On the Evolution of the Firm Size Distribution: Facts and Theory

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Using a comprehensive data set of Portuguese manufacturing firms, we show that the firm size distribution is significantly right-skewed, evolving over time toward a lognormal distribution. We also show that selection accounts for very little of this evolution. Instead, we propose a simple theory based on financing constraints. A calibrated version of our model does a good job at explaining the evolution of the firm size distribution. (*JEL* L11)

Since the seminal study by Robert Gibrat (1931), several authors have looked at the patterns of firm growth and their implications for the firm size distribution (FSD): Peter E. Hart and Sigbert J. Prais (1956), Herbert A. Simon and Charles P. Bonnini (1958), Edwin Mansfield (1962), Yuji Ijiri and Simon (1964, 1977), among others. Conventional wisdom received from these studies has held that expected firm growth rates are independent of size (Gibrat's Law), and that the FSD is stable and approximately lognormal. However, recent empirical evidence (David S. Evans, 1987; Bronwyn H. Hall, 1987), based on more complete data sets than used in the past, shows that the relation between growth and size is not constant but rather decreasing. This suggests that the distribution of firm size in more complete sets of data may evolve over time and differ from a lognormal distribution. Yet, none of the studies that focused on the FSD examined empirically its *evolution,* either at the industry or at the economywide level.[1]

The goal of this paper is twofold. First, to derive some stylized facts concerning the FSD and its evolution over time (Section I). Second, to propose a theoretical explanation for the observed stylized facts, namely one that is based on financing constraints (Section III). The data sources we use in the paper are mainly from Portuguese manufacturing firms. However, as we argue below, several of the features of the Portuguese data sets are consistent with findings from other countries.

The main findings of this investigation are the following: First, the data suggest that the distribution of the logarithms of firm size *of a given cohort* is very skewed to the right at time of birth, and gradually evolves towards a more symmetric distribution. In particular, the data are consistent with this distribution converging towards a lognormal distribution. The *total* firm size distribution, in turn, is fairly stable over time, and somewhat skewed to the right.

One possible explanation for this pattern is selection, especially if we consider that exit rates are higher among smaller firms. However, the data shows that selection only accounts for a very small fraction of the evolution of firm size, most of the observed changes in the FSD being due to the evolution of the distribution of the survivors of a given cohort.

The model we offer to account for this evolution assumes that a firm's initial size is the minimum of its desired size and the entrepreneur's wealth constraint, whereas mature surviving firms are not financially constrained. This implies that the evolution of the size distribution is determined by firms ceasing to be

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[1] John Sutton (1998) presents a theory with implications for the evolution of the size distribution of a given cohort (among other implications); but the empirical test of his theory does not include dynamic data. Patrick McCloughan (1995) simulates the evolution of the FSD using alternative assumptions about the relationship between size and growth, and about the entry and exit processes. His analysis, however, focuses on concentration as a summary measure of the FSD, rather than on the whole distribution. See Sutton (1997) for additional references.

financially constrained. Calibrating the model to replicate the first three moments of the estimated distribution, we find that it does a good job at explaining the evolution of the FSD.

I. Stylized Facts

In this section, we present a number of stylized facts concerning the firm size distribution of Portuguese manufacturing firms. Some of these facts confirm previous results obtained from data for other countries. This is reassuring, as it suggests that our choice of data for Portuguese firms is not very restrictive. Some other facts presented in this section, namely the evolution of the FSD, go somewhat beyond what was presented in previous work. In fact, presenting these "novel" stylized facts, as well as a theory to explain them, is the main goal of the paper.

A. *The Firm Size Distribution: An Overview*

In this subsection we analyze the size distribution of firms operating in Portuguese manufacturing in 1991. Two sets of data are used. The first data set was obtained from a private firm, IF4, that collects balance sheet data from firms that are legally required to publicly report their accounts. These are typically the largest firms in the economy.[2] Restricting to those firms operating in manufacturing for which data on employment is available, a sample of 587 firms results. This sample is the kind of sample that has typically been used in previous work on the firm size distribution, typically based on U.S. data.[3]

The second data source is a survey conducted by the Portuguese Ministry of Employment–*Quadros do Pessoal* (*QP*). This is a comprehensive survey, covering all firms employing paid labor in the economy. This makes the data set a very good source for the study of the FSD, at least if we restrict our attention to firms employing paid labor; in manufacturing, it records 33,678 firms. An additional advantage of this data source is that it includes firm-level information on the number of employees. The main weakness of this source is that, since it has mainly labor-related data, the number of employees is the only available measure of firm size.

We estimate density distributions based on these two data sets. Since we are particularly interested in comparing the actual distribution of firm sizes to the lognormal distribution, we use the logarithm of employment as a measure of firm size. Rather than imposing a particular form on the FSD, we use nonparametric estimation methods, which provide a very convenient way of estimating the density without imposing much structure on the data.[4]

The first set of estimates is produced using the sample of firms with publicly available information. As can be seen in Figure 1, the firm size distribution is reasonably symmetric, bell-shaped, and in fact similar in shape to the normal distribution (or, rather, the lognormal distribution, as the x-axis is on a log scale). In fact, excluding one outlying observation, the Jarque and Bera test yields a value of 0.719, based on which one cannot reject normality. Broadly speaking, this result is in line with much of the previous work on the FSD, which, like Figure 1, is based on data sets of firms with publicly available data.

Consider now the density estimated from the more complete set of manufacturing firms. Figure 2 shows that, in contrast with the previous

[2] In fact, this data is mainly used to produce a list of the top 1,000 firms, published every year by a Portuguese newspaper. The data is also digitally available, including the entire set of firms for which public information exists.

[3] This suggests that there is nothing special about the Portuguese economy as regards the firm size distribution. Further evidence is given in the Appendix, where we show that similar patterns can be found in various countries. We also show that the main features found for manufacturing as a whole can also be found in particular industries.

[4] We used a kernel density smoother. Using this method, each point of the estimated density function is obtained as a weighted sum of the data frequencies in the neighborhood of the point being estimated. The weighting function is typically a probability density function (p.d.f.), the normal density in our case. Varying the width of the neighborhood at each point (the bandwidth parameter) allows for control of the degree of smoothing in the estimated density (Bernard W. Silverman, 1986). For facilitating comparisons across distributions, all the densities presented in the paper were estimated using the same bandwidth of 0.5. Estimation with different bandwidths or kernels did not produce qualitatively different results.

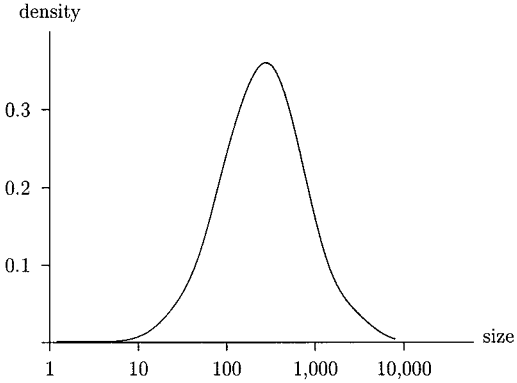


FIGURE 1. FIRM SIZE DISTRIBUTION, BASED ON EMPLOYMENT DATA FROM THE IF4 DATA SET

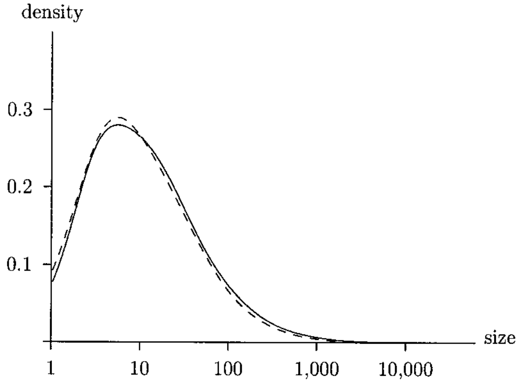


FIGURE 2. FIRM SIZE DISTRIBUTION IN 1983 (SOLID LINE) AND 1991 (DASHED LINE), BASED ON EMPLOYMENT DATA FROM THE *QUADROS DO PESSOAL* DATA SET

plot, the shape is clearly different from the normal, rather suggesting that the complete FSD is probably far more skewed than what the previous literature has posited. The figure also suggests that the firm size distribution is fairly stable over time. In fact, even though most firms existing in 1991 did not exist in 1983, the densities for these two years are remarkably similar.

Figures 1 and 2 show that the lognormal is not a good description of the FSD when the entire population of firms is considered, although it fits reasonably well the population of firms for which accounting data is publicly available. An obvious implication is that the firms for which public information is available are not a random sample from the total population. More interestingly, the results also reveal that this set of firms is neither the right tail of the whole distribution. Rather, the data seem to result from a sampling process in which larger firms are selected with increasing probability in such a way that the resulting product distribution is close to a lognormal.

B. *Firm Age and the Firm Size Distribution*

The previous subsection characterized the distribution of the population of firms in a given period. In this subsection, we are interested in the distribution of firm size by age.

There are two ways of analyzing the effect of age on the FSD. The first one is to use a cross-section of firms for which there is information on age and to analyze the FSD for groups of different ages.[5] Although our data set does not include firm age, it does include a variable that can be used as proxy for age, namely the longest tenure in the firm. Based on this proxy, we divided firms into the following age groups as measured by the longest tenure: 1, 2–4, 5–9, 10–19, 20–29, and 30 or more years.[6] Naturally, the longest tenure is a lower bound for firm age; it cannot be expected to be an accurate measurement of age. But since the discrepancy is especially significant for older firms and a residual class of 30 or more years is considered, we expect our age classes to be approximately correct.[7]

Figure 3 plots the nonparametric estimate of the FSD for each of the age groups. The plots clearly indicate that age plays an important part in the process of shaping the FSD. The size distribution for very young firms is highly

[5] The advantage of this approach is that it allows the use of the complete sample, covering the whole spectrum of firm ages. The disadvantage is that the groups are highly heterogeneous, for they include firms that were created at different times and subject to different selection processes.

[6] For the sake of brevity, we will refer to the above categories as "age groups."

[7] An additional source of error results from the way the data is collected: The information on the longest tenure concerns the calendar year in which the person joined the firm, and the survey is referred to the month of March; therefore, our one-year-old firms are indeed firms with less than three months of age. (This actually enables us to look at the size distribution of very young firms.) Likewise, firms in the 2–4 group are firms aged from three months to three years and three months, and so on.

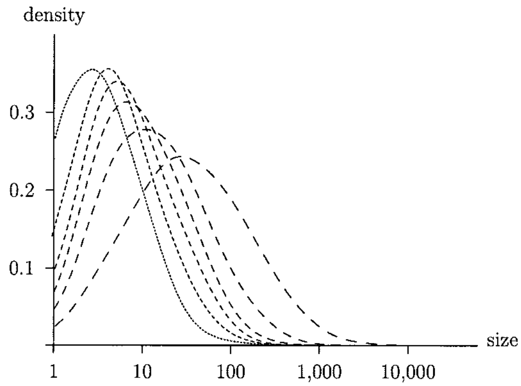


FIGURE 3. FIRM SIZE DISTRIBUTION BY AGE GROUP, BASED ON EMPLOYMENT DATA FROM THE *QUADROS DO PESSOAL* DATA SET

Note: Longer dash sizes correspond to older firms.

concentrated in small values and is far more skewed than the overall firm size distribution. As firms age, the distribution moves towards the right-hand side. The mode increases, the right tail becomes thicker and the left tail thinner, and the degree of skewness decreases significantly.

A parametric quantification of the evolution shown in Figure 3—based on the extended generalized gamma distribution—is given in Table 1. If firm size (s) follows an extended generalized gamma distribution, then $w = (\ln s - \mu)/\sigma$ has p.d.f.

$$\begin{cases} |q|(q^{-2})^{q^{-2}}\exp(q^{-2}(qw - \exp(qw)))/\Gamma(q^{-2}) \\ \quad q \neq 0 \\ (2\pi)^{-1/2}\exp(-\frac{1}{2}w^2) \\ \quad q = 0 \end{cases}$$

where $\Gamma(t) = \int_0^\infty x^{t-1}e^{-x}\,dx$ is the gamma function. One of the advantages of this distribution is that it can assume different forms depending on the value of the shape parameter q. For $q = 0$, w follows the standard normal distribution or, equivalently, s follows a lognormal distribution. For $q < 0$, the density of w is positively skewed (long tail at the right); for $q > 0$, the density is negatively skewed.[8]

TABLE 1—FIRM SIZE DISTRIBUTION BY AGE COHORT

Age group	μ	σ	q
All firms	1.762	1.185	−0.707
	(0.011)	(0.005)	(0.015)
Age ≤ 1	0.738	0.778	−1.000
	(0.066)	(0.036)	(0.146)
Age 2–4	1.322	0.953	−0.566
	(0.019)	(0.009)	(0.031)
Age 5–9	1.693	1.021	−0.426
	(0.021)	(0.009)	(0.032)
Age 10–19	1.975	1.092	−0.346
	(0.021)	(0.009)	(0.031)
Age 20–29	2.386	1.236	−0.323
	(0.033)	(0.014)	(0.042)
Age ≥ 30	3.362	1.499	−0.118
	(0.034)	(0.015)	(0.037)

Notes: Standard errors are in parentheses. Results for parametric estimations are based on the extended generalized gamma distribution. For $q = 0$, the extended generalized gamma reduces to a lognormal distribution.

The values in the table clearly show that the FSD becomes more symmetric as firms age. However, it is noticeable that, even for firms aged over 30, the distribution of (log) size is still far from symmetric. In fact, the left tail in Figure 3 is still thicker than the right tail; alternatively the value of q from Table 1 is still significantly different from zero.[9] Obviously, there is no reason to assume that the process towards a symmetric distribution reaches its steady state at the age of 30, but 30 years seems to be quite a respectable age for a firm, an age that only a small minority of entrants is likely to achieve. Our results thus seem to suggest that the lognormal distribution would fit the size distribution of firms for a small minority of firms only.

II. Selection

In the previous section, we studied the evolution of the firm size distribution by classifying firms by age. Strictly speaking, this only characterizes the evolution of the firm size

[8] This is the parameterization suggested by Vern Farewell and Ross L. Prentice (1977). See also Jerry F. Lawless (1980) for thorough presentation of this parameterization. The plot of the densities estimated with the extended generalized gamma is in the Appendix. The densities are very similar to the nonparametric ones.

[9] Note that exits do occur; otherwise, the concentration of firms on the left tail would be even greater.

distribution under the assumption that the basic conditions are constant over time. An alternative for studying the effect of age on the FSD is to use longitudinal data, specifically, to identify cohorts of firms and follow their evolution over time.[10] The disadvantage of using longitudinal data is that samples are much smaller and, in most cases, only a limited life span can be covered. One advantage is to reduce the heterogeneity found in cross-section, age-based data. An additional advantage, as we will see below, is to allow for a simple test of selection as an explanation of the evolution of the FSD.

We consider the cohort of firms that entered Portuguese manufacturing in 1984 and follow them until 1991. The effect of aging is evaluated by comparing the FSD's of this set of firms in 1984 and 1991. From the 2,651 firms identified as new in 1984, only 1,031 were still active in 1991. This leads to three different distributions of interest. The first one is the distribution of all entrants in 1984; the second one, the distribution of survivors in 1991; and the third one, the size distribution in 1984 of those firms that survived until 1991.

Figure 4 plots these three distributions. As in the previous figure, it is clear that the FSD after seven years (in 1991) is clearly less skewed than the FSD at birth. However, unlike in Figure 3, one can now identify two sources for this evolution. First, as of 1984 the total sample is more skewed than the sample of those firms that survive until 1991: selection plays a role in the shift of the FSD. Second, within the sample of survivors, the 1984 distribution is more skewed than the 1991 one: aging also plays a role in this shift.

Figure 4 shows that selection explains a very small part of the evolution of the firm size distribution. We believe this to be an important result, especially considering how much the theoretical literature, beginning with Boyan Jovanovic (1982), relies on selection as a main determinant of industry evolution.[11]

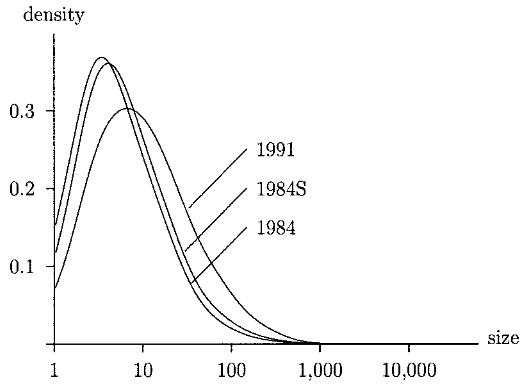


Figure 4. Size Distribution of the 1984 Cohort of Entrants: Densities Based on 1984 and 1991 Data as Well as 1984 Data for the Firms that Survived Through 1991

In the next section, we use the data from the 1984 cohort to investigate an alternative explanation for the evolution of the FSD: financing constraints.

III. Financing Constraints

Several authors (e.g., Steven M. Fazzari et al., 1988) have convincingly shown that financial constraints are a significant determinant of firms' investment decisions. In particular, this seems true for young firms (Evans and Jovanovic, 1989).[12] In this section, we consider the relevance of financing constraints for the evolution of the FSD. As we will see, financing constraints can to some extent explain the increased skewness in the size distribution that is typically observed in young cohorts of firms.

Suppose that financing constraints are especially relevant for young firms. Then, even if the long-run size distribution for a given cohort is close to symmetric, we should observe a significant skew to the right during the first periods,

[10] In the *QP* data set, each firm is identified by a specific code number. By comparing identification numbers in a sequence of years, we are able to identify entries and exits, and in particular we are able to follow a given cohort of firms.

[11] Thomas F. Cooley and Vincenzo Quadrini (2001) are an important exception to this rule.

[12] See also Robert Cressy (1996) and Bin Xu (1998).

that is, a large mass of small firms. Among this mass of small firms, some are small because they want to be small on efficiency grounds, whereas others are small because they are financially constrained. In future periods, when financing constraints cease to be binding, the latter will grow to their optimal size, thus giving rise to a more symmetric distribution of firm size.

A. *Model*

To formalize this intuition, consider the following two-period model of a competitive industry.[13] Suppose that each firm's efficiency, measured by θ, is constant in both periods. Moreover, assume that each entrepreneur is endowed with initial wealth, $w(\mathbf{z}, \varepsilon)$, where $\mathbf{z}$ is a vector of observed attributes and ε a random shock. Without loss of generality, assume that $w(\mathbf{z}, \varepsilon)$ is measured in firm size units; that is, $w(\mathbf{z}, \varepsilon)$ is the maximum capacity (measured in number of employees) that the entrepreneur can build given his or her wealth. This implies that actual first period size is the minimum of $s^*(\theta)$ and $w(\mathbf{z}, \varepsilon)$, where $s^*(\theta)$ is optimal size. In the second period, the firm is no longer subject to financing constraints and so actual size is equal to optimal size, $s^*(\theta)$. To summarize, we have $s^1 = \min\{w, s^*\}$ and $s^2 = s^*$, where s^t is size in period t.

In order to test the explanatory power of this simple model, we attempted to calibrate it using the set of 1984 entrants who survived until 1991, a total of 515 observations. Our first and second periods are given by 1984 and 1991, respectively. We thus have, for $i = 1, \ldots, 515$,

$$(1)\quad s_i^{84} = \min\{s_i^*, w(a_i, \varepsilon_i)\}$$
$$s_i^{91} = s_i^*.$$

Unfortunately, we do not have information on the entrepreneur's wealth. We know, however, the entrepreneur's age and education level, two potential proxies for wealth. Based on the econometric analysis presented in the next subsection, we decided that age, not education level, is likely to be a good proxy for wealth. Moreover, anecdotal evidence suggests that, even if the entrepreneur's private wealth is not sufficient to produce s^*, the desired output level may be possible based on family or other personal contacts. This amounts to saying that the effective w in equation (1) is not necessarily the entrepreneur's wealth, rather the value of the wealth he or she has access to.

Based on these considerations, we calibrated the following model of firm i's initial size:

$$(2)\quad s_i^{84} = \begin{cases} \min\{s_i^{91}, \beta a_i^2 e^{\varepsilon_i}\} \\ \quad \text{with probability } \alpha/a_i \\ s_i^{91} \\ \quad \text{with probability } 1 - \alpha/a_i \end{cases}$$

where a_i is age, α and β constants, and ε_i is a normally distributed shock with zero mean and variance σ^2. In words, equation (2) states that, with probability $1 - \alpha/a_i$, the entrepreneur has sufficient personal contacts so that he or she is not financially constrained. With probability α/a_i, initial size is the minimum of the desired size (on efficiency grounds) and the entrepreneur's personal wealth. Personal wealth, in turn, is a stochastic function of the entrepreneur's age.

In the remainder of this section, we first present the empirical results on the relation between entrepreneur's age, education, and firm size. Based on these results, we decided to use age as a proxy for wealth. Finally, we present the results from the model calibration as well as some measures of model fit.

B. *Age, Education, and Wealth*

As mentioned above, no data is available on entrepreneurs' wealth. The best we can aim at is finding good proxies for wealth. Under the hypothesis that our model is correct, namely that financing constraints are binding for young firms but not for mature ones, a good proxy for wealth should be such that it explains the output level of young firms but not the output level of mature firms.

[13] This model shares some of the features of Jovanovic's (1982) theory of industry evolution.

With this idea in mind, we estimate a series of equations explaining the log of firm size at each age of the 1984 cohort. As explanatory variables, we consider, in addition to industry dummies, the following two variables: the entrepreneur's education level and the entrepreneur's age. Education is a proxy for human capital: entrepreneurs endowed with better education should have better abilities and thus a greater θ, which translates into a greater output. Age has a double effect. After controlling for education, age is a proxy for labor market experience, which should raise efficiency. On the other hand, age is also a proxy for the existence of liquidity constraints, as potential entrepreneurs become wealthier as they grow older.

We are able to identify a sample of 515 firms surviving until 1991 for which there is complete data on age and education of all persons classified as "business owners." Using this sample, we attempt to distinguish the two effects of age by estimating the series of equations referred to above. If age is mostly reflecting liquidity constraints, then its effect should be important at birth but should vanish over time. If, on the contrary, age measures ability, then its effect should persist.[14]

Results from several regressions are shown in Table 2.[15] The results suggest that the effect of age is important mostly during startup. When firms are aged seven, their size is no longer influenced by the entrepreneur's age. This can be directly read from columns (1) and (3) in

TABLE 2—AGE OF ENTREPRENEURS AND FINANCIAL CONSTRAINTS

	(1)	(2)	(3)
1984 log(age)	0.594		0.587
	(0.122)		(0.124)
Age class 1		−0.354	
		(0.085)	
Age class 2		−0.134	
		(0.083)	
Age class 3		−0.018	
		(0.081)	
Years of education			0.058
			(0.001)
σ	0.594	0.603	0.631
	(0.035)	(0.035)	(0.032)
q	−1.138	−1.132	−1.043
	(0.169)	(0.168)	(0.149)
log-likelihood	−573.733	−572.494	−581.372
1991 log(age)	0.132		0.158
	(0.208)		(0.209)
Age class 1		−0.068	
		(0.122)	
Age class 2		0.218	
		(0.124)	
Age class 3		0.129	
		(0.119)	
Years of education			0.125
			(0.014)
σ	0.926	0.919	0.939
	(0.029)	(0.029)	(0.029)
q	−0.167	−0.186	−0.156
	(0.121)	(0.123)	(0.117)
log-likelihood	−693.604	−690.458	−700.580

Notes: Number of observations: 515; standard errors are in parentheses. Estimation includes 25 3-digit industry dummies and [in columns (1) and (2)] ten education dummies.

[14] An alternative interpretation is that entrepreneur's age is a good proxy for previous experience, which determines efficiency at time of start-up; and that, as firms grow, the effect of firm-specific experience swamps that of previous general experience, so that entrepreneur's age ceases to be a relevant determinant of efficiency. In other words, efficiency is eventually mainly determined by years of firm-specific experience, and this is the same for all firms in the same age cohort, regardless of the entrepreneur's age. Under this alternative explanation, the results that follow should be interpreted as pertaining to firms that are "experience constrained," rather than cash constrained, at time of start-up.

A test between the two alternative interpretations could be performed if we had a measure of the importance of previous experience for operating in each industry. The idea is that the importance of previous experience may vary across industries, whereas the effect of cash constraints is likely to apply equally across industries. Unfortunately, we have no data to proxy the importance of previous experience in each industry. However, when we include interaction terms between the age variables and industry dummies in the regressions reported in Table 2, we find no significant increase in the value of the likelihood function.

[15] Estimation was performed under the assumption that firm size follows an extended generalized gamma distribution. For briefness, we only present results from the first and the last years in our sample. Results for intermediate years are themselves intermediate. The age classes considered in the regressions are the following. Class 1: entrepreneur's birth date after 1950 (124 observations; average log size 1.465); Class 2: 1944–1949 (123; 1.840); Class 3: 1935–1943 (136; 1.983); Class 4: birth date prior to 1934 (132; 2.135).

Table 2. The t-statistic associated with the effect of age is above 4 in 1984 and below 1 in 1991. Also note that the point estimates in columns (1) and (3) are much smaller for 1991 than for 1984, and that in column (2) there is a monotonic increase in the coefficients associated with the different age classes for 1984 (the omitted class is the one with the oldest entrepreneurs) while this does not hold for 1991. The economic significance of the estimates is also considerable: in 1984, a firm owned by a young entrepreneur (group 1) would start up with a size approximately 30 percent lower than a firm owned by an old entrepreneur (group 4).[16]

Differently from the entrepreneur's age, the entrepreneur's level of education has an effect on firm size both at time of birth and afterwards. In fact, the coefficients associated with the level of education in the two periods are statistically different from zero. However, unlike age, the estimated coefficient in 1991 is significantly greater than the one in 1984. Together with the results for age, the results for the level of education suggest that, while education is proxying efficiency, age is to a large extent proxying cash constraints.

C. *Simulation/Calibration*

For each triplet (α, β, σ), we generated about one million observations of the estimated 1984 distribution according to (2).[17] We calibrated the values of (α, β, σ) in order to match the first three moments of the actual 1984 distribution (mean, variance, and skewness).[18]

The calibrated values are $\alpha = 26.15$, $\beta = 0.00460815$, and $\sigma = 0.344$. These values imply that the probability of being financially constrained goes down from 100 percent (entrepreneurs younger than 26.15) to 33 percent (80-year-old entrepreneurs), whereas the average maximum size allowed by financing constraints goes up from 1.8 employees (20-year-old entrepreneur) to 29.5 employees (80-year-old entrepreneur). For a fuller set of values, see Table 3.

TABLE 3—ENTREPRENEUR'S AGE AND FINANCING CONSTRAINTS ACCORDING TO CALIBRATED MODEL ($\alpha = 26.15, \beta = 0.00460815, \sigma = 0.344$)

Age at firm start-up	Probability of being constrained	Expected size if constrained
20	1.00	1.8
30	0.87	4.2
40	0.65	7.4
50	0.52	11.5
60	0.44	16.6
70	0.37	22.6
80	0.33	29.5

In order to get a visual impression of the fit, we estimated the 1984, 1991, and 1984E densities as before, that is, nonparametrically. The results can be found in Figure 5, where the 1984E density, the one corresponding to the calibrated model, is plotted in dashes. The fit seems good, although the implied 1984E distribution displays a longer right tail than the actual one.[19]

An additional measure of fit can be obtained by considering an alternative "minimalist" model where the only difference between 1984 and 1991 is proportional growth. Specifically, we calibrate an alternative model $s_i^{84} = \alpha s_i^{91}$, where α is calibrated to match the actual 1984 average size. In order to compare the two models, we compute the following measure of *FIT*:

[16] An alternative explanation for the drop in the age coefficient might be that all entrepreneurs are seven years older in 1991 than in 1984, and that age is mainly significant among young entrepreneurs. However, a restricted regression, excluding those firms whose entrepreneurs in 1984 are younger than the youngest entrepreneur in 1991 (25 years old) as well as those whose entrepreneurs in 1991 are older than the oldest in 1984 (81 years old), yields similar results.

[17] Specifically, we generated 2,000 times 515 observations.

[18] In addition to a quadratic equation, we also attempted to calibrate a linear and a cubic equation. The best results were obtained with a quadratic equation.

[19] As a reference, the coefficients of the μ, σ, and q parameters of the extended generalized gamma are 1.42 (0.06), 0.71 (0.03), −1.07 (0.13); 1.76 (0.04), 0.82 (0.03), −0.64 (0.08); 2.05 (0.13), 1.08 (0.002), −0.47 (0.12) for the 1984, 1984E, and 1991 distributions, respectively. The estimates for the 1984E distribution were obtained from 100 bootstrap samples of size 515 from the whole set of simulated firms.

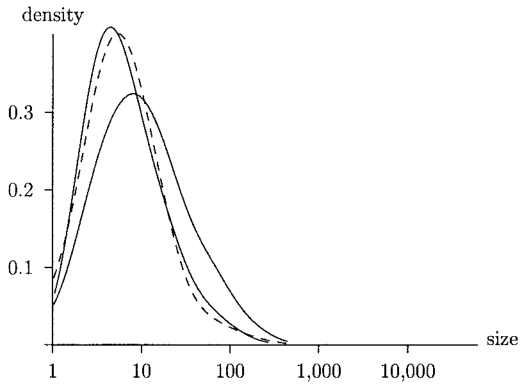


FIGURE 5. FINANCIAL CONSTRAINTS AND FIRM SIZE DISTRIBUTION: DENSITIES BASED ON ACTUAL DATA FOR 1984 AND 1991 (SOLID LINES) AND CALIBRATED 1984 DATA (DASHED LINE)

$$FIT = 1 - \frac{\sum_{i=1}^{N} (\hat{f}(i) - f(i))^2}{\sum_{i=1}^{N} (\tilde{f}(i) - f(i))^2}$$

where $\hat{f}$ is the frequency of the calibrated distribution, $\tilde{f}$ is the frequency of the "minimalist" model, and f is the actual frequency. We consider two possible frequency classes. First, we consider each number of employees to correspond to one class. Second, we consider class limits such that each class includes at least five observations (of the 515 actual observations).[20] The values of *FIT* are 0.72 and 0.74, respectively, suggesting that the choice of frequency classes is not very important. In summary, our model explains 70 to 75 percent of the variance not explained by a "minimalist" model of proportional growth.[21]

[20] This leads to classes with the following upper bounds: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 18, 20, 22, 24, 26, 29, 39, 49, 59, 89, ∞.

[21] Recall that our calibration is based on matching the first three moments of the actual 1984 distribution. An alternative criterion would be to maximize *FIT*, as defined above.

IV. Final Remarks

Past conventional wisdom has held that expected firm growth rates are independent of size (Gibrat's Law), and that the firm size distribution is stable and approximately lognormal. Recent empirical evidence, however, shows that the first of these facts does not hold when considering more complete data sets than those used in the past. In this paper, we show that the second fact—a lognormal distribution of firm size—also fails to hold in more complete data sets. Rather, the FSD seems quite skewed to the right but evolving over time toward a more symmetric one.

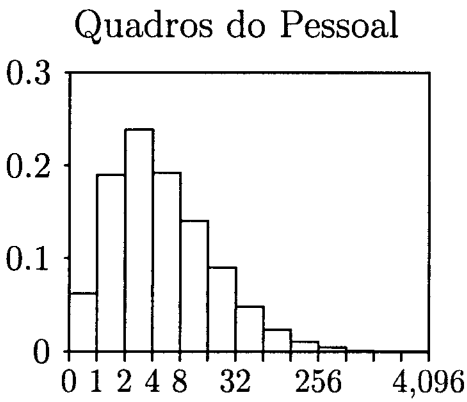
We propose an explanation for this behavior of the FSD, one that is based on financing constraints. Although our model is somewhat stylized, it does a reasonable job at accounting for the evolution of the firm size distribution. A promising line for future research is to incorporate more complex models of firm dynamics, both in terms of the evolution of optimal size (cf. Jovanovic, 1982; Hugo A. Hopenhayn, 1992; Richard Ericson and Ariel Pakes, 1995) and the source and impact of financing constraints (cf., Rui Albuquerque and Hopenhayn, 2001; Cooley and Quadrini, 2001).

APPENDIX

This Appendix contains additional information on the firm size distribution not included in the main text. This includes histograms of the raw data based on which nonparametric densities are estimated, international comparisons of the firm size distribution, and firm size distribution at the (5-digit) industry level.

Histograms from Raw Data.—Figure A1 includes the histograms of firm size from the two data sets referred to in the text: IF4 and *Quadros do Pessoal.* By comparison with Figures 1 and 2 in the main text, we conclude that the nonparametric density estimation is fairly accurate.

International Comparisons.—One possible limitation of our stylized facts is that they pertain



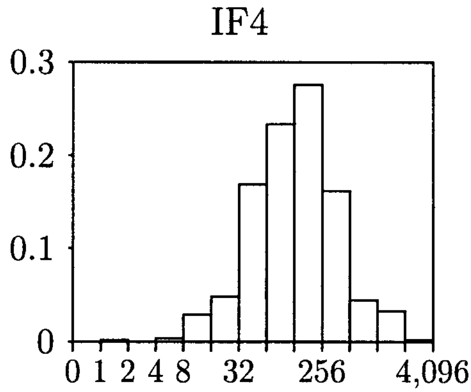


FIGURE A1. FIRM SIZE DISTRIBUTION IN PORTUGAL BASED ON TWO DIFFERENT DATA SETS

to a very special economy (Portugal). However, international comparisons of the total firm size distribution suggest that Portugal is not very different from other countries in this respect. Figure A2 depicts the firm size distribution in six different countries. All distributions have relatively similar shapes. Notably, the most similar distributions correspond to the two countries that are the most different in size (Portugal and the United States). In any event, the data does not give any credibility to the idea that Portugal is a special country in terms of the firm size distribution.

Firm Size Distribution by Age Group: Parametric Estimation.—In the main text, it is claimed that the parametric estimation based on the extended generalized gamma distribution produces results which are qualitatively similar to those of nonparametric estimation. Figure A3 confirms this claim.

Firm Size Distribution by Sector.—All of the analysis in the paper is conducted at the level of the manufacturing sector. Whenever possible, industry variables, down to the 5-digit level, are used. The advantage of using aggregate data is that more data points are available—and nonparametric estimation requires a large number of data points. However, lest it might be thought that the stylized facts reported in the paper depend on the level of aggregation, we present here results for selected 5-digit industries. The criteria for selecting these particular sectors is the number of firms: for most other industries, the number is insufficient for nonparametric estimation.

Figure A4 presents the 1984 and 1991 densities for six selected sectors. Comparison to Figure 2 in the text suggests that the qualitative features found in the complete data set are indeed found in sectoral data as well. Figure A5 replicates Figure 4 in the main text for the same sectors. Again, the claim that the selection effect explains very little of the evolution of the firm size distribution is confirmed at the sector level. Finally, Table A1 presents results for the parametric estimation based on the extended generalized gamma distribution which are very similar to those presented in Table 1 in the main text.

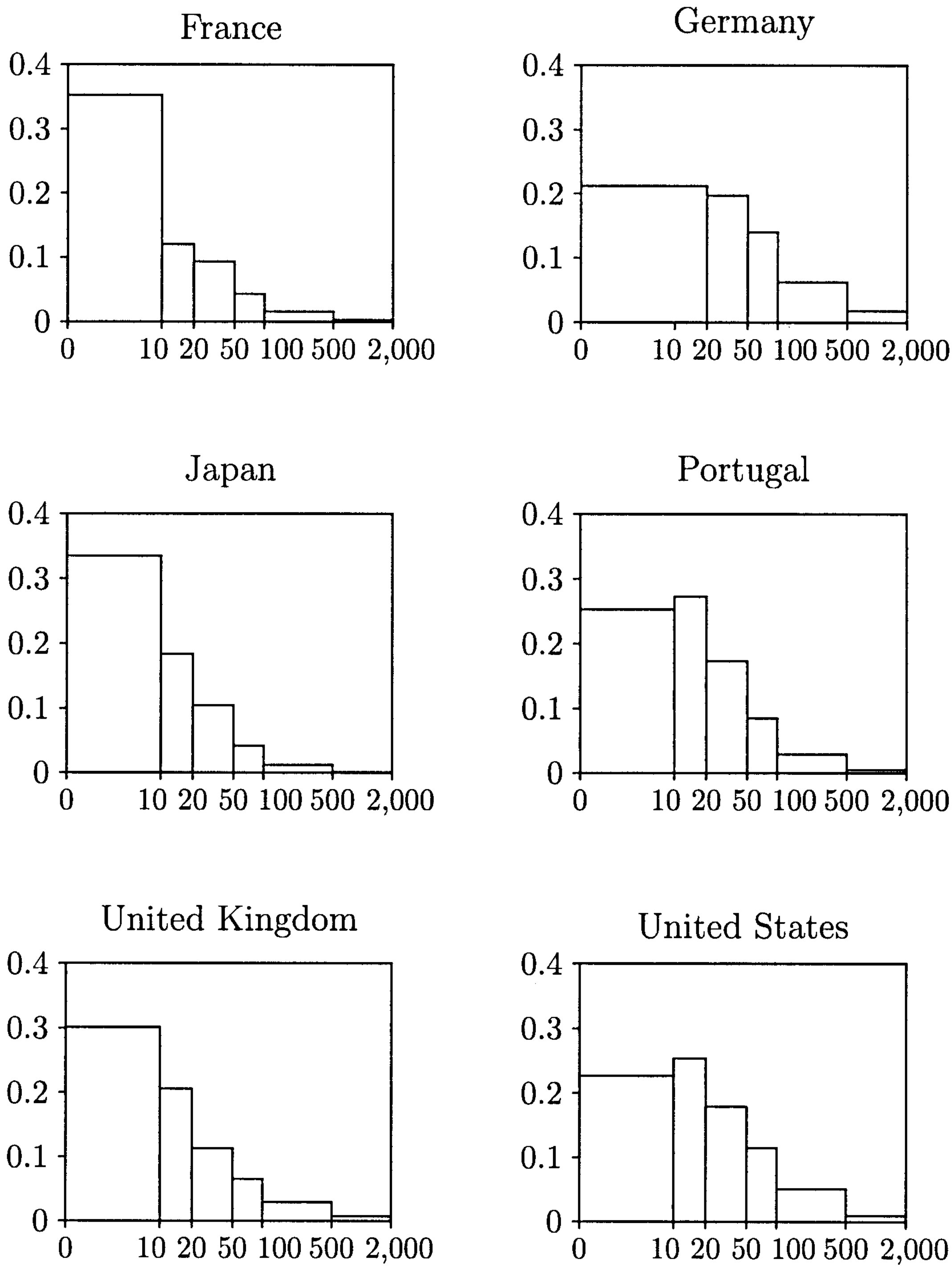


FIGURE A2. FIRM SIZE DISTRIBUTION IN SELECTED COUNTRIES

Notes: Figures for Portugal are based on *QP*. Figures for other countries are based on Bart van Ark and Eric Monnikhof (1996).

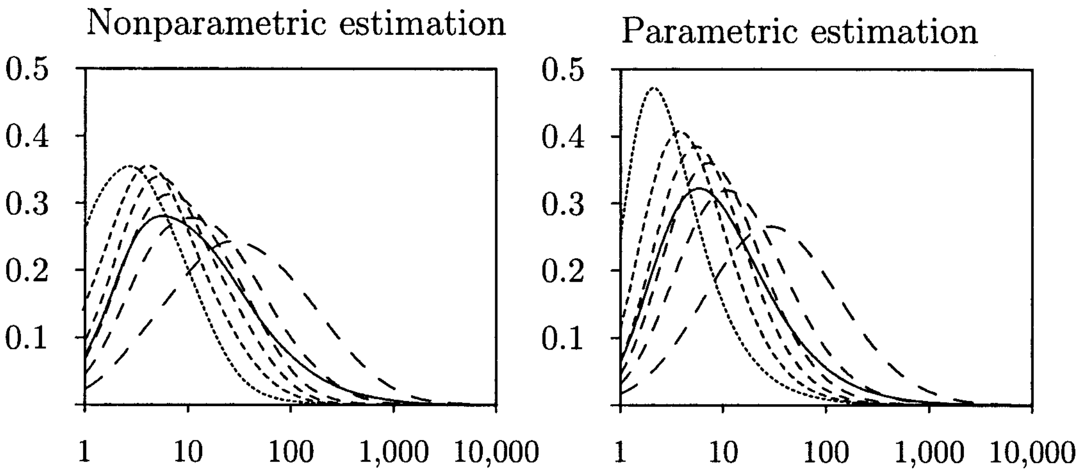


FIGURE A3. FIRM SIZE DISTRIBUTION BY AGE: COMPARISON OF NONPARAMETRIC AND PARAMETRIC ESTIMATION

Note: Dashed lines correspond to age group densities, older firms with longer dash size; solid line refers to the distribution of all firms.

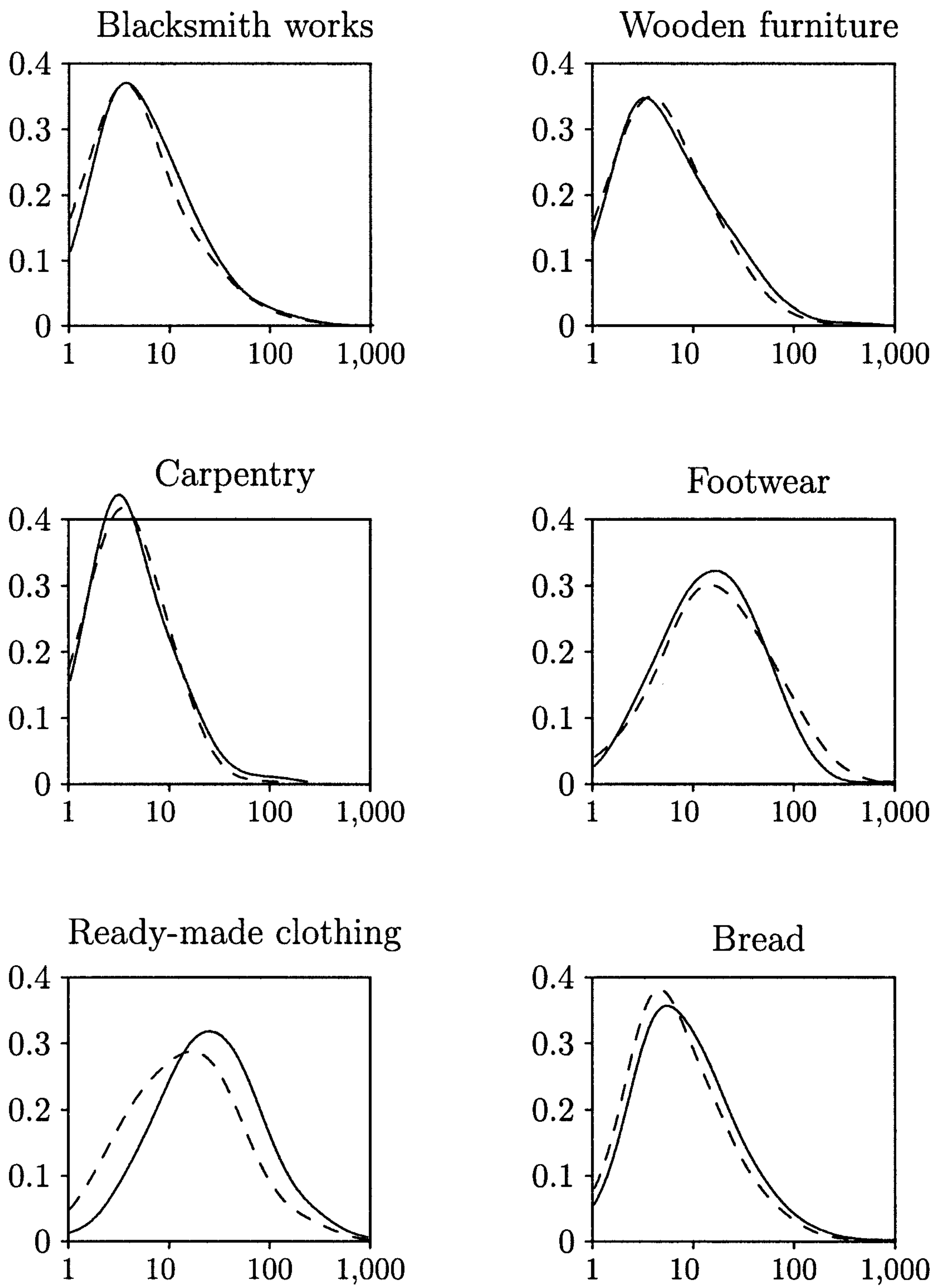


FIGURE A4. TOTAL FIRM SIZE DISTRIBUTION IN SELECTED SECTORS IN 1984 (SOLID LINE) AND 1991 (DASHED LINE)

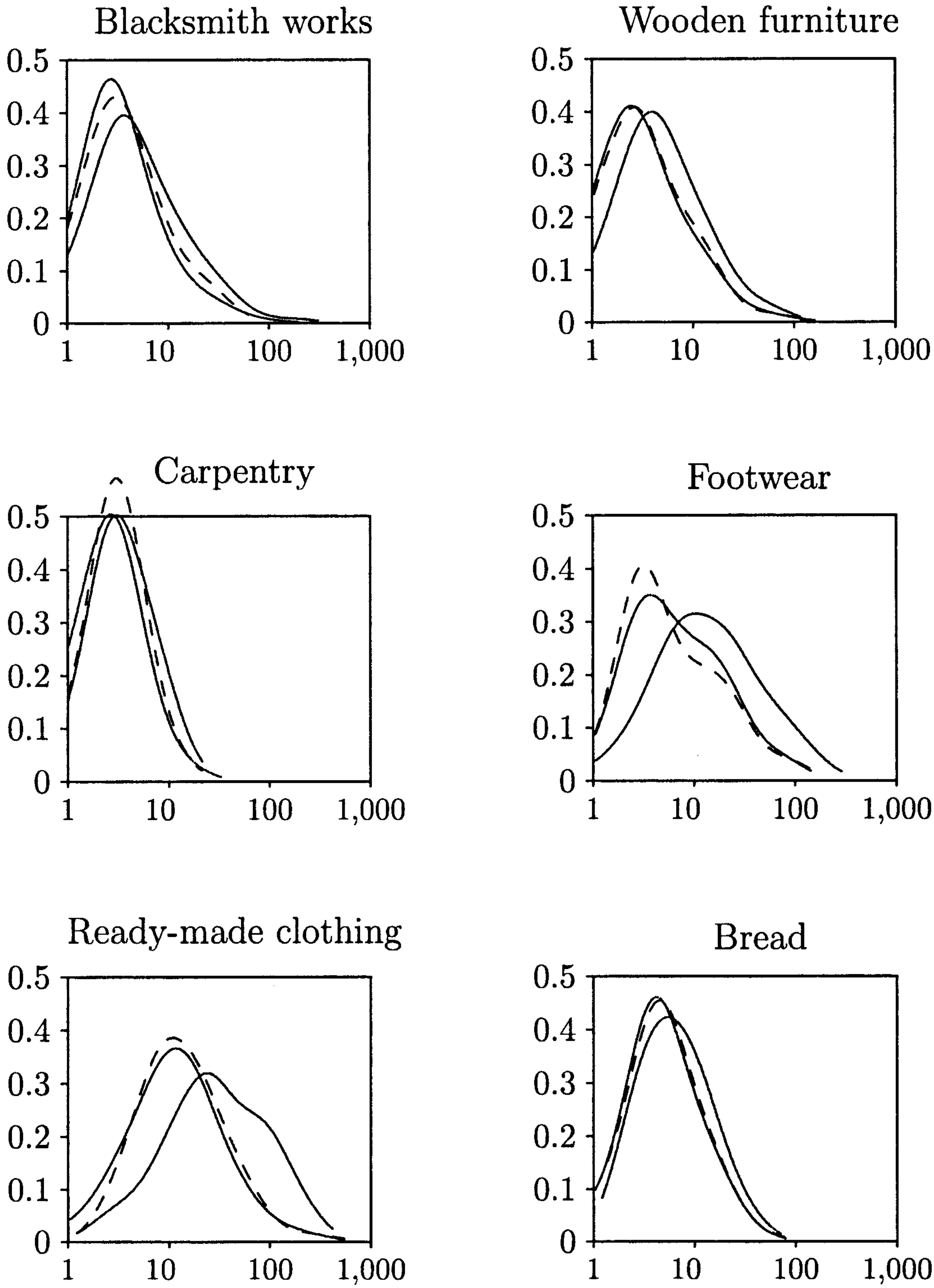


FIGURE A5. FIRM SIZE DISTRIBUTION OF THE 1984 COHORT IN SELECTED SECTORS: TOTAL 1984 AND 1991 DENSITIES (SOLID LINES) AND 1984 DENSITY OF SURVIVORS (DASHED LINE)

TABLE A1—PARAMETRIC ESTIMATION OF THE FIRM SIZE DISTRIBUTION ACCORDING TO THE GENERALIZED GAMMA DISTRIBUTION FOR SELECTED 5-DIGIT INDUSTRIES

	1984			1991		
Industry	μ	σ	q	μ	σ	q
Bread	1.73	0.91	−0.73	1.64	0.96	−0.54
	(0.04)	(0.02)	(0.07)	(0.04)	(0.02)	(0.06)
Ready-made clothing	3.10	1.23	−0.09	2.60	1.27	−0.17
	(0.05)	(0.02)	(0.07)	(0.03)	(0.01)	(0.04)
Footwear	2.68	1.18	−0.12	2.71	1.27	−0.21
	(0.07)	(0.03)	(0.09)	(0.06)	(0.03)	(0.07)
Carpentry	1.16	0.79	−0.77	1.06	0.81	−0.67
	(0.04)	(0.02)	(0.07)	(0.03)	(0.02)	(0.07)
Wooden furniture	1.27	0.93	−0.80	1.29	0.97	−0.63
	(0.04)	(0.02)	(0.06)	(0.03)	(0.01)	(0.05)
Blacksmith works	1.32	0.91	−0.85	1.21	0.93	−0.78
	(0.03)	(0.02)	(0.06)	(0.03)	(0.02)	(0.05)

Note: Standard errors are in parentheses.

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