Merger policy in open economies

Pedro P. Barros and Luís Cabral*

Universidade Nova de Lisboa, Lisbon, Portugal

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We extend Farrell and Shapiro's (1990) analysis of horizontal mergers to the case of an open economy. We show how the rules for approving a merger ought to be adapted to account for the fact that the regulator is only concerned with domestic welfare, that is, ignores the effect of the merger on foreign firms and consumers. We also explore the consequences of this externality in a model of a 'single market' which includes consumers and producers of different countries. In particular, we provide conditions under which a decentralized process of evaluating merger proposals à la Farrell-Shapiro can survive the externality mentioned above.

1. Introduction

The oligopoly theory literature on merger analysis has, for the most part, considered the case of a closed economy, that is, a well defined geographic market consisting of a number of producers and consumers, all belonging to the same country.1 Merger analysis then consists of looking at the effects of a proposed merger on total welfare, defined as the sum of consumers' and producer's surpluses (possibly a weighted sum).

This kind of analysis seems inappropriate for the study of economies where a great deal of competition in domestic markets is provided by foreign competition, either in the form of imports or foreign investment. The focus of this paper is precisely the normative analysis of mergers in open economies. In particular, we extend Farrell and Shapiro's (1990) analysis based on the concept of the external effect of a merger to the case of open economies.

We first consider the case of one single economy which is subject to foreign competition. The issue is how the rules for approving a merger ought to be adapted if the regulator were only concerned with domestic welfare (which excludes profits earned by foreign firms).

Correspondence to: Luís Cabral, Universidade Nova de Lisboa, Faculdade de Economia, Travessa Estevao Pinto, 1000 Lisboa, Portugal.

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1Exceptions to this characterization include the work of Landes and Posner (1981), Fisher (1987), and Schmalensee (1987).
The analysis is then generalized to the case of a 'single market' which includes consumers and producers of different countries. We characterize a second level of external effect, which corresponds to the welfare change not accounted for by each individual country. We also provide conditions under which a decentralized process of evaluating merger proposals à la Farrell–Shapiro can survive this second sort of externality.

2. The Farrell–Shapiro model

Recently, Farrell and Shapiro (1990) have put forward a simple equilibrium model which generalizes and extends some of the previous ideas in the literature on merger evaluation with theoretical foundations in oligopoly theory. Since most of our paper is based on theirs, we will describe some of their results in greater detail.

Following what was hinted at in the previous literature, Farrell and Shapiro (1990) take the view that, if a merger is proposed, then it must be profitable for the firms that are willing to be part of it. Therefore, it is a sufficient condition for a merger to be welfare enhancing that the 'external' effect (the effect on consumers’ surplus plus profits by firms not participating in the merger) be positive.

In order to evaluate the external effect of a merger, Farrell and Shapiro (1990) consider a given merger as a sequence of infinitesimal mergers, each corresponding to a change dQ in total quantity. They are able to show that

\[ dW - d\Pi_I = -\frac{\partial P}{\partial Q} \left( Q_I - \sum_{i \in I} \lambda_i q_i \right) dQ, \]

where

\[ -\frac{dq_i}{dQ} = \lambda_i = -\frac{\partial P/\partial Q + q_i \partial^2 P/\partial Q^2}{\partial^2 C_i/\partial q_i^2 - \partial P/\partial Q}, \]

where \( W \) denotes welfare and \( \Pi \) profits; \( I \) is the set of firms participating in the merger ('insiders') and \( O \) is the set of all other firms ('outsiders' to the merger); \( q_i \) is firm \( i \)'s quantity, \( Q = \sum q_i \), \( P(Q) \) is the inverse demand function, and \( C_i(q_i) \) is firm \( i \)'s cost function.

The value of \( \lambda_i \) is closely related to the slope of the reaction function, which is given by \( dq_i = -\lambda_i/(1 + \lambda_i) \ dQ \). It can be shown that: (i) \( \lambda_i > 0 \); (ii)

\[ \text{In most of the analysis, it is assumed that a merger implies an increase in equilibrium price. Farrell and Shapiro (1990) claim that 'rather impressive synergies - learning, or economies of scale - are typically necessary for a merger to reduce prices' (p. 114). Throughout the paper, we will maintain the same assumptions as Farrell and Shapiro (1990).} \]
$\lambda_i = 1$ if demand and cost are linear functions; (iii) $0 < \lambda_i < 1$ if marginal cost is non-decreasing and demand is sufficiently convex (specifically, if $\frac{\partial^2 P}{\partial Q^2} > \frac{\partial^2 C_i}{\partial q_i^2}/q_i$).

Eq. (1), together with some stability conditions, implies that a necessary and sufficient condition for the external effect to be positive is that

$$s_I < \sum_{i \in O} \lambda_i s_i,$$

where $s_i = q_i/Q$, $s_I = \sum_{i \in I} s_i$.

Inequality (3) provides a useful criterion for merger analysis in the context of a closed economy. In the next section, we derive a formula for the case of an open economy.

3. Extension to an open economy

The first extension we will consider is that of an economy which is open to entry by foreign firms and/or imports (the crucial feature is that there is some foreign supply in a domestic oligopolistic market). The departure from Farrell and Shapiro's (1990) analysis results from considering a regulator who is only concerned with domestic welfare, defined as the sum of consumer's surplus and domestic firms' profits. Let $I$ be the set of domestic firms participating in the merger, $O$ the set of domestic firms not participating in the merger, $D \equiv I \cup O$, and $F$ the set of foreign firms. Domestic welfare is then given by

$$W = \int_0^Q P(x) \, dx - P(Q)Q + P(Q)Q_D - \sum_{i \in D} C_i(q_i),$$

where $Q_D$ is total quantity produced by domestic firms: $Q_D = \sum_{i \in D} q_i$ (a similar notation will be used for the quantity produced by firms in $I$, $O$, and $F$).

Firm $i$'s profits are given by $\Pi_i = P(Q)q_i - C_i(q_i)$. We assume $\Pi_i$ to be a concave function for all $i$. As in Farrell and Shapiro (1990), we assume that a Cournot equilibrium holds both before and after the merger. This assumption puts our focus on the effects of changing the number of competitors in the market.\footnote{Farrell and Shapiro (1990, p. 116) state sufficient conditions for the external welfare effect of a merger, computed as a sequence of infinitesimal mergers, to have the same signal as that of the infinitesimal merger considered in (1). These are that $\frac{\partial^3 P}{\partial Q^3} \geq 0$, $\frac{\partial^2 C_i}{\partial q_i^2} \leq 0$, $\forall i \in O$, in the relevant ranges. In what follows, we will assume that these conditions are satisfied.}

The first-order (sufficient) conditions for profit maximization (from which equilibrium quantities result) are then given by

\footnotetext{The assumption of Cournot behavior is made mainly for expositional reasons. The extension to a general conjectural variations model is straightforward. In a still more general framework one can also consider a change in the oligopoly solution resulting from the merger.}
As we saw in the preceding section, Farrell and Shapiro's (1990) analysis is conducted in terms of the external effect of a merger, that is, the effect on consumers and on firms not participating in the merger. Since we are only concerned with domestic welfare, the external effect now only includes the impact on consumers and on domestic firms not participating in the merger. Our main result in this section establishes a necessary and sufficient condition for the external effect of a merger to be positive, and thus a sufficient condition for the impact of the merger on domestic welfare to be positive.

**Proposition 1.** The external effect of an infinitesimal merger is positive if and only if

\[ s_I + s_F < \sum_{i \in F} \lambda_i s_i, \]

where \( s_F = \sum_{i \in F} s_i \).

**Proof.** Differentiating \( W \), we get

\[ dW = -\frac{\partial P}{\partial Q} dQ + P(Q) dQ_d + \frac{\partial P}{\partial Q} dQ_d dQ - \sum_{i \in \emptyset} \frac{\partial C_i}{\partial q_i} dQ_i + dC_t, \]

where \( dC_t \) stands for total cost effect on merging firms (including any efficiency gains accruing from the merger).

The change in the merging firms' profits is given by

\[ d\Pi_I = P(Q) dQ_d + \frac{\partial P}{\partial Q} Q_d dQ - dC_t. \]

The external effect of an infinitesimal merger is then given by

\[ dW - d\Pi_I = -\frac{\partial P}{\partial Q} (Q + Q_d - Q_d) dQ + \sum_{i \in \emptyset} \left( P - \frac{\partial C_i}{\partial q_i} \right) dQ_i. \]

From the first-order condition (5) and the implicit function theorem we have

\[ \frac{\partial q_i}{\partial Q} = -\frac{\partial P/\partial Q + q_i \partial^2 P/\partial Q^2}{\partial P/\partial Q - \frac{\partial^2 C_i/\partial q_i^2}{\partial Q}} = -\lambda_i, \]

and thus \( dq_i = -\lambda_i dQ \). Furthermore, also from the first-order condition (5),
Therefore, (9) can then be re-written as
\[
\frac{dW - d\Pi}{dQ} = - \frac{\partial P}{\partial Q} (Q_i + Q_F) dQ + \sum_{i \in O} \frac{\partial P}{\partial Q} \lambda_i q_i dQ.
\]  
Rearranging terms, we get
\[
\frac{dW - d\Pi}{dQ} = \frac{\partial P}{\partial Q} \left( Q_i + Q_F - \sum_{i \in O} \lambda_i q_i \right) dQ.
\]

Since by assumption (cf. footnote 2) the merger implies a reduction in total market output, that is, \(dQ < 0\), the necessary and sufficient condition for \(dW - d\Pi\) to be positive is given by
\[
Q_i + Q_F - \sum_{i \in O} \lambda_i q_i < 0,
\]
from which (6) follows immediately. □

It can be easily shown that the result also applies to the case of a merger between domestic and foreign firms, or between foreign firms only. In the latter case, since the merging firms' profits do not enter the domestic welfare function, the condition that the external effect be positive is a necessary and sufficient condition for the total welfare effect to be positive.

Lacking knowledge of the exact values of \(\lambda_i\), some information regarding the demand and cost functions might be helpful in finding (weaker) conditions for a positive external effect. If marginal cost in non-decreasing and the demand function is sufficiently convex, then \(0 < \lambda_i < 1\), and \(s_t + s_F < \sum_{i \in O} s_i\), or simply \(s_t + s_F < 50\%\) (recall that \(s_t + s_F + s_O = 1\)), becomes a necessary condition. If both the demand and the cost functions are linear, then \(\lambda_i = 1\) and \(s_t + s_F < 50\%\) is equivalent to the condition in Proposition 1.\(^5\)

It is interesting to note that, contrary to what might at first be expected, foreign competition makes it actually more difficult for a merger to be approved [cf. (6)], assuming that the external effect is the criterion to be used. To see why, notice that the external effect can be divided into two

\(^5\)As an application of the above results, consider the recent merger between three major firms in the Portuguese insurance market, UAP Portugal, Aliança Seguradora and Garantia. The value \(s_t + s_F\) is in this case 50.18\% (life insurance) and 38.25\% (non-life insurance). Assuming demand is sufficiently convex and marginal cost non-decreasing, this suggests that the external effect of the merger would be negative in the case of life insurance; closer scrutiny of the merger would thus be warranted according to the external effect criterion.
terms. First, there is a decline in consumer surplus which results from the price increase. Second, there is an increase in profits by the firms not participating in the merger. This increase results from both an increase in price and an increase in quantity. But, in terms of domestic welfare, this second term is lower the greater the market share held by foreign firms. Hence, the external effect will more likely be negative when foreign firms hold a large market share.

It is important to note that this result is not inconsistent with the idea that foreign competition increases the competitiveness of the domestic market. In fact, it can be shown that if the number of foreign firms, as well as their market share, are both very large, then the price effect of a merger will be very small, and so will the absolute value of the external welfare effect. In other words: additional foreign competition may imply a negative external effect, but it also implies that its absolute value is lower. In terms of merger policy, the apparent inconsistency between the two ideas can be resolved by saying that when the absolute value of the external effect is very small, one should attach greater importance to the efficiency gains of a merger, that is, one should not follow the external effect criterion too literally.

Finally, we should note that, while the analysis has so far looked at the case of an importing economy, it is also possible to consider the extension to an exporting economy. One can easily show that the condition for a positive external effect is in this case more easily satisfied. The idea is that the gains in terms of domestic firms' profits more than outweigh the losses in terms of domestic consumer's surplus (consumers are very few, compared with the size of domestic firms). A small, open, exporting economy (e.g., Switzerland, in various sectors) should in general have a more lenient policy towards mergers — but for a very different reason than the small, open, importing economy.

4. Merger policy in Europe

The analysis in the previous section indicates that, in an open economy, there may still be welfare effects not accounted for by the domestic policymaker's external effect (namely, the effect on foreign firms' profits and on foreign consumers' surplus). This externality is one of the most compelling arguments in favor of a supra-national merger authority. In this and in the following sections, we develop a model of merger policy by a supra-national authority. In particular, we consider the case of a single market which includes several countries, each with its own merger authority. A good example — in fact, the example which motivates the analysis that follows — is the European Common Market.

Anticipating the creation of the European Single Market, a wave of European mergers, typically between firms of different European countries,
has now been going on for a while. Despite all this activity, it is still unclear which role will be played by the European Commission in terms of merger policy [cf. Jacquemin (1990)]. Merger policy was not explicitly dealt with in the Treaty of Rome. Rather, it has traditionally been guided by a comprehensive interpretation of Articles 85 and 86 of the Treaty. More recently, however, a new regulation was approved as an attempt to create the basis for an EC-wide merger policy (Regulation No. 4064/89, which became effective in 1990).

Unlike the United States, the new EC Merger Guidelines make no explicit reference to concentration levels that trigger closer scrutiny or the interdiction of a proposed merger. Rather, they define a series of thresholds above which a given merger is to be scrutinized by the Commission and below which decisions are delegated to national authorities. Our focus in the analysis that follows is precisely on the role played by the Commission vis-à-vis the role played by national merger authorities.

5. A model of a single market

Our stylized view of a single market is a measure of consumers whose preferences aggregate into a demand function $D(p)$ for some given homogeneous good, and a finite set of producers with cost functions given by $C_i(\cdot)$. What distinguishes this model from the case considered by Farrell and Shapiro (1990) is that both consumers and firms can be of different nationalities, and thus welfare measures will be different depending on whether one takes the perspective of the market as a whole or of a given constituent country in particular. As we will see, this crucially depends on the relation between each country's share in total demand (denoted by $d_k$) and in total supply (denoted by $s_k$).

We assume there exists a community-wide merger authority in addition to one in each member country. The central and the national authorities differ, first of all, in their objective functions: the former maximizes total welfare, whereas the latter's objective is national welfare (the sum of consumers' and producers' surplus for national consumers and producers).

6. External external effect

The external effect from the perspective of a given country will in general differ from the external effect from the community's perspective. The difference between these external effects constitutes an external external effect. Its characterization is an important step towards discussing the role played by a supra-national merger authority vis-à-vis national merger authorities.

We begin by deriving the condition for the external effect of a merger between firms within country $k$, from that country's perspective, to be positive.
**Proposition 2.** The external effect of an infinitesimal merger, from country k's perspective, is positive if and only if

\[ d_k - s_k + s_t - \sum_{j \in O_k} \lambda_j s_j < 0, \]  

which is also equivalent to

\[ d_k - s_k + \sum_{j \in O_{-k}} \lambda_j s_j + s_t - \sum_{j \in O} \lambda_j s_j < 0. \]  

**Proof.** Domestic welfare in country k is given by

\[ W_k = d_k \left( \int_0^Q P(x) \, dx - P(Q)Q \right) + P(Q)Q_k - \sum_{j \in O_k} C_j(q_j). \]  

Differentiating, we get

\[ dW_k = -d_k \frac{\partial P}{\partial Q} Q \, dQ + \frac{\partial P}{\partial Q} Q_k \, dQ + P \, dQ_k - \sum_{j \in O_k} \frac{\partial C_j}{\partial q_j} \, dq_j - dC_t. \]  

The variation in the merging firms' profits is in turn given by

\[ d\Pi_f = P \, dQ_t + \frac{\partial P}{\partial Q} Q_t \, dQ - dC_t. \]  

Therefore, the variation in the external effect is given by

\[ dW_k - d\Pi_f = -d_k \frac{\partial P}{\partial Q} Q \, dQ + s_k \frac{\partial P}{\partial Q} Q \, dQ - s_t \frac{\partial P}{\partial Q} Q \, dQ + \sum_{j \in O_k} \lambda_j s_j \frac{\partial P}{\partial Q} Q \, dQ \]

\[ = \left( d_k - s_k + s_t - \sum_{j \in O_k} \lambda_j s_j \right) \left( - \frac{\partial P}{\partial Q} Q \, dQ \right), \]

from which the first part of the proposition follows. The second part follows from the fact \( O = O_k \cup O_{-k} \).

Expressions (15) and (16) constitute generalizations of Farrell and Shapiro's (1990) results. The first equation, in particular, is useful for studying the divergence between the community's and each individual country's perspectives on merger policy. Recall that, since we assume the single market is an

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6The case considered by Farrell and Shapiro (1990) corresponds to \( d_k = s_k = 1 \) and \( O_{-k} = \emptyset \). Proposition 1 is also a particular case of Proposition 2, corresponding to \( d_k = 1, s_k = 1 - s_t \).
isolated market, Farrell and Shapiro's (1990) condition applies, that is, the external effect from the community's perspective is positive if and only if
\[ s_i - \sum_{j \in \mathcal{O}} \lambda_j s_j < 0. \]  
(21)

Comparison of (21) and (15) shows two sources of divergence between the community and country \( k \) regarding the size of the external effect. The first one is the additional term \((d_k - s_k)\) which appears in the second equation. The proof of Proposition 2 shows that this term results from the effect of the merger in the equilibrium price, an effect which is negative in consumer surplus and positive in firm revenues. From country \( k \)'s perspective, only country \( k \) consumers and country \( k \) firms matter. If \( d_k = s_k \), then both the positive and the negative effects in country \( k \) are proportionally smaller than the community's, so that country \( k \)'s external effect is not biased relative to the community's (on the count of the price effect). If \( d_k > s_k \), however, then country \( k \) is too 'conservative' from the community's perspective; country \( k \)'s external effect is smaller than the community's. On the contrary, if \( d_k < s_k \), then country \( k \)'s external effect is larger than the community's; country \( k \)'s decision are thus likely to be too biased (from the community's perspective) in favor of allowing a merger (assuming the policy is to allow mergers when the external effect is positive).

The second source of divergence between the community and country \( k \) regarding the size of the external effect is the difference in the last terms of (21) and (15). The proof of Proposition 2 shows that these terms result from the effect of the merger in the equilibrium quantities of non-merging firms, an effect which is positive (since \( \lambda_j > 0 \)). In the case of the community as a whole, the summation is taken over \( j \in \mathcal{O} \), whereas in the case of country \( k \) it is only taken over \( j \in \mathcal{O}_k \). The difference corresponds to the fact that country \( k \) ignores the effect of the merger on foreign firms. Since this effect is always positive, the bias is always in the direction of country \( k \) being too 'conservative' from the community's perspective.

7. Decentralized merger policy

Despite the divergence between national and community-wide external effects of a merger, it may be in the community's interest to decentralize decisions on mergers to the relevant national authorities. In the EEC Regulation of 1989, this is done based on a number of minimum thresholds below which decisions are to be taken at the country level. The implicit idea is that the Commission's staff can only handle a certain number of cases a year, and so they may as well choose the most important ones in terms of total size. However, this criterion ignores two important aspects. First, the decision to take up a case should reflect the desire to internalize all welfare
effects, in particular the external external effect of a merger; and, as we have seen, the size of this effect depends on the size of the merger as well as on market shares, in a way that is not reflected by the EEC Regulation thresholds. Second, while the community’s and each country’s perspectives may differ, the community’s policy should, as much as possible, use the information contained in each country’s decision (of allowing or not allowing a merger).

Suppose a merger is proposed between a set of firms in country $k$. Assume that each national authority $k$ possesses information on market shares and on the values of $\lambda_j$ (for all $j$ belonging to country $k$); and suppose that a decision is made by country $k$’s merger authority based on the criterion of allowing the merger if and only if the external effect is positive. The central authority has information on market shares but not on the values of $\lambda_j$. The question we now try to answer is: How should the central merger authority use the information contained in the decision by country $k$’s merger authority?

By analogy with hypothesis testing in statistics, we can set this as a problem of Type I and Type II errors. A Type I error occurs when the central authority upholds a decision by country $k$’s authority when in fact it should not do so. By contrast, a Type II error occurs when a decision by country $k$’s authority is not upheld when in fact it should have. For example, if the central authority (implicitly) gives approval to a merger which was given approval by country $k$, but the community-wide external effect is negative, then a Type I error is incurred.

Fig. 1 depicts the possibility of Type I errors as a function of the merging firms’ markets shares, $s_j$, and country $k$’s net imports share, $d_k - s_k$. The condition for a positive community-wide external effect, $s_l - \sum_{j \in O_k} \lambda_j s_j < 0$, is satisfied by all points below the horizontal thick line. In turn, the condition for a positive external effect in country $k$, $d_k - s_k + s_l - \sum_{j \in O_k} \lambda_j s_j < 0$, is satisfied by all points below the downward sloping thick line. The two lines intersect at point $X$, which coordinates are given by

$$X = \left( - \sum_{j \in O_k} \lambda_j s_j, \sum_{j \in O_k} \lambda_j s_j \right).$$

Four regions are defined by the two lines. Each region is identified by a pair of letters: lowercase $a$ and $r$ denote ‘approval’ and ‘rejection’ by country $k$’s authority, respectively; capital $A$ and $R$, on the other hand, denote ‘approval’ and ‘rejection’ by the central authority, respectively. Assuming that the

\[This seems consistent with the assumption that evaluating merger proposals involves large costs in terms of staff time (e.g., data gathering) needed to obtain the values of $\lambda_j$. If the central authority had access to the same information as the national authorities, then there would be no reason for merger policy to be decentralized at all.\]
central merger authority follows the policy of upholding the national authorities' decisions, then a Type I error occurs in regions $aR$ and $rA$. In the first case, a merger proposal is accepted even though the community-wide external effect is negative. In the second case, a merger proposal is rejected even though the community-wide external effect is positive.

What are the implications of this analysis in terms of the community's merger policy? Qualitatively, the idea is that if country $k$ is a large net exporter (low $d_k - s_k$), then the central authority should be careful about upholding a positive decision by country $k$'s merger authority. Likewise, if country $k$ is a large net importer (high $d_k - s_k$), then the central authority should be careful about upholding a negative decision by country $k$'s authority.

An interesting example of these ideas is the recent (negative) EC ruling on the proposed takeover by Aérospatiale SNI-Alenia and Selenia SpA of De Havilland, a Canadian aircraft manufacturer owned by Boeing. This was the first instance in which a merger or takeover was ruled out by the European Commission since Regulation No. 4064/89 was approved (cf. EC Bulletin, Supplement 2/90). The European Commission's decision was subject to strong criticism by the French and Italian governments. The ruling was based on the analysis of market concentration: the merger would give Aérospatiale-Alenia/De Havilland 67% of the EC market for commuting aircraft and about 50% of the world market. While demand data were not readily available, if we assume demand to be sufficiently convex, $s_i < 50\%$ is a necessary condition for an EC-wide positive external effect. This condition is not satisfied, so the likely effect would be a decrease in EC welfare if the merger were carried out. It is also clear that $d_k - s_k$ is low, which justifies the divergence between central and national authorities. France is a net exporter of aeronautic products ($s_e > d_e$), which means that those effects internal to the
country are mainly the positive ones, while the negative ones are, in a significant portion, borne out by foreign consumers. The data is thus consistent with a positive external effect for France and a negative one for the community as a whole.

The qualitative idea stated above can be made more precise. Notice that to the right of point $X$ a positive decision by country $k$ implies that the community-wide external effect is (also) positive; whereas to the left of point $X$ a negative decision by country $k$ implies that the community-wide external effect is (also) negative. Although the central authority does not know the exact coordinates of point $X$, sufficient conditions can be found which imply that we are to the left or to the right of point $X$. We can thus find sufficient conditions for a Type-I-error-free decentralized process.

**Proposition 3.** Suppose that country $k$ is a net importer, that is, $d_k > s_k$. If the external effect of a merger between firms in country $k$ is positive from the point of view of country $k$, then it is also positive from the point view of the community as a whole.

**Proof.** The proof is trivial by direct inspection of Fig. 1. Since the first coordinate of $X$ is negative, $d_k - s_k > 0$ implies that we are to the right of point $X$. Furthermore, a positive external effect from country $k$'s perspective means that we are below the downward sloping thickline. Together, these facts imply that we are in one of the no-Type-I-error regions. □

**Proposition 4.** Suppose that $0 \leq \lambda_j \leq 1$ (the demand function is convex or not too concave: marginal costs are non-decreasing) and that $d_k < 2s_k - 1$. If the external effect of a merger between firms in country $k$ is negative from the point of view of country $k$, then it is also negative from the point of view of the community as a whole.

**Proof.** The proof is analogous to that of the previous proposition. Notice that $d_k < 2s_k - 1$ is equivalent to $d_k - s_k < -(1 - s_k)$ and that $1 - s_k = \sum_{j \in O_k} s_j > \sum_{j \in O_k} \lambda_j s_j$ by the assumption that $0 \leq \lambda_j \leq 1$. □

8. Cross-border mergers

Many of the mergers which have recently taken place in Europe involve firms from different countries. While the results in the previous section were set with respect to a merger within one country, they can be extended to the case of multi-country mergers. In this section, we assume there is a merger between firms of two different countries. Three cases are considered, regarding the evaluations of each national authority: two positive evaluations; two negative evaluations; one positive and one negative. The proofs for the first two cases, analogous to those in the previous section, are omitted.
Proposition 5. Suppose that countries \( k \) and \( l \) are, on aggregate, net importers, that is, \( d_k + d_l > s_k + s_l \). If the external effect of a merger between firms in countries \( k \) and \( l \) is positive from the point of view of each country, then it is also positive from the point of view of the community as a whole.

Proposition 6. Suppose that \( 0 \leq \lambda_j \leq 1 \) (the demand function is sufficiently convex; marginal costs are non-decreasing) and that \( d_k + d_l < 2(s_k + s_l) - 1 \). If the external effect of a merger between firms in countries \( k \) and \( l \) is negative from the point of view of each country, then it is also negative from the point of view of the community as a whole.

We are left with the case when the external effects for each of the two countries have opposite signs. The question is then whether the central authority should uphold the decision made by one of the central authorities. The following proposition, which is actually valid for the general case of a merger between firms in any number of countries, provides the answer to the question. An extra piece of notation is necessary at this stage. Let \( I_k \) be the set of merging firms belonging to country \( k \) and \( I_{-k} \) the set of merging firms not belonging to country \( k \), that is, \( I = I_k \cup I_{-k} \).

Proposition 7. (i) If \( d_k > s_k + s_{I_{-k}} \) and the external effect of a merger is positive from the point of view of country \( k \), then it is also positive from the point of view of the community as a whole.

(ii) If \( 0 \leq \lambda_j \leq 1 \), \( d_k < 2s_k - 1 + s_{I_{-k}} \), and the external effect of a merger is negative from the point of view of country \( k \), then it is also negative from the point of view of the community as a whole.

Proof. A derivation similar to that in the proof of Proposition 2 establishes that the external effect from the point of view of country \( k \) is positive if and only if

\[
d_k - s_k + s_{I_k} - \sum_{j \in O_k} \lambda_j s_j < 0, \tag{23}
\]

which in turn is equivalent to

\[
d_k - s_k - s_{I_{-k}} + s_I - \sum_{j \in O_k} \lambda_j s_j < 0, \tag{24}
\]

since by definition \( s_I = s_{I_k} + s_{I_{-k}} \). The remainder of the proof is analogous to that of the previous results.

\[ \square \]

9. Total welfare effects

A merger policy based on the concept of the external effect minimizes the
possibility of a welfare-reducing merger proposal being approved. However, it often results in the rejection of welfare-enhancing merger proposals.

This fact has already been recognized by Farrell and Shapiro (1990), who considered a figure similar to fig. 2. The horizontal axis measures the size of the external effect, whereas the vertical axis measures the internal effect. Points above the downward sloping line $\Delta W = 0$ correspond to welfare-enhancing mergers, whereas points to the right of the vertical axis correspond to positive external effects. We thus have three regions to consider. Regions $A$ and $C$ correspond to correct decisions based on the external effect test. In fact, the sign of the external effect is the same as the sign of the total welfare change. In region $B$, however, a decision based on value of the external effect would lead to undue rejection of the merger proposal.

A decentralized merger policy based on the concept of the external effect induces additional sources of error. Even in a favorable case for the decentralized process described in the previous sections, the possibility of error by the central authority is increased. Fig. 3 depicts one such case, namely the case when $d_k > s_k$ and $\Delta W_k - \Delta \Pi_I > 0$ (for a merger between firms in country $k$). As we have seen, if $d_k > s_k$, then $\Delta W_k - d \Pi_I > 0$ implies that
\( \Delta W - \Delta \Pi_f > 0 \). Hence, the line \( \Delta W_k = 0 \) must be to the right of \( \Delta W = 0 \) and the line \( \Delta W_k - \Delta \Pi_f = 0 \) to the right of the vertical axis.

We have now six regions to consider. The only error-free regions are now \( A' \) and \( C \); by contrast, in region \( B' \cup B'' \cup A'' \cup A''' \) and error of undue rejection occurs. There are two ways the central authority can reduce this error. One is to obtain a correct estimate of the external effect. This will reduce the size of the error to \( B' \cup B'' = B \), the case considered by Farrell and Shapiro (1990). The second one is to obtain a correct estimate of the internal effect of the merger. This will reduce the size of the error to \( B'' \cup A''' \), an area that results from the external effect.

10. Conclusion

In this paper, we extend Farrell and Shapiro's (1990) equilibrium analysis of horizontal mergers to the case of an open economy. We show how the rules for approving a merger ought to be adapted to account for the fact that the regulator is only concerned with domestic welfare. We also explore the consequences of this externality in a model of a 'single market' which includes consumers and producers of different countries. In particular, we provide conditions under which a decentralized process of evaluating merger proposals à la Farrell–Shapiro can survive the externality above mentioned.

Recent decisions by the European Commission on cases of mergers and acquisitions have made it clear that the political, administrative, and even economic costs of making decisions on an ad hoc basis are far too high. There is a need for a clear set of rules designating when the central authority should intervene and how it should decide. Our paper is an attempt at contributing to this effort.

References