Playing roulette

Consider first an American roulette wheel. Let $W$ be the amount of winnings. The probability distribution for winnings is

$$W = \begin{cases} 1 & \text{with probability } 18/38 \\ -1 & \text{with probability } 20/38. \end{cases}$$

This implies that the expected payoff is

$$E(W) = (1) \left( \frac{18}{38} \right) + (-1) \left( \frac{20}{38} \right) = -0.0526.$$ 

That is, on average the player loses 5.26¢ on each play. The variance of the winnings is

$$V(W) = \left[ 1 - (-0.0526) \right]^2 \left( \frac{18}{38} \right) + \left[ -1 - (-0.0526) \right]^2 \left( \frac{20}{38} \right) = 0.9972,$$

which means that $SD(W) = \sqrt{0.9972} = 0.999$. Note that this is also the result using the shortcut formula

$$V(W) = (1)^2 \left( \frac{18}{38} \right) + (-1)^2 \left( \frac{20}{38} \right) - (-0.0526)^2 = 0.9972.$$ 

Calculations for the European roulette wheel are done in the same way, except that the probability distribution for the winnings is different:

$$W = \begin{cases} 1 & \text{with probability } 18/37 \\ -1 & \text{with probability } 19/37. \end{cases}$$

This implies that the expected payoff is

$$E(W) = (1) \left( \frac{18}{37} \right) + (-1) \left( \frac{19}{37} \right) = -0.027.$$ 

That is, on average the player loses 2.7¢ on each play. The variance of the winnings is

$$V(W) = \left[ 1 - (-0.027) \right]^2 \left( \frac{18}{37} \right) + \left[ -1 - (-0.027) \right]^2 \left( \frac{19}{37} \right) = 0.9993,$$

which means that $SD(W) = \sqrt{0.9993} = 0.9996$. 

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