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Optimal pricing of the Portuguese telephone service

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A first attempt is made at finding the optimal two-part tariff for Portuguese telephone service. A theoretical model of consumer demand is presented and the two-part tariff which maximizes social welfare is derived. The model is then calibrated using Portuguese data and previous econometric studies. The optimal long-run two-part tariff consists of an access charge that is close to twice the marginal cost of access and a usage charge 25% above the marginal cost of use. The level of the optimal two-part tariff depends mainly on the marginal cost of taxation and on an index of aversion to inequality. The structure of the optimal two-part tariff depends mainly on the ratio between the marginal consumer's usage level and the average usage level. Although the cost estimates are very tentative, the results suggest that the 1986 usage charge was greater than optimum. No similar conclusion can be drawn with respect to the access charge.

I. INTRODUCTION

The literature on two-part tariffs dates back to 1946, when R. Coase suggested a two-par tariff to solve the 'marginal cost controversy'. The issue was how to reconcile efficient pricin (i.e. marginal-cost pricing) with the requirement of a balanced budget. If there are economie of scale, e.g. a large fixed cost and a constant marginal cost, then marginal-cost pricin implies a loss equal to the fixed cost F. Coase suggested that this loss be covered by an acces fee equal to F/n, where n is the number of customers.

If the total number of consumers is not constant, but rather depends on the access an usage charges, then Coase's solution is not optimal. On the other hand, even if the number o consumers does not change with tariffs, distributional concerns may warrant a tarif structure different from that proposed by Coase. In general, the optimal solution consists o a positive access fee and a usage charge which may be above or below marginal cost. O (1971), Feldstein (1972b), and Ng and Weissner (1974) presented the main results on th optimal structure of two-part tariffs.[1]

[1]There is also a voluminous theoretical literature on the efficiency and distribution effects of margina cost pricing and two-part tariffs, from a general equilibrium perspective. See Vohra (1988) an references therein.

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Despite the significant developments of the theoretical literature, few empirical applications have been attempted. Feldstein (1972a), Alleman (1977), and Mitchell (1978) are three exceptions. This paper makes a first attempt at finding the optimal two-part tariff for the Portuguese telephone service. The model differs from that of Feldstein in that it considers an elastic demand for access instead of a fixed number of subscribers. On the other hand, it differs from those of Mitchell and Alleman in that it derives the optimal two-part tariff and accounts for efficiency and distribution effects. In fact, it has been argued that optimal public sector pricing and taxation ought to 'be seen in an integrated approach where both distribution of income and allocation of resources are considered' (Bös, 1984, p. 166).[2] An attempt is made to incorporate these ideas by maximizing an 'adjusted social welfare' function, which will be defined below.

The paper is organized as follows. Section II introduces a theoretical model of consumer demand for telephone services and derives the welfare maximizing two-part tariff. The model is similar to that of Brown and Sibley (1985: Section 4.5 and Appendix), but is adapted in order to reflect both efficiency and distribution concerns. Section III explains how the model was calibrated using Portuguese data and econometric studies. The main numerical results are presented in Section IV. Section V concludes the paper.

II. THE MODEL

Utility and demand

It is assumed there is a population of potential telephone users. Each user has income y and a utility function $U(d, du, x)$, where: $d=1$ if the consumer has access to the telephone network and $d=0$ otherwise; u is a measure of telephone usage (e.g. number of phone calls); and x is the numeraire good.[3]

Each consumer solves the problem

$$\max_{d,u,x} U(d, du, x) \text{ s.t. } d(P_A + P_U u) + x = y \tag{1}$$

where P_A is the access charge and P_U the usage charge. Suppose that $u^* \equiv u(P_U, y-P_A)$ and $x^* \equiv x(P_U, y-P_A)$ are the optimal demands conditional on $d=1$. Clearly, $d=1$ if and only if $U(1, u^*, x^*)$ is greater or equal to $U(0,0,y)$, the utility of a consumer who is disconnected from the network. Denote the indirect utility function by $V(P_U, y-P_A)$, and define y' as the indifferent consumer's income level, i.e.

$$V(P_U, y'-P_A) = U(0,0,y'). \tag{2}$$

It is assumed that both x and u are normal goods, i.e. have non-negative income elasticities. This implies that a consumer prefers joining the network if and only if y is greater than y'. Assuming that income is distributed with density $f(y)$, demand for access (A) and demand for usage (U) are thus given by

$$A = \int_{y'}^{\infty} f(y)\, dy \tag{3}$$

[2] See also Bös (1986), Feldstein (1972a, 1972b, 1974).

[3] The appearance of d as an independent argument of the utility function reflects the fact that consumers may derive utility from the simple fact of being connected to the telephone network. This is an instance of what is known as 'option demand' (cf. Taylor, 1980).

and

$$U = \int_{y'}^{\infty} u(y) f(y)\, dy \tag{4}$$

Costs

It is assumed that there is a fixed cost F and constant marginal costs both for connection and for use. That is, the cost function has the form

$$C(A, U) = F + C_A A + C_U U. \tag{5}$$

where C_A and C_U are the marginal costs of access and use, respectively.

Social surplus

The optimal solution maximizes the adjusted social welfare

$$W = CS + \lambda PS, \tag{6}$$

where CS and PS are the consumer and the producer's surplus, respectively:

$$CS = \int_{y'}^{\infty} \omega(y)\, V(P_U, y-P_A) f(y)\, dy \tag{7}$$

where $\omega(y)$ is the welfare weight of a consumer with income y,

$$PS = \int_{y'}^{\infty} [u(P_U)(P_U - C_U) + P_A - C_A] f(y)\, dy, \tag{8}$$

and λ is a parameter which reflects the relative value of producer surplus.

It is not considered, as is usual in the literature, that the firm is subject to a budget constraint. Rather it is assumed that a positive profit in the provision of telephone service is essentially a substitute for government taxation, and similarly a negative profit has to be financed by means of government taxation. Maximizing total surplus subject to a budget constraint would thus be a 'third-best' solution, to use Ng's (1977) terminology. Public pricing being a substitute for government taxation, the two instruments should be used together in an optimal way, thus achieving the second-, not the third-best.[4]

We should note that the maximization of social welfare subject to a budget constraint is formally analogous to the maximization of adjusted social welfare: in the former case, we would be maximizing a Lagrangean function like Equation 6, where $\lambda - 1$ would be the Lagrange multiplier.

Optimal solution

The first-order conditions for the maximization of Equation 6 are given by

$$(P_U) - \int_{y'}^{\infty} \alpha(y) u(P_U, y-P_A) f(y)\, dy - \lambda [u(P_U, y'-P_A)(P_U - C_U)$$

$$+ P_A - C_A] f(y') \frac{\partial y'}{\partial P_U} = 0 \tag{9}$$

[4] This includes, as will be shown, using public pricing as a means for income distribution.

and

$$(P_A)\int_{y'}^{\infty}\alpha(y)f(y)dy+\lambda\int_{y'}^{\infty}f(y)dy-\lambda[u(P_U,y'-P_A)(P_U-C_U)$$
$$+P_A-C_A]f(y')\frac{\partial y'}{\partial P_A}=0 \quad (10)$$

where $\alpha(y)$ is the monetary welfare weight of a consumer with income y.[5]

Rearranging Equation 10, we get

$$\frac{P_A-C_A+u'(P_U-C_U)}{P_A}=\left(1-\frac{\alpha_A}{\lambda}\right)\frac{1}{\varepsilon_A} \quad (11)$$

where $u'\equiv u(P_U,y'-P_A)$ is the indifferent consumer's demand for use, $\alpha_A\equiv\int_{y'}^{\infty}\alpha(y)f(y)dy$ is the averaged social marginal utility of income of telephone users, and $\varepsilon_A\equiv-(\partial A/\partial P_A)(P_A/A)$ is the price elasticity of demand for access.

Differentiating Equation 2 and applying Roy's Lemma gives

$$\frac{\partial y'}{\partial P_U}=\frac{\partial y'}{\partial P_A}u' \quad (12)$$

Dividing Equation 9 by Equation 10 and using Equation 12, gives

$$-U+\alpha U-\alpha(P_U-C_U)\frac{\partial U}{\partial P_U}=u'(\alpha A-A) \quad (13)$$

(Recall Equations 3 and 4.) Some algebra manipulation finally leads to

$$\frac{P_U-C_U}{P_U}=\left[1-\frac{\alpha_U}{\lambda}-\left(1-\frac{\alpha_A}{\lambda}\right)\frac{u'}{\bar{u}}\right]\frac{1}{\varepsilon_U} \quad (14)$$

where $\alpha_U\equiv\int_{y'}^{\infty}\alpha(y)u(y)f(y)dy$ is the averaged social marginal utility of income of telephone users weighted by $u(y)$, $\varepsilon_U\equiv-(\partial U/\partial P_U)(P_U/U)$ is the price elasticity of demand for use, and $\bar{u}\equiv U/A$ is the average demand for use.

As can be seen from Equations 11 and 14, the optimal two-part tariff has the same nature as Ramsey–Boiteux optimal pricing: both the access charge P_A and the usage charge P_U are inversely proportional to the elasticity of demand for access and demand for use, respectively.

The numerator on the left-hand side of Equation 11 reflects the profit from an additional user in the network. In general, this would be given by the difference between price and marginal cost, as in Equation 14. However, when a new consumer joins the network, not only P_A-C_A is gained but also the profit arising from additional demand for use, $u(P_U-C_U)$.

[5]Equations 9 and 10 are only approximate first-order conditions, for variations of the marginal utility of income are ignored. The qualitative features of the results are not affected by this simplifying assumption, and the quantitative impact is likely to be small.

From the right-hand side of Equations 11 and 14, it can be seen that the optimal tariff depends on the relative value of money held by the government (λ) or by telephone users (α_A and α_U). The higher λ is, or the lower α_A and α_U are, the higher the optimal P_A and P_U will be.[6] Notice that the values of α_A and α_U affect not only the level, but the structure of the optimal tariff as well. If telephone usage is concentrated in consumers with low weight on social welfare [low $\alpha(y)$], then a higher margin should be put on P_U as opposed to P_A. The value of λ depends on the efficiency and equity effects of increased government taxation used in financing the telephone company. In the extreme case where distributive concerns are ignored, the value of λ equals one plus the marginal efficiency cost of raising an extra tax dollar.

Finally, the optimal usage charge also depends on the ratio between the marginal consumer's use and average use ($\beta\equiv u'/\bar{u}$). To quote Brown and Sibley (1985):

> The effect of this adjustment term is to force the [public firm] to take account of the fact that when [P_U] is increased, not only does consumption fall, but participation [i.e. demand for access] declines, too. This keeps the usage charge [P_U] at a lower markup over marginal cost [C_U] than would otherwise be the case. (p. 96)

If u' is very small, then variations in P_U have a relatively small impact on the marginal consumer's surplus. Therefore, the demand for access is relatively insensitive to changes in P_U. In the optimum, this relative monopoly power should be taken advantage of by increasing P_U, just as in standard Ramsey pricing.

III. CALIBRATION OF THE MODEL

The main purpose of this paper is to make a first attempt at finding the optimal two-part tariff for the Portuguese telephone service. This section describes the way in which the model previously presented was calibrated with Portuguese data. The process of calibration is a mixture of applying the results of previous econometric studies, adjusting parameters so as to replicate the values of the 'benchmark' year (1986), and a certain dose of 'wild guessing'.

First, some assumptions are made that allow Equations 11 and 14 to be written as a function of parameters which can be more readily calibrated:

(A1) Income is distributed with a Pareto distribution, i.e.

$$f(y)=\theta y^{0\theta}y^{-(1+\theta)} \quad (15)$$

where y^0 is the minimum income level.

(A2) The welfare weights $\alpha(y)$ are derived from the isoelastic social welfare function

$$W=\int_{y^0}^{\infty}k'y^{1-\delta}f(y)dy \quad (16)$$

which, normalizing the average $\alpha(y)$ to be one, yields

$$\alpha(y)=\frac{\delta+\theta}{\theta}\left(\frac{y^0}{y}\right)^{\delta} \quad (17)$$

[6]In the limit, if $\lambda=\infty$, then the optimization problem reduces to that solved by Oi (1971).

(A3) The income elasticity of telephone usage is constant:

$$u(P_U, y-P_A)=v(P_U,P_A)y^{\eta_U}. \tag{18}$$

Based on these assumptions, Equations 11 and 14 can be rearranged to get

$$M_U \equiv \frac{P_U-C_U}{C_U}=\left[1-\left(\frac{1}{\theta-\eta_U}-\frac{1}{\theta+\delta-\eta_U}\frac{\alpha_A}{\lambda}\right)\frac{\eta_U}{\varepsilon_U}\right]^{-1}-1 \tag{19}$$

and

$$M_A \equiv \frac{P_A-C_A}{C_A}=\left[1-\left(1-\frac{\alpha_A}{\lambda}\right)\frac{1}{\varepsilon_A}\right]^{-1}\left[1-M_U\beta\frac{1-\sigma}{\sigma}\right]-1 \tag{20}$$

where

$$\alpha_A = A^{\delta/\theta}, \tag{21}$$

$$\theta=\frac{\eta_U}{1-\beta} \tag{22}$$

$\beta \equiv u'/\bar{u}$, and σ is the fraction of access costs on total variable costs.[7]

In order to solve Equations 19–22, a total of nine values are needed, including demand parameters, cost parameters, policy parameters, and benchmark values.

(i) **Demand parameters:** ε_A, ε_U, η_U, **and** β. The first three values were obtained based on the studies by Cabral (1985), Melo (1985) and Pereira (1987). Concerning the value of β, note that from Equation 12

$$u'=\frac{\dfrac{\partial y'}{\partial P_U}}{\dfrac{\partial y'}{\partial P_A}}=\frac{\dfrac{\partial A}{\partial P_U}}{\dfrac{\partial A}{\partial P_A}}=\frac{\varepsilon_{AU}P_A}{\varepsilon_A P_U}, \tag{23}$$

where $\varepsilon_{AU} \equiv \partial A\, P_U/\partial P_U A$. Based on the benchmark values of P_A, P_U and $\bar{u}$, and again on the results of Melo (1985) and Pereira (1987), the value of β can be obtained.

(ii) **Cost parameter:** σ. Unfortunately, very little is known about the cost structure of Portuguese telecommunications. We base ourselves on the study by Park and Mitchell (1987) and other studies referenced therein.

(iii) **Policy parameters:** δ and λ. The value of δ should be interpreted in the following manner. Suppose there are two consumers, one with income y such that $\alpha(y)=1$ and the other with double that income. The value of δ measures the amount society is willing to sacrifice in order to transfer one dollar from the rich to the poor consumer. λ, on the other hand, is the value of one dollar held by the government. If the government budget had no distributive impact, λ would be one plus the marginal cost of taxation. In general, its value is expected to be less than that, but still greater than one.

(iv) **Benchmark values:** A and $\bar{u} \equiv U/A$.

For most important parameters, a central value and two alternative ones, one higher and one lower are considered. Therefore, when presenting the results, a central scenario (the 'base-case') and several alternative ones are considered. Table 1 presents the values of the parameters for which different values were used.

[7]The derivation of these results is available from the author upon request.

Table 1. *Value of parameters considered in simulations*

	Central	High	Low
ε_A	0.35	0.45	0.25
ε_U	0.50	0.60	0.40
η_U	1.00	1.25	0.75
β	0.75	0.85	0.65
σ	0.50	0.75	0.25
λ	1.25	1.40	1.10
δ	0.25	0.50	0.00

IV. RESULTS

The optimal margins for the base-case (first column of Table 1) are $M_A=119\%$ and $M_U=23\%$: in the optimum, the access charge should be roughly twice the marginal cost, while the usage charge should be relatively close to marginal cost.

The values on Table 2 and the arrows in Fig. 1 give an idea of how sensitive the results are to changes in some of the parameters used in the simulations. Each line of Table 2 gives the optimal values of M_A and M_U as a function of a given parameter while keeping the other parameters at their base-case levels. So, for example, if the elasticity of access is low (i.e. $\varepsilon_A=0.25$, from Table 1), then the optimal two-part tariff is $M_A=541\%$ and $M_U=23\%$. Figure 1 shows graphically how the optimal values of M_A and M_U change as each parameter changes from the base-case value to the high value. Based on these results, it is concluded that:

1. The optimal value of P_U tends to be close to marginal cost and is relatively insensitive to changes in most parameters other than β.
2. The optimal value of P_A, on the contrary, seems to be very sensitive to changes in various parameters, especially the marginal cost of taxation λ and the access elasticity ε_A. Access fees are a good substitute for taxation, and thus a high marginal cost of taxation elsewhere implies a high optimal value of P_A.
3. The structure of the optimal two-part tariff is mainly affected by the value of β, while the level depends chiefly on the values of λ and δ.

Table 2. *Optimal margins as a function of various parameters*

	M_A		M_U	
	Low	High*	Low	High
ε_A	541	61	23	23
ε_U	105	128	30	18
η_U	124	116	25	21
β	85	131	52	9
σ	29	149	23	23
λ	32	453	14	31
δ	106	136	15	30

* Cf. Table 1.

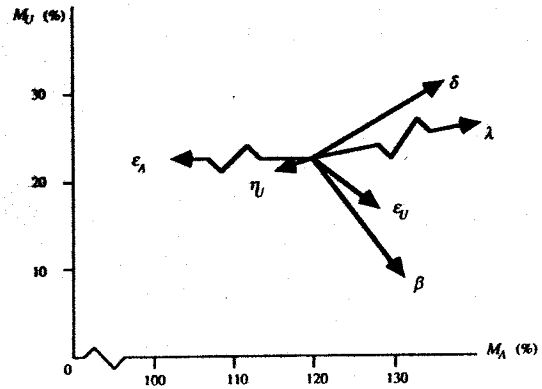

Fig. 1. Variation in optimal margins as a function of an increase in some parameters

Another issue of interest is to compare the optimal margins with the actual margins implicit in the benchmark values (1986). In order to do so, we would have to know the actual values of C_A and C_U in 1986, which we do not. Based on the scale elasticity estimated by Seabra (1985), it is assumed that variable costs are 70% of total cost. The analysis is then made contingent on the value of σ, the fraction of access costs on total variable costs. The results on Table 3 suggest that, for a fairly wide range of values of σ, the actual 1986 value of M_U was higher than optimum. As for M_A, the results are ambiguous. In the more likely case that $\sigma = 50\%$, they suggest that the 1986 access charge was lower than optimum.

Table 3. *Optimum margins and actual 1986 margins as a functions of σ*

	M_A		M_U	
σ	Optimum	Actual	Optimum	Actual
25%	29	204	23	71
50%	119	52	23	156
75%	149	1	23	412

V. FINAL REMARKS

It should be stressed that this paper is just a first attempt at finding the optimal two-part tariff for the Portuguese telephone service. There are several aspects in which the present analysis can be improved. The first one is quality of data, especially cost data. It has been shown that the optimal value of the toll charge is close to marginal cost. A more reliable estimate of marginal cost would thus be of great use.

The main results seem to be very sensitive to changes in parameters which are imperfectly known. Therefore, an important task from a policy viewpoint is to estimate the welfare costs of implementing the 'optimal' two-part tariff based on the erroneous values of those parameters (Heyman *et al.*, 1988).

Another area for further work is the disaggregation of demand by types. Though demand was considered to be homogeneous, it is known that the patterns of demand can vary between urban and rural populations, local and long-distance traffic, and so on. Insofar as it is possible to discriminate between these different types of demand, one should contemplate the possibility of a multiple-part tariff, with varying access and toll charges.

It should finally be noted that the analysis of this paper refers to optimal pricing from a long-run perspective. It would be expected that the results would be different in the short-run, both because of demand conditions (e.g. different elasticities) and supply conditions (e.g. capacity constraints.)

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