securities.”

Within this noisy rational expectations framework, where all traders process information in a fully Bayesian fashion, our results show that NIB trading may cause equilibrium prices to exhibit a post-announcement drift following public disclosures such as earnings reports. When prices contain NIB supply (or demand) noise prices do not capture all information present in contemporaneous public information. Some traders may buy and sell securities without conditioning their decisions on information related to the firm’s fundamental value. We show that this can cause prices to exhibit fluctuations and to diverge from the security’s true value. These two factors jointly result in traders’ inferences about fundamental values drawn from prices to become less precise, hence increasing the total risk faced by risk-averse traders. This additional risk tempers traders’ demands for the risky asset. In a setting with multiple rounds of trading, this is manifested in traders taking smaller positions in earlier rounds in anticipation of future opportunities to trade with more information at their disposal, thereby retarding the aggregation of their private information into prices. However, in later rounds, as increased information becomes available and the precision of their information increases and future trading opportunities dwindle, they react more fully to differences between their private beliefs about asset values and observed market prices. Consequently, their demands and the corresponding prices adjust only gradually to their private and public information. Prices therefore exhibit greater sensitivity to NIB supply in the early trading rounds (i.e., prices have to adjust more to equilibrate a given level of excess supply) relative to later trading rounds. Thus, NIB effects are gradually filtered out of prices over many rounds of trades causing prices to slowly but systematically evolve to reflect more closely the underlying values of assets.

To illustrate, suppose a positive earnings surprise increasing expectations regarding the underlying asset’s value is observed. This surprise would indicate that expectations prior to the receipt of the signal, on average, were too low. Within our context, such reduced expectations imply either a positive NIB supply or that the earnings signals were too low in earlier periods. Because prices are noisy, and traders continue to attach positive weight to their prior beliefs, earlier periods’ effects are eliminated only gradually from equilibrium prices. As a result, price increases both when the positive surprise is observed and, on average, in the future. While traders are aware of the systematic relationships between earnings surprises and future price changes, they behave as if they do not exploit this correlation structure. The reason is that that when prices are noisy, risk aversion, combined with the possibility of betting in the future with more information, stops them from betting (taking positions in risky assets) with full confidence (demands are attenuated). Consequently, equilibrium prices in early rounds of trade only partially reflect traders’ expectations and the systematic relationships do not get “bid away.” From these observations, we then proceed to derive a general relation between publicly available information and the equilibrium price process. Specifically, we determine the correlation between the surprise component of the current public signal and future price changes and show that this association is positive. We show that the direction of future price changes can be predicted based on the current earnings surprise (the "earnings drift" as empirically documented in several studies; see Bernard and Thomas, 1990). However, it is important to emphasize again that this is not a trading opportunity anomalously ignored by investors. On the contrary, market participants rationally hold off on investing, preferring to wait until more information becomes available. The drift appears to be a trading opportunity only to outsiders such as empirical researchers who, not being able to observe the market's true expectations, cannot properly adjust for the risk injected by NIB trading. Indeed, the equilibrium price function we derive from our noisy rational expectations model generates a methodological implication for the "appropriate" measurement of "abnormal returns" by the empiricist. That is, expected returns, relative to which "abnormal returns" are quantified, should be conditioned both on price and non-price data (such as earnings surprises). Abnormal return measures derived in this fashion will no longer exhibit a post announcement drift "anomaly." These results regarding equilibrium pricing in rational expectations markets may be reconciled with Bernard and Thomas's (1990) findings of investor underreaction to earnings and Abarbanell and Bernard's (1992) evidence on underreaction to analysts' forecasts. Our results might provide one explanation for the following question raised in Abarbanell and Bernard (1992) as to why the market would ignore recent quarterly earnings realizations in forming expectations about future earnings, while at the same time incorporating at least some other information that predicts future earnings (Beaver, Lambert and Morse, 1980). Our analysis demonstrates equilibrium prices that exhibit precisely this puzzling combination: Under a noisy rational expectations framework, security prices will both lead and lag earnings. Empirical results consistent with this are provided by Ball and Bartov (1995).

A considerable body of literature in economics, finance, and accounting is predicated on noisy rational expectations models. The earliest study directly related to our results is Bray (1981) which demonstrates that some traditional benchmarks of market efficiency do not hold inside the rational expectations paradigm (specifically, that the futures price may not be an unbiased estimator of the spot price). Brown and Jennings (1989) examines the role of technical trading in the market and shows that the current price may not be sufficient for past prices. A more recent study, Brennan and Cao (1995), also shows that past prices may be informative in a model with multiple rounds of trade. However, none of these studies has explored the relationship between price evolution and public information.

Several studies in the accounting literature examine the role of public information within the noisy rational expectations framework including Kim and Verrecchia (1991, 1991a, 1994, 1997) and Grundy and McNichols (1989). These papers construct a three-round rational expectations equilibrium in contrast to our four-round equilibrium. There are other differences in the way the information signals are structured between each of these papers and ours; the most critical difference is that our paper focuses on the role of the supply uncertainty in the evolution of the equilibrium as opposed to the role of information signals. Another significant point concerns the structure of the equilibrium. Grundy and McNichols (1989) construct two distinct equilibria that differ in the way traders learn from prices in the second (and subsequent) rounds of trade. In the first equilibrium, the time zero price together with subsequent public signals spans the information contained in prices that are realized in subsequent periods. The equilibrium constructed in Kim and Verrecchia (1991) also has this property. In contrast, in the second equilibrium, traders learn something from prices at each round of trade in addition to the information contained in the public information signal. Our analysis covers both these possibilities. The general case corresponds to an equilibrium where traders learn from the price at every round whereas the limit case yields the equilibrium where the time zero price, together with public signals, contains the information in prices that are realized after observing the public signal.

To summarize, while prior literature has demonstrated that prices may not follow a martingale process within the noisy rational expectations framework, none of these studies shows, as ours does, that prices fail to fully reflect public information signals. In addition, our analysis focuses on the relationship between a general NIB process and price evolution rather than the relationship between prices and the information structure. We formally demonstrate that NIB trading by itself can generate semi-strong inefficiency and also derive closed-form expressions for the post-announcement drift that are consistent with observed empirical properties.

The paper proceeds as follows. Section two describes our model. Section three and four provide our results concerning the correlation between earning surprises and the post-announcement drift (semi-strong inefficiency). Section five discusses the implications of our research and its relation to prior empirical studies. Conclusions are provided in Section six and detailed calculations are supplied in the appendices.

1. The Model

Empirical studies document that positive (respectively, negative) information shocks concerning risky securities generate both abnormal positive (respectively, negative) contemporaneous returns, and abnormal positive (respectively, negative) returns in succeeding periods. To formally derive such price behavior in equilibrium, we require a multi-period economy with risky assets and many traders who anticipate a series of public disclosures. In addition, each public disclosure must be accompanied by a round of trade and a new disclosure-contingent price. We construct such a model using the noisy rational-expectations framework. Our model has an additional feature whereby prices are subject to a new NIB shock every period. Thus, in each period, additional information, obtained either from the public signal or through the market clearing price, moves the expectation of traders closer to x whereas the NIB shock distorts the market clearing price.

We consider a four-date, three-trading-rounds, noisy rational expectations equilibrium model of trading and prices with a risky asset and many traders. The risky asset has a normally distributed liquidating value of x units (per share), with mean 0 and variance σ_x^2. Each trader has access to three sources of information about the liquidating value of the risky asset, x. The details of the information process are as follows. Each trader acquires private information, y_{i_0}, before markets open for trade. Private signals about asset values are identically and independently distributed across traders, and given by:

$$y_{i_0} = x + \varepsilon_i$$

where $\varepsilon_i \sim N(0, \sigma_{\varepsilon_i}^2)$, and are independently distributed from x.[2]

For simplicity, we assume that the precision of the private signal is identical across investors, which implies that $\sigma^2_{\varepsilon_i} = \sigma^2_{\varepsilon_j} = \sigma^2_{\varepsilon}$ for all i and j. One round of trade then takes place, with the equilibrium price P_0 at time t_0 providing an additional source of information. The demands for the risky and riskless securities are chosen to maximize the expected utility of end-of-final-period-wealth with the knowledge among traders that there will be further rounds of trading following anticipated future public disclosures available at time t_1 and time t_2. The public information disclosure at time 1 and time 2, denoted by y_1 and y_2, are defined respectively as:

$$y_1 = x + u_1 \quad \text{and} \quad y_2 = x + u_2.$$

The public signals reflect the liquidating value with noise u_1 and u_2 respectively, in which $u_i \sim N(0, \sigma^2_u)$, $i = 1, 2$. The noise terms are identically, but not necessarily independently, distributed, that is, we allow for the possibility of correlation between u_1 and u_2. The risky asset yields a payoff of x at time $t = 3$, when the final period wealth W_{i_3} is consumed. W_{i_3} therefore consists of the returns on investment in the risky asset in periods 0, 1 and 2. Denoting these investment levels by z_{i_0}, z_{i_1}, and z_{i_2}, the realized returns on these holdings are: $z_{i_0}(P_1 - P_0)$, $z_{i_1}(P_2 - P_1)$ and $z_{i_2}(x - P_2)$ respectively. Therefore, ending wealth W_{i_3} can be expressed as:

$$W_{i_3} = z_{i_0}(P_1 - P_0) + z_{i_1}(P_2 - P_1) + z_{i_2}(x - P_2).$$

Traders' utility functions are negative exponential in end-of-final-period wealth W_{i_3}, with a constant absolute risk aversion coefficient ρ_i expressed as:

$$EU\big(W_{i_3}\big) = -E\big[\exp\big(-\rho_i\big\{z_{i_0}(P_1 - P_0) + z_{i_1}(P_2 - P_1) + z_{i_2}(x - P_2)\big\}\big)\big]. \tag{1}$$

Each trader has access to private and public information sources. Private information, y_{i_0}, is acquired at time $t = 0$; public announcement y_1 is available at $t = 1$; and public announcement y_2 at $t = 2$.

In addition, the equilibrium prices P_0, P_1, and P_2 (at times $t = 0$, 1, 2, respectively) also provide information to the traders. Therefore, the information sets available to individual i at times $t = 0$, 1, 2, denoted by I_{i_0}, I_{i_1}, and I_{i_2}, respectively, are given by:

$$I_{i_2} = \big\{y_{i_0}, y_1, y_2, P_0, P_1, P_2\big\}; \quad I_{i_1} = \big\{y_{i_0}, y_1, P_0, P_1, \big\}; \quad I_{i_0} = \big\{y_{i_0}, P_0, \big\}.$$

The model is one of noisy rational expectations where aggregate supply is uncertain. As is standard in rational-expectations models (see, for instance, Admati, 1985), we assume that all variables are jointly normal and that the equilibrium we seek involves price functions that are linear in signals and aggregate supply of the risky asset. We shall also assume, as is common in rational expectations studies, a "large" economy where individual traders are price-takers; the average of the traders' private information is the true underlying asset value x and the average of the trader's net demands (or supplies) is equal to the per-capita excess supply (or demand) Z_t.[3]

The linear rational expectations structure requires that the aggregate per-capita supply, Z_t, are distributed jointly normal and independent of all other information variables. Given the joint normality, we can always write aggregate supply of the risky asset at time $t = 0, 1, 2$ in general as:

$$Z_0 \sim N\left(0, \sigma^2_{Z_0}\right); \quad Z_1 = b_{1,0}Z_0 + \tau_1, \quad \text{and} \quad Z_2 = b_{2,0}Z_0 + b_{2,1}\tau_1 + \tau_2 \tag{2}$$

where τ_1 and Z_0 are independent and τ_2 is independent of Z_0, τ_1.

As formally introduced below in Assumptions 2 and 3, we impose certain restrictions on the $b_{t,k}$ coefficients in equation (2) that ensure that the noise process has desirable economic properties. Under the restriction of Assumptions 2 that requires $b_{2,0}, \leq b_{1,0} \leq 1$ and $b_{z,0} < b_{1,0} \leq 1$, the effect of the NIB shock in a given period decays over time. Assumption 3 imposes a further condition that the variance of the noise process is stationary, and makes it possible to derive closed-form solutions.

The supply of the risky asset, Z_t, should be interpreted as the number of shares made available for trade by NIB traders; i.e., it represents the number of shares that NIB traders choose not to hold in a given period, and hence should be absorbed by the informed, non-NIB traders. The specification in Assumption 2 accords the temporal evolution of Z_t a considerable degree of generality. For example:

$b_{t,k} = 0$ implies that the number of shares NIB traders make available for trade is independently determined each period;

on the other hand, $b_{t,k} = 1$ implies that the change in the number of shares NIB traders make available for trade in independently determined each period;

and finally, $b_{t,k} \in (0, 1)$ implies that the fraction $b_{t,k}$ of shares made available for trade (and hence not held) by NIB traders in period k continues to be made available for trade in period t, in addition to a random change.

The determination of an equilibrium begins with a specification of beliefs regarding the relationship between information and prices. The key assumption is that these conjectures are linear and may be written, using suitable coefficients A_{ij}, as:

$$\begin{aligned}
P_0 &= A_{01}x - A_{02}Z_0, \\
P_1 &= A_{11}x + A_{12}y_1 + A_{13}P_0 - A_{14}\tau_1, \\
P_2 &= A_{21}x + A_{22}y_1 + A_{23}y_2 + A_{24}P_0 + A_{25}P_1 - A_{26}\tau_2.
\end{aligned} \tag{3}$$

The solution then proceeds through backward optimization beginning with the determination of price and trader-demands following the second public announcement. We then solve for individual demands in earlier periods, treating future demands as random variables. Then we use market clearing to derive equilibrium prices. Finally, we ensure that the equilibrium prices are consistent with the original beliefs regarding the structure of prices (as in 3 above). However, the solution procedure is technically complex and it is difficult to rule out (multiple) equilibria, some of which are supported by implausible beliefs. For this reason, we restrict attention to equilibria where the price evolution follows the plausible pattern of evolving towards the true fundamental value as summarized in our first assumption below.

Assumption 1 (Belief Evolution) *The coefficients in equation (3), A_{ij}, are all positive.*

The key point is that the condition, $A_{t1} \geq 0$ ensures that the information in the market clearing price P_t concerning x, on average, is positively correlated with x. For example,

$$E[x \mid y_1, y_2, P_0, P_1, P_2] - E[x \mid y_1, y_2, P_0, P_1] = A_{21}(x - E[x \mid y_1, y_2, P_0, P_1])$$

is positively (or zero) correlated with x *as long as* $A_{21} \geq 0$. The requirement that the other coefficients, A_{tj}, $j \geq 2$ be positive states that equilibrium prices behave "normally" in the sense that they have a positive relation with information signals and a negative relation with the excess supply (see also the solution in Section C).

The main results in the paper can be derived with very minor restrictions as stated below on the general linear structure of NIB trading outlined in equation (2) for three-trading dates $t = 0, 1, 2$.

Assumption 2 (Structure of NIB Supply) *The aggregate uncertain supply shocks decay over time, that is, in the notation of equation (2), we have:*

$$(i) \quad b_{2,0} < b_{1,0} \leq 1 \quad \textit{and} \quad b_{2,1} \leq 1 \tag{4}$$

The conditions in Assumption 2 require that the effect of the NIB shock in a given period decay over time. This restiction, in conjunction with Assumption 1, is sufficient to ensure the existence of a positive drift. We need a stronger condition in order to calculate an explicit equilibrium. The condition needed for deriving an explicit equilibrium is that the NIB supply follow an AR1 process as described (for three dates) in the next assumption.

Assumption 3 (The AR1 Assumption) *The aggregate uncertain supply is given as follows:*

$$Z_0 \sim N\left(0, \sigma_{Z_0}^2\right); \quad Z_1 = bZ_0 + \left(\sqrt{1-b^2}\right)\tau_1, \quad \textit{and} \quad Z_2 = bZ_1 + \left(\sqrt{1-b^2}\right)\tau_2 \tag{5}$$

where $0 \leq b \leq 1$ *and* τ_t *are i.i.d and* $\sigma^2(\tau_t) = \sigma^2(Z_0)$ *for every* t, $t = 1, 2$.

For comparability with equation (2), it suffices to note that we have rescaled τ_j as $\sqrt{1-b^2}\tau_j$ in order to set the variance of Z_0 and τ_j equal. Note that if $b = 0$, we get Z_t being i.i.d whereas the other extreme, $b = 1$ yields $Z_t = Z_0$ for every t.

Summarizing, the assumptions ensure the existence of a positive post-announcement drift consistent with empirical predictions. Assumption (1) is an economics-based restriction requiring that traders believe: (i) on average, the observed equilibrium price change in any period is towards the underlying fundamental value x and (ii) excess supply exerts downward pressure on prices. Assumption 2, is a very minor exogenous restriction on the NIB trading process. The equilibrium that we use in our detailed calculations is derived under Assumption 3 which is a special case of Assumption 2. This equilibrium *endogenously satisfies* the requirements of Assumption 1. Indeed, properties (i), and (ii) of Assumption 1 seem so intuitive that it is hard to imagine equilibria where they fail, especially in the context of actual data.[4]

Our first result, Proposition 1, states a general relationship between unexpected earnings (as defined in the next section) and unexpected returns inside a noisy rational expectations model assuming only that prices are linear (that is, without requiring either Assumptions 1 or 2). The next result, Proposition 2, shows that this association is *positive* provided Assumptions 1 and 2 hold. Our last result, Proposition 3, derives a closed form expression for the drift using the structure in Assumption 3 and shows that the equilibrium satisfies the requirements of Assumption 1, and hence, leads to a positive earnings drift. The comparitive statics and simulation are based on this closed form solution. We emphasize that our results on a positive association between period 1 earnings surprise and period 2 unexpected returns hold for *any* equilibrium derived under the relatively weak restrictions of Assumptions 1 and 2. Alternatively, imposing *only* Assumption 3, we are able to derive an equilibrium solution exhibiting a positive drift (Proposition 3).[5]

2. Noisy Rational Expectations Equilibrium

The purpose of this section is to develop the relationship between prices and demands arising from utility maximizations under a rational expectations structure. We then use these first-order conditions directly in establishing the existence of an earnings drift. These results are general and hold for any equlibrium that satisfies Assumption 1. Additionally, we show that the drift is positive when Assumption 2 is satisfied. The next step is to obtain a closed form solution under the hypothesis in Assumption 3. We use this closed-form solution for a discussion of the properties of the post-announcement drift and to construct a numerical simulation that matches the observed level of the post-announcement drift.

2.1. Equilibrium Prices in Period t = 2

Standard results show that there exists a noisy rational-expectations equilibrium at time 2. The equilibrium price P_2 and associated optimal demand configuration $\{z_{i_2}\}$ are given by:

$$z_{i_2} = \frac{1}{\rho_i V_2}\left[E\left(x \mid I_{i_2}\right) - P_2\right] \tag{6}$$

where V_2 is the variance of $E(x \mid I_{i_2})$. This conditional expectation is linear in the information variables. The conditional expectation can be evaluated directly (see Appendix A), and aggregating across traders and applying market clearing (that is, setting average demand equal to the per-capita noisy supply), yields a solution:

$$P_2 = A_{21}x + A_{22}y_1 + A_{23}y_2 + A_{24}P_0 + A_{25}P_1 - A_{26}\tau_2 \tag{7}$$

that is consistent with the conjectures stated earlier.

2.2. Equilibrium Prices in Period t = 1

The derivation of time 1 and time 0 equilibrium asset demands, z_{i_1} and z_{i_0}, and the associated equilibrium price functions, P_1 and P_0, are more complex. To satisfy the requirements of the rational expectations equilibrium model, traders must fully anticipate the consequences of the forthcoming public announcements, y_1 and y_2, and future trading opportunities when they determine their initial portfolios at time 0 and time 1. We show in Appendix A that for suitable coefficients c, and α_{11}, a backwards utility maximization results in the following equation:

$$z_{i_1} - c\bar{z}_{i_2} = \frac{1}{\rho_i \alpha_{11}} \left[\left(E\left(P_2 \mid I_{i_1} \right) - P_1 \right) \right]. \tag{8}$$

Because $E[P_2 \mid I_{1i}]$ and $\bar{z}_{i_2} = E[z_{i_2} \mid I_{i_1}]$ are linear in the available information $\{y_{i_0}, y_1, P_0\}$, applying the market clearing condition yields a solution:

$$P_1 = A_{11}x + A_{12}y_1 + A_{13}P_0 - A_{14}\tau_1. \tag{9}$$

2.3. *Equilibrium Prices in Period t* = 0

Individual i's time 0 optimal demand for the risky asset is obtained through a procedure similar to that used for determining time 1 demands. However, both the demands at time 1 and at time 2 have to be treated as random variables. Nevertheless, it is possible to show that the associated equilibrium price is of the same form as the conjectured relation:

$$P_0 = A_{01}x - A_{02}Z_0 \tag{10}$$

Details of the solution process are discussed in Appendix C.

2.4. *Orthogonality Results*

We begin with a result that is invoked several times in our analysis. For ease of reference, we label it as a lemma.

Lemma 1 (Conditional Expectation and Independence) *Let* $\tilde{P}, \tilde{Y}_1, \ldots, \tilde{Y}_n$ *be any collection of random variables measurable with respect to a common underlying sigma algebra* $\mathcal{F}$. *Then the conditional expectation* $E[\tilde{X} \mid \tilde{Y}_1, \ldots, \tilde{Y}_n]$ *satisfies the following condition:* $\tilde{X} - E[\tilde{X} \mid \tilde{Y}_1, \ldots, \tilde{Y}_n]$ *is independent of* $Y_1, \ldots, Y_n$ *and* $\mathrm{Cov}\{X - E[X \mid Y_1, \ldots, Y_n], Y_j\} = 0$ *for every* j. *As a consequence,* $\mathrm{Cov}\{X, Y_j\} = \mathrm{Cov}\{E[X \mid Y_1, \ldots, Y_n], Y_j\}$ *for every* j.

The analytical technique we employ combines the consequences of utility maximizing behavior represented by equations (6) and (8) with the orthogonality result in Lemma 1. Specifically, if $\xi_1 \in I_{i_1}$ is any time 1 public information variable, then ξ_1 lies in every traders' information set at time 1, we must have:

$$\mathrm{Cov}\left\{ P_2 - E\left[P_2 \mid I_{i_1} \right], \xi_1 \right\} = 0$$

From this Lemma it follows that:

$$\mathrm{Cov}\left\{ P_2 - E\left[P_2 \mid I_{i_1} \right], \xi_1 \right\} = 0 \quad \text{for } \xi_1 = y_1, P_1 \text{ or } P_0 \tag{11}$$

We shall combine equations (8) and (11) to arrive at a characterization of the earnings drift—a phenomenon that has been widely documented (see Foster et al., 1984 or Bernard and Thomas, 1990). We begin by defining such a drift precisely within the rational expectations framework.

3. Post-Announcement Earnings Drift

This section develops our main results and shows that a current earnings surprise has a predictable correlation with future price changes. Specifically, we show that if unexpected future price changes are measured using only market data, they will exhibit a positive correlation with current earnings surprises. The analysis is developed as follows. We define precisely the notion of unexpected prices and earnings within the context of our model. Then, using the orthogonality result in Lemma 1 and the equilibrium price and demand equations, we derive the sign of the post-announcement drift in a general setting. Subsequently, we use an explicit equilibrium derivation to determine a closed-form expression for the post-announcement drift to derive empirical implications.

3.1. Earnings Surprise and Unexpected Price Changes

First, we define the notion of an unexpected price change relative to all price data available at time 1, namely $\{P_1, P_0\}$.[6] The total price change, $P_2 - P_1$, can be decomposed into expected and unexpected components, denoted by EP_2 and UP_2 respectively, where $EP_2 = E[P_2 \mid P_1, P_0]$ and $UP_2 = P_2 - E[P_2 \mid P_1, P_0]$. The main result that we seek to establish is that the unexpected price change at *time* 2 computed based on price data, UP_2, is positively correlated with the unexpected component of the public disclosure at *time* 1, denoted by UE_1 (as defined below).

The unexpected component of the earnings signal at time 1, UE_1 represents the surprise in the earnings signal relative to past public information. At time 0, the publicly available information set consists solely of the price, P_0. Thus, the unexpected component of the public report at time 1 is given by: $UE_1 = y_1 - E[y_1 \mid P_0]$.[7] Therefore, our primary goal is to derive an expression for:

$$\text{Cov}\{UE_1, UP_2\} = \text{Cov}\{y_1 - E[y_1 \mid P_0], P_2 - E[P_2 \mid P_1, P_0]\}.$$

3.2. Results

The first step in our analysis is to refine the covariance relation between unexpected earnings and unexpected price change. From Lemma 1, because UP_2 is orthogonal to the information set $\{P_1, P_0\}$ and $E[y_1 \mid P_1, P_0]$ lies in the information set $\{P_1, P_0\}$, $\text{Cov}\{E[y_1 \mid P_1, P_0] - E[y_1 \mid P_0], UP_2\} = 0$. It follows that:

$$\text{Cov}\{UE_1, UP_2\} = \text{Cov}\{y_1 - E[y_1 \mid P_0], UP_2\} = \text{Cov}\{y_1 - E[y_1 \mid P_0, P_1], UP_2\} \quad (12)$$

The quantity, $y_1 - E[y_1 \mid P_1, P_0]$, measures the incremental information in earnings relative to the "market data" available at time 1, $\{P_1, P_0\}$. Equation (12) shows that for determining the relationship between the unexpected price change UP_2 and unexpected earnings UE_1, it suffices to establish the relation between the "incremental information component" $UE_1 - E[UE_1 \mid P_1, P_0] = y_1 - E[y_1 \mid P_1, P_0]$ and UP_2. One inference may be made immediately from this observation—if $\{P_0, P_1\}$ were sufficient for y_1, then $y_1 - E[y_1 \mid P_1, P_0] \equiv 0$ and $\text{Cov}\{UE_1, UP_2\} = 0$. In other words, a post-announcement drift can arise only when NIB trading forces the market information set $\{P_0, P_1\}$ to be insufficient for the earnings signal y_1. This fundamental point is captured in our first result that establishes the role of NIB trading for the earnings drift.

Proposition 1 (Accounting Information and Future Prices) *The incremental information about the "unexpected" price change at time 2, $UP_2 = P_2 - E[P_2 \mid P_1, P_0]$ provided by the earnings surprise at time 1, $UE_1 = y_1 - E[y_1 \mid P_0]$ is characterized through the following formula:*

$$\text{Cov}\{UE_1, UP_2\} = \rho\alpha_{11}\text{Cov}\{y_1 - E[y_1 \mid P_1, P_0], Z_1 - cZ_2, \}$$

where $\rho, \alpha_{11} > 0$.

Proof: See Appendix B. ■

To understand the formal proofs of this argument that are provided in our propositions, consider a setting where there has been a positive "unexpected" realization of y_1, that is, a setting in which $UE_1 > 0$. Intuitively, because the realization of y_1 is greater than that anticipated based on time 0 information P_0, expectations of the underlying value, x, based on y_1, P_1 and P_0, will exceed, on average, the expectation based on P_0 and P_1 alone. Such a setting suggests that observed prices have been depressed due to positive excess NIB supply, and hence, reduced both the conditional expected value of y_1 resulting in the earnings surprise and also the expectations regarding the asset value x.

Given that there exists a monotone relation between realizations of y_1 and NIB supply (conditional on prices), a positive earnings drift is a direct consequence. An increased NIB supply will be inferred from higher (than expected) levels of y_1. First, consider the possibility that this increased supply is due to the persistent component, Z_0. Given the greater precision of information at time 1, and hence the higher confidence of traders in their expectations regarding future prices, demand for the asset intensifies at time 1 relative to time 0. Because demand intensifies and the expectations regarding x conditional on the time 1 signal, are, on average, the same as that at time 0, a trader who demanded a positive quantity at time 0, will, on average, demand an additional positive quantity at time 1. In sum, the effect of the NIB shock Z_0 is to induce a positive correlation in demand across periods.

This is equivalently manifested in a temporal decay of the impact of a given level of NIB supply on the equilibrium market clearing price (that is, a reduction in the magnitude of the coefficient on Z_t in the equilibrium price function). In turn, this leads to a positive correlation between Z_0 and $P_1 - P_0$ (as can be explicitly calculated in the complete equilibrium given in equations (15)).

In essence, the NIB shocks Z_t plays two roles. A direct role is related to the need of the market to absorb the excess supply, Z, which results in an impact on equilibrium market prices, this impact is captured by the coefficient on Z in the equilibrium price function. The second role—an indirect one—is related to the change in beliefs caused by the fact that the shock is unobservable. Both the direct (supply) and indirect (noise) effects interact to induce the positive correlation in demands across periods and, hence, a post-announcement drift.

Next, suppose that the increased supply is attributable mainly to the period-specific component, that is, from $\tau_1 > 0$. Because τ_2 is independent of τ_1 and has mean zero (i.e., the τ's are mean reverting), a lower level of period-specific excess supply should be expected on average in the next period resulting also in a price rise. An analogous argument shows that a negative τ_1 in the first period implies a price decline in the second period (due to the relative increase in supply).

The earnings drift derived in Proposition 1 may be broken down into components based on the persistent and transitory factors in the supply uncertainty. Given the structure in Assumption 2, we have:

$$\begin{aligned}
&\rho\alpha_{11}\mathrm{Cov}\{Z_1 - cZ_2, y_1 - E[y_1 \mid P_1, P_0]\}\\
&\quad = \rho\alpha_{11}(\mathrm{Cov}\{(b_{1,0} - cb_{2,0})Z_0, y_1 - E[y_1 \mid P_1, P_0]\})\\
&\qquad + \rho\alpha_{11}(1 - cb_{2,1})(\mathrm{Cov}\{\tau_1, y_1 - E[y_1 \mid P_1, P_0]\}). \qquad (13)
\end{aligned}$$

The term $\mathrm{Cov}\{Z_0, P_1 - P_0\}$ is multiplied by the factor $\rho\alpha_{11}$. The risk-tolerance factor ρ is positive by definition whereas α_{11} is related to the posterior variance of x at time 1 and is also positive. From Assumption 2, $b_{1,0} > b_{2,0}$ and $b_{2,1} \leq 1$; so, if the magnitude of the constant c is less than 1, the right-hand-side in (13) is positive. Indeed, for any equilibrium that follows the "normal" pattern of evolution as defined in Assumption 1, it can be shown that $c < 1$ (Appendix A.3). Therefore, under the pattern of belief and price evolution characterized in Assumption 1, the equilibrium necessarily leads to a positive earnings drift as summarized in the next result.

Proposition 2 (Positive Earnings Drift) *Suppose that the conditions of Assumptions 1 and 2 both hold. Then any resulting equilibrium exhibits a positive earnings drift.*

Proof: See Appendix B. ■

The intuition that is captured in the proof is that for fixed levels of P_1 and P_0, a higher realization of y_1 is associated, on average, with a greater level of NIB supply, Z_1. This may be understood as follows. An increase in y_1 while P_1 and P_0 remain unchanged necessarily indicates *both* a higher level of x (based on the fact that y_1 is unexpectedly high) and a higher level of Z_1 (because P_1 is being held constant despite the higher y_1). This positive association, in turn, leads to a positive relation between earnings surprise and future unexpected returns.

4. A Specific Equilibrium Solution

It is possible to derive one specific equilibrium solution with prices explicitly determined in terms of the parameters. Such an explicit solution is useful for comparative statics and it may be used to analyze trading volume (see Kim and Verrecchia, 1991; and Dontoh and Ronen, 1993). We use this particular equilibrium solution for obtaining expressions for the magnitude of the drift in terms of the underlying parameters and for our calibration. We recall that the only assumption needed for deriving the equilibrium is that Z_t follows an AR1 process (Assumption 3). The equilibrium then exhibits the property of a positive post announcement earnings drift with the following demands:

$$\begin{aligned}
z_{i_2} &= \frac{1}{\rho_i}\left[s\left(y_{i_0} - b^2 q_0 - b(1-b)q_1 - (1-b)q_2\right)\right] \\
z_{i_1} &= \frac{1}{\rho_i}\left[s\left(y_{i_0} - bq_0 - (1-b)q_1\right)\right] \\
z_{i_0} &= \frac{1}{\rho_i}\left[s\left(y_{i_0} - q_0\right)\right]
\end{aligned} \tag{14}$$

and prices

$$\begin{aligned}
P_2 &= V_2\left(\frac{s_u}{1+r}(y_1+y_2) + (s_0 + b^2 s)q_0 + (s_1 + b(1-b)s)q_1 + (s_2 + (1-b)s)q_2\right) \\
P_1 &= K_1(s_u y_1 + s_0 q_0 + s_1 q_1) + [K_1 - T(1-b)]s(bq_0 + (1-b)q_1) \\
P_0 &= K_0(s_0 + s)q_0 + K_1 s_0 q_0 + [K_1 - T(1-b)]sbq_0;
\end{aligned} \tag{15}$$

where

$$s_x = \frac{1}{\sigma_x^2}; \quad s = \frac{1}{\sigma_\epsilon^2}; \quad s_u = \frac{1}{\sigma_{u_1}^2} = \frac{1}{\sigma_{u_2}^2}; \quad s_{Z_0} = s_Z = \frac{1}{\sigma_{Z_0}^2} = s_{\tau_t} = s_\tau$$

and

$$q_0 = x - \frac{\rho}{s}Z_0; \quad q_1 = x - \frac{\rho}{s}\left(\sqrt{\frac{1+b}{1-b}}\right)\tau_1; \quad q_2 = x - \frac{\rho}{s}\left(\sqrt{\frac{1+b}{1-b}}\right)\tau_2 \tag{16}$$

are "incremental information in prices" variables, where ρ is the average level of risk-tolerance (see Appendix A) and b is the autocorrelation parameter in NIB trading; s_0 and s_1 are the associated "precisions" as follows:

$$s_0 = \frac{s^2 s_Z}{\rho^2}; \quad s_1 = \frac{s^2(1-b)s_\tau}{\rho^2(1+b)} = s_2 \tag{17}$$

and

$$\begin{aligned}
V_2 &= \frac{1+r}{(1+r)(s_x + s + s_0 + s_1 + s_2) + 2s_u}; \quad V_1 = \frac{1}{s_x + s + s_0 + s_1 + s_u}; \\
V_0 &= \frac{1}{s_x + s + s_0}
\end{aligned} \tag{18}$$

denote the posterior variances of x at time t.

The above constitute a specific equilibrium where $K_1 > T > 0$ and $K_0 > 0$. The details of how this equilibrium is obtained are provided in the appendices; the exact values of the constants K_0, K_1, T are provided in Appendix C. Apart from providing comparability with earlier studies, this equilibrium has the additional desirable property of being "continuous" at $b = 0$. That is, setting $b = 0$ in the solutions in (15) yields the standard equilibrium with constant noise Z_0 and no transitory shocks τ_t.

Under the equilibrium price structure described in (15), Assumption 1 may be verified directly. An expression for the earnings drift is provided in the next proposition. While the exact expression involves many terms and is hard to interpret directly, it may be computed exactly and its properties as functions of the parameters can be graphed (see Figures 1–3). These graphs show, for example, that the magnitude of the drift increases in the level of NIB trading and decreases in the precision of private information.

Proposition 3 (Equilibrium Properties of the Earnings Drift) *In the specific equilibrium determined in our model, the covariance between unexpected earnings at time 1, UE_1, and*

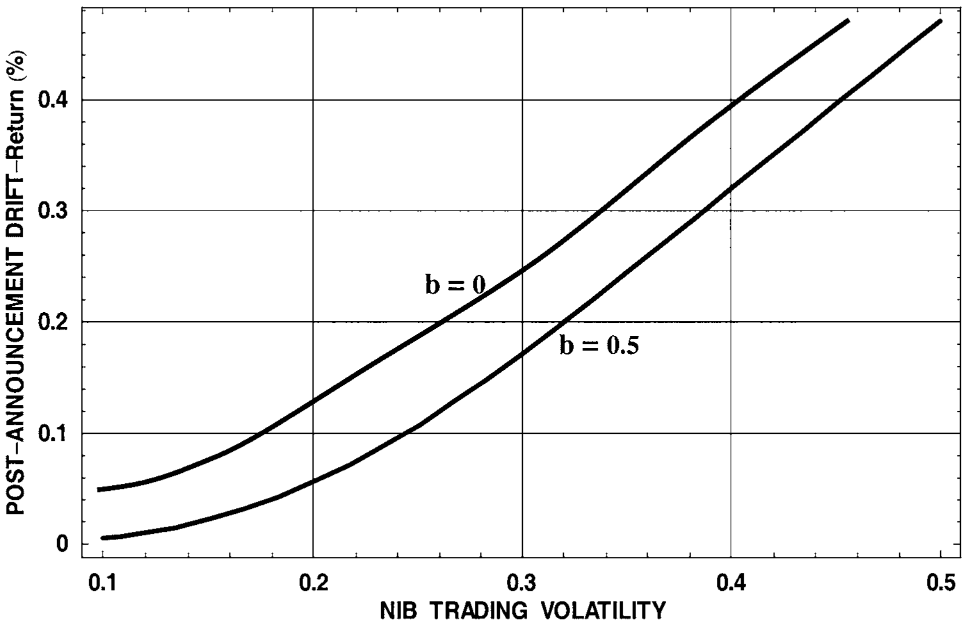

Figure 1. Effect of NIB trading volatility on post-announcement drift.
For purposes of the simulation, we assumed that $\sigma_\tau^2 = \sigma_z^2$ and that $\sigma_u^2 = \sigma_\epsilon^2$ (see discussion in paper), where $s_u = \frac{1}{\sigma_u^2}$, $s_\epsilon = \frac{1}{\sigma_\epsilon^2}$ are the respective precisions of the public signal and individuals' private information, and σ_z^2 and σ_τ^2 are respectively the variances of the aggregate per capita supply Z and the change in the aggregate per capita supply τ. Parameter values for σ_u^2, σ_ϵ^2, ρ—the risk aversion parameter and σ_x^2—the variance of the liquidating value, were set according to the empirically estimated values $\sigma_u^2 = \sigma_\epsilon^2 = 0.7474$, $\sigma_x^2 = 0.1225$ and $\rho = 5.3$ (see Table 1 and Appendix D). The intertemporal correlation coefficient between public signals r was set at its midpoint value of 0.5. Finally, to investigate the effect of the autoregressive decay parameter b on the drift, the simulation was run both at $b = 0$ reflecting intertemporally independent NIB trading noise, and also at its mid-point value of 0.5.

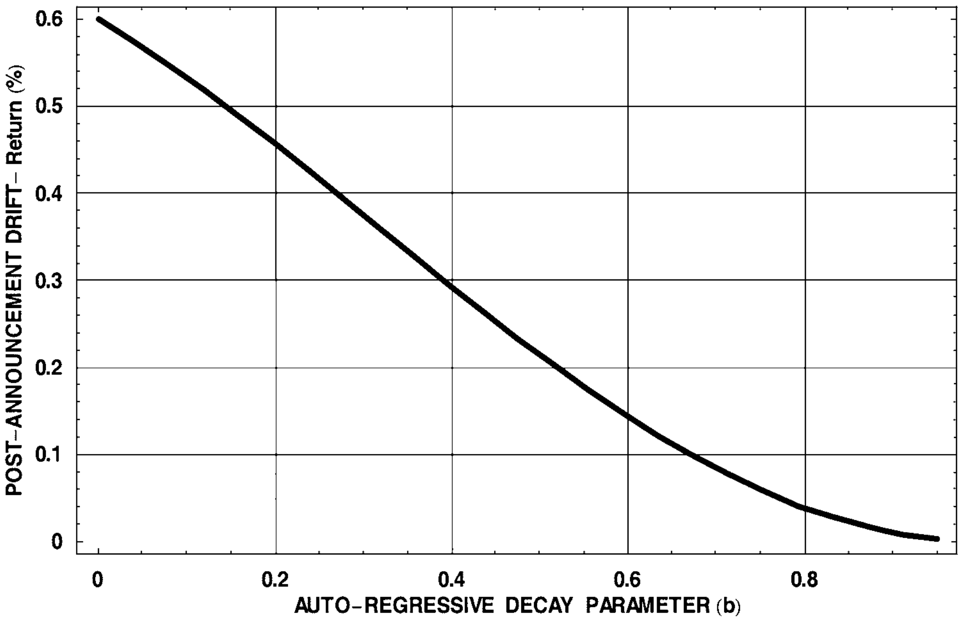

Figure 2. Effect of auto-regressive decay parameter on post-announcement drift.
For purposes of the simulation, we assumed that $\sigma_\tau^2 = \sigma_z^2$ and that $\sigma_u^2 = \sigma_\epsilon^2$ (see discussion in paper), where $s_u = \frac{1}{\sigma_u^2}$, $s = \frac{1}{\sigma_\epsilon^2}$ are the respective precisions of the public signal and the precision of individuals' private information, and σ_z^2 and σ_τ^2 are respectively variances of the aggregate per capita supply z and the change in the aggregate per capita supply τ. Parameter values for $\sigma_u^2, \sigma_\epsilon^2, \rho$—the risk aversion parameter and σ_x^2—the variance of the liquidating value, were set according to the empirically estimated values $\sigma_u^2 = 0.7474$, $\sigma_x^2 = 0.1225$ and $\rho = 5.3$ (see Table 1 and Appendix D) and the autoregressive decay parameter b was set at zero to reflect intertemporal independent NIB trading noise. The intertemporal correlation coefficient between public signals r was set at its midpoint value of 0.5. Lastly, σ_z^2 was set at 0.018. This is equivalent to an NIB volatility level of 0.19 which is consistent with the empirically estimated 3-day CAR of 0.65% post-announcement drift.

the unexpected price change at time 2, UP_2 is given by:

$$\mathrm{Cov}\{UE_1, UP_2\} = \frac{\rho}{s}(1 - bc)\left(\alpha b \sigma_{Z_0}^2 + \beta\left(\sqrt{1 - b^2}\right)\sigma_{\tau_1}^2\right) \geq 0$$

where α β are positive constants and $0 \leq c \leq 1$

Proof: See Appendix B. ■

If $b = 1$ and $Z_t = Z_0$ for every t, it can be shown that $c = 1$ and the expression for the drift becomes zero. However, for any $b < 1$, the drift is strictly positive. In particular, if $b = 0$ (i.e., the NIB noise is independent across periods), the drift is strictly positive.

Before concluding our formal analysis, we analyze the relationship between the second-period earnings surprise, UE_2, and the unexpected return in period three, UP_3. We first

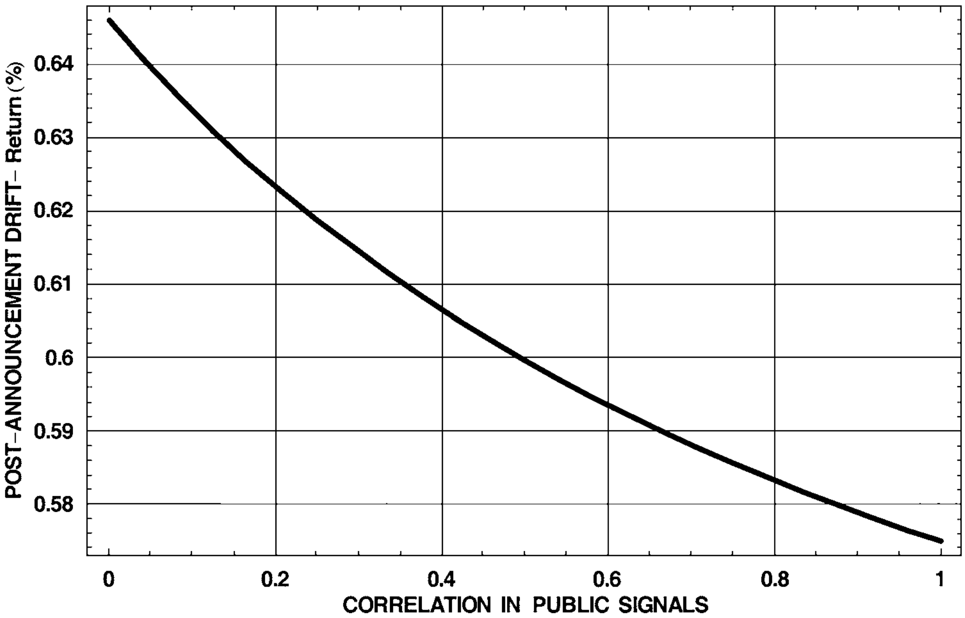

Figure 3. Effect of correlation in public signals on post-announcement drift.
For purposes of the simulation, we assumed that $\sigma_\tau^2 = \sigma_z^2$ and that $\sigma_u^2 = \sigma_\epsilon^2$ (see discussion in paper), where $s_u = \frac{1}{\sigma_u^2}$, $s = \frac{1}{\sigma_\epsilon^2}$ are the respective precisions of the public signal and the precision of individuals' private information, and σ_z^2 and σ_τ^2 are respectively variances of the aggregate per capita supply z and the change in the aggregate per capita supply τ. Parameter values for σ_u^2, σ_ϵ^2, ρ—the risk aversion parameter and σ_x^2—the variance of the liquidating value, were set according to the empirically estimated values $\sigma_u^2 = \sigma_\epsilon^2 = 0.7474$, $\sigma_x^2 = 0.1225$ and $\rho = 5.3$ (see Table 13 and Appendix D) and the autoregressive decay parameter b was set at zero to reflect intertemporal independent NIB trading noise. Lastly, σ_z^2 was set at 0.018. This is equivalent to an NIB volatility level of 0.19 which is consistent with the empirically estimated 3-day CAR of 0.65% post-announcement drift.

define these quantities:

$$UE_2 = y_2 - E[y_2 \mid y_1, P_1, P_0]$$
$$UP_3 = x - E[x \mid P_2, P_1, P_0].$$

Again, we can demonstrate quite generally that when the coefficient of y_2 in the conditional expectation $E[x \mid y_2, y_1, P_2, P_1, P_0]$ is positive, then

$$\text{Cov}\{UE_2, UP_3\} = K \times \text{Var}(y_2 \mid y_1, P_2, P_1, P_0) > 0 \quad \text{for some constant } K > 0.$$

Further, in the context of the specific equilibrium that we establish, we can show that the positive covariance between the earnings surprise and unexpected price change in the next period is robust across alternative specifications of earnings surprise. For instance, define $\widehat{UE}_2 = y_2 - E[y_2 \mid y_1]$, i.e., let earnings surprise be calculated based only on past earnings realization; then $\text{Cov}\{\widehat{UE}_2, UP_3\} > 0$. This specification of earnings surprise is consistent with Bernard and Thomas's use of a seasonal random walk model of earnings expectations in their documentation of post-announcement drift.[8] Again, the intuitive explanation is that when $\widehat{UE}_2 > 0$, the expected value of x increases at time 2. However, the time 2 price, P_2, only reflects a portion of these increased expectations. In addition, the time 1 price, P_1, which is based on y_1, is lower than that implied by y_2. The combined effect is that the expected value of x based on y_2 is, on average, greater than the expected value computed based on P_0, P_1 and P_2. Thus, positive values of $\widehat{UE}_2$ imply, on average, a price increase in the future. An exact computation showing that $\text{Cov}\{\widehat{UE}_2, UP_3\} > 0$ is provided in Appendix C.2.

5. Implications for Empirics

Abarbanell and Bernard (1992) point to several studies (Rendleman, Jones and Latane, 1982; Bernard and Thomas, 1990; Freeman and Tse, 1989; and Wiggins, 1991) that attempt to explain the post-announcement drift in terms of investors' failure to appreciate the extent to which changes in current quarterly earnings signal future changes in earnings. They conclude that stock prices reflect:

> ... at least partially a naive earnings expectation: one based on a seasonal random walk, where expected earnings are simply earnings for the corresponding quarter from the previous year *and that* ... the evidence in both Bernard and Thomas (1990) and Wiggins (1991) is consistent with market participants maintaining earnings expectations that are 'too close' to a naive seasonal random walk ... (p. 1184).

We find that a rational anticipation of the correlation structure does not suffice to remove the post-announcement drift (see Figures 1, 2). Therefore, the existence of a drift does not necessarily imply that traders fail to anticipate the correlation structure in public signals—the ability to predict future price changes based on public signals cannot by itself be taken as evidence of market irrationality.

The next three sections provide more specific details of how security-specific risk affects the pattern of price evolution. We consider (i) the levels of intertemporal variations in NIB supply, that is, changes in the level of b; (ii) the degree of correlation in public signals, that is, changes in the level of r; and (iii) noise in private information signals, that is, changes in the level of σ_ϵ^2, or equivalently, s. For each factor, we analyze the effects on the magnitude and direction of the price and earnings drifts. While analytical derivations are possible, the form of the parameters of the price function are algebraically quite complex. For this reason, we use numerical calibration to demonstrate the effects of varying the three features of economic interest outlined above.

5.1. Variance of Supply Shocks and Drift

We first examine the effects of an increase in the level of NIB volatility. The post-announcement drift, as expected, increases with the level of NIB volatility (Figure 1). Intuitively, as prices become more "noisy" due to an increase in the variance of NIB supply, they capture less of the value-relevant information present in accounting earnings leading to an increase in $\text{Var}(y_1 \mid P_1, P_0)$. Because a smaller fraction of the information is captured by concurrent prices, contemporaneous earnings provide more information about underlying values and become more correlated with future price changes leading to a monotone relationship between the level of NIB volatility and the earnings drift. Our second graph examines the effects of increases in b, that is, of reducing the impact of the period specific component, $\sqrt{1-b^2}\tau_t$, while increasing the persistent component, bZ_0. In this case, the earnings drift decreases in b (see Figure 2). The intuition is that the systematic component gets filtered out of prices more readily than the period specific component. Therefore, a greater weight on the systematic component of NIB (while the total NIB variance is held constant) results in prices closer to fundamental values, and thus, a lower post-announcement drift.

5.2. Correlation of Public Signals and Drift

The magnitudes of the price and earnings drifts are affected by the covariance structure of successive public announcements. The correlation between u_1 and u_2 captures the notion that successive public signals may have built in dependencies. For instance, if our public signals are interpreted as quarterly earnings reports, it is conceivable that the noise in these reports is positively correlated (systematic noise due to accounting procedures) or negatively correlated (arising from transitory events that reverse in the next period).

The magnitude of the earnings drift decreases in the level of correlation between public signals, r (Figure 3). Intuitively, the more positive the correlation between the public signals, the less the incremental information we can expect to infer (about the value of a risky asset) from the second public signal. Reduced incremental information from the second public signal leads to a less-attenuated demand in the first period as there is less need for postponing part of the demand until the second signal is received. This reduced demand attenuation in the current period results in a smaller drift. In contrast, when correlation is high and negative, a large amount of uncertainty is resolved in the second period (perfect negative correlation reveals x). Consequently, a large portion of the price adjustment occurs in period 2 giving rise to a high correlation between current earnings signals and future price changes, that is, a larger earnings drift. This economic rationale suggests that the earnings drift should be a monotone decreasing function of correlation in the noise components of the public signal, a finding that is documented in Figure 3.

5.3. Information Precision and Drift

As shown in Figure 4, the earnings drift exhibits an inverted 'U' shape in the precision of private information, $s = 1/\sigma_\epsilon^2$. When private information precision is low, the trader relies more on the public information and the current price reflects the public signal to a greater extent. Therefore, the *incremental* information in the public signal about future price movements beyond that already reflected in current prices, is low. Consequently, the result is a lower earnings drift. By contrast, as the precision of private information increases, the trader is more confident about underlying security values. As a result, NIB volatility will induce a lower demand attenuation and prices will more completely reflect information about the underlying value, x even before the public signal is received. Consequently, this will leave less room for prices to drift in the future as the price process converges to x. This explains why the two ends of the graph in Figure 4 are close to zero resulting in an inverted 'U' shape.

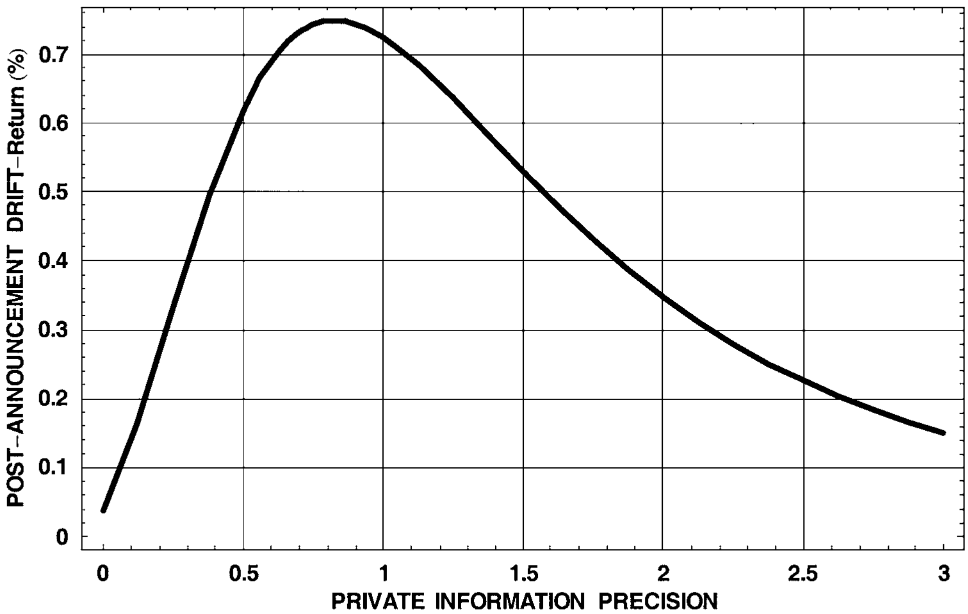

Figure 4. Effect of information precision on post-announcement drift.
For purposes of the simulation, we assumed that $\sigma_\tau^2 = \sigma_z^2$ and that $\sigma_u^2 = \sigma_\epsilon^2$ (see discussion in paper), where $s_u = \frac{1}{\sigma_u^2}$, $s = \frac{1}{\sigma_\epsilon^2}$ are the respective precisions of the public signal and the precision of individuals' private information, and σ_z^2 and σ_τ^2 are respectively variances of the aggregate per capita supply z and the change in the aggregate per capita supply τ. Parameter values for σ_u^2, σ_ϵ^2, ρ—the risk aversion parameter and σ_x^2—the variance of the liquidating value, were set according to the empirically estimated values $\sigma_u^2 = 0.7474$, $\sigma_x^2 = 0.1225$ and $\rho = 5.3$ (see Table 1 and Appendix D) and the autoregressive decay parameter b was set at zero to reflect intertemporally independent NIB trading noise. The intertemporal correlation coefficient between public signals r was set at its midpoint value of 0.5. Lastly, σ_z^2 was set at 0.018. This is equivalent to an NIB volatility level of 0.19 which is consistent with the empirically estimated 3-day CAR of 0.65% post-announcement drift.

If we assume that a larger analysts' following, and/or a larger institutional interest in a given company's stock implies greater precision of the private signals acquired on that company, our model would predict a smaller earnings drift for those companies at least for some ranges. These are indeed the findings of Bhushan (1994).

5.4. *Implications for Volume*

A solution for the equilibrium holding levels, z_{i_t}, within our model is provided in equations (A.27) and (A.29) of Appendix C. These equations imply:

$$z_{i_2} - z_{i_1} = \frac{\rho}{\rho_i}\left(\sqrt{1-b^2}\tau_2 - (1-b)\left(bZ_0 + \sqrt{1-b^2}\tau_1\right)\right).$$

That is, trading is generated both through changes in the supply of the asset (i.e., $\sqrt{1-b^2}$ $(\tau_2 - \tau_1)$) and for informational reasons captured by the effect of past noise, Z_0. A direct examination shows that the expression for trading volume is related to, but not identical with, the change in prices. In our setting, it is possible to have trading volume even if there is zero price change.[9] Dontoh and Ronen (1993) consider a situation where differential interpretations of signals is possible. This is unlike our model in which signals are interpreted homogeneously. Many of the derivations in our model survive unchanged if private information is assumed to have differing precisions or if public signals are interpreted differentially. In particular, the basic analytical technique developed in our paper and Propositions 1 and 2 hold quite generally regardless of the assumptions concerning private signal precision and the interpretation of signals. Overall, the conclusions of our study, which focus on the post-announcement behavior of prices and not on trading volume, do not change significantly as a result of perturbations of the model that have significant implications for trading volume.

As discussed in this section, our theoretical predictions derived from the rational expectations model are strikingly similar to the observed patterns of price behavior. As a final consistency check, we performed a calibration combining the mathematical expressions derived in this paper with actual observed market parameters. The results of this calibration show that the level of NIB trading required to generate the observed level of post-announcement drift is approximately 1.5% of trading volume, which seems a fairly plausible estimate. Even if this estimate exceeds the actual level of NIB trading, our calibration study confirms that NIB trading could account for a significant portion of the empirically documented post-announcement drift.

5.5. Calibration of the Postannouncement Drift

Does the theoretical model presented above account for the observed magnitude of post-announcement drift? Regardless of the portion of the observed drift it explains, our model provides insights regarding the phenomenon of delayed responses to public announcements and apparent semi-strong market inefficiency. Nonetheless, it is useful to identify parameter values that would lead our theoretically predicted drift to match the empirically observed drift. To this end, we estimate the drift for a sample of 10,400 firm-quarters of data for NYSE/AMEX firms for 1984–1986, using criteria for inclusion and methodology similar to those employed by Bernard and Thomas (1989, Section 3) (hereafter, BT89).

Following Bernard and Thomas (1990) (hereafter, BT90), we form ten portfolios for each calendar quarter, based on the SUE (standardized unexpected earnings; for definition, see BT90) deciles of firms announcing earnings within that quarter. We focus on the three-day market reactions to earnings announcements in the subsequent quarter (which, as in BT90, includes the period from two days prior through to the disclosure date itself, as reported in Compustat). The measure of abnormal returns is the size-adjusted return described in BT89. Daily abnormal returns for a given firm are obtained by subtracting from its return the return on the same-size-decile portfolio (NYSE/AMEX, based on the January 1 market value of equity). These daily abnormal returns are cumulated over time and summed across firms to obtain portfolio cumulative returns. The cumulative three-day abnormal return for the subsequent quarter for each decile portfolio, multiplied by the appropriate price to convert returns to price differences, becomes our measure of drift. We focus on the

Table 1. Parameter values obtained from data.

Variance of underlying per-share value (σ_x^2)	0.1225
Noise in public information (σ_u^2)	0.7474
Three-day abnormal return (Proposition 2)	0.0065
Coefficient of risk-aversion (ρ)	5.3

subsequent quarter as the closest empirical counterpart to the model's predicted $E[UP \mid (y_1 - E(y_1 \mid P_0))]$ (see Proposition 2). We investigate only the extreme deciles by computing the subsequent-quarter cumulative abnormal return of the portfolio, long in the highest decile and short in the lowest decile.

To carry out the calibration, we estimate the parameters involved in Proposition 3. We estimate σ_u^2 on the basis of the variance of first-differences of quarterly earnings, using the model-postulated process $y = x + u$, in which y is taken to represent quarterly earnings multiplied by an appropriate price/earnings multiple. (This adjustment is necessary because in our model, y denotes a signal about values rather than earnings). As a consequence, σ_u^2 is half the variance of first-differenced quarterly earnings scaled by the square of the price/earnings ratio. The variance of private information, σ_ϵ^2, is assumed equal to σ_u^2. ρ is calculated using the risk-free rate, the market return, and the per-capita wealth committed to equity by traders (see Appendix D). The resulting values are documented in Table 1. We simulate the drift setting $b = 0$ (which makes the NIB supply i.i.d across periods). We do not construct proxies for the remaining parameters σ_Z^2 and σ_τ^2. However, we can judge the consistency of our theory with empirical observations by asking whether the values of σ_Z and σ_τ that equate the simulated drift with the level of drift present in actual data fall within a plausible range. We find that the volatility of NIB trading required to make the documented drift in prices conform with the theoretical derivation of our model involves a small fraction of the observed volume volatility (see Appendix D). In addition, the expression for the postannouncement drift derived from our model is quite sensitive to the level of NIB shocks (see Figure 1).

We note that both data restrictions and intrinsic difficulties prevent exact statistical estimates of the parameters reflected in the expression for the drift derived in Proposition 3. Setting $b = 0$ ensures that q_0, q_1 and q_2 have an identical form (see 16) and illustrates the fact that the derivation of a positive drift does *not* require positive correlation in NIB trades. The value, $\rho = 5.3$, is derived in Appendix D.1 using market data but involves some assumptions regarding traders' utility function in wealth. Under these circumstances, the reported values involve considerable levels of judgment. They are intended to be illustrative, rather than precise, estimates. However, we can conjecture that the postannouncement drift calculated from our model will be similar in magnitude to the observed drift even if NIB trading constitutes only a small fraction of the total variation in trading volume. We therefore conclude that it is possible to reconcile the observed drift with equilibrium price behavior within a multi-period, rational-expectations market.

6. Conclusion

The earnings drift, which is often advanced as an example of market irrationality, can arise even if traders act rationally on their information. Specifically, within a multi-period rational expectations model where the presence of (contemporaneously unobservable) share supply variations affects equilibrium prices, there is a positive correlation between the unexpected component of current public signals and future price changes. Such a correlation arises from the fact that while prices reveal private information that cannot be found in public signals, non-information based trading distorts the information content of prices relative to the implications of both private and public information. As a consequence, both prices and public signals have to be considered as distinct sources of information in the decision process of rational traders. Under these circumstances, perfectly rational and efficient markets may appear semi-strong inefficient and slow to respond to earnings announcements.

Our findings thus suggest that the robustness of earnings-based "anomalies" may be a rational outcome of varying uncertain share-supply. Also, these "anomalies" cannot be eliminated through risk adjustments based purely on price data. In other words, our results suggest that the computation of expected returns relative to which abnormal returns are measured should incorporate accounting information in addition to price data.

By providing a specific equilibrium solution, our analysis also leads to other interesting predictions regarding the magnitude of the post-announcement drift. In particular, we demonstrate that the magnitude of market underreaction to public signals may be linked to the noise introduced by supply shocks, the precision of information and the correlation in public signals. The properties of our theoretical solutions correspond well with the findings in several different empirical studies. Thus, our study provides one coherent theoretical basis for interpreting results pertaining to the characteristics of the post-announcement earnings drift.

Appendix

A. Deriving the Equilibrium

We begin by outlining the technical result that underlies all our computations. Let Θ_n represent coordinates in $\Re^n$, $\mathcal{A}$ a positive definite symmetric $n \times n$ matrix, $\mathcal{Q}_n$ an arbitrary n-vector and C some constant. Then the integral:

$$\int_{\Re^n} \exp\left\{-\tfrac{1}{2}(\Theta_n' \mathcal{A} \Theta_n + 2\mathcal{Q}_n' \Theta_n + 2C)\right\} d\Theta_n$$

may be evaluated as:

$$\sqrt{(2\pi)^n |\mathcal{A}|} \exp\left(\tfrac{1}{2}\mathcal{Q}_n' \mathcal{A}^{-1} \mathcal{Q}_n - C\right) \tag{A.1}$$

This evaluation is based on factoring the positive definite symmetric matrix $\mathcal{A}$ as $\Phi\Phi'$ and setting $\mathcal{U}_n = \Phi'\Theta_n + \Phi^{-1}\mathcal{Q}_n$ (Anderson, 1971).

The derivation of the equilibrium involves working backwards starting with period 2 and ending at period 0. We therefore begin by applying this technique to the decision problem faced by a typical trader i at the second period.

A.1. Derivation of Time 2 Risky-Asset Demands

The general method outlined above is needed mainly for calculating time 1 demands. However, we begin by applying it at time 2, where more direct procedures are available, so we can illustrate the technique in a univariate setting. The final wealth of trader i (where we have taken the initial endowment to be 0), W_{i_3}, is given by:

$$\begin{aligned} W_{i_3} &= z_{i_0}(P_1 - P_0) + z_{i_1}(P_2 - P_1) + z_{i_2}(x - P_2) \\ &= z_{i_0}(P_1 - P_0) + z_{i_1}(P_2 - P_1) + z_{i_2}\big(E\big(x \mid I_{i_2}\big) - P_2\big) + z_{i_2}\big(x - E\big(x \mid I_{i_2}\big)\big) \end{aligned}$$

and trader i's decision problem at time 2 is to maximize $E[-\exp(\rho_i W_{i_3}) \mid I_{i_2}]$ by suitably choosing z_{i_2}.

$$\begin{aligned} &E\big(-\exp\big\{-\rho_i W_{i_3}\big\} \mid I_{i_2}\big) \\ &\quad = \frac{-1}{|\Omega_2|^{\frac{1}{2}} 2\pi^{\frac{1}{2}}} \int_{-\infty}^{\infty} \exp\left\{-\rho_i\big[W_{i_2} + z_{i_2}\big(E\big[x \mid I_{i_2}\big] - P_2\big) + z_{i_2}\Theta_{i_2}\big] - \frac{1}{2}\big[\Theta_{i_2}'\Omega_2^{-1}\Theta_{i_2}\big]\right\} d\Theta \end{aligned}$$

where:

$$\Theta_{i_2} = x - E\big(x \mid I_{i_2}\big), \Omega_2 = V_2 = \operatorname{var}\big(x \mid I_{i_2}\big), W_{i_2} = z_{i_0}(P_1 - P_0) + z_{i_1}(P_2 - P_1).$$

Setting $\mathcal{A}_2 = \Omega_2^{-1}$, $\mathcal{Q}_{i_2} = \rho_i z_{i_2}$ and $C_{i_2} = W_{i_2} + z_{i_2}(E[x \mid I_{i_2}] - P_2)$ and applying (A.1),

$$E\big[-\exp\big(W_{i_3}\big) \mid I_{i_2}\big] = -\exp\big(\tfrac{1}{2}\rho_i^2 z_{i_2}^2 V_2 - C_{i_2}\big). \tag{A.2}$$

Collecting the terms that involve z_{i_2}, the problem reduces to:

$$\max_{z_{i_2}} \quad \tfrac{1}{2}\rho_i^2 V_2 z_{i_2}^2 - \rho_i z_{i_2}\big(E\big[x \mid I_{i_2}\big] - P_2\big).$$

This yields the solution in equation (6): $z_{i_2} = (\rho_i V_2)^{-1}(E(x \mid I_{i_2}) - P_2)$. In order to compute $E(x \mid I_{i_2})$, define:

$$\begin{aligned} q_0 &\equiv \frac{1}{A_{01}}(P_0) = x - B_0 Z \\ q_1 &\equiv \frac{1}{A_{11} - \frac{A_{14}}{B_0}}\left(P_1 - A_{12}y_1 - A_{13}P_0 - \frac{A_{14}}{B_0}q_0\right) = x - B_1\tau_1 \\ q_2 &\equiv \frac{1}{A_{21} - \frac{A_{26}}{B_0}}\left(P_2 - A_{22}y_1 - A_{23}y_2 - A_{24}P_0 - A_{25}P_1 - \frac{A_{26}}{B_0}q_0\right) = x - B_2\tau_2 \end{aligned} \tag{A.3}$$

where

$$B_0 = \frac{A_{02}}{A_{01}}, \quad B_1 = \frac{A_{14}B_0}{A_{11}B_0 - A_{14}}, \quad B_2 = \frac{A_{26}B_0}{A_{21}B_0 - A_{26}}.$$

The q_i can be used to replace P_i in the information sets, i.e., $I_{i_0} = \{y_{i_0}, q_0\}$, $I_{i_1} = \{y_{i_0}, y_1, q_0, q_1\}$, $I_{i_2} = \{y_{i_0}, y_1, y_2, q_0, q_1, q_2\}$. Using this specification, the joint distribution of x with time 2 information variables is multivariate normal with variance-covariance matrix:

$$\sum = \begin{bmatrix} & x & y_{i_0} = x + \varepsilon_i & y_1 = x + u_1 & y_2 = x + u_2 & q_0 = x - B_0 Z & q_1 = x - B_1\tau_1 & q_2 = x - B_1\tau_2 \\ x & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 \\ y_{i_0} & \sigma_x^2 & \sigma_x^2 + \sigma_\varepsilon^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 \\ y_1 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 + \sigma_u^2 & \sigma_x^2 + r\sigma_u^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 \\ y_2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 + r\sigma_u^2 & \sigma_x^2 + \sigma_u^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 \\ q_0 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 + B_0^2\sigma_z^2 & \sigma_x^2 & \sigma_x^2 \\ q_1 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 + B_1^2\sigma_t^2 & \sigma_x^2 \\ q_2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 & \sigma_x^2 + B_2^2\sigma_t^2 \end{bmatrix}.$$

Let $\sum_{12} \equiv \sum_{21}' = (\sigma_x^2, \sigma_x^2, \sigma_x^2, \sigma_x^2, \sigma_x^2, \sigma_x^2)$ and $\sum_{22}$ be the 6×6 matrix obtained from $\sum$ above by eliminating the first row and first column. Then the coefficients of the conditioning information variables are determined by $\sum_{12}\sum_{22}^{-1}$ while the posterior variance of x at time 2, V_2 satisfies $\operatorname{Var}[x - E(x \mid I_{i_2})] = \sigma_x^2 - \sum_{12}\sum_{22}^{-1}\sum_{21}$. Denoting the respective precisions by:

$$s = \frac{1}{\sigma_{\epsilon_i}^2}; \quad s_u = \frac{1}{\sigma_u^2}; \quad s_{q_0} = \frac{1}{B_0^2\sigma_{Z_0}^2}; \quad s_{q_1} = \frac{1}{B_1^2\sigma_\tau^2}; \quad s_{q_2} = \frac{1}{B_2^2\sigma_\tau^2},$$

the coefficients in the conditional expectation may be evaluated as:

$$E\left[x \mid I_{i_2}\right] = V_2\left[s y_{i_0} + \frac{s_u}{1+r} y_1 + \frac{s_u}{1+r} y_2 + s_{q_0} q_0 + s_{q_1} q_1 + s_{q_2} q_2\right] \tag{A.4}$$

with the posterior variance of x, V_2, given by:

$$V_2 = \frac{1+r}{2s_u + (1+r)\left(s_x + s + s_{q_0} + s_{q_1} + s_{q_2}\right)}. \tag{A.5}$$

Averaging (6) over i and using market clearing now leads to:

$$P_2 = V_2\left[sx + \frac{s_u}{1+r} y_1 + \frac{s_u}{1+r} y_2 + s_{q_0} q_0 + s_{q_1} q_1 + s_{q_2} q_2\right] - \rho V_2 Z_2. \tag{A.6}$$

It is also worth deriving an expression for z_{i_2}. Comparing equations (A.4) and (A.6) and using equation (6), it follows that

$$z_{i_2} = \frac{\rho}{\rho_i} s\left(y_{i_0} - x\right) + \frac{\rho}{\rho_i} Z_2 = \frac{\rho}{\rho_i}(s\epsilon_i + Z_2). \tag{A.7}$$

A.2. The First-Period Price and Demands

Trader i's decision problem at time 1 concerns the holding z_{i_1} which will maximize expected terminal wealth based on the information available at time 1. Using iterated expectations, $E(-\exp\{-W_{i_3}\} \mid I_{i_1}) = E(E(-\exp\{-W_{i_3}\} \mid I_{i_2})I_{i_1})$. Using the expression in (A.2) and substituting for the optimal z_{i_2},

$$E\left(-\exp\left\{-\rho_i W_{i_3}\right\} \mid I_{i_1}\right)$$
$$= \frac{-1}{\left|\Omega_{i_1}\right|^{\frac{1}{2}} 2\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left\{-\rho_i\left[W_{i_1} + z_{i_1}(P_2 - P_1) + \frac{1}{2}\rho_i V_2 z_{i_2}^2\right] - \frac{1}{2}\left[\Theta'\Omega_{i_1}^{-1}\Theta\right]\right\} d\Theta$$

where $W_{i_1} = z_{i_0}(P_1 - P_0)$, $\Theta_i = \{P_2 - E(P_2 \mid I_{i_1}), z_{i_2} - E(z_{i_2} \mid I_{i_1})\}$ are random variables orthogonal to I_{i_1}. Ω_{i_1} denotes their joint variance-covariance matrix, that is,

$$\Omega_{i_1} = \begin{pmatrix} \operatorname{var}\left(P_2 \mid I_{i_1}\right) & \operatorname{Cov}\left\{z_{i_2}, P_2 \mid I_{i_1}\right\} \\ \operatorname{Cov}\left\{z_{i_2}, P_2 \mid I_{i_1}\right\} & \operatorname{var}\left(z_{i_2} \mid I_{i_1}\right) \end{pmatrix} \text{ denoted by } \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix}.$$

In order to solve the maximization problem, first note that:

$$\rho_i z_{i_1}(P_2 - P_1) + \tfrac{1}{2}\rho_i^2 V_2 z_{i_2}^2 = \rho_i z_{i_1}(P_2 - \bar{P}_{2i}) + z_{i_1}(\bar{P}_{2i} - P_1) + \tfrac{1}{2}\rho_i^2 V_2\left[\left(z_{i_2} - \bar{z}_{i_2}\right) + \bar{z}_{i_2}\right]^2. \tag{A.8}$$

Let:

$$\mathcal{A}_{i_1} = \Omega_{i_1}^{-1} + \begin{bmatrix} 0 & 0 \\ 0 & \rho_i^2 V_2 \end{bmatrix}, \quad \mathcal{A}_{i_1}^{-1} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{12} & \alpha_{22} \end{bmatrix} \tag{A.9}$$

$$\bar{z}_{i_2} = E\left(z_{i_2} \mid I_{i_1}\right); \quad \bar{P}_{2i} = E\left[P_2 \mid I_{i_1}\right] \tag{A.10}$$

$$\mathcal{Q}_{i_1} = \left\{\rho_i z_{i_1}, \rho_i^2 V_2 \bar{z}_{i_2}\right\} \tag{A.11}$$

$$C_{i_1} = \rho_i z_{i_1}(\bar{P}_{2i} - P_1) + \tfrac{1}{2}\rho_i^2 V_2 \bar{z}_{i_2}^2 + \rho_i W_{i_1}. \tag{A.12}$$

Individual i's first period demand optimization problem can be expressed as:

$$\max_{z_{i_1}} \frac{-1}{\left|\Omega_{i_1}\right|^{\frac{1}{2}} 2\pi} \int_{\Theta_i} \exp\left\{-\tfrac{1}{2}\left(\Theta_i'\mathcal{A}_{i_1}\Theta_i + 2\mathcal{Q}_{i_1}'\Theta_i + 2C_{i_1}\right)\right\} d\Theta_i. \tag{A.13}$$

Because $\Omega_{i_1}^{-1}$ is positive definite while the matrix $[\begin{smallmatrix} 0 & 0 \\ 0 & \rho_i^2 V_2 \end{smallmatrix}]$ is positive (semi) definite, their sum $\mathcal{A}_{i_1}$ is automatically positive definite. Therefore, from (A.1), the maximization problem may be transformed to:

$$\max_{z_{i_1}} -\frac{\left|\mathcal{A}_{i_1}\right|^{\frac{1}{2}}}{\left|\Omega_{i_1}\right|^{\frac{1}{2}}} \exp\left(\tfrac{1}{2}\mathcal{Q}_{i_1}'\mathcal{A}_{i_1}^{-1}\mathcal{Q}_{i_1} - C_{i_1}\right).$$

After substituting back for $\mathcal{A}_{i_1}^{-1}$, $\mathcal{Q}_{i_1}$, and C_{i_1}, and gathering terms in z_{i_1}, the problem reduces to:

$$\max_{z_{i_1}} - \exp\left\{-\rho_i\left[z_{i_1}(\bar{P}_{2i} - P_1) - \rho_i^2 V_2 \alpha_{12}\bar{z}_{i_2} z_{i_1} - \tfrac{1}{2}\rho_i \alpha_{11} z_{i_1}^2\right]\right\}.$$

Maximizing and solving for z_{i_1} yields: $(\bar{P}_{2i} - P_1) - \rho_i^2 V_2 \alpha_{12}\bar{z}_{i_2} - \rho_i \alpha_{11} z_{i_1} = 0$. This equation may be rewritten as:

$$z_{i_1} - c\bar{z}_{i_2} = \frac{\bar{P}_{2i} - P_1}{\rho_i \alpha_{11}} \quad \text{where } c = -\frac{\rho_i V_2 \alpha_{12}}{\alpha_{11}}. \tag{A.14}$$

A.3. Conditions Ensuring c < 1

Because of the random NIB supply shock $\sqrt{1-b^2}\tau_2$ that affects the second-period price, traders might like to "insure" against the noise in the second-period price by acquiring some fraction of their expected time 2 demand at time 1. By definition, c is this reflection of the expected second-period holding on first-period demand (see equation (A.14)). Intuitively, it seems reasonable that the expected demand would be split across periods, i.e., c will be some fraction $0 < c < 1$. The fact that $c < 1$ holds quite generally as we show below. We also demonstrate, in the next section, that the explicit solution for c that we determine within our equilibrium does indeed lie between 0 and 1 and is the same for every trader (see equation (A.31)).

The first step is to calculate the coefficients α_{ij}. Denoting determinants by $|\cdot|$ and using standard matrix inversion formulae, equation (A.9) yields:

$$\begin{aligned} \alpha_{11} &= \frac{\sigma_{11} + \rho_i^2 V_2 |\Omega_{i_1}|}{|\Omega_{i_1}||\mathcal{A}_{i_1}|} & \alpha_{12} &= \frac{\sigma_{12}}{|\Omega_{i_1}||\mathcal{A}_{i_1}|} \\ \alpha_{21} &= \frac{\sigma_{12}}{|\Omega_{i_1}||\mathcal{A}_{i_1}|} & \alpha_{22} &= \frac{\sigma_{22}}{|\Omega_{i_1}||\mathcal{A}_{i_1}|} \end{aligned} \tag{A.15}$$

The denominators in the expressions above are positive because the matrices Ω_{i_1} and $\mathcal{A}_{i_1}$ are both positive definite. Therefore, when $\sigma_{12} \geq 0$ (this is the non-intuitive case), equation (A.14) implies that $c < 0$ and automatically less than 1. Now consider the alternative case where $\sigma_{12} \leq 0$ and $c > 0$. To establish that $c < 1$ in this case, we first note that (A.4) implies that:

$$\begin{aligned} E[x \mid I_{i_2}] - E[x \mid I_{i_1}] &= V_2 s_u(y_2 - E[y \mid I_{i_1}]) + V_2 s_2(q_2 - E[q_2 \mid I_{i_1}]) \\ P_2 - \bar{P}_{2i} &= V_2 s_u(y_2 - E[y \mid I_{i_1}]) + V_2(s_2 + s(1-b))(q_2 - E[q_2 \mid I_{i_1}]) \end{aligned}$$

Next, Assumption 1 implies that the variances and covariances of all the quantities above are positive (critically, that q_2 is positively correlated with x). It follows that:

$$\mathrm{Cov}\{P_2 - \bar{P}_{2i}, E[x \mid I_{i_2}] - E[x \mid I_{i_1}]\} > 0 \tag{A.16}$$

Given this inequality, equation (A.14) leads to

$$c = -\frac{\rho_i V_2 \alpha_{12}}{\alpha_{11}} = -\frac{\rho_i V_2 \sigma_{12}}{\sigma_{11} + \rho_i^2 V_2 \left|\Omega_{i_1}\right|} \geq -\frac{\rho_i V_2 \sigma_{12}}{\sigma_{11}} \tag{A.17}$$

$$\begin{aligned}
\rho_i V_2 \sigma_{12} &= \operatorname{Cov}\left\{P_2 - \bar{P}_{2i}, \rho_i V_2\left(z_{i_2} - \bar{z}_{i_2}\right)\right\} \\
&= \operatorname{Cov}\left\{P_2 - \bar{P}_{2i}, \left(E\left[x \mid I_{i_2}\right] - E\left[x \mid I_{i_1}\right]\right) - (P_2 - \bar{P}_{2i})\right\} \\
&= \operatorname{Cov}\left\{P_2 - \bar{P}_{2i}, \left(E\left[x \mid I_{i_2}\right] - E\left[x \mid I_{i_1}\right]\right)\right\} - \operatorname{Var}(P_2 - \bar{P}_{2i}) \\
&> -\operatorname{Var}(P_2 - \bar{P}_{2i}) = \sigma_{11}
\end{aligned} \tag{A.18}$$

Equation (A.18) may be restated as $-\rho_i V_2 \sigma_{12} < \sigma_{11}$ and combining with (A.17) yields $c < 1$. We note that although $\sigma_{12} = \operatorname{Cov}\{z_{i_2}, P_2 - \bar{P}_{2i} \mid I_{i_1}\}$ and $\sigma_{22} = \operatorname{Var}(z_{i_2} \mid I_{i_1})$ depend on ρ_i, the quantities

$$\left|\Omega_{i_1}\right|\left|\mathcal{A}_{i_1}\right| = 1 + \rho^2 V_2 \operatorname{Var}\left(sx - Z_2 \mid I_{i_1}\right) \quad \rho_i V_2 \sigma_{12} = \rho V_2 \operatorname{Cov}\left\{sx - Z_2, P_2 \mid I_{i_1}\right\}$$

are identical across traders as a result of the fact that the equilibrium price is independent of individual preferences and information. Thus, c is the same for every trader as is derived explicitly in Appendix B, (A.31).

B. Proofs of Propositions

Proof of Proposition 1: Because $y_1 - E[y_1 \mid P_1, P_0]$ is orthogonal to the set $\{P_1, P_0\}$ while $E[P_2 \mid P_1, P_0]$ is measurable with respect to $\{P_1, P_0\}$, we have (see Lemma 1):

$$\begin{aligned}
\operatorname{Cov}\{y_1 - E[y_1 \mid P_1, P_0], P_2 - E[P_2 \mid P_1, P_0]\} &= \operatorname{Cov}\{y_1 - E[y_1 \mid P_1, P_0], P_2\} \\
&= \operatorname{Cov}\{y_1 - E[y_1 \mid P_1, P_0], P_2 - P_1\}.
\end{aligned} \tag{A.19}$$

Next, because $y_1 - E[y_1 \mid P_1, P_0]$ lies in the information set I_{i_1}

$$\operatorname{Cov}\left\{y_1 - E[y_1 \mid P_1, P_0], P_2 - E\left[P_2 \mid I_{i_1}\right]\right\} = 0 \tag{A.20}$$

Combining these two results with equations (12) we arrive at:

$$\operatorname{Cov}\{UE_1, UP_2\} = \operatorname{Cov}\left\{y_1 - E[y_1 \mid P_1, P_0], E\left[P_2 \mid I_{i_1}\right] - P_1\right\}.$$

Applying 8, we obtain:

$$\frac{1}{\rho_i}\operatorname{Cov}\{UE_1, UP_2\} = \alpha_{11}\operatorname{Cov}\left\{z_{i_1} - c\bar{z}_{i_2}, y_1 - E[y_1 \mid P_1, P_0]\right\}.$$

Noting that $z_{i_2} - \bar{z}_{i_2}$ is orthogonal to time 1 variables, we obtain:

$$\frac{1}{\rho_i}\operatorname{Cov}\{UE_1, UP_2\} = \alpha_{11}\operatorname{Cov}\left\{z_{i_1} - cz_{i_2}, y_1 - E[y_1 \mid P_1, P_0]\right\} \tag{A.21}$$

and averaging across traders completes the proof. ■

Proof of Proposition 2: First, $Z_1 - cZ_2 = (b_{1,0} - cb_{2,0})Z_0 + (1 - cb_{2,1})\tau_1 + c\tau_2$. Second $E[y_1 \mid P_1, P_0] = E[y_1 \mid \hat{q}_1, q_0]$ where $\hat{q}_1 = (1/A_{12})[P_1 - A_{13}P_0] = y_1 + (A_{11}/A_{12})q_1$. This conditional expectation may be explicitly calculated:

$$E[y_1 \mid P_1, P_0] = E[y_1 \mid \hat{q}_1, q_0] = \beta\hat{q}_1 + \alpha q_0$$

where α and β are determined through the matrix equation:

$$[\mathrm{Cov}(y_1, q_0), \mathrm{Cov}(y_1, \hat{q}_1)] \cdot \begin{bmatrix} \mathrm{Var}(q_0) & \mathrm{Cov}(q_0, q_1) \\ \mathrm{Cov}(q_0, q_1) & \mathrm{Var}(\hat{q}_1) \end{bmatrix}^{-1} = [\alpha, \beta]. \tag{A.22}$$

Writing $A = A_{11}/A_{12}$, we have: $\mathrm{Cov}(y_1, q_0) = \sigma_x^2$; $\mathrm{Cov}(y_1, \hat{q}_1) = (1 + A)\sigma_x^2 + \sigma_u^2$; $\mathrm{Var}(q_0) = \sigma_x^2$; $\mathrm{Cov}(q_0, q_1) = \sigma_x^2$ and $\mathrm{Var}(\hat{q}_1) = (1 + A)^2\sigma_x^2 + \sigma_u^2 + A^2\sigma_\tau^2$. Therefore, the coefficients α and β are given by:

$$\alpha = \frac{[(1 + A)^2 - (1 + A)]\sigma_x^4 + A^2\sigma_u^2\sigma_\tau^2}{\Delta}$$
$$\beta = \frac{A\sigma_x^4 + \sigma_u^2\sigma_x^2 + \left[(1 + A)(A_{02}/A_{01})^2\sigma_x^2 + \sigma_u^2\right]\sigma_{Z_0}^2}{\Delta} \tag{A.23}$$

where Δ is the determinant of the variance-covariance matrix of $\hat{q}_1$ and q_0. It follows that α and β are both positive. Noting that both x and y_1 are uncorrelated with Z_t and that q_0 and q_1 are uncorrelated with τ_2, we obtain

$$\begin{aligned} \mathrm{Cov}\{y_1 - E[y_1 \mid P_1, P_0], Z_1 - cZ_2\} &= \mathrm{Cov}\{-E[y_1 \mid P_1, P_0], Z_1 - cZ_2\} \\ &= \mathrm{Cov}\{-\alpha q_0 - \beta\hat{q}_1, b_{1,0} - cb_{2,0})Z_0 \\ &\quad + (1 - cb_{2,1})\tau_1\} \\ &= \alpha(A_{02}/A_{01})(b_{1,0} - cb_{2,0})\sigma_{Z_0}^2 \\ &\quad + \beta(A_{14}/A_{11})(b_1 - cb_{2,1})\sigma_\tau^2. \end{aligned} \tag{A.24}$$

The expression on the last line in (A.24) is positive given Assumptions 1 and 2. ■

Proof of Proposition 3: The proof of Proposition 2 can be made more specific here as the coefficients are known exactly.

$$\frac{A_{02}}{A_{01}} = \frac{\rho}{s}\frac{A_{14}}{A_{11}} = \frac{\rho}{s}\sqrt{\frac{1-b}{1+b}} \qquad A = \frac{A_{11}}{A_{12}} = \frac{s_1}{s_u} + \frac{Ts(1-b)^2}{V_1[1 + Ts(1-b)]s_u}$$
$$c = \frac{V_2(s(1-b))V_1 + V_2(s(1-b)^2)(V_1 + s^{-2})}{(V_1 - V_2) + 2V_2(s(1-b)V_1 + V_2(s(1-b)^2)(V_1 + s^{-2})} \tag{A.25}$$

Further, under Assumption 3, $b_1 = \sqrt{1 - b^2}$, $b_{1,0} = b$ and $b_{2,0} = b^2$ Substituting these values into (A.23) and (A.24) gives an expression for the drift which can be plotted easily as a function of the parameters $b, \sigma_v^2, r, \sigma_{Z_0}^2 = \sigma_{\tau_t}^2$. These graphs are presented in Figures 1–4. ■

C. An Equilibrium Solution

In this section we outline how to calculate a complete multi-period equilibrium that yields the prices in equation (15) of the paper. The procedure involves assuming that conjectures are as in (15) and then verifying that the resulting prices are the same as the conjectures. Recalling the definitions,

$$q_0 = x - \frac{\rho}{s} Z_0; \quad q_1 = x - \frac{\rho}{s}\left(\sqrt{\frac{1+b}{1-b}}\right)\tau_1; \quad q_2 = x - \frac{\rho}{s}\left(\sqrt{\frac{1+b}{1-b}}\right)\tau_2$$

and assuming for the moment that the information set $\{y_1, y_2, P_0, P_1, P_2\}$ is informationally equivalent to $\{y_1, y_2, q_0, q_1, q_2\}$, we obtain (using equation (6) and integrating over i)

$$\begin{aligned} P_2 &= V_2\left[sx + s_0 q_0 + s_1 q_1 + \frac{s_u}{1+r}(y_1 + y_2)\right] - \rho(b^2 Z_0 + b\sqrt{(1-b^2)}\tau_1 + \sqrt{(1-b^2)}\tau_2) \\ &= V_2\left[\frac{s_u}{1+r}(y_1 + y_2) + (s_0 + b^2 s)q_0 + (s_1 + b(1-b)q_1 + s(1-b)q_2\right] \end{aligned} \quad \text{(A.26)}$$

verifying the solution for P_2. Substituting back the solution for P_2 into (6) results in:

$$z_{i_2} = \frac{s}{\rho_i}\big(y_{i_0} - b^2 q_0 - b(1-b)q_1 - (1-b)q_2\big) \quad \text{(A.27)}$$

Conditioning on I_{i_1}, these solutions imply

$$\begin{aligned} P_2 - \bar{P}_{2i} &= V_2\left[\frac{s_u}{1+r}\big(y_2 - E\left[y_2 \mid I_{i_1}\right]\big) + (s_2 + s(1-b))(q_2 - \bar{q}_{2i})\right] \\ z_{i_2} - \bar{z}_{i_2} &= \frac{\rho}{\rho_i} s(1-b)(q_2 - \bar{q}_{2i}) \end{aligned} \quad \text{(A.28)}$$

To calculate the variance-covariance matrix associated with the variables in (A.28), we first record the posterior variances of x given information set I_{i_t} (under the conjectures in equation (15) of the paper):

$$V_2 = \frac{1+r}{2s_u + (1+r)(s_x + s + s_{q2} + s_{q1} + s_{q0})}; \quad V_1 = \frac{1}{s_u + s_x + s + s_{q0} + s_{q1}};$$
$$V_0 = \frac{1}{s_x + s + s_{q0}}$$

This allows us to calculate the parameters σ_{jk} and α_{jk} (see A.9, A.15) and substituting into (A.14) yields:

$$\rho_i\left[\alpha_{11} z_{i_1} - \alpha_{12}\bar{z}_{i_2}\right] = E\left[P_2 \mid I_{i_1}\right] - P_1.$$

The explicit formula for α_{ij} (available from authors upon request) combined with applying market clearing yields:

$$P_1 = K_1(s_u y_1 + s_{q0})q_0 + sbq_0 + s_{q_1}q_1 + s(1-b)q_1 + T(s(1-b))(bq_0 + (1-b)q_1)$$
$$z_{i_1} = \frac{s}{\rho_i}\left(y_{i_0} - bq_0 - (1-b)q_1\right)$$
$$c = \frac{V_2 V_1 s(1-b) + V_2 s^2(1-b)^2[V_1 + 1/s_2]}{V_1 - V_2 + 2V_2 V_1 s(1-b) + V_2 V_1 s^2(1-b)^2(1/s_2)} \tag{A.29}$$

where:

$$T = \frac{V_2(1 - V_1 s(1-b))}{1 + V_2 V_1(s(1-b))^2 + \rho V_2(1-b^2)\sigma_\tau^2}$$
$$K_1 = V_1[1 + T(s(1-b))] \tag{A.30}$$

Notice that $0 < T < V_2$ as $1 > V_1 s \geq V_1 s(1-b)$ (and the denominator of (A.30) is greater than 1. With K_1 as in (A.30), the first equation in (A.29) may be rewritten as:

$$P_1 = K_1(s_u y_1 + s_{q_0}q_0 + s_{q1}q_1) + [K - 1 - T(1-b)]\, s\, [bq_0 + (1-b)q_1]$$

This verifies the time 1 price and demands. We note that in this equilibrium solution, $P_2 - \bar{P}_{2i}$ and $x - E[x \mid I_{i_1}]$ are positively correlated and that $c \leq 1$. In fact,

$$0 < c = 1. \tag{A.31}$$

We next provide the details of the derivation of z_{i_0} within this equilibrium.

C.1. Time 0 Prices and Demands

By the process of integration outlined at times 1 and 2, trader i's random wealth (at $t = 0$) is given by:

$$\exp(-\rho_i)\left[-\frac{1}{2\pi\left|\Omega_{i_0}\right|^{\frac{1}{2}}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left\{-\rho z_{i_0}(P_1 - P_0) - \rho_i^2 K_1 z_{i_1}^2 - \frac{1}{2}\left(1 - \rho_i^2 V_2\alpha_{22}\right)\rho_i^2 V_2 \bar{z}_{i_2}^2 - \frac{1}{2}\Theta'_{i_0}\Omega_{i_0}^{-1}\Theta_{i_0}\right\} d\Theta_{i_0}\right]$$

in which $\Theta_{i_0} = \{P_1 - E(P_1|I_{i_0}), z_{i_1} - E[z_{i_1}|I_{i_0}]\}$ is conditioned on I_{i_0} with variance-covariance matrix denoted by Ω_{i_0}. An important detail in evaluating this integral is the fact that in this particular equilibrium, $\bar{z}_{i_2} = E[z_{i_2}|I_{i_0}$ is a linear combination of $z_{i_1} - \bar{z}_{i_1}$ and $P_1 - E[P_1 \mid I_{i_0}]$. Integrating the expression above, differentiating with respect to z_{i_0} and setting equal to 0 yields:

$$[a_{11}\lambda^2 + 2a_{21}\lambda\mu + a_{22}\mu^2]z_{i_0} + c_0\bar{z}_{i_1} - d_0\overline{\bar{z}_{i_2}} = E\left[P_1 \,\middle|\, I_{i_0}\right] - P_0 \tag{A.32}$$

where:

$$a_{11} = V_0 - V_1, \quad a_{12} = V_0(1+b) - V_1 + \frac{\rho^2}{s^2}\frac{1-b}{1+b}\sigma_\tau^2,$$
$$a_{22} = V_0(1+b)^2 - V_1 + \frac{\rho^2}{s^2}\frac{1-b}{1+b}\sigma_\tau^2$$
$$\lambda = K_1(s(1-b)-b), \quad \mu = [1 + T(s(1-b))]$$

Substituting for the coefficients a_{ij} and solving results in:

$$P_0 = K_0(s_0 q_0 + s q_0) + K_1 s q_0 + [K_1 - T(1-b)]sbq_0 \tag{A.33}$$

where:

$$K_0 = a_{11}\lambda^2 + 2a_{12}\lambda\mu + a_{22}\mu^2.$$

Because the equilibrium P_0 is as conjectured, the price structure in (15) constitutes a genuine equilibrium.

C.2. Alternative Measures of Earnings Surprise and Drift

The computation is not amenable to orthogonality arguments and detailed algebra is unavoidable. Define:

$$\hat{q}_2 = \left(\frac{1}{A_{21} + A_{22} + A_{23} + A_{25}}\right)(P_2 - A_{23}q_0) = x + \beta_0(u_1 + u_2) + \beta_1 B_1 \tau_1 + \beta_2 B_2 \tau_2$$
$$\hat{q}_1 = \left(\frac{1}{A_{11} + A_{12}}\right)(P_1 - A_{13}q_0) = x + \alpha_0 u_1 - \alpha_1 B_1 \tau_1; \quad \hat{q}_0 = q_0 = x - B_0 Z_0 \tag{A.34}$$

Then each $\hat{q}_i$ is of the form $x + \eta_i$ where η_i is uncorrelated with x while: (i) $\text{Cov}\{\eta_0, \eta_i\} = 0$ for $i = 1, 2$ and (ii) $\text{Cov}\{\eta_1, \eta_2\} = \alpha_0\beta_0\sigma_u^2(1+r) + \alpha_1\beta_1 B_1^2\sigma_\tau^2$. Let $\hat{s}_i$ denote the respective precisions of $\hat{q}_i$ (where $\hat{s}_0 = s_{q_0}$ is used for notational symmetry). Then:

$$E[x \mid P_2, P_1, P_0] = E[x \mid \hat{q}_2, \hat{q}_1, \hat{q}_0]$$
$$= \hat{V}_2\left[\frac{(1 - \hat{m}\hat{s}_1)\hat{s}_2}{1 - m^2 s_1 s_2}\hat{q}_2 + \frac{(1 + m\hat{s}_2)\hat{s}_1}{1 - m^2 s_1 s_2}\hat{q}_1 + \hat{s}_0\hat{q}_0\right] \tag{A.35}$$

where $m = \text{Cov}\{\eta_1, \eta_2\}$ and $\hat{V}_2$ is the posterior conditional variance which is given by:

$$\frac{1}{\hat{V}_2} = \frac{(1 + m\hat{s}_1)\hat{s}_2}{1 - m^2\hat{s}_1\hat{s}_2} + \frac{(1 + m\hat{s}_2)\hat{s}_1}{1 - m^2\hat{s}_1\hat{s}_2} + \hat{s}_0 + s_x \tag{A.36}$$

In addition,

$$\widehat{UE}_2 = y_2 - E[y_2 \mid y_1] = y_2 - \left(\frac{\sigma_x^2 + r\sigma_u^2}{\sigma_x^2 + \sigma_u^2}\right)y_1 \tag{A.37}$$

Thus, $\widehat{UE}_2$ is independent of y_1 by definition (see Lemma 1) and also of Z_0 and τ_i by assumption (that x, u_i and supply uncertainty are all mutually jointly-independent). Consequently, using the expressions in (A.34) and dividing the numerator and denominator of each covariance by $\sigma_x^2\sigma_u^2$ yields:

$$\begin{aligned}
&\mathrm{Cov}\{\widehat{UE}_2, \hat{q}_0\} = \mathrm{Cov}\{\widehat{UE}_2, x\} = \frac{(1-r)}{s_u + s_x} \quad \mathrm{Cov}\{\widehat{UE}_2, \hat{q}_1\} = \alpha_1 \frac{(1-r)}{s_u + s_x} \\
&\mathrm{Cov}\{\widehat{UE}_2, \hat{q}_2\} = \frac{(1-r)(\beta_0 + \beta_1 + \beta_2)}{s_u + s_x}
\end{aligned} \tag{A.38}$$

Substituting for α_1, β_i from the equilibrium solution into (A.38) allows us to explicitly determine the covariance terms above. Next, substituting from (A.35) we obtain

$$\begin{aligned}
\mathrm{Cov}\{\widehat{UE}_2, UP_3\} &= \mathrm{Cov}\{\widehat{UE}_2, x - E[x \mid \hat{q}_2, \hat{q}_1, \hat{q}_0\} \\
&= \mathrm{Cov}\{\widehat{UE}_2, x\} - \left(\frac{\hat{V}_2(1 + m\hat{s}_1)\hat{s}_2}{1 - m^2\hat{s}_1\hat{s}_2}\right)\mathrm{Cov}\{\widehat{UE}_2, \hat{q}_2\} \\
&\quad - \left(\frac{\hat{V}_2(1 + m\hat{s}_2)\hat{s}_1}{1 - m^2\hat{s}_1\hat{s}_2}\right)\mathrm{Cov}\{\widehat{UE}_2, \hat{q}_1\} - \hat{V}_2\hat{s}_0\mathrm{Cov}\{\widehat{UE}_2, x\}
\end{aligned} \tag{A.39}$$

Substituting back from the equilibrium price to determine the coefficients α_i and β_i yields a complete expression for the correlation. The algebraic closed form expression is not interpretable. We provide a graph of the correlation for $r \in [0, 1]$ with the other parameters set to the values $b = \frac{1}{\sqrt{2}} s_x = s_u = s_z = s_\tau = 1$ (see Figure 4).

D. Calibrated Drift: Required NIB Volatility

This appendix describes the estimation of the parameters used in the numerical calibration. The numerical calibration was conducted for a combined portfolio consisting of long positions in the portfolio with the highest unexpected earnings and the portfolio with the lowest unexpected earnings. All calculations are based on 1985 data obtained from the 1993 NYSE Fact Book and the 1993 Federal Reserve Board Funds Flow, the 1990 NYSE Institutional Investors Fact Book, and 1990 NYSE share ownership data.

D.1. NIB Volatility Percentage

To verify if the calibrated values fall within a plausible range relative to empirical estimates, we first derive an estimate of the risk-aversion parameter. However, it must be noted that utility functions are defined only up to monotone linear transformations, that is, the functions $U(w)$ and $U(aw + b)$ represent exactly the same set of preferences for any positive constants a and b. With this reservation, using a utility function $-\exp(-\rho w)$ for wealth w, we obtain a value for ρ by equating the expected utility of investing in the market with the certainty equivalent generated by the risk-free rate. This leads to the following equation:

$$\frac{1}{\sqrt{2\pi}\sigma_m}\int -\exp[-\rho(1+r)]\exp\left[-\frac{1}{2}\left(\frac{r-r_m}{\sigma_m^2}\right)\right]dr = -\exp[-\rho(1+r_f)]$$

where r_m is the quarterly market return, r_f is the quarterly riskfree rate (estimated from quarterly T-bills), σ_m^2 is the variance of the quarterly market return. The estimated values of these parameters are $r_m = 0.042958$, $r_f = 0.0237$, and $\sigma_m^2 = 0.007218$. The equation above yields a solution $\rho = 2(\frac{r_m - r_f}{\sigma_m^2})$ and substituting the numerical estimates results in a risk-aversion parameter of:

$$\rho = 2\left(\frac{0.042958 - 0.0237}{0.007218}\right) = 5.33.$$

We next derive the relationship between observed per-capita trading volume and NIB trading. Assume that individuals per capita asset demands are driven by new information (belief changes) and NIB considerations. The per capita change in asset demands attributable to NIB considerations is $Z_t - Z_{t-1} = (b-1)Z_{t-1} + \sqrt{1-b^2}\tau_t$. Define $\mathcal{VOL}$ as total trading volume. Hence, per capita trading volume can be decomposed into information based trading, denoted by I, and NIB based trading as follows:

$$\frac{1}{N}\mathcal{VOL} = I + (b-1)Z_{t-1} - \sqrt{1-b^2}\tau_t$$

where N is the estimated daily number of traders. Assuming that I and τ are uncorrelated and noting that $\sigma_{Z_0} = \sigma_{Z_t} = \sigma_{\tau_t} = \sigma_\tau, \forall t$, we obtain

$$\frac{1}{N^2}\sigma_{\mathcal{VOL}}^2 = \sigma_I^2 + [(b-1)^2 + (1-b^2)]\sigma_\tau^2 = \sigma_I^2 + 2(1-b)\sigma_\tau^2.$$

Our aim is to estimate the total volatility (left-hand-side) from empirical data and the level of $\sigma_\tau^2 = \sigma_Z^2$ required to match the empirically observed drift with the calibration. We can then compare these levels to see how much of the total volatility has to be attributed to NIB trading in order to match the empirically documented earnings drift.

To estimate the total per-capita daily trading volatility, we used the following data:

Average number of transactions per day: 78,926

Average number of transactions per day per decile: 7,893

Since the number of traders cannot exceed the number of trades, the minimum value for (daily) per-capita volatility was $\sqrt{\frac{\sigma_{vol}^2}{2(7{,}893)^2}}$. For the highest and lowest decile, these estimates were:

$$\sqrt{\frac{14{,}012{,}329{,}633}{(7{,}893)^2}} = 15; \quad \text{and} \quad \sqrt{\frac{18{,}873{,}487{,}582}{(7{,}893)^2}} = 17.4, \text{ respectively.}$$

The average value was 16.2 and this yields a minimum estimate for the total per-capita volatility. To the extent that there are fewer traders than the number of trades, per-capita volatility will increase and the percentage of NIB volatility required to sustain the observed drift will decrease.

The final step in our calibration is to solve for the level of NIB volatility needed to sustain the observed drift given the parameter estimates discussed above. To this end, we solve the following equation:

$$UP_2 = \frac{\text{Cov}\{UP_2, UE_1\}}{\text{Var}(UE_1)} UE_1 = F\left[\sigma_Z^2, \sigma_\tau^2\right] UE_1$$

where $\text{Cov}\{UP_2, UE_1\}$ is as in Proposition 3 and

$$\text{Var}(UE_1) = \text{Var}(y_1 - E[y_1 \mid P_0]) = \frac{1}{s_0 + s_x} + \sigma_u^2.$$

Based on the estimate of risk-aversion of $\rho = 5.33$ the level of NIB volatility needed to solve this equation (with $\sigma_\tau^2 = \sigma_Z^2$) was found to be 0.2 (see Figure 1). Thus, the NIB volatility required to sustain the drift is approximately 1.25% (i.e., 0.2/16.2) of the *minimum* estimate of the total per-capita volatility. Increasing or decreasing the value of ρ by a factor of 10 gave a maximum estimate of NIB trading of 12.5% for sustaining the drift.

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Notes

1. For example, Abarbanell and Bernard (1992) suggest that anomalous stock price behavior around earnings announcements is caused by a failure to infer correctly the distribution of future earnings based on past earnings (i.e., a failure in rational inference).
2. It is conceivable that traders may acquire a second private signal prior to the second-trading round. We do not model this possibility explicitly. Introducing such a second private signal should not have any qualitative implications for the properties of the earnings drift within our model. To see this intuitively, consider the case where the second private signal perfectly reveals x. Since any private signal acquired prior to the announcement of the public signal will merely enhance the precision of each trader's personal beliefs about x, and since the drift has been shown to exist irrespective of the precision of the traders' information, the acquisition of a second private signal should not qualitatively alter our inferences. For purposes of parsimony, therefore, it seems desirable to omit the introduction of a second private signal, which would unnecessarily complicate the model.
3. If there are N traders, in equilibrium, the average per-capita noisy supply (or demand), Z_t, satisfies $Z_t = \frac{1}{N}\sum_{i=1}^{n} z_{i_t}$ where z_{i_t} denotes trader i's demand in period t. It is standard to extend this approach to a continuum

of traders and write $Z_t = \int_0^1 z_{i_t}\, di$ and $x = \int_0^1 y_{i0}\, di$ despite some associated technical complications (see Judd, 1985). We note that our general result, Proposition (1), will hold with finitely many traders as long as individual traders behave as price-takers.

4. We cannot formally rule out the existence of equilibria based on non-intuitive conjectures such as prices evolving away from fundamental values.
5. It can be demonstrated that a post announcement drift for the setting with two trading rounds and only one public announcement exists with properties similar to those derived in the text. However, the solution procedure for deriving the drift in this simpler setting is virtually identical to that in the text where we consider a framework with three trading rounds and two public signals. Empirical research primarily focuses on the quantification of the post announcement drift upon consecutive quarterly earnings announcements. The setting with three rounds of trading and two public signals is therefore more closely aligned with the setting analyzed in the empirical literature than the two-trading-round, one public signal setting. Because it is closer descriptively to empirical studies and is no harder to solve than the two-trading round one public signal setting, we use the three-trading round and two public signals setting as our main analytic framework. For space considerations, we do not include the solution for the simpler two-round one-signal setting in the paper, but we can make it available to interested readers.
6. The alternative is to use: (ii) all price and public information signals available at time 1, namely $\{P_1, P_0, y_1\}$. The point of our analysis is to show that unexpected price movements based soley on market data, i.e., prices, results in an association with unexpected earnings change.
7. Notice that P_0 represents the "average" of private beliefs regarding x at time 0 and that UE_1 represents the additional information in the earnings signals relative to this "average."
8. In contrast, the prior specification that we used (see Section 3.1.) $UE_1 = y_1 - E[y_1 \mid P_0]$ is consistent with an empiricist's use of consensus analyst forecasts as expected earnings. Arguably, analysts condition their forecasts on observable market information such as observed prices (in our context, P_1 and P_0).
9. This contrasts with Kim and Verrechia (1991) where the price change is proportional to trading volume. Further discussion regarding price and volume is provided in Kandel and Pearson (1995) and Dontoh and Ronen (1993).

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