Silent or Salient?
Perks and Perils of Performance Posting

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Abstract

Many firms in the U.S. spend more on their sales force than they do on any other marketing spend. Thus, improving sales force performance is of paramount importance. A controversial way is to post performance (i.e., display everyone’s performance), now done with ease on social platforms due to advances in information technology. On one hand, posting performance encourages social comparison and competition, though it may discourage low-end performers. On the other hand, not posting performance may encourage greater effort from sales agents to pull ahead or avoid falling behind, if they are unaware of how others are doing. The result of these opposing factors is, *prima facie*, unclear. I study the effectiveness of performance posting using theory and experiments. In a game-theoretic model of incomplete information about agents’ abilities, I allow a firm to control the precision of social comparison by choosing whether to post performance. Firstly, I find that a firm should not post performance when agents’ abilities are sufficiently homogenous, as this prevents a low-ability (high-ability) agent from being overly discouraged (overly complacent). In contrast, a firm should post performance when agents’ abilities are sufficiently heterogeneous, as a low-ability (high-ability) agent puts in more effort from the social pressure to avoid lagging by too much (to maintain the lead). Secondly, some social comparison helps performance posting but too much hurts, due to tradeoffs between how much a high-ability (low-ability) agent scales up (down) effort, i.e., there is a non-monotonic relationship between social comparison and the effectiveness of performance posting. Thirdly, I find that the differential profit from posting increases (decreases) when the financial compensation is unattractive (attractive). In other words, firms with less attractive remuneration should pay more attention to performance posting. Next, I demonstrate the empirical validity of these propositions using ongoing experiments. Together, my theoretical and empirical results provide guiding principles on when a firm can benefit from performance posting.

Keywords: Performance Posting; Social Comparison; Sales Force; Competitive Strategy; Game Theory

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1 Motivation

Each year, firms in the U.S. spend $800 billion on sales force compensation and hire at least 20 million people to carry out their sales function (Zoltners et al. 2008). Given that many businesses spend more on their sales force than they do on any other marketing activity, improving the performance of the sales force is of paramount importance. Yet, managing the sales force is often an overlooked area in marketing research.

To increase sales performance, a firm has several levers. The most obvious lever studied in the marketing literature is sales force compensation, and includes designing of optimal compensation plans (e.g., Basu et al. 1985), accounting for heterogeneity in the sales force (e.g., Raju and Srinivasan 1996, Rao 1990, Misra, Nair and Daljord 2013), estimating the effectiveness of compensation plans (e.g., Chung, Steenburgh and Sudhir 2013, Misra and Nair 2011), and using sales contests in the form of rank order tournaments to motivate efforts (Kalra and Shi 2001). Another lever is territory allocation (e.g., Lodish 1975, Zoltners and Sinha 2005), i.e., how to allocate sales force across different geographies and industries, and how this interacts with sales compensation (e.g. Caldieraro and Coughlan 2009).

More recently, progressive firms are exploring the use of social incentives to increase sales force performance. These include the use of social pressure (Steenburgh and Ahearne 2012) and forced rankings (Ahearne et al. 2012). In the same vein, there exists another lever to increase sales performance – one that is intuitively important to managers, relatively inexpensive to implement, potentially controversial, but not well-studied – posting performance in a social setting within a group. When is it profitable for firms to post performance of their sales force agents versus not post performance? On one hand, posting performance might encourage social comparison and competition in the workplace, but it might also have the unintended effect of discouraging low-end performers. On the other hand, not posting performance might yield better aggregate performance if employees are unaware of how others are doing, and therefore, put in more effort to pull ahead or avoid falling behind.

Even though the question of displaying everyone’s performance (or not) is straightforward, the result of these opposing factors is unclear. There is also lack of guidance from existing literature to this question, which has examined how relative performance affects compensation decisions (e.g., Kalra and Shi 2001, Lazear and Rosen 1981, Lim et al. 2009), but not when it is actually profitable for firms to make available such information given potential information asymmetries in the performance of others.

Furthermore, this question is not only important, but also increasingly relevant given recent advances in information technology. Enabled by the proliferation of social platforms, relatively low-cost software such
as RepTivity, Hoopla and LevelEleven now integrate with customer relationship management databases provided by companies like salesforce.com. This allows for interfaces designed to share accurate information about the performance of sales agents in a group, as seen in Figure 1.

![Salesforce CRM interface](image1)

Figure 1: Firms Can Choose Whether to Provide Updates of Everybody’s Sales Performance (Source: RepTivity)

The increasing ease of posting performance has not, however, translated into an across-the-board adoption. Anecdotally, performance posting has received mixed reviews among sales agents and managers. Some find performance posting useful, citing it as “a public expression of progress, sales goals and objectives”, without which, sales agents “lack the visibility into where they are in relation to the team and to their peers”\(^1\). Others decry performance posting as “childish” and “useless”. Given that performance posting affects the morale, effort and consequently, performance, we need more than anecdotal evidence of its value or lack thereof. More rigorous research can help firms assess the value of performance posting, and decide when to let sales agents know (“salient”) or not know (“silent”) their performance in a social setting.

How then should a firm decide when to post or not post performance? A firm whose objective is to maximize aggregate performance needs to consider how interdependencies among sales agents, specifically, how knowing one another’s performance, influence the collective outcome. To address this, I set up a stylized model of a single firm and two sales agents in a sequential game of incomplete information. I allow for agents’ abilities to be private information that is drawn from a distribution which is common knowledge. In my model, the firm moves first and pre-commits to performance posting (or not), and the agents then simultaneously decide how much effort to exert in order to generate sales. I then solve for a steady-state equilibrium which arises from the strategic interdependency between a firm’s decision to post performance and the amount of effort agents put in.

\(^1\)http://www.mediasalestoday.com/should-managers-use-leaderboards-to-motivate-sales-teams/
Using my model, I develop several theoretical propositions on how performance posting affects a firm’s profit. I find that if agents’ abilities are sufficiently homogenous, a firm obtains higher profit by not posting performance. But if there is sufficient heterogeneity in agents’ abilities, a firm obtains higher profit by posting performance. Also, the extent of social comparison between the agents and the attractiveness of the financial compensation moderate the effectiveness of performance posting. In particular, social comparison has a non-monotonic effect on the effectiveness of performance posting – some social comparison is good but too much is bad, and an attractive financial incentive reduces the differential profit between posting performance versus not. To test whether my model holds up, I examine the empirical validity of these propositions using ongoing experiments. Taken together, my theoretical and empirical results suggest that firms need to carefully consider the interaction of multiple factors before making their decision on whether to carry out performance posting.

The remainder of this paper is as follows. In Section 2, I cover the various literatures related to the question I have posed. I then specify in Section 3 a game-theoretic formulation of the problem and solve for a firm’s profit under the presence and absence of performance posting. Comparing the difference in profit between both cases, I explain when it is optimal for a firm to post performance. In Section 4, I present ongoing experiments designed to test whether the predictions from my model hold up. Finally, I conclude in Section 5 with extensions and future work.

Naturally, performance posting is used not only to motivate a sales force, but is also practiced in other domains such as education\textsuperscript{2} and healthcare\textsuperscript{3}. Given that my model examines the building blocks of performance (arising from one’s ability and motivation to put in effort) in a social setting (leading to social comparison of performance), and focuses on the decision of another player (e.g., the firm or a social planner) on whether to make available information about performance (knowing that this affects the degree and outcome of social comparison), I hope that my analytical and empirical insights can apply to broader contexts and provide a modest first step towards a general understanding as to when performance posting can be effective.

\textsuperscript{2}http://articles.latimes.com/2013/aug/01/local/la-me-ln-teachers-ratings–20130801
\textsuperscript{3}Take for example how public posting has been considered as a way to motivate primary care providers to improve their performance (http://www.intrahealth.org/page/motivating-providers-by-posting-performance-data), although it is less clear whether performance posting works always, where policy-makers are currently considering when to use performance posting.
2 Literature Review

More broadly, performance posting relates to the social effects of information disclosure. Performance posting is, after all, a practice that informs one about the performance of others – itself an under-studied but important area in the sales force literature (Amaldoss et al. 2008) – and also informs others about one’s performance. In this section, I draw from several literatures (marketing, economics, psychology, neuroscience and education) and organize the findings into two main themes. In Section 2.1, I examine how social comparison affects behavior. In Section 2.2, I examine the literature on how a firm can provide social information to influence behavior, which, like performance posting, is a form of mechanism design. In Section 2.3, I explain the gap in our understanding of when performance posting is effective, and how my paper intends to bridge this.

2.1 Social Incentives Matter

The literature on social comparison often begins with Festinger (1954), whose seminal paper discusses the drive for self-evaluation within an individual which necessitates comparison with other people, i.e., we look to others for information that will help us evaluate ourselves on dimensions relating to abilities or skills. But the broader idea relating to social motivation goes back even further to Triplett (1898) who considers social facilitation and finds that bicycle racers had faster times when they raced with others than when they raced alone, children reeled in fishing lines faster when performing alongside another child as compared to when they were reeling on their own, and that the “bodily presence of another contest participating simultaneously” can “liberate latent energy not ordinarily available”, i.e., increase competitive motivation.

In addition to motivation, social comparison also serves more direct purposes such as self-enhancement (Wood 1989). Complemented by recent papers in neuroscience, evidence from functional magnetic resonance imaging (fMRI) show how social comparison triggers motivation-related processes in the brain during a rewards-related activity (Fliessbach et al. 2007), providing neurological support for the commonly-observed phenomena of people wanting to do well relative to others (Bunnk and Gibbons 2007). Going one step further, emotions that arise from social comparison may also be a primary source of utility: Dvash et al. (2010), also using fMRI, finds that participants who lost money expressed joy and gloating if the other player had lost more money, and participants who won money expressed envy if the other player had made more money. Said differently, even when a person loses money, merely adding information about another person’s greater loss may increase certain brain activations to a point similar to those of an actual gain. This body
of evidence provides some motivation for how absolute and relative measures enter into a person’s primary utility function, a finding which economists are now using to resolve certain paradoxes in the economics of happiness (Clark et al. 2008).

2.2 The Enlightened Firm

Given that social comparison matters in a real and direct way, firms can take certain actions if they know how people engage in social comparison, whether this is in a firm-to-consumer setting, e.g., a firm using social norms to encourage consumers’ participation in environmental conservation (Goldstein, Cialdini and Griskevicius 2008), or in a firm-to-employee setting, e.g., a firm changing the proportion of winners to losers in a sales contest to yield greater effort from its sales force when it knows that employees exhibit disutility of losing to others (Lim 2010). Relating to performance posting, I will focus on the literature relating to a firm-to-employee setting, and specifically, how a firm can influence an employee’s behavior by providing information about how other employees perform at a similar task.

2.2.1 Performance Feedback

While there is limited work in the sales force literature on the role of performance feedback, there is empirical work on the effects of performance feedback in education and management settings. In education, Azmat and Iriberri (2010) study the effect of a high school providing performance feedback privately to students, and find that even though students are rewarded only for absolute performance, providing performance feedback about how they fared relative to the class average increases students’ grades by 5%. The authors attribute this improvement to the stimulating of competitive preferences. In another setting, Kuhnne and Tymula (2012) find that people put in more effort and expect to obtain a better rank when told that they may learn about their ranking, suggesting that feedback about relative performance modifies one’s self-esteem. The findings from both of these studies are directionally consistent with Blanes i Vidal and Nossol (2011) who find that grocery packers at a warehouse who received private information about how they fared relative to others displayed a long-term increase in productivity. Here, the authors find that this is best explained by relative concerns stemming from social comparison.

Given the benefits of relative performance feedback, why do we not observe every firm using it to boost performance? Perhaps the outcome is not as rosy as what previous studies seem to suggest. For example, Barankay (2012) offers contrasting evidence that there can be negative effects of relative performance
feedback. In a randomized workplace experiment, he finds that removing private rank incentives actually increases sales performance – mostly that of male employees – by 11%, even though rankings do not convey any direct and additional financial benefit. Taken together, these studies show that relative performance feedback does affect performance, though it is not immediately clear whether this has a positive or negative effect.

Given that relative performance feedback has an impact on performance even when feedback has no direct effect on financial incentives (incentives in the abovementioned studies are either fixed or piece-rate), it is no surprise then that relative performance feedback has an effect when it does affect financial incentives (as is the case of contests or tournament incentives). Hannan, Krishnan and Newman (2008) explore the effect of relative performance feedback on both piece-rate and tournament incentives, and find that relative performance feedback improves the average performance of participants in a piece-rate incentive but hurts the average performance of participants in a tournament incentive if the feedback is sufficiently precise. Freeman and Gelber (2010) examine a tournament in which they vary information about participants’ performance and prize structure. They find that when subjects knew the performance distribution in an earlier round, those lower in the distribution did better with multiple prizes than they did with the single prize condition, but those higher in the distribution performed similarly with multiple prizes as they did under the single prize condition. But when subjects did not know the performance distribution in an earlier round, the single and multiple prize treatments produced similar outcomes across the distribution, thus suggesting that the lack of relative performance information meant that they did not scale back effort in the single prize treatment. In other words, it may help a firm to provide or withhold such information if it elicits greater effort from agents – a finding that resonates with work on interim performance evaluation in dynamic tournaments (e.g., Ederer 2010).

### 2.2.2 Social Effects of Performance Posting

Performance posting, unlike relative performance feedback given in private, tends to be social in nature. In other words, performance posting not only informs oneself about others’ performance, it also informs others about one’s performance. To the extent that a sales person may care what others think about his or her performance when it is posted, performance posting may create social incentives (e.g., pride) or disincentives (e.g., social pressure, embarrassment) if people know that their performance is being observed or monitored by others. This may be especially relevant in a group task: Lount and Wilk (2014) find an interaction effect between performance posting and group work: when individual performance was (not) publicly posted in
the workplace, employees in a group did better (worse) than when working alone, which the authors termed social laboring (loafing).

The social or public nature of performance posting also brings our work closer to the literature on peer effects and its influence on productivity. Falk and Ichino (2006) find that subjects who worked on the same task at the same time in the same room had not only smaller standard deviations in output (compared to those who worked alone) but also higher output even though the financial incentive for a subject in either case was the same and independent of output. In another example, Mas and Moretti (2009) examine how the productivity of a focal worker in a supermarket chain goes up when highly productive coworkers are introduced, with the focal worker’s effort being positively related only to the productivity of other workers who see the focal worker, but not those who do not see the focal worker. This offers an additional richness to the findings of Bandiera et al. (2005) who find that workers produce higher output in piece-rate versus relative incentives because they internalize the negative externality of their actions when they can monitor others and be monitored. Given that peer effects exist, and in a variety of ways, a firm may then take into consideration the aggregate output of all agents arising from these interactions. This is what Roels and Su (2014) does in the specific context of social comparison, in which they show that social planners need to consider how ahead-seeking (behind-averse) behavior of agents leads to output polarization (clustering), and that one can mitigate these effects either by providing the full reference distribution of outputs or by assigning players into uniform rather than diverse reference groups.

2.3 Building on the Literature

Having gone through the various literatures that relate to my research question, I will now briefly describe my research contribution. Using a theoretical model, I will disentangle the effects of social comparison from the availability of information, rather than assume that social comparison does not take place in the absence of information about how others are doing (e.g., Kuhnen and Tymula 2012). In other words, I model the effects of social comparison both in the presence and absence of performance information. This allows me to study when it is optimal for a firm to use performance posting to elicit the greatest amount of collective effort from a sales force who works individually on a sales task but functions broadly as part of a sales team.

In my model, I build on the findings from the above literature in the following ways. First, people care not only about absolute incentives but also about relative incentives in a primary and direct way. I will thus enrich the psychology of the agents to include social interdependencies via my utility specification. This extends previous work (Bandiera et al. 2005) which examined the separate effects of absolute and relative
incentives on behavior. Second, firms—knowing that people do care about both absolute and relative incentives—can take certain actions to fulfill its objective of profit maximization. I explore the context of performance posting as a relevant firm action and how it affects the effort levels which agents exert. Third, it is not clear whether information about how others do relative to themselves helps or hurts both the firm and the agents; different studies provide mixed evidence and also contingencies. I hope to reconcile the mixed evidence using my model.

In the next section, I will lay out a theoretical model that incorporates factors related to performance posting. Specifically, I will consider the potential heterogeneity in the abilities of agents, the extent of social comparison or competitiveness among agents, and the structure of the financial incentive.

3 Model

The road map for this section is as follows. I will first set up my theoretical model in Section 3.1 and then present the results for when performance posting yields higher profit for the firm in Section 3.2. Within Section 3.1, I will lay out the players, how performance is determined, the objectives of the players, and the sequence of the game. I then solve for the equilibrium that arises in this game, and lay out in Section 3.2 the comparative statics for the equilibrium, followed by the outcomes when the firm chooses not to post performance versus when it chooses to post performance. Table 1 lists all parameters and decision variables that appear in my model. Figure 2 in Section 3.1.5 shows how the dynamic game unfolds.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>Exogenous price per unit of sales to firm</td>
</tr>
<tr>
<td>$w$</td>
<td>Commission rate per unit of sales chosen by firm and received by agents</td>
</tr>
<tr>
<td>$s$</td>
<td>Decision of firm whether to post performance ($s = 1$) or not ($s = 0$)</td>
</tr>
<tr>
<td>$a_i$</td>
<td>Ability of agent $i$</td>
</tr>
<tr>
<td>$a_j$</td>
<td>Ability of agent $j$</td>
</tr>
<tr>
<td>$a_H$</td>
<td>Level of high ability in the population</td>
</tr>
<tr>
<td>$a_L$</td>
<td>Level of low ability in the population</td>
</tr>
<tr>
<td>$p$</td>
<td>Probability that an agent is of high ability</td>
</tr>
<tr>
<td>$e_i$</td>
<td>Effort put in by agent $i$</td>
</tr>
<tr>
<td>$e_j$</td>
<td>Effort put in by agent $j$</td>
</tr>
<tr>
<td>$c$</td>
<td>Cost of effort for agents</td>
</tr>
<tr>
<td>$k$</td>
<td>Extent of social comparison or competitiveness between agents</td>
</tr>
</tbody>
</table>

Table 1: Parameters and Decision Variables
3.1 Game Set-Up

3.1.1 Players

In business-to-business settings, it is common practice for a firm to hire agents to obtain sales for the firm. I begin with a simple model comprising of three players: a single firm and two sales agents, \( i \) and \( j \). Assume that \( i \) and \( j \) work on a similar and comparable task for the firm. Going forward, I will use \( i \) as the focal agent and a similar case always applies to \( j \).

I assume that each agent has an inherent ability parameter \((a_i > 0, a_j > 0)\) which is private information, i.e., known only to himself or herself, and not known to either the other agent or to the firm. That said, the distribution of agents’ abilities is common knowledge: an agent is of high ability \((a_H)\) with probability \(p\) and low ability \((a_L)\) with probability \((1 - p)\). This gives rise to four possible scenarios: \(\{a_i = a_H, a_j = a_H\}\), \(\{a_i = a_H, a_j = a_L\}\), \(\{a_i = a_L, a_j = a_H\}\) and \(\{a_i = a_L, a_j = a_L\}\) with respective probabilities of \(p^2\), \(p(1 - p)\), \((1 - p)p\) and \((1 - p)^2\).

3.1.2 Performance of Agents

Agent \(i\)’s performance is a function of his or her ability parameter \(a_i\) and effort decision \(e_i \geq 0\). In addition to ability being unobservable to the other agent and the firm, the same applies for an agent’s effort decision, i.e., effort is not observable to the other agent and the firm. The firm does observe, however, the performance of both agents that transpire from each agent’s combination of ability and effort. I allow for substitutability between ability and effort in which performance is the sum of \(i\)’s ability and effort, \(a_i + e_i\). This is a specification commonly used in the literature (e.g., Ederer 2010, Kuhnen and Tymula 2012).

A firm, in order to achieve high levels of performance, would ideally like to hire agents with high ability levels who also put in high levels of effort. However, neither ability nor effort is observable by the firm. Furthermore, I assume that abilities of agents are independent draws from a known distribution that reflect the inherently random nature of abilities, despite best efforts by a firm to screen for ability. A firm, in order to maximize profit, seeks to maximize agents’ aggregate performance and specifically chooses the action (whether to post performance or not) in order to maximize agents’ aggregate effort. I will now specify the firm’s objective.

3.1.3 Firm’s Objective

The firm’s objective is to maximize the following profit function:
\[ \pi = (P - w) \cdot [(a_i + e_i) + (a_j + e_j)] \]  

(1)

In equation 1, \( P \) is an exogenous price net of variable costs which the firm obtains per unit of sales, and \( w \) is an exogenous commission paid by the firm to the agents per unit of sales performance achieved. \( (P - w) \) is thus the margin per unit of sales for the firm, where \( P > w \).

I assume that the performances of \( i \) and \( j \) sum to give the aggregate performance of the firm, \((a_i + e_i) + (a_j + e_j)\). In other words, there is no direct substitutability or complementarity between the performances of both agents. The firm’s objective is to maximize profit by choosing whether to post performance \((s = 1)\) or not \((s = 0)\). To maximize profit given exogenous margins \((P - w)\), a firm’s objective is then to maximize aggregate performance \((a_i + e_i) + (a_j + e_j)\). To do so, a firm would ideally like to have agents with high abilities putting in high levels of effort. That said, the ability of an agent is determined by a random draw from the distribution of abilities, and is therefore not within control of the firm. In essence, then, the firm’s objective is to maximize aggregate effort \(e_i + e_j\), and seeks to do so by deciding whether to post or not post performance. I will elaborate on this decision of the firm, \( s \), shortly.

3.1.4 Agents’ Utility Function

I now turn to the agents’ utility functions. I enrich the psychology of agents to include social interdependencies, i.e., I assume that agents are motivated not only by financial incentives, but also obtain utility from social comparison. This builds not only on decades of research on social comparison in social psychology (e.g., Bunnk and Gibbons 2007, Festinger 1954, Wood 1989) but also a growing body of literature in neuroscience (e.g., Dvash et al. 2010, Fliessbach et al. 2007) which show the direct effects of social comparison on reward processing in the brain. Assuming that the individual rationality constraints for the agents are satisfied and that the agents remain in the firm, i.e., there is no attrition (or hiring for that matter), then agent \( i \) chooses effort level \( e_i \) to maximize his or her utility given by:

\[ U_i(e_i) = w \cdot (a_i + e_i) + k \cdot \frac{a_i + e_i}{(a_i + e_i) + (a_j + e_j)} - \frac{e_i^2}{2} \]  

(2)

Equation 2 shows \( i \)'s utility being made up of three additively separable terms. The first two represent utility arising from income and social comparison respectively, and the third represents disutility arising from cost of effort.

Based on \( i \)'s performance, an agent receives a financial compensation of \( w \cdot (a_i + e_i) \) from the firm, i.e., financial compensation is the product of the commission rate per unit of performance and performance. Social
comparison is specified as \( k \cdot (\frac{a_i + e_i}{a_i + e_i + (a_j + e_j)}) \) and comprises of two components. \( \frac{a_i + e_i}{(a_i + e_i) + (a_j + e_j)} \) represents the performance or remuneration share of \( i \), and has the desirable property that marginal gains are smaller in magnitude than marginal losses of the same magnitude. This comes about because of the concavity of the social comparison function. Performance share itself is moderated by the extent of a common social comparison parameter \( k \geq 0 \) which represents a “culture of competitiveness” (Garcia et al. 2013) between the two agents, with a higher \( k \) representing a higher level of competition between both agents. Finally, I operationalize the assumption of convex costs using a quadratic cost function, \( \frac{e_i^2}{2} \).

**Firm Does Not Post Performance.** Since this is a game of incomplete information, then a firm not posting performance \((s = 0)\) means that the focal agent will not know about the performance of the other agent. In that case, the expected utility of \( i \) will depend on social comparison with the performance of an “average” agent:

\[
EU_i(e_i|s = 0) = w \cdot (a_i + e_i) + k \cdot \left( \frac{a_i + e_i}{a_i + e_i + (a_j + e_j)} \right) - \frac{e_i^2}{2} \tag{3}
\]

**Firm Posts Performance.** On the other hand, if the firm posts performance \((s = 1)\), this means that with probability \( p \), \( i \)'s utility will be

\[
U_i(e_i|s = 1) = w \cdot (a_i + e_i) + k \cdot \left( \frac{a_i + e_i}{a_i + e_i + (a_j=H + e_j=H)} \right) - \frac{e_i^2}{2} \tag{4}
\]

And with probability \( 1 - p \), \( i \)'s utility will be:

\[
U_i(e_i|s = 1) = w \cdot (a_i + e_i) + k \cdot \left( \frac{a_i + e_i}{a_i + e_i + (a_j=L + e_j=L)} \right) - \frac{e_i^2}{2} \tag{5}
\]

Thus, the expected utility of \( i \) will be:

\[
EU_i(e_i|s = 1) = w \cdot (a_i + e_i) + p \cdot k \left( \frac{a_i + e_i}{a_i + e_i + (a_j=H + e_j=H)} \right) + (1 - p) \cdot k \left( \frac{a_i + e_i}{a_i + e_i + (a_j=L + e_j=L)} \right) - \frac{e_i^2}{2} \tag{6}
\]

### 3.1.5 Sequence of Events

I now describe the sequence of events in my dynamic game of incomplete information shown in Figure 2. Throughout the game, I assume that the following are common knowledge: (i) the distribution of agents’ abilities; (ii) commission rate \( w \); and (iii) the extent of social comparison or competitiveness between the
agents as represented by $k$. All of these are exogenous parameters in my model.

In stage one, agents receive private information from nature about their own ability types (either $a_H$ or $a_L$), which are independent draws from the ability distribution. In stage two, the firm maximizes profit, with the margin per unit of sales remaining constant at $(1 - w)$, i.e. the firm maximizes profits by maximizing aggregate sales $(a_i + e_i) + (a_j + e_j)$ and does so by pre-committing to post or not post performance. In stage three, agents choose efforts to maximize utilities based on their private information about their own abilities and also on the firm’s decision $s$. In stage four, the firm observes each agent’s performance after the agent chooses effort, regardless of whether it carries out performance posting. But if the firm had pre-committed in stage two to post performance ($s = 1$), each agent also learns precisely about the other agent’s sales performance; if the firm pre-commits in stage two to not post performance, then each agent makes the best possible inference that the other agent is of average ability with a corresponding effort and performance level. In other words, $i$ thinks that $j$‘s performance is what an average agent would achieve and $j$ thinks likewise of $i$‘s performance in the absence of performance posting. I then solve for the unique equilibrium in this game of incomplete information.

It is worth highlighting that in my model, regardless whether the firm posts performance or not, both agents are inclined to compare his or her performance to the other agent’s performance as moderated by $k$, the competitiveness between the agents. In other words, I disentangle social comparison – a key aspect of human nature – from the availability of information about others’ performance. This allows us to understand the effects of social comparison on effort when a firm can control the availability or precision of information about others’ performance.

I model this as a non-repeated “one-shot” game to understand the steady-state equilibrium which arises from the firm’s decision on whether to post or not post performance. In other words, I am interested in the long-run effects of posting or not posting performance.

### 3.2 Theoretical Results

From the firm’s perspective, there are four possible scenarios that can arise regarding the ability outcomes of both agents: $\{a_i = a_H, a_j = a_H\}$, $\{a_i = a_H, a_j = a_L\}$, $\{a_i = a_L, a_j = a_H\}$ and $\{a_i = a_L, a_j = a_L\}$ with respective probabilities of $p^2$, $p(1 - p)$, $(1 - p)p$ and $(1 - p)^2$. In addition, agents of different ability types respond differently when the firm posts or not posts performance. I solve for the Nash equilibrium effort and performance in each of these scenarios and integrate it out across the four scenarios. I do this for both

---

4Here is where I depart from Kuhnen and Tymula (2012) which assumes that the absence of information about the other agent means that social comparison does not happen. In my model, I disentangle the existence of social comparison from the availability of information: the only difference between both cases is the precision with which agents are able to make an inference about the other agent’s performance.
Whenever $i$ decides how much effort to put in against $j$, there exists a unique Nash equilibrium in non-negative efforts given by:

$$
e_i^*(a_i, a_j, k, w) = \frac{(a_i^2 + a_i a_j + 2k + 3a_i w + a_j w + 2w^2)\sqrt{\lambda_i} + (w - a_i)\lambda_i}{2\lambda_i}$$

where $\lambda_i = a_i^2 + a_j^2 + 4a_j w + 2a_i(a_j + 2w) + 4(k + w^2)$.

In deciding how much effort to put in, the key difference between the cases of posting versus not posting performance is the precision with which $i$ knows $a_j$, and consequently, $j$’s performance. I will elaborate on this further.

**Comparative Statics.** A profit-maximizing firm seeks to maximize agents’ aggregate performance. Given exogenous ability draws for the agents, the firm decides whether to post performance or not in order to maximize agents’ aggregate effort. I lay out this decision in Section 3.2.4. But first, here is how each agent’s optimal effort varies with the parameters of my model. In the following paragraphs, I provide the intuition of the comparative statics on the optimal effort in equation 7. The expressions for all comparative statics and their boundary conditions, going forward, are provided in Appendix A.

I begin with the commission rate $w$. Taking the first derivative of $e_i^*$ with respect to $w$, I observe that
\[ \frac{\partial e_i^*}{\partial w} > 0 \] for the entire parameter space. This means that the firm can increase agents’ efforts as long as it increases the commission rate, which is intuitive as agents put in more effort when there is more financial incentive to do so. When I take the second derivative of \( e_i^* \) with respect to \( w \), I observe that \[ \frac{\partial^2 e_i^*}{\partial w^2} > 0, \] i.e., there is increasing return to effort with respect to the commission rate under any of the following conditions: (i) \( i \) has lower ability and there is some presence of competitiveness \( k \); or (ii) \( i \) has substantially higher ability \( a_i \), the commission rate \( w \) is small and competitiveness \( k \) is higher than some threshold level; or (iii) \( i \) has substantially higher ability \( a_i \), the commission rate \( w \) is sufficiently large and there is presence of competitiveness \( k \). Otherwise, effort is increasing and concave in the commission rate. Intuitively, there are increasing returns in \( i \)'s effort from an increase in the commission rate if (i) \( i \)'s ability is much lower than \( j \)'s and thus more room for marginal gains in effort from an increase in commission rate \( w \) as long as there is some competitiveness \( k \); or (ii) \( i \)'s ability is much higher than \( j \)'s, the commission rate \( w \) is sufficiently low, and competitiveness \( k \) is sufficiently high such that the marginal increase in utility from competitiveness can compensate for the lack of financial incentives; or if (iii) \( i \)'s ability is much higher than \( j \)'s, the commission rate \( w \) is large enough, beyond any competitiveness effect arising from heterogeneity in abilities, such that the marginal increase in utility from financial incentives is worth the incremental increase in effort.

Next, \[ \frac{\partial e_i^*}{\partial k} > 0 \] for the entire range of the parameter space, but is convex, i.e., \[ \frac{\partial^2 e_i^*}{\partial k^2} > 0, \] only if the following conditions are jointly met: \( i \) has higher ability and the difference in abilities is sufficiently large, the commission rate is sufficiently small, \( and \) if competitiveness is lower than a threshold level. Otherwise, there will be decreasing returns to optimal effort \( e_i^* \) as \( k \) increases. Intuitively, agents will always put in more effort as competitiveness increases, but there are increasing returns to competitiveness only if \( i \) has higher ability, gets an increase in utility from the competitiveness aspect because \( k \) is fairly low to begin with, and the commission rate \( w \) is small relative to heterogeneity in abilities such that the commission rate in itself is insufficient to motivate \( i \) to put in high levels of effort to begin with. Notice also that if \( i \) has lower ability, there is \textit{never} any increasing returns to optimal effort. This is because while an agent with lower ability will always find it worthwhile to increase effort if competitiveness \( k \) increases – just as an agent with higher ability would – being of lower ability also means that the return from competitiveness is limited by the agent of higher ability also putting in more effort, thus resulting in decreasing returns. Another point worth noting is that the conditions under which \[ \frac{\partial^2 e_i^*}{\partial k^2} > 0 \] and \[ \frac{\partial^2 e_i^*}{\partial w^2} > 0 \] are mutually exclusive – notice that both can never be convex at the same time for \( i \), but it can be the case that \[ \frac{\partial^2 e_i^*}{\partial k^2} > 0 \] and \[ \frac{\partial^2 e_i^*}{\partial w^2} < 0, \] or \[ \frac{\partial^2 e_i^*}{\partial k^2} < 0 \] and \[ \frac{\partial^2 e_i^*}{\partial w^2} > 0. \] This will be an important consideration for the firm as it seeks to maximize aggregate effort through its decision on whether to post performance or not, as we will see later.

I now carry out comparative statics of effort with respect to the ability of the focal agent. Taking the first
derivative of $e_i^*$ with respect to $a_i$, I find that $\frac{\partial e_i^*}{\partial a_i} < 0$ for $k > 0$, i.e. $i$’s optimal effort is always decreasing in his or her own ability. This is because ability and effort are substitutes in the agents’ production of sales output. Furthermore, $\frac{\partial^2 e_i^*}{\partial a_i^2} > 0$ for $k > 0$, which means that the corresponding decrease in $i$’s effort is greater than the decrease in $i$’s ability. This is because of the convexity assumption for cost of effort.

Last, and most importantly, I move on to the comparative statics of effort with respect to the ability of the other agent. This is most crucial to my question as the firm’s decision to post performance affects the availability of information relating to an agent’s belief about the other agent’s ability and performance, and thus influences the amount of effort an agent puts in. Notice first that $\frac{\partial e_i^*}{\partial a_j} < 0$ if $0 < a_i < a_j$ and $k > 0$. But if $a_i > a_j$ and $k > 0$, then $\frac{\partial e_i^*}{\partial a_j} > 0$. In other words, if $i$ is the agent with higher (lower) ability, his or her optimal action is to scale up (down) effort as $j$’s ability increases. This result is similar to a common finding in contest theory amongst heterogeneous players, in which a weaker player (of lower ability) experiences “discouragement” and will cut back on costly effort as it becomes relatively unprofitable to beat the stronger player (of higher ability), while the stronger player will be more passive or “complacent” unless $s/he faces another player of similar strength (Dechenaux et al. 2012). Furthermore, $\frac{\partial^2 e_i^*}{\partial a_j^2} > 0$ only if (i) $i$ has sufficiently lower ability than $j$; (ii) the commission rate $w$ is sufficiently small; and (iii) $k$ is sufficiently small. This means that an increase in optimal effort by an agent of higher ability is always concave in the other agent’s ability, while a decrease in optimal effort by the agent of lower ability is convex only if the above conditions are met. If both agents have the same ability, then optimal effort is always decreasing in ability.

3.2.1 If Performance is Not Posted

If performance is not posted, both agents will not know the realized performance of the other agent at stage four of the game. In this case, the only inference for social comparison that agent $i$ can make is that $s/he is facing an agent of average ability, $E_i[a_j] = pa_H + (1 - p)a_L$, who puts in average-type level of efforts. In this case, $i$’s optimal effort is given by:

$$e_i^*(a_i, E_i[a_j], k, w|s = 0) = \frac{(w - a_i)\lambda_i + (a_i^2 + a_iA + 2k + 3a_iw + Aw + 2w^2)\sqrt{\lambda_i}}{2\lambda_i}$$

where $A = pa_H + (1 - p)a_L$, $\lambda_i = a_i^2 + A^2 + 4Aw + 2a_i(A + 2w) + 4(k + w^2)$.

To provide some intuition on how $i$’s optimal effort changes with respect to the various parameters in the case of $s = 0$, I carry out the following comparative statics.
Agent $i$ is of High Ability: Assume that $i$ receives a private high ability draw, $a_H$, and puts in optimal effort $e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)$. Examining how optimal effort varies with respect to $a_H$, I find that

$$\frac{\partial e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial a_H} < 0 \quad \text{and} \quad \frac{\partial^2 e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial a_H^2} > 0$$

as long as $k > 0$. This means that $i$ scales back effort at a decreasing rate with respect to $a_H$. Thus, in the absence of performance posting, an agent with high ability puts in less optimal effort as $a_H$ increases. I also find that

$$\frac{\partial e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial k} > 0 \quad \text{and} \quad \frac{\partial^2 e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial k^2} < 0$$

for the entire parameter space as long as $k > 0$. In other words, $i$ increases effort as $a_H$ increases. This is due to the competitiveness effect, and this increase is at a decreasing rate.

With probability $p$, $j$ is also of high ability $a_H$, and $i$ will scale back on effort as $a_H$ increases. With probability $(1 - p)$, $j$ is of low ability and if so, $i$ decreases effort due to a reduction in the competitiveness effect, i.e., the high ability agent being complacent. These two effects act in the same direction, but the relative magnitudes of the effects are themselves moderated by $p$ when there is incomplete information about abilities. As $p$ decreases, or as $(1 - p)$ increases, and in comparing with an average agent, the decrease in the competitiveness effect becomes more dominant for high ability $i$ as it becomes less likely that the other agent is also of high ability. Probability $p$ directly moderates how much effort $i$ puts in when comparing to an agent of average ability. I find that

$$\frac{\partial e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial p} > 0 \quad \text{and} \quad \frac{\partial^2 e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial p^2} < 0$$

as long as $k > 0$, which means that $i$ will put in more optimal effort but at a decreasing rate as the expected ability of $j$ increases.

I also do comparative statics for the competitiveness parameter $k$. I find that

$$\frac{\partial e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial k} > 0$$

for the entire parameter space. Thus, an agent of high ability will always put in more effort as competitiveness $k$ increases. I also find that

$$\frac{\partial^2 e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial k^2} > 0$$

if all of these conditions are satisfied: (i) if the level of high ability $a_H$ is sufficiently high; (ii) the difference between $a_H$ and $a_L$ is sufficiently large; (iii) probability $p$ is not too large; and (iv) $k$ is not too high.

Likewise, for parameter $w$, I find that

$$\frac{\partial e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial w} > 0$$

for the entire parameter space. Thus, an agent of high ability will always put in more effort as the commission rate increases. I also find that

$$\frac{\partial^2 e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial w^2} > 0$$

in any one of the following conditions: (i) $a_H$ is very small and $k > 0$; (ii) the difference between $a_H$ and $a_L$ is sufficiently large, $p$ is sufficiently small, and $k$ is sufficiently competitive; (iii) $p$ is sufficiently large; or (iv) the difference between $a_H$ and $a_L$ is sufficiently small and $k > 0$.

Agent $i$ is of Low Ability: Assume that $i$ receives a private low ability draw, $a_L$, and puts in optimal effort $e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)$. Examining how optimal effort varies with respect to $a_H$, I find
that \( \frac{\partial c_i^*(a_i=a_L,E_i[a_j],k,w|s=0)}{\partial a_H} < 0 \) for the entire parameter space, i.e., \( i \) is always discouraged. In addition, \( \frac{\partial^2 c_i^*(a_i=a_L,E_i[a_j],k,w|s=0)}{\partial a_H^2} > 0 \) if all of these conditions are met: (i) \( a_H \) is sufficiently larger than \( a_L \); (ii) \( p \) is sufficiently large; and if (iii) \( k \) is sufficiently small.

Next, I find that \( \frac{\partial c_j^*(a_i=a_L,E_i[a_j],k,w|s=0)}{\partial a_L} < 0 \) for the entire parameter space. This means that an agent with low ability will put in less optimal effort as \( a_L \) increases. Furthermore, \( \frac{\partial^2 c_j^*(a_i=a_L,E_i[a_j],k,w|s=0)}{\partial a_L^2} > 0 \) for the entire parameter space, i.e. this decrease in optimal effort is convex. If both agents are of low ability, then it makes sense to scale back effort as \( a_L \) increases, and this comes solely from the substitutability between ability and effort. If the other agent is of high ability, then increasing \( a_L \) actually closes the gap in abilities between \( i \) and an agent of average ability.

Next, \( p \) moderates the increase in the average ability of \( j \) in the case of no posting. Specifically, I find that \( \frac{\partial c_j^*(a_i=a_L,E_i[a_j],k,w|s=0)}{\partial p} < 0 \) for the entire parameter space and \( \frac{\partial^2 c_j^*(a_i=a_L,E_i[a_j],k,w|s=0)}{\partial p^2} > 0 \) under certain conditions: (i) \( a_H \) is sufficiently larger than \( a_L \); (ii) \( p \) is sufficiently large; and (iii) \( k \) is sufficiently small. This means that an agent with low ability will put in less optimal effort if \( j \)'s average ability increases, and this decrease is convex under the abovementioned conditions.

For \( k \), I find that \( \frac{\partial c_j^*(a_i=a_L,E_i[a_j],k,w|s=0)}{\partial k} > 0 \) and \( \frac{\partial^2 c_j^*(a_i=a_L,E_i[a_j],k,w|s=0)}{\partial k^2} < 0 \) for the entire parameter space. This means that an agent with low ability will always exert more effort as competitiveness increases, but at a decreasing rate. Comparing this with the earlier analysis, only an agent with higher ability will have increasing returns to optimal effort under certain conditions but this is never the case for an agent with lower ability. This is because the return from competitiveness for an agent of lower ability is limited by the agent of higher ability also putting in more effort, thus resulting in decreasing returns.

For commission rate \( w \), I find that \( \frac{\partial c_j^*(a_i=a_L,E_i[a_j],k,w|s=0)}{\partial w} > 0 \) for the entire parameter space and \( \frac{\partial^2 c_j^*(a_i=a_L,E_i[a_j],k,w|s=0)}{\partial w^2} < 0 \) as long as \( k > 0 \). This differs from the earlier case where there can be increasing returns to an increase in commission rate \( w \) even if \( i \) has lower ability, either when there is some presence of competitiveness \( k \), or if the commission rate \( w \) is sufficiently large and there is presence of competitiveness \( k \). The difference in this case – when abilities are private information and the firm does not post performance – arises because comparison with an average agent means that \( i \) who is of lower ability is already putting in more effort arising from the competitiveness effect, such that any increase in the commission rate \( w \) will not yield increasing returns.

**Aggregation of Optimal Efforts from Firm’s Perspective.** From the firm’s perspective, there are four possible scenarios regarding the realized abilities of the agents: \( \{a_i = a_H, a_j = a_H\} \), \( \{a_i = a_H, a_j = a_L\} \), \( \{a_i = a_L, a_j = a_H\} \), and \( \{a_i = a_H, a_j = a_H\} \). Each of these scenarios come with the respective probabilities
of \( p^2, p(1-p), (1-p)p \) and \( (1-p)^2 \). I can solve for the aggregate effort in equilibrium, given by \( e_{s=0}^* = e_i^*(a_i, E_i[a_j], k, w|s = 0) + e_j^*(E_j[a_i], a_j, k, w|s = 0) \). The expression for \( e_{s=0}^* \) is given by:

\[
\begin{align*}
((1-p)^2(D + B(2k + a_L^2(2-p) + w(pa_H + 2w) + a_L(pa_H + (4-p)w))))/ \\
(4k + a_L^2(2-p)^2 + 2a_L(2-p)(pa_H + 2w) + (pa_H + 2w)^2) + (p^2(EC))/ \\
e_{s=0}^* = (a_L^2(1-p)^2 + a_H^2(1+p)^2 + 2a_H(1+p)(a_L(1-p) + 2w) + 4a_L(1-p)w + 4(k + w^2)) + \\
(1-p)p(D + B(2k + a_L^2(2-p) + w(pa_H + 2w) + a_L(pa_H + (4-p)w)))/ \\
(4k + a_L^2(2-p)^2 + 2a_L(2-p)(pa_H + 2w) + EC)/C^2
\end{align*}
\]

where \( A = pa_H + (1-p)a_L, \)

\( B = \sqrt{4k + a_L^2(2-p)^2 + 2a_L(2-p)(pa_H + 2w) + (pa_H + 2w)^2}, \)

\( C = \sqrt{a_L^2(1-p)^2 + a_H^2(1+p)^2 + 2a_H(1+p)(a_L(1-p) + 2w) + 4a_L(1-p)w + 4(k + w^2)}, \)

\( D = -a_L^2 - 4a_Hk - 2a_H^2A - a_LA^2 - 3a_L^2w + 4kw - 2a_LAw + A^2w + 4Aw^2 + 4w^3, \)

\( E = -a_H^2 - 4a_Hk - 2a_H^2A - a_HA^2 - 3a_H^2w + 4kw - 2a_HAw + A^2w + 4Aw^2 + 4w^3 + (2k + a_H^2(1+p) + w(a_L - pa_L + 2w) + a_H(a_L - pa_L + (3+p)w)). \)

Given the aggregate effort in equilibrium, the firm’s profit when it does not post performance is given by:

\[
\pi_{s=0}^* = (P - w)e_{s=0}^* \tag{10}
\]

### 3.2.2 If Performance is Posted

Given that the expected utility of \( i \) is linearly separable in the scenarios of the other agent being either of high or low ability, and weighted by \( p \) and \( (1-p) \) respectively, I first solve for the case in which \( i \) is of high ability, and anticipates \( j \) to be of high ability as well. If so, \( i \)'s optimal effort is:

\[
e_i^*(a_i = a_H, a_j = a_H, k, w|s = 1) = \frac{F + 4G^{3/2}}{8G} \tag{11}
\]

where \( F = -4a_H^3 - 4a_Hk - 4a_H^2w + 4kw + 4a_Hw^2 + 4w^3, \)

\( G = a_H^2 + k + 2a_Hw + w^2. \)

If \( i \) is of high ability, and anticipates \( j \) to be of low ability, then \( i \)'s optimal effort is:
\( e_i^*(a_i = a_H, a_j = a_L, k, w|s = 1) = \frac{H + J\sqrt{M}}{2M} \) (12)

where \( H = -a_H^3 + 2a_Ha_L - a_Ha_L^2 - 4a_Hk - 3a_H^2w - 2a_Ha_Lw + a_L^2w + 4kw + 4a_Lw^2 + 4w^3 \),
\( J = a_H^2 + a_Ha_L + 2k + 3a_Hw + a_Lw + 2w^2 \),
\( M = a_H^2 + 2a_Ha_L + a_L^2 + 4k + 4a_Hw + 4a_Lw + 4w^2 \).

Putting both equations 11 and 12 together, I obtain \( i \)'s optimal effort if \( i \) is of high ability:

\( e_i^*(a_i = a_H, k, w|s = 1) = p\left(\frac{F + 4G^{3/2}}{8G}\right) + (1 - p)\left(\frac{H + J\sqrt{M}}{2M}\right) \) (13)

I do the same, assuming \( i \) is of low ability and anticipates \( j \) to be of high ability:

\( e_i^*(a_i = a_L, a_j = a_H, k, w|s = 1) = \frac{Q + R\sqrt{M}}{2M} \) (14)

where \( M = a_H^2 + 2a_Ha_L + a_L^2 + 4k + 4a_Hw + 4a_Lw + 4w^2 \),
\( Q = -a_H^2a_L - 2a_Ha_L^2 - a_L^3 - 4a_Lk + a_H^2w - 2a_Ha_Lw - 3a_L^2w + 4kw + 4a_Hw^2 + 4w^3 \),
\( R = a_Ha_L + a_L^2 + 2k + a_Hw + 3a_Lw + 2w^2 \).

And if \( i \) is of low ability and anticipates \( j \) to be of low ability:

\( e_i^*(a_i = a_L, a_j = a_L, k, w|s = 1) = \frac{T + 4U^{3/2}}{8U} \) (15)

where \( T = -4a_L^3 - 4a_Lk - 4a_L^2w + 4kw + 4a_Lw^2 + 4w^3 \),
\( U = a_L^2 + k + 2a_Lw + w^2 \).

Putting both equations 14 and 15 together, I obtain \( i \)'s optimal effort if \( i \) is of low ability:

\( e_i^*(a_i = a_L, k, w|s = 1) = p\left(\frac{Q + R\sqrt{M}}{2M}\right) + (1 - p)\left(\frac{T + 4U^{3/2}}{8U}\right) \) (16)

**Aggregation of Optimal Efforts from Firm’s Perspective.** From the firm’s perspective, again, there are four possible scenarios with regards to the realized abilities of the agents: \( \{a_i = a_H, a_j = a_H\} \), \( \{a_i = a_H, a_j = a_L\} \), \( \{a_i = a_L, a_j = a_H\} \), and \( \{a_i = a_H, a_j = a_H\} \). Each of these scenarios come with the respective probabilities of \( p^2 \), \( p(1 - p) \), \( (1 - p)p \) and \( (1 - p)^2 \). Solving for the aggregate effort in equilibrium,
\[ e_{s=1}^* = e_i^*(a_i, a_j, k, w|s = 1) + e_j^*(a_i, a_j, k, w|s = 1), \]

I obtain:

\[ e_{s=1}^* = w - V + p^2X + Y - 2pY + p^2Y + pZ - p^2Z \]  

(17)

where \( V = pa_H + (1 - p)a_L \),
\[ X = \sqrt{a_H^2 + 2a_Hw + k + w^2}, \]
\[ Y = \sqrt{a_L^2 + 2a_Lw + k + w^2}, \]
\[ Z = \sqrt{a_H^2 + a_L^2 + 4(k + a_L + w^2) + 2a_H(a_L + 2w)}. \]

Given the aggregate effort in equilibrium, the firm’s profit when it posts performance is given by:

\[ \pi_{s=1}^* = (P - w)e_{s=1}^*. \]  

(18)

### 3.2.3 Comparing Aggregate Effort Across No Posting versus Posting

Figure 3 summarizes the intuition from the comparative statics laid out in Sections 3.2.1 and 3.2.2. This figure represents the directional change (but not magnitude) of optimal effort that \( i \) puts in against \( j \) under the various scenarios of ability draws, as we compare between the cases of a firm not posting versus posting performance.

If \( i \) is of low ability \( a_L \), and does not know whether \( s/he \) is going up against a low- or high-ability agent in \( j \), for the case in which performance is not posted, then the amount of effort that \( s/he \) puts in (two units) will lie between the cases in which performance is posted and the realized performance of \( j \) is either that of a low-ability agent (three units, due to more intense competition) or a high-ability agent (one unit, due to discouragement).

If \( i \) is of high ability \( a_H \), and does not know whether \( s/he \) is going up against a low- or high-ability agent in \( j \), for the case in which performance is not posted, then the amount of effort that \( s/he \) puts in (five units) will lie between the cases in which performance is posted and the realized performance of \( j \) is either that of a low-ability agent (four units due to complacency) or a high-ability agent (six units due to more intense competition).

From the firm’s perspective, it considers the amount of effort each type of agent will put in under the four possible scenarios \( \{a_i = a_H, a_j = a_H\}, \{a_i = a_H, a_j = a_L\}, \{a_i = a_L, a_j = a_H\} \text{ and } \{a_i = a_L, a_j = a_L\} \), and integrates it out across these scenarios with respective probabilities of \( p^2, p(1 - p), (1 - p)p \text{ and } (1 - p)^2 \). It does this for each case of no posting versus posting, and then compares the profits. I will now lay out the
key propositions.

\[
\begin{array}{c|c|c}
\text{No Posting} \\
\text{Self (i) / Other (j)} & a_L & a_H \\
\hline
a_L & \hline
a_H & \hline
\end{array}
\]

\[
\begin{array}{c|c|c}
\text{Posting} \\
\text{Self (i) / Other (j)} & a_L & a_H \\
\hline
a_L & \hline
a_H & \hline
\end{array}
\]

Figure 3: How Effort Levels Compare Across No Posting versus Posting

3.2.4 Firm’s Decision

The three propositions I present in this section stem from the firm’s comparison of its profit under the decision to not post performance \((π_{s=0})\) and to post performance \((π_{s=1})\). In Proposition 1, I describe how the firm’s decision to post performance depends on the heterogeneity of abilities – a function of not only \(a_L\) and \(a_H\) but also \(p\). Propositions 2 and 3 then examine how the effectiveness of performance posting vary as a function of social comparison \(k\) and the commission rate \(w\).

**Proposition 1: Performance posting does better for the firm when there is sufficient heterogeneity in agents’ abilities.**

In Figure 4, I show the difference in profit between posting versus not posting and how it varies across different values of \(p\). First, notice in the case of limited heterogeneity \((a_H > a_L)\) that performance posting hurts for all values of \(p\). The intuition is that when there is limited heterogeneity, a firm does better by not posting performance because it prevents a low-ability agent from being overly discouraged, and at the same time, ensures a high-ability agent from being overly complacent. This yields more effort from both types of agents compared to the case when a firm posts performance – but only if the heterogeneity in agents’ abilities is sufficiently small.

When there is sufficient heterogeneity in terms of the potential difference in ability levels \((a_H \gg a_L)\), a firm does better by posting performance because the low-ability agent will put in more effort to avoid being too far behind. At the same time, a high-ability agent will want to maintain his or her lead. There is another dimension to heterogeneity – it is not only the difference in \(a_H\) versus \(a_L\), but also the probability that an agent is of high ability, \(p\). When \(p\) is close to 0 or 1, notice that a firm does worse by posting performance.
This goes back to Proposition 1 in which there needs to be sufficient heterogeneity for performance posting to yield higher profit. When \( p \) is close to 0, it is likely that both agents are of low ability, and therefore, there is limited heterogeneity. Similarly, when \( p \) is close to 1, it is likely that both agents are of high ability, and therefore, again, there is limited heterogeneity. In both of these cases, notice in Figure 4 that performance posting does worse.

In the extremes, when \( p \) is 0 or 1, there is no longer any difference in profit between posting and not posting performance. This is because there is complete information in these two extremes: it is common knowledge that both agents are either of low or high ability, and therefore no information gained whether the firm posts or does not post performance.

![Difference in Profit Between Posting and Not](image)

**Figure 4**: Heterogeneity in Abilities Leads to Effectiveness of Posting

**Proposition 2**: As the extent of social comparison or competitiveness between agents increases, there is a non-monotonic effect on the effectiveness of performance posting.

Figure 5 depicts graphically what happens as I increase the extent of social comparison or competitiveness \( k \) between agents using comparative statics, conditional on a firm deciding to post performance if there is sufficient heterogeneity. First, as \( k \) increases from low to medium, performance posting does better for moderate values of \( p \). The reason for this is that under medium levels of social comparison or competitiveness, the amount of effort a high-ability agent puts in will more than compensate for the decline in effort by a low-ability agent, thus making aggregate profit higher. However, when \( k \) increases from medium to high, notice that not only does the peak in the profit difference drop, but there are also now larger regions of \( p \) in which performance posting does worse. The reason for this is that when \( k \) is high, the low-ability agent gets overly discouraged to a point that even an increase in effort by a high-ability agent will no longer sufficiently
compensate for the decline in effort by a low-ability agent. As a result, aggregate profit for the firm becomes lower when a firm posts performance.

Figure 5: Social Comparison Helps Posting, But Only Up to a Certain Point

**Proposition 3:** *As commission rate increases, there is a smaller difference in profit between performance posting and not.*

Figure 6 depicts graphically what happens as I increase the commission rate, conditional on a firm deciding to post performance if there is sufficient heterogeneity. Notice that when the commission rate increases, the peaks and troughs of the difference in profit between posting and not posting gets compressed towards zero. In other words, the stakes for this question are lower when the firm is paying well. The intuition for this is that when a firm is paying well, an agent will be motivated to put in more effort simply because the financial incentive is attractive. This is because social comparison matters less when the money is good, and so, the difference in profit between posting and not posting performance becomes smaller. Said differently, the stakes for this decision are higher when the firm is not paying well in a low commission setting. The implication for this is that firms that have narrow margins (and therefore less leeway to increase the commission rate) may find performance posting to have potentially more perks (or more perils).

Along the same lines, there is also some anecdotal evidence that during recessionary environments or when sales are slow-moving, firms are more inclined to consider posting of performance in order to motivate employees to work harder. My model shows that firms are indeed thinking in the right direction. That said, it is even more important to ensure that the firm is making an optimal decision with regards to performance posting, as the stakes for getting this question right become much higher.

5 [http://www.insightsquared.com/2014/05/25-reasons-your-sales-results-are-struggling/]
4 Lab Experiment

Having established several theoretical propositions, I now test their empirical validity using ongoing lab experiments. At this point, I have motivated my research question in the context of the sales force within marketing. That said, my model is more generalizable – it addresses the fundamentals of performance in many other contexts which involve ability and effort combining to give performance.

I now show the generalizability and relevance of my model in a real-effort task undertaken by university undergraduates who solve double-digit multiplication problems (e.g., 89 x 73, 56 x 39) for bonus payments. In my experiment, participants first go through a practice round and then an actual round of 10 questions. A similar task had been used in previous literature to elicit real effort from participants (e.g., Brugen and Strobel 2007). I first describe the setup of the experiment in Section 4.1 and then present the results from ongoing experiments in Section 4.2.

4.1 Description

130 members of a university subject pool were recruited to participate in a web-based computer task, and received $10 for an hour’s worth of time in the lab. The number of participants was determined in advance based on the number of subjects signed up to participate in a single week-long lab session afforded to me given the lab schedule. Throughout the task, participants identified themselves only via their lab ID, and performance posting in the treatment condition was done via their lab IDs. In other words, the identity of participants are not revealed to one another.

One would imagine that the social effects of performance posting in the treatment condition would be stronger if I used actual names (e.g., Lim 2010). Thus, the results of performance posting in my experiment were possibly a lower bound.
To ensure that the task itself was incentive-compatible, participants received an attractive bonus payoff ranging from $0.25 to $1.00 for each problem answered correctly. There were 20 problems altogether, and participants could earn substantial bonus payments. Each session was designed either as a treatment condition with performance posting of scores from every participant in the lab, or a control condition where there was no performance posting.

Participants first did the practice round in which they solved 10 randomly-generated double-digit multiplication problems with a time limit per problem. Before the start of the practice round, participants were informed about the distribution of practice round scores from previous participants who had done the same task in a pre-test. Participants then went on to do one multiplication problem at a time. Please see Figure 7 for a screen-shot of each multiplication problem. Once the time limit was up for each question, the program proceeded to the next question regardless of whether an answer was provided. Participants needed to key in the correct answer by the time limit in order to receive a point for the problem. After finishing all 10 multiplication problems, the program provided a score (out of 10) at the top of the screen, and a recap of which questions the participants answered correct or incorrectly.

At this point, depending on whether the participants were in the treatment (or control) condition, participants were informed (not informed) that there will be performance posting of their results by lab ID at the end of each round. Participants then proceeded to the actual round. If participants were in the treatment condition (performance posting), the lab assistant obtained the scores of all participants electronically and proceeded to list the ranked scores of every participant with the lab ID as the identifier. This process took around five minutes. If participants were in the control condition (no performance posting), they remained in their seats for five minutes. Finally, participants receive their financial payout for both practice and actual rounds (consisting of both participation fee and bonus earnings).

![What is 74 x 38?](image)

Figure 7: Clock Counting Down as Participants are Given Limited Time to Solve a Series of Multiplication Questions
4.2 Treatments and Experimental Results

4.2.1 Heterogeneity Affects Effectiveness of Performance Posting

To test Proposition 1, I need to manipulate the heterogeneity of participants’ abilities, and examine if performance posting resulted in a greater increase in performance during the actual round (over the practice round) for the case in which there was a greater heterogeneity in abilities. Given that participants in each session are undergraduate students from the same institution, the distribution of abilities should remain fairly constant\(^7\). Since it is challenging to manipulate the abilities of participants coming from the same institution, I manipulated the difficulty of the task instead. In the experiment described above, I set up two types of tasks – a difficult task (for which the time limit to solve each question is 15 seconds) and an easy task (for which the time limit to solve each question is 18 seconds). These two time limits ensured that different degrees of heterogeneity in performance were obtained. I assume here that the greater heterogeneity in performance for the easy task arises from a greater heterogeneity in ability, i.e. there is no difference in heterogeneity in effort between both tasks.

Figure 8 shows the distribution in performance of the practice round, i.e., before the effects of the treatment and control were in place. There was greater heterogeneity in performance \((n = 66, \text{mean}= 5.25, \text{s.d.}= 3.24)\) in the practice round of the easy task compared to the performance of the practice round of the difficult task \((n = 64, \text{mean}= 3.20, \text{s.d.}= 2.66)\), with the standard deviation of the performance in the easy task being higher than that of performance in the difficult task. Doing the Levene’s test for equality of variances, I find that they are statistically different for the easy versus the difficult task \((W = 4.17, p = 0.043)\). I also do the Kolmogorov-Smirnov test and find that both distributions are statistically different \((D = 0.299, p = 0.006)\).

Based on Proposition 1, my prediction is that performance posting will result in better performance for the easy task with greater heterogeneity, relative to the case of no performance posting. However, not posting performance will result in better performance for the difficult task with less heterogeneity, relative to the case of performance posting. As observed in Figure 9, participants in aggregate improved by 3.1% without performance posting and by 18.1% when performance was posted for the easy task when there is a more heterogeneous distribution. On the other hand, for the less heterogeneous distribution in the difficult task, participants in aggregate improved by 17.4% without performance posting, and by 9.7% when performance was posted.

I carry out statistical tests using the absolute differences of the participants between the practice and

\(^7\)As a next step, I would like to run this task across different universities where there may be an inherently different distribution of abilities.
actual round. For the easy task with greater heterogeneity, the simple main effect of this improvement is statistically significant ($F = 5.18, p = 0.024$), while for the difficult task with less heterogeneity, the simple main effect of this improvement is not statistically significant ($F = 1.62, p = 0.204$). Most importantly, the crossover interaction between the degree of heterogeneity and the presence or absence of performance posting is statistically significant ($F = 6.27, p = 0.013$), thus giving empirical validity for Proposition 1.

At both tasks, participants improved from the practice to actual round, with a greater improvement at the difficult task on average. Improvements from the practice to the actual round may be due to task
familiarity with the pen-and-paper multiplication task, efficiency in entering the answer on the computer interface, or some form of learning how to optimize usage of time for a question (e.g., knowing that if time is running out for a question, it is better to wait for the next question to appear than to try and beat the clock for the current question). Participants may also improve in their ability to answer math questions under time constraints. Even though participants improved from the practice to actual round, this improvement is controlled for in the 2 (homogenous/difficult versus heterogeneous/easy task) x 2 (post versus not post performance) between-subjects factorial design.

4.2.2 Subsequent Experiments

In ongoing experiments, I test Propositions 2 and 3 in the following ways. To test how social comparison or competitiveness affects optimal efforts and profit under performance posting (Proposition 2), I will keep task difficulty constant and manipulate social comparison or competitiveness. I do this by inserting a message before the math task to increase the competitiveness of participants and emphasize the importance of putting in more effort if they hope to win. Variation of this message can be achieved using different font sizes and colors to evoke different levels of competitiveness.

![The competition is heating up! If you hope to win, you will have to put in effort.
Are you up for the challenge?](image)

Figure 10: Example of Manipulating Competitiveness in Math Task

To test my last proposition, I vary the size of the bonus payment per correct answer. According to my model, Proposition 3 predicts that as I increase the size of the bonus payment, there will be a smaller difference between the treatment (posting performance) and the control (not posting performance).

5 Conclusion and Future Directions

5.1 Conclusion

The sales force is one of the most important functions in marketing. Many businesses spend more on their sales force than they do on any other marketing function. Annual sales force expenditures in the U.S.
to several times that of other marketing spends. More than 20 million people are employed in sales just in the U.S. alone. Increasing the performance of the sales force thus promises substantial profits for firms.

Motivating and managing the performance of sales force is one of the key factors. I focus on performance posting – a practice used by firms but under-studied in the literature despite its potential importance. Enabled by advances in information technology, performance posting is now relatively costless to implement, though not readily adopted across industries. I explore potential tradeoffs of performance posting, and prescribe circumstances under which performance posting can have its perks or perils.

To solve this research question, I use a blend of theory and empirics. Motivated by a growing body of evidence in psychology, economics and neuroscience, I enrich the psychology of the agents by specifying a utility function that contains both financial payoffs and social interdependencies in performance. Using a game-theoretic model of incomplete information consisting of two agents, I allow each agent’s ability to be private information, unknown to the firm and other agent. I first examine how optimal effort of each agent changes with or without performance posting, and solve for the profit under these two different policies. I then compare the difference in profit for the firm, and derive propositions on when performance posting does better for the firm. I develop three key propositions. First, if agents’ abilities are sufficiently homogenous, a firm does better without posting performance. On the contrary, if agents’ abilities are sufficiently heterogeneous, a firm does better with performance posting. Second, I find that the extent of social comparison or competitiveness of the agents has a non-monotonic effect on the firm’s profit if a firm decides to post performance. Lastly, I find that a more attractive financial incentive reduces the difference in profits between the presence and absence of performance posting. Using ongoing lab experiments, I demonstrate the empirical validity of my propositions, and in doing so, also show the generalizability of my model to contexts outside of the sales force.

My contribution to the research is twofold. To date, social comparison has been modeled as non-existent in the absence of information about others’ performance, but this runs counter to a growing body of evidence in social psychology that social comparison is a part of human nature. My first contribution is to methodologically disentangle social comparison from the availability of information about others’ performance. This allows us to understand the effects of social comparison when a firm can control the precision of information about others’ performance.

My other contribution is to answer a substantive question that is of managerial interest and importance, which hitherto had not been examined in past literature. Based on my analytical and empirical results, we now have guiding principles on when a firm benefits from performance posting. Going one step further, beyond sales force management in marketing, the same set of guidelines can be applied to other domains such as education and healthcare which are currently considering when to use performance posting to increase
5.2 Extensions and Managerial Implications

5.2.1 Extensions

There are several extensions I will like to consider. First, for the theoretical model, I can explore different ways of formulating performance in terms of abilities and efforts as robustness checks. My model focuses on abilities and efforts being additive, and therefore, substitutes. I can relax this assumption and allow for complementarity between ability and effort, i.e., ability influences an agent’s marginal benefit of effort. Also, there are different ways to specify social comparison by the agents.

I can also allow for substitutability or complementarity between the production functions of agents when aggregating individual production into the sales of the firm, by introducing an interaction term between the performances of both agents, weighted by a parameter that is either negative (i.e., substitutable) or positive (i.e., complementary). This is especially relevant in team-based tasks where there are interdependencies between the performance of agents. I can also consider the effects of different compensation schemes and whether there are other types of social comparison information that a firm can post. Another interesting extension is whether a firm can do better in profits by engaging in selective posting (e.g., post only the top 25% of performers) or post performance in broader bands.

For future work relating to performance posting, I will like to explore the effect of biased inferences. We know from the literature that people may (i) be biased in the way they update their beliefs given the signals they receive (Mobius et al. 2013); (ii) asymmetrically process objective information about oneself (Eil and Rao 2011); or (iii) misestimate their performance relative to others (e.g., Kruger 1999). These may have interesting implications for when a firm should use performance posting.

5.2.2 Managerial Implications

While I focus on performance posting, there are two broader applications to my findings. First, performance posting is used not only in sales and marketing, but also in other domains like education (e.g., teacher posting student scores, department chairs posting teaching ratings of faculty members), healthcare (e.g., success rates of medical operations), hygiene (Jin and Leslie 2003) and even the design of online reputation systems (Dellarocas 2010). I hope that my findings provide a set of guidelines for managers considering the
use of performance posting, both within marketing and also in other disciplines.

Second, I hope that my results are a first step towards understanding the efficacy of other similar practices such as an open salary policy in which employees can look up anyone’s salary or bonus. For example, Whole Foods has an open salary policy\(^8\), not just within a same hierarchy of employees, but across all hierarchies including the CEO level. To properly address the question of whether firms should adopt an open salary policy, social incentives such as equity and fairness will also need to be considered, along with the proper reference set for social comparison. Altogether, I hope that my theoretical and empirical results provide a modest step towards addressing these important managerial questions.

\(^8\) http://www.businessinsider.com/whole-foods-employees-have-open-salaries-2014-3)
References


Appendix A: Comparative Statics

A.1 Comparative Statics for Symmetric Agents

We begin with the case in which agents have same abilities. With probability \( p^2 \), both agents will have high ability \( a_i = a_j = a_H \), and equation 7 simplifies to the following:

\[
e^*(a_H, k, w) = \frac{\sqrt{a_H^2 + k + 2a_H w + w^2} + w - a_H}{2}
\]  

(19)

Likewise, with probability \((1 - p)^2\), both agents will have low ability \( a_i = a_j = a_L \), and equation 7 then simplifies to the following:

\[
e^*(a_L, k, w) = \frac{\sqrt{a_L^2 + k + 2a_L w + w^2} + w - a_L}{2}
\]  

(20)

In either case, let the optimal effort for symmetric agents be given by \( e^*(a, k, w) = \frac{\sqrt{a^2 + k + 2aw + w^2 + w - a}}{2} \), where \( a \) can represent \( a_H \) or \( a_L \).

Taking the first derivative of \( e^* \) with respect to \( a \), we obtain:

\[
\frac{\partial e^*}{\partial a} = \frac{w + a}{2\sqrt{a^2 + k + 2aw + w^2}} - \frac{1}{2}
\]  

(21)

Since \( k \geq 0 \), \( w + a \) can never be greater than \( \sqrt{a^2 + k + 2aw + w^2} \) since \( \sqrt{a^2 + k + 2aw + w^2} = \sqrt{(a + w)^2 + k} \), i.e. \( \frac{w + a}{\sqrt{a^2 + k + 2aw + w^2}} \leq \frac{1}{2} \). \( \therefore \frac{\partial e^*}{\partial a} \leq 0 \).

Taking the first derivative of \( e^* \) with respect to \( w \), we obtain:

\[
\frac{\partial e^*}{\partial w} = \frac{w + a}{2\sqrt{a^2 + k + 2aw + w^2}} + \frac{1}{2}
\]  

(22)

Again, since \( k \geq 0 \), \( \frac{w + a}{\sqrt{a^2 + k + 2aw + w^2}} \leq \frac{1}{2} \). \( \therefore \frac{\partial e^*}{\partial w} > 0 \).

Taking the first derivative of \( e^* \) with respect to \( k \), we obtain:

\[
\frac{\partial e^*}{\partial k} = \frac{1}{4\sqrt{a^2 + k + 2aw + w^2}}
\]  

(23)

Again, since \( k \geq 0 \), \( w > 0 \) and \( a > 0 \), \( \sqrt{a^2 + k + 2aw + w^2} \) has to be positive and real. \( \therefore \frac{\partial e^*}{\partial k} > 0 \).
A.2 Comparative Statics for Asymmetric Agents

Taking the first derivative of \( e_i^* \) with respect to \( a_i \), we obtain:

\[
\frac{\partial e_i^*}{\partial a_i} = \frac{(a_i^3 + a_j^3 + a_i^2(3a_j + 6w - \sqrt{\lambda}) - a_j^2(-6w + \sqrt{\lambda}) - 4(k + w^2)(-2w + \sqrt{\lambda}) + \lambda_i(3a_j^2 + 6k + 12a_jw + 12w^2 - 2a_j\sqrt{\lambda} - 4w\sqrt{\lambda}) + 2a_j(k + 6w^2 - 2w\sqrt{\lambda})}{2\lambda_i^{3/2}}
\]  \hspace{1cm} (24)

where \( \lambda_i = a_i^2 + a_j^2 + 4a_jw + 2a_i(a_j + 2w) + 4(k + w^2) \).

Taking the second derivative of \( e_i^* \) with respect to \( a_i \), we obtain:

\[
\frac{\partial^2 e_i^*}{\partial a_i^2} = \frac{(-6a_j - 4a_j - 6w - (2a_i + 2a_j + 4w)^2\mu_i}{4\lambda_i^{3/2}} + \frac{(2a_i + 2a_j + 3w)(2a_i + 2a_j + 4w)\mu_i + 2\sqrt{\lambda_i}}{2\lambda_i} - \frac{(2a_i + 3w)\sqrt{\lambda_i}}{\lambda_i^2} + ((2a_i + 2a_j + 4w)^2(\xi_i + \mu_i\sqrt{\lambda_i})/\lambda_i^3 - \xi_i + \mu_i\sqrt{\lambda_i}/\lambda_i^2}
\]  \hspace{1cm} (25)

where \( \lambda_i = a_i^2 + a_j^2 + 4a_jw + 2a_i(a_j + 2w) + 4(k + w^2) \), \( \mu_i = a_i^2 + a_i a_j + 2k + 3a_iw + a_jw + 2w^2 \), \( \xi_i = -a_i^3 - 2a_i^2a_j - a_i a_j^2 - 4a_i k - 3a_i^2 w - 2a_i a_j w + a_j^2 w + 4kw + 4a_j w^2 + 4w^3 \).

Taking the first derivative of \( e_i^* \) with respect to \( a_j \), we obtain:

\[
\frac{\partial e_i^*}{\partial a_j} = \frac{(a_i - a_j)k}{(a_i^2 + a_j^2 + 4a_jw + 2a_i(a_j + 2w) + 4(k + w^2))^{3/2}}
\]  \hspace{1cm} (26)

Taking the second derivative of \( e_i^* \) with respect to \( a_j \), we obtain:

\[
\frac{\partial^2 e_i^*}{\partial a_j^2} = \frac{-2k(2a_i^2 - a_j^2 - a_j w + a_i(a_j + 5w) + 2(k + w^2))}{(a_i^2 + a_j^2 + 4a_jw + 2a_i(a_j + 2w) + 4(k + w^2))^{5/2}}
\]  \hspace{1cm} (27)

Conditions under which \( \frac{\partial^2 e_i^*}{\partial a_j^2} > 0 \):

(i) \( 0 < a_i < \frac{a_j}{2} \), and \( 0 < w < a_j - 2a_i \), and \( 0 < k < \frac{1}{2}(-2a_i^2 - a_i a_j + a_j^2 - 5a_i w + a_j w - 2w^2) \).

Taking the first derivative of \( e_i^* \) with respect to \( w \), we obtain:

\[
\frac{\partial e_i^*}{\partial w} = \frac{(a_i^3 + a_j^3 + 4(k + w^2)(2w + \sqrt{\lambda_i}) + 4a_jw(3w + \sqrt{\lambda_i}) + a_j^2(6w + \sqrt{\lambda_i}) + a_j^2(3a_j + 6w + \sqrt{\lambda_i}) + a_j^2(3a_j + 6w + \sqrt{\lambda_i}) + a_i(3a_j^2 + 8k + 12a_jw + 12w^2 + 2a_j\sqrt{\lambda_i} + 4w\sqrt{\lambda_i})}{2\lambda_i^{3/2}}
\]  \hspace{1cm} (28)

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where \( \lambda_i = a_i^2 + a_j^2 + 4a_iw + 2a_i(a_j + 2w) + 4(k + w^2) \).

Taking the second derivative of \( e_i^* \) with respect to \( w \), we obtain:

\[
\frac{\partial^2 e_i^*}{\partial w^2} = \frac{8k \left( -a_i^2 + 2a_j^2 + a_i(a_j - w) + 5a_iw + 2(k + w^2) \right)}{\lambda_i^{5/2}} \tag{29}
\]

where \( \lambda_i = a_i^2 + a_j^2 + 4a_iw + 2a_i(a_j + 2w) + 4(k + w^2) \).

Conditions under which \( \frac{\partial^2 e_i^*}{\partial w^2} > 0 \):
(i) \( 0 < a_i < 2a_j \) and \( k > 0 \); or
(ii) \( a_i > 2a_j \) and \( 0 < w < a_i - 2a_j \) and \( k > \frac{1}{2} \left( a_i^2 - a_i a_j - 2a_j^2 + a_iw - 5a_jw - 2w^2 \right) \); or
(iii) \( a_i > 2a_j \) and \( w > a_i - 2a_j \) and \( k > 0 \).

Taking the first derivative of \( e_i^* \) with respect to \( k \), we obtain:

\[
\frac{\partial e_i^*}{\partial k} = \left( \frac{4(w - a_i) + \frac{2\mu_i}{\sqrt{\xi}} + 2\sqrt{\lambda_i}}{2 \lambda_i} \right) / 2 \lambda_i -
\]

\[
(2(-a_i^3 - 2a_i^2a_j - a_i a_j^2 - 4a_i k - 3a_i^2 w - 2a_i a_j w + a_j^2 w + 4kw + 4a_jw^2 + 4w^3 +
\mu_i \sqrt{\lambda_i}) / \lambda_i^2 \tag{30}
\]

where \( \lambda_i = a_i^2 + a_j^2 + 4a_iw + 2a_i(a_j + 2w) + 4(k + w^2), \mu_i = a_i^2 + a_i a_j + 2k + 3a_iw + a_jw + 2w^2. \)

Taking the second derivative of \( e_i^* \) with respect to \( k \), we obtain:

\[
\frac{\partial^2 e_i^*}{\partial k^2} = \frac{-\frac{4\mu_i}{\lambda_i^{5/2}} + \frac{4}{\lambda_i} - \frac{4(4w - 4a_i + \frac{2\mu_i}{\sqrt{\lambda_i}} + 2\sqrt{\lambda_i})}{\lambda_i^2}}{2 \lambda_i} \tag{31}
\]

where \( \lambda_i = a_i^2 + a_j^2 + 4a_iw + 2a_i(a_j + 2w) + 4(k + w^2), \mu_i = a_i^2 + a_i a_j + 2k + 3a_iw + a_jw + 2w^2. \)

Conditions under which \( \frac{\partial^2 e_i^*}{\partial k^2} > 0 \):
(i) \( a_i > 2a_j, 0 < w < a_i - 2a_j \) and \( 0 \leq k < \frac{1}{2} \left( a_i^2 - a_i a_j - 2a_j^2 + a_iw - 5a_jw - 2w^2 \right) \)

A.3 Comparative Statics for the Case where Performance is Not Posted

A.3.1 Agent with Low Ability

\[
\frac{\partial e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial a_H} =
\]

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\[
\frac{\partial^2 e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial a_L^2} = \\
\frac{2k p^2(-2k + a_H^2 p^2 + a_L^2(-2 - p + p^2) + a_H p w - 2w^2 - a_L(a_H p (-1 + 2p) + (4 + p))}{(4k + a_L^2(-2 - p)^2 - 2a_L(-2 - p)(a_H p + 2w) + (a_H p + 2w)^2)^{5/2}} - \frac{\partial e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial a_H^i} = \\
\frac{(32)}{
\frac{(-3) a_L^2 - 4k - 2a_L^2(-1 - p) - 4a_L(a_L(1 - p) + a_H p) - 2a_L(1 - p)(a_L(1 - p) + a_H p)(a_H p) - \frac{2(1 - p) + 2a_H a_L^2 p^2 + a_H^2 p^2 - 2a_H a_L p^2 + a_L^2 p^2 - 4a_L w + a_H p w - a_L p w - 2w^2)}{(4k + a_L^2(-2 + p)^2 - 2a_L(-2 - p)(a_H p + 2w) + (a_H p + 2w)^2)^{5/2}}}
\]

Conditions under which \( \frac{\partial^2 e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial a_L^2} > 0 \):

(i) \( a_H > 2a_L + w, \) and \( -\frac{a_L - w}{a_H + a_L} < p \leq 1, \) and \( 0 < k < \frac{1}{2}(-2a_L^2 + a_H a_L p - a_L^2 p^2 - 2a_H a_L p^2 + a_L^2 p^2 - 4a_L w + a_H p w - a_L p w - 2w^2) \).

\[
\frac{\partial^2 e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial a_H^i} = \\
\frac{(32)}{
\frac{(-3) a_L^2 - 4k - 2a_L^2(-1 - p) - 4a_L(a_L(1 - p) + a_H p) - 2a_L(1 - p)(a_L(1 - p) + a_H p)(a_H p) - \frac{2(1 - p) + 2a_H a_L^2 p^2 + a_H^2 p^2 - 2a_H a_L p^2 + a_L^2 p^2 - 4a_L w + a_H p w - a_L p w - 2w^2)}{(4k + a_L^2(-2 + p)^2 - 2a_L(-2 - p)(a_H p + 2w) + (a_H p + 2w)^2)^{5/2}}}
\]

Conditions under which \( \frac{\partial^2 e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial a_L^2} > 0 \):

(i) \( a_H > 2a_L + w, \) and \( -\frac{a_L - w}{a_H + a_L} < p \leq 1, \) and \( 0 < k < \frac{1}{2}(-2a_L^2 + a_H a_L p - a_L^2 p^2 - 2a_H a_L p^2 + a_L^2 p^2 - 4a_L w + a_H p w - a_L p w - 2w^2) \).

where \( \lambda = a_L^2 + 4k + 2a_L(a_L(1 - p) + a_H p) + (a_L(1 - p) + a_H p)^2 + 4a_L w + 4(a_L(1 - p) + a_H p) w + 4w^2, \) \( \xi = a_L^2 + 2k + a_L(a_L(1 - p) + a_H p) + 3a_L w + (a_L(1 - p) + a_H p) w + 2w^2. \)
\begin{equation}
\frac{\partial^2 e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial k^2} = \frac{-2(2k + 2a_L^2 p^2 + a_L^2(2 - 5p + 2p^2) + 5a_L p w + 2w^2 + a_{L}(a_{H}(5 - 4p) + (4 - 5p)w))}{(4k + a_L^2(-2 + p)^2 - 2a_L(-2 + p)(a_{H}p + 2w) + (a_{H}p + 2w)^2)^{5/2}}
\end{equation}

where \( \lambda = a_L^2 + 4k + 2a_L(a_L(1 - p) + a_{H}p) + (a_L(1 - p) + a_{H}p)^2 + 4a_L w + 4(a_L(1 - p) + a_{H}p) w + 4w^2, \)
\( \xi = a_L^2 + 2k + a_L(a_L(1 - p) + a_{H}p) + 3a_L w + (a_L(1 - p) + a_{H}p) w + 2w^2. \)

\begin{equation}
\frac{\partial e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial w} = \frac{(-3a_L^2 + 4k - 2a_L(a_L(1 - p) + a_{H}p) + (a_L(1 - p) + a_{H}p)^2 + 8(a_L(1 - p) + a_{H}p) w + 12w^2 + ((4a_L + 4(a_L(1 - p) + a_{H}p)) + 8w)w)}{(3a_L + a_L(1 - p) + a_{H}p + 4w)^{\sqrt{\lambda}}/(2\lambda) - ((4a_L + 4(a_L(1 - p) + a_{H}p)) + 8w)(a_L^2 - 4a_L k - 2a_L(1 - p) + a_{H}p) - a_{L}(a_L(1 - p) + a_{H}p)^2 - 3a_L^2 w + 4kw - 3a_L(a_L(1 - p) + a_{H}p) w + (a_L(1 - p) + a_{H}p)^2 w^2 + 4w^3 + \xi \sqrt{\lambda}^2)/(2\lambda^2)}
\end{equation}

where \( \lambda = a_L^2 + 4k + 2a_L(a_L(1 - p) + a_{H}p) + (a_L(1 - p) + a_{H}p)^2 + 4a_L w + 4(a_L(1 - p) + a_{H}p) w + 4w^2, \)
\( \xi = a_L^2 + 2k + a_L(a_L(1 - p) + a_{H}p) + 3a_L w + (a_L(1 - p) + a_{H}p) w + 2w^2. \)

\begin{equation}
\frac{\partial^2 e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial w^2} = \frac{8k(2k + 2a_L^2 p^2 + a_L^2(2 - 5p + 2p^2) + 5a_L p w + 2w^2 + a_{L}(a_{H}(5 - 4p) + (4 - 5p)w))}{(4k + a_L^2(-2 + p)^2 - 2a_L(-2 + p)(a_{H}p + 2w) + (a_{H}p + 2w)^2)^{5/2}}
\end{equation}

\begin{equation}
\frac{\partial e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial p} = \frac{(-2(a_H - a_L)a_L^2 - 2(a_H - a_L) a_L(a_L(1 - p) + a_{H}p) - 2(a_H - a_L) a_L w + 2(a_H - a_L)(a_L(1 - p) + a_{H}p) w + 4(a_H - a_L) w^2 + ((2(a_H - a_L)a_L + 2(a_H - a_L)(a_L(1 - p) + a_{H}p) + 4(a_H - a_L)w)w)}{(a_H - a_L) a_L + (a_H - a_L)w\sqrt{\lambda}^2)/(2\lambda) - ((2(a_H - a_L)a_L + 2(a_H - a_L)(a_L(1 - p) + a_{H}p) + 4(a_H - a_L)w)(-a_L^2 - 4a_L k - 2a_L(1 - p) + a_{H}p) - a_{L}(a_L(1 - p) + a_{H}p)^2 - 3a_L^2 w + 4kw - 2a_L(a_L(1 - p) + a_{H}p) w + (a_L(1 - p) + a_{H}p)^2 w^2 + 4(a_L(1 - p) + a_{H}p) w^2 + 4w^3 + \xi \sqrt{\lambda}^2)/(2\lambda^2)}
\end{equation}

where \( \lambda = a_L^2 + 4k + 2a_L(a_L(1 - p) + a_{H}p) + (a_L(1 - p) + a_{H}p)^2 + 4a_L w + 4(a_L(1 - p) + a_{H}p) w + 4w^2, \)

40
\[ \xi = a_L^2 + 2k + a_L(a_L(1-p) + a_Hp) + 3a_Lw + (a_L(1-p) + a_Hp)w + 2w^2. \]

\[ \frac{\partial^2 e^*_i(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial p^2} = \]

\[ \frac{2(a_H - a_L)^2 k(-2k + a_H^2 p^2 + a_L^2 (-2 - p + p^2) + a_Hpw - 2w^2 - a_L(a_Hp(-1 + 2p) + (4 + p)w))}{(4k + a_H^2(-2 + p) - 2a_L(-2 + p)(a_Hp + 2w) + (a_Hp + 2w)^2)^{5/2}} \]

Conditions under which \( \frac{\partial^2 e^*_i(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial p^2} > 0; \)

(i) \( a_H > 2a_L + w \) and \( \frac{a_L - w}{a_H + a_L} < p \leq 1 \) and

\[ 0 < k < \frac{1}{2}(-2a_H^2 + a_Ha_Lp - a_L^2p^2 - 2a_Ha_Lp^2 + a_H^2p^2 - 4a_Lw + a_Hpw - a_Lpw - 2w^2). \]

A.3.2 Agent with High Ability

\[ \frac{\partial e^*_i(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial a_H} = \]

\[ (-3a_H^2 - 4k - 2a_H^2 - 4a_H(a_L(1-p) + a_Hp) - 2a_Hp(a_L(1-p) + a_Hp) - (a_L(1-p) + a_Hp)^2 - 6a_Hw - 2a_Hpw - 2(a_L(1-p) + a_Hp)w + 2p(a_L(1-p) + a_Hp)w + 4pw^2 + ((2a_H + 2a_Hp + 2(a_L(1-p) + a_Hp) + 2p(a_L(1-p) + a_Hp) + 4w + 4pw) (a_H^2 + 2k + a_H(a_L(1-p) + a_Hp) + 3a_Hw + (a_L(1-p) + a_Hp)w + 2w^2))/(2\sqrt{\lambda}) + (2a_H + a_HL(1-p) + 2a_Hp + 3w + pw)\sqrt{\lambda}]/(2\lambda) - ((2a_H + 2a_Hp + 2(a_L(1-p) + a_Hp) + 2p(a_L(1-p) + a_Hp) + 4w + 4pw) (-a_H^2 - 4a_Hk - 2a_H^2(a_L(1-p) + a_Hp) - a_H(a_L(1-p) + a_Hp)^2 - 3a_Hw + 4k - 2a_H(a_L(1-p) + a_Hp)w + (a_L(1-p) + a_Hp)^2w + 4(a_L(1-p) + a_Hp)w^2 + 4w^3 + (a_H^2 + 2k + a_H(a_L(1-p) + a_Hp) + 3a_Hw + (a_L(1-p) + a_Hp)w + 2w^2)\sqrt{\lambda}))/(2\lambda^2) \]

where \( \lambda = a_H^2 + 4k + 2a_H(a_L(1-p) + a_Hp) + (a_L(1-p) + a_Hp)^2 + 4a_Hw + 4(a_L(1-p) + a_Hp)w + 4w^2 \)

\[ \frac{\partial^2 e^*_i(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial a_H^2} = \]

\[ \frac{(2k(1+p)(a_H^2 p(1+p)^2 + a_L^2(-1+p)^2(3+p) - a_L(-0 + 8p + p^2))w - a_H(1+p)(a_H(-3+p+2p^2) - (3+p)w) - 2(-3+p)(k + w^2))}{(a_H^2(-1+p)^2 + a_H^2(1+p)^2 - 2a_H(1+p)(a_L(-1+p) - 2w) - 4a_L(-1+p)w + 4(k+w^2))^{5/2}} \]

\[ \frac{\partial e^*_i(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial a_L} = \]
where \( \lambda = a^2_H + 4k + 2a_H(a_L(1 - p) + a_H p) + (a_L(1 - p) + a_H p)^2 + 4a_H w + 4(a_L(1 - p) + a_H p)w + 4w^2 \)

\[
\frac{\partial^2 e^*_i(a_i = a_H, E_i|a_j], k, w|s = 0)}{\partial a^2_L} =
\frac{-2(k(1-p)^2(-a^2_H(-1+p)^2+a^2_H(2+p-p^2))}{(a_L(-1+p)+a_H(a_L-3a_Lp+2a_H^2w+5w-pw)+2(k+w^2))}/(\sqrt{\lambda}+2\sqrt{\lambda})/(2\lambda)-
\]

\[
\frac{\partial e^*_i(a_i = a_H, E_i|a_j], k, w|s = 0)}{\partial k^2} =
\frac{-(2a_H(a_L-3a_Lp+2a_H^2w+5w-pw)+2(k+w^2))}{(a^2_H(-1+p)^2+a^2_H(1+p)^2-2a_H(a_L(-1+p)-2w)-4a_L(-1+p)+4a_H w + 4(k+w^2))^{5/2}}
\]

where \( \lambda = a^2_H + 4k + 2a_H(a_L(1 - p) + a_H p) + (a_L(1 - p) + a_H p)^2 + 4a_H w + 4(a_L(1 - p) + a_H p)w + 4w^2 \)

\[
\frac{\partial^2 e^*_i(a_i = a_H, E_i|a_j], k, w|s = 0)}{\partial k^2} =
\frac{-(2(2a_H^2(-1+p)^2+a_H^2(-1+p+2p^2)-5a_L(-1+p)+a_L(a_L^2+3a_Lp-4a_H^2w^2)-w+5p)+(k+w^2))}{(a^2_L(-1+p)^2+a^2_H(1+p)^2-2a_H(1+p)(a_L(-1+p)-2w)-4a_L(-1+p)+4a_H w + 4(k+w^2))^{5/2}}
\]

Conditions under which \( \frac{\partial^2 e^*_i(a_i = a_H, E_i|a_j], k, w|s = 0)}{\partial w} > 0 \):

(i) \( a_H > 0 \) and \( w > 0 \), \( 0 < a_L < \frac{a_H-w}{2} \) and \( 0 < p < \frac{a_H-2a_H-w}{2a_H} \)

\[
0 \leq k < \frac{1}{2}(a^2_H-a_Ha_L-2a^2_L-a^2_Hp-3a_Ha_Lp+4a^2_Hp-2a^2_Hp^2+4a_Ha_Lp^2-2a^2_Lp^2+a_Hw-5a_Lw-5a_Hpw+5a_Lpw-2w^2).
\]

\[
\underline{\text{42}}
\]
\[-3a_H^2 + 4k - 2a_H(a_L(1 - p) + a_Hp) + (a_L(1 - p) + a_Hp)^2 + 8(a_L(1 - p) + a_Hp)w + 12w^2 + ((4a_H + 4(a_L(1 - p) + a_Hp) + 8w)\]
\[(a_H^2 + 2k + a_H(a_L(1 - p) + a_Hp) + 3a_Hw + (a_L(1 - p) + a_Hp)w + 2w^2)/(2\sqrt{\lambda}) + \]
\[(3a_H + a_L(1 - p) + a_Hp + 4w)\sqrt{\lambda}/(2\lambda) - \]
\[(4a_H + 4(a_L(1 - p) + a_Hp) + 8w)(-a_H^2 - 4a_Hk - 2a_H^2(a_L(1 - p) + a_Hp) - (a_L(1 - p) + a_Hp)^2 + 3a_H^2w + 4kw - 2a_H(a_L(1 - p) + a_Hp)w + (a_L(1 - p) + a_Hp)^2w + 2w^2 + \]
\[(a_H^2 + 2k + a_H(a_L(1 - p) + a_Hp) + 3a_Hw + (a_L(1 - p) + a_Hp)w + 2w^2)\sqrt{\lambda})/(2\lambda^2) \]

where \( \lambda = a_H^2 + 4k + 2a_H(a_L(1 - p) + a_Hp) + (a_L(1 - p) + a_Hp)^2 + 4a_Hw + 4(a_L(1 - p) + a_Hp)w + 4w^2 \)

\[
\frac{\partial^2 e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial w^2} = \]

\[
(8k(2a_H^2(-1 + p)^2 + a_H^2(-1 + p + 2p^2) - 5a_L(-1 + p)w + a_H(a_L + 3a_Lp - 4a_Lp - w + 5kw + 2(k + w^2)))/(a_L^2(-1 + p)^2 + a_H^2(1 + p)^2 - 2a_H(1 + p)(a_L(-1 + p) - 2w) - 4a_L(-1 + p)w + 4(k + w^2))^{5/2} \]

Conditions under which \( \frac{\partial^2 e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial w^2} > 0 \):

(i) \( 0 < a_H \leq w \) and \( k > 0 \), or

(ii) \( a_H > w \) and \( 0 < a_L < \frac{a_H - w}{2} \) and \( 0 \leq p < \frac{a_H - 2a_L - w}{2} \) and

\[ k > \frac{1}{5}(a_H^2 - a_H a_L - 2a_L^2 - a_H^2 - 3a_H a_L p + 4a_L p - 2a_H^2 p^2 + 4a_H a_L p^2 - 2a_L^2 p^2 + a_H w - 5a_L w - 5a_L p w + 5a_L p w - 2w^2) \)

or

(iii) \( \frac{a_H - 2a_L - w}{2} \leq p \leq 1 \) and \( k > 0 \), or

(iv) \( \frac{a_H - w}{2} \leq a_L < a_H \) and \( k > 0 \).

\[
\frac{\partial e^*_i(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial p} = \]

\[
(-2a_H^2(a_H - a_L) - 2a_H(a_H - a_L)(a_L(1 - p) + a_Hp) - 2a_H(a_H - a_L)w + 2(a_H - a_L)(a_L(1 - p) + a_Hp)w + 4(a_H - a_L)w^2 + ((2a_H(a_H - a_L) + 2(a_H - a_L)(a_L(1 - p) + a_Hp) + 4(a_H - a_L)w)w + (a_H^2 + 2k + a_H(a_L(1 - p) + a_Hp) + 3a_Hw + (a_L(1 - p) + a_Hp)w + 2w^2)/(2\sqrt{\lambda}) + (a_H - a_L)w \sqrt{\lambda}/(2\lambda) - ((2a_H(a_H - a_L) + 2H - a_H)(a_L(1 - p) + a_Hp) + 4(a_H - a_L))w)
\]
\[-a_H^2 - 4a_Hk - 2a_H^2(a_L(1 - p) + a_Hp) - a_H(a_L(1 - p) + a_Hp)^2 - 3a_H^2w + 4kw - 2a_H(a_L(1 - p) + a_Hp)w + (a_L(1 - p) + a_Hp)w^2 + 4(a_L(1 - p) + a_Hp)w^2 + 4w^3 + \]
\[(a_H^2 + 2k + a_H(a_L(1 - p) + a_Hp) + 3a_Hw + (a_L(1 - p) + a_Hp)w + 2w^2)\sqrt{\lambda})/(2\lambda^2) \]

where \( \lambda = a_H^2 + 4k + 2a_H(a_L(1 - p) + a_Hp) + (a_L(1 - p) + a_Hp)^2 + 4a_Hw + 4(a_L(1 - p) + a_Hp)w + 4w^2 \)

\[
\frac{\partial^2 e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial p^2} = \]

\[
(2a_H - a_L)^2k(a_H^2(-1 + p)^2 + a_H^2(-2 + p + p^2) + a_H(w - pw) - a_H(a_L - 3a_Lp + 2a_Hp^2 + 5w - pw) - 2(k + w^2))/(a_H^2(-1 + p)^2 + a_H^2(1 + p)^2 - 2a_H(1 + p)(a_L(-1 + p) - 2w) - 4a_L(-1 + p)w + 4(k + w^2))^{5/2} \]
A.4 Comparative Statics for the Case where Performance is Posted

A.4.1 Agent with Low Ability

\[
\frac{\partial e_i^*(a_i = a_L, k, w|s = 1)}{\partial a_H} = \frac{(-a_H + a_L)kp}{\mu^{3/2}}
\]  

(52)

where \( \mu = a_H^2 + a_L^2 + 4a_Lw + 2a_H(a_L + 2w) + 4(k + w^2) \).

\[
\frac{\partial e_i^*(a_i = a_L, k, w|s = 1)}{\partial a_L} = \]

\[
\frac{1}{2}(2(-1 + p)(a_L + w)(-a_L + w + \sqrt{\lambda})/\lambda + (1 - p)(-3a_L^2 - k - 2a_Lw + w^2 + 3(a_L + w)\sqrt{\lambda})/\lambda + \]

\[
(p(-a_H^2 - 4a_Ha_L - 3a_L^2 - 4k - 2a_Hw - 6a_Lw + (a_H + a_L + 2w)(a_H^2 + 3a_Lw + a_H(a_L + w) + 2(k + w^2))/\sqrt{\mu} + (a_H + 2a_L + 3w)/\sqrt{\mu})/\mu - 2p(a_H + a_L + 2w)^3 + 3(a_L + w)^3 + 3(a_L + a_H(a_L + w) + 2(k + w^2))/\sqrt{\mu}))/\mu^2)
\]

(53)

where \( \lambda = a_H^2 + k + 2a_Lw + w^2 \),

\( \mu = a_H^2 + a_L^2 + 4a_Lw + 2a_H(a_L + 2w) + 4(k + w^2) \).

\[
\frac{\partial e_i^*(a_i = a_L, k, w|s = 1)}{\partial k} = \]

\[
\frac{1}{2}((-1 + p)(-a_L + w + \sqrt{\lambda})/\lambda + (-1 + p)(3a_L^2 + 3k + 6a_Lw + 3w^2 - 2a_L\sqrt{\lambda} + 2w\sqrt{\lambda})/2\lambda^{3/2} + 2p(-2a_L + 2w + 3a_L + 3a_L^2 + a_H(a_L + w) + 2(k + w^2))/\sqrt{\mu} + \sqrt{\mu})/\mu - (4p(-a_H^2a_L - a_Ha_L^2 - a_H^3 - 4a_Lk + a_H^2w - 2a_Ha_Lw - 3a_L^2w + 4kw + 4a_Hw^2 + 4w^3 + (a_L^2 + 3a_Lw + a_H(a_L + w) + 2(k + w^2))/\sqrt{\mu}))/\mu^2)
\]

(54)

where \( \lambda = a_H^2 + k + 2a_Lw + w^2 \),

\( \mu = a_H^2 + a_L^2 + 4a_Lw + 2a_H(a_L + 2w) + 4(k + w^2) \).

\[
\frac{\partial e_i^*(a_i = a_L, k, w|s = 1)}{\partial w} = \]

\[
\frac{1}{2}(2(-1 + p)(a_L + w)(-a_L + w + \sqrt{\lambda})/\lambda + (1 - p)(-3a_L^2 - k + 2a_Lw + 3w^2 + 3(a_L + w)\sqrt{\lambda})/\lambda + \]

\[
(p(a_H^2 - 2a_Ha_L - 3a_L^2 + 4k + 8a_Hw + 12w^2 + 2(a_H + a_L + 2w)(a_H^2 + 3a_Lw + a_H(a_L + w) + 2(k + w^2)))/\sqrt{\mu} + (a_H + 3a_L + 4w)/\sqrt{\mu}))/\mu - (4p(a_H + a_L + 2w)(-a_H^2a_L - 2a_Ha_L^2 - a_H^3 - 4a_Lk + a_H^2w - 2a_Ha_Lw - 3a_L^2w + 4kw + 4a_Hw^2 + 4w^3 + (a_L^2 + 3a_Lw + a_H(a_L + w) + 2(k + w^2))/\sqrt{\mu}))/\mu^2)
\]

(55)
where \( \lambda = a_l^2 + k + 2a_L w + w^2 \),
\( \mu = a_H^2 + a_L^2 + 4a_L w + 2a_H(a_L + 2w) + 4(k + w^2) \).

\[
\frac{\partial e_i^*(a_i = a_L, k, w|s = 1)}{\partial \mu} = \left( -a_H^2 - 2a_H a_L - a_L^2 - 4a_H k + a_H^2 w - 2a_H a_L w - 3a_L^2 w + 4k w + 4a_H w^2 + 4w^3 + \right)
\left( \lambda + 3a_L w + a_H(a_L + w) + 2(k + w^2) \right) \sqrt{\mu} / \mu \quad (56)
\]

where \( \lambda = a_L^2 + k + 2a_L w + w^2 \),
\( \mu = a_H^2 + a_L^2 + 4a_L w + 2a_H(a_L + 2w) + 4(k + w^2) \).

A.4.2 Agent with High Ability

\[
\frac{\partial e_i^*(a_i = a_H, k, w|s = 1)}{\partial a_H} =
\frac{1}{2}\left( -2p(a_H + w) \left( -a_H + w + \sqrt{\lambda} \right) / \lambda + p \left( 3a_H^2 - k - 2a_H w + w^2 + 3a_H w \sqrt{\lambda} \right) / \lambda + \right)
(1 - p) \left( 3a_H^2 - 4a_H a_L - a_L^2 - 4k - 6a_H w - 2a_L w + \right)
\left( a_H + a_L + 2w \right) \left( a_H^2 + 2k + w(l + 2w) + a_H(a_L + 3w) \right) \sqrt{\mu + (2a_H + a_L + 3w) / \mu} /
\left( a_H^3 + 2k + w(a_L + 2w) + a_H(a_L + 3w) \right) \sqrt{\mu} / \mu \quad (57)
\]

where \( \lambda = a_H^2 + k + 2a_H w + w^2 \),
\( \mu = a_H^2 + a_L^2 + 4a_L w + 2a_H(a_L + 2w) + 4(k + w^2) \).

\[
\frac{\partial e_i^*(a_i = a_H, k, w|s = 1)}{\partial a_L} =
- (a_H - a_L)(1 + p) \left( a_H - a_L \right) \left( -2 + p \right) / \mu^{3/2} \quad (58)
\]

where \( \mu = a_H^2 + a_L^2 + 4a_L w + 2a_H(a_L + 2w) + 4(k + w^2) \).

\[
\frac{\partial e_i^*(a_i = a_H, k, w|s = 1)}{\partial k} =
(a_H^2 p \sqrt{\mu} - 4(k + w^2)(-2\sqrt{\lambda} + 2p \sqrt{\lambda} - p \sqrt{\mu} +
\left( a_H^2 (4\sqrt{\lambda} - 3p \sqrt{\lambda} + 2p \sqrt{\mu}) + \left( 4a_L w(3\sqrt{\lambda} - 3p \sqrt{\lambda} + 2p \sqrt{\mu} + \right) \right) a_H(4w(\sqrt{\lambda} - p \sqrt{\lambda} + \sqrt{\mu}) + a_L(4\sqrt{\lambda} - 3p \sqrt{\lambda} + 2p \sqrt{\mu}))) / \mu^{3/2} \sqrt{\lambda} \quad (59)
\]

where \( \lambda = a_H^2 + k + 2a_H w + w^2 \),
\( \mu = a_H^2 + a_L^2 + 4a_L w + 2a_H(a_L + 2w) + 4(k + w^2) \).
\[
\frac{\partial e^*_i(a_i = a_H, k, w| s = 1)}{\partial w} = \\
\frac{1}{2}(-2p(a_H + w)(-a_H + w + \sqrt{\lambda})/\lambda + \\
p(-a_H^2 + k + 2a_H w + 3w^2 + 3(a_H + w)\sqrt{\lambda})/\lambda + \\
((1 - p)(-3a_H^2 - 2a_H a_L + a_L^2 + 4k + 8a_L w + 12w^2 + \\
2(a_H + a_L + 2w)(a_H^2 + 2k + w(a_L + 2w) + a_H(a_L + 3w))/\sqrt{\lambda} + \\
(3a_H + a_L + 4w)\sqrt{\mu})/ \\
\mu - (4(1 - p)(a_H + A_L + 2w) \\
(-a_H^3 - 2a_H^2 a_L - a_H a_L^2 - 4a_H k - 3a_H^2 w - 2a_H a_L w + a_L^2 w + 4k w + 4a_L w^2 + 4w^3 + \\
(a_H^2 + 2k + w(a_L + 2w) + a_H(a_L + 3w))\sqrt{\mu})/\mu^2)
\]

where \(\lambda = a_H^2 + k + 2a_H w + w^2,\)
\(\mu = a_H^2 + a_L^2 + 4a_L w + 2a_H(a_L + 2w) + 4(k + w^2).
\]

\[
\frac{\partial e^*_i(a_i = a_H, k, w| s = 1)}{\partial p} = \\
\frac{1}{2}(-a_H + w + \sqrt{\lambda} - \\
(-a_H^3 - 2a_H^2 a_L - a_H a_L^2 - 4a_H k - 3a_H^2 w - 2a_H a_L w + a_L^2 w + 4k w + 4a_L w^2 + 4w^3 + \\
(a_H^2 + 2k + w(a_L + 2w) + a_H(a_L + 3w))\sqrt{\mu})/\mu
\]

where \(\lambda = a_H^2 + k + 2a_H w + w^2,\)
\(\mu = a_H^2 + a_L^2 + 4a_L w + 2a_H(a_L + 2w) + 4(k + w^2).
\]