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Simultaneous entry and welfare☆

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Abstract

I examine the welfare properties of free entry under conditions of simultaneous entry. Specifically, I consider the second-best problem of influencing the number of entrants while taking as given firm behavior upon entry. I consider two alternative models of simultaneous entry: grab-the-dollar entry and war-of-attrition entry. I show that, if entry costs are low, then the results from previous models of sequential entry are fairly robust to the possibility of simultaneous entry. If however entry costs are high, then the welfare effects of free entry depend delicately on the nature of the entry game being played.

JEL classification: L1; L6; L9

Keywords: Entry; Welfare

1. Introduction

Economists frequently presume that free entry is desirable from a social welfare point of view. In fact, free entry is one of the conditions underlying the First Welfare Theorem. However, it is well known that, once one of the conditions for the theorem is removed, the other conditions are no longer necessarily desirable (Lipsey and Lancaster, 1956). Specifically, a number of authors have shown that, once we abandon the assumption of competitive pricing behavior, free entry may no longer be a good thing. See Williamson (1968), Spence (1976), Dixit and Stiglitz (1977),

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von Weizsäcker (1980), Perry (1984), Mankiw and Whinston (1986) and Suzumura and Kiyone (1987).[1]

One common feature to all of these models is the assumption that entry is a sequential process. Although this is the reasonable assumption in some cases, there are other cases when the assumption of simultaneous entry seems more reasonable. For example, in 1968 Lockheed and McDonnell Douglas simultaneously entered in the market for wide-body aircraft, leading both firms to huge losses (Bluestone et al., 1981; Newhouse, 1988; Cabral, 2001). More recently, British Satellite Broadcasting and Sky Television simultaneously attempted to enter the British satellite TV industry, though eventually BSB agreed to a merger that essentially left Sky under control (*The Economist*, November 10, 1990; Brandenburger and Ghemawat, 1994; Ghemawat, 1994).

In this paper, I examine the welfare properties of free entry under conditions of simultaneous entry. Like many of the previous authors, I consider the second-best problem of influencing the number of entrants while taking as given firm behavior upon entry. I do not model the post-entry game explicitly. Rather, I consider the degree of post-entry competition as one of the parameters of interest.

In addition to the model of sequential entry, which I include as a reference point, I introduce two alternative models of simultaneous entry: grab-the-dollar entry and war-of-attrition entry. The grab-the-dollar entry model assumes that entry is an instantaneous process and that entry decisions are simultaneously made in each period.[2] The war-of-attrition entry model assumes that entry takes time, i.e., in order to enter firms must spend a certain amount of resources over a period of time.

The comparison of the three alternative models of entry reveals striking similarities and striking differences as well. If entry costs are low, then all three entry models imply similar predictions, namely: if market competition is weak, then entry incentives are excessive, whereas tough market competition implies insufficient entry incentives. If entry costs are high, however, then free entry has very difficult implications across entry models: under the sequential model, free entry implies the second-best; under grab-the-dollar entry, there is insufficient entry; and under war-of-attrition entry there is excess entry. In summary, if entry costs are low, then the results from previous models of sequential entry seem fairly robust. If however entry costs are high, then the welfare effects of free entry depend delicately on the entry game being played.

The structure of the paper is as follows. In Section 2, I lay out the main common features of the various entry models. In Sections 3–5, I present the specific results for each of the entry models. In Section 6, I compare these results, highlighting both the commonalities across models as well as the differences. Section 7 considers the extension to the *n* potential entrant case, and Section 8 concludes the paper.

[1] A related line of research considers the welfare effects of changing the number of *potential* entrants. For example, Vives (1988) shows that entry may decrease but social welfare never goes down with more potential entrants.

[2] The assumption that firms make decisions simultaneously need not be taken literally. The important assumption is that each firm makes a decision not knowing what the other firm's decision is/was.

2. Preliminaries

My basic framework consists of a market for a homogeneous product and two potential entrants. If only one firm enters, then it earns profits π_1. If two firms enter, then each gets π_2. I will treat π_2 as an index of toughness of product market competition, ranging from $\pi_2 = 0$ (Bertrand competition) to $\pi_2 = \frac{1}{2}\pi_1$ (perfect collusion). Finally, in order to enter each firm must pay a sunk entry cost K. (Both π_1 and π_2 are gross of entry costs.) In order to make the problem interesting, I assume that the entry cost is strictly positive but small enough that there is room for at least one firm: $0 < K < \pi_1$.

Gross social surplus (not net of entry costs) is given by μ_1 if there is only one entrant and μ_2 if there are two entrants. Notice that, for given demand and cost curves, there is a relation between μ_2 and π_2. In order to make the problem interesting, I assume that consumer surplus is strictly positive, so that $\mu_1 > \pi_1$. Notice, moreover, that my assumption of product homogeneity implies that $\mu_2 \geqq \mu_1$ and that, if $\pi_2 = \frac{1}{2}\pi_1$ (perfect collustion), then $\mu_2 = \mu_1$.

For future purposes, it is useful to define $K^* \equiv \mu_2|_{\pi_2=0} - \mu_1$. This is the maximum increase in gross social surplus from the addition of a second competitor. It is therefore the maximum value of sunk cost such that society would consider a second entrant. A summary of the paper's notation can be found in Table 1.

Under the standard model of sequential entry, the notions of insufficient and excess entry are fairly straightforward: if the equilibrium number of entrants is greater than the socially optimal number of entrants, then we say there is excess entry; conversely, if the equilibrium number of entrants is lower than the socially optimal number of entrants, then we say there is insufficient entry. Under simultaneous entry, however, an equilibrium is given by a probability distribution over possible numbers of entrants. Comparison with the social optimum is then more difficult. I therefore propose the following definition of insufficient and excess entry:

Definition 1. We say there is excess entry if there exists an entry tax that strictly increases social welfare. We say there is insufficient entry if there exists an entry subsidy that strictly increases social welfare.

Notice that this definition is consistent with the idea of insufficient and excess entry under the standard sequential entry model. If the equilibrium number of entrants is greater than the socially optimal number of entrants, then there exists a tax such that the number of entrants is reduced down to the socially optimal number (excess entry). If the equilibrium number of entrants is lower than the socially optimal number of entrants, then there exists a subsidy such the number of entrants increases up to the socially optimal number (insufficient entry).

The focus of this paper is on the welfare effects of free, simultaneous entry. I will consider two separate models of simultaneous entry: grab-the-dollar entry (Section 4) and war-of-attrition entry (Section 5). For comparison purposes, I will start with a review of the sequential entry model in the next section. Finally, in Section 6, I will compare the implications of the different entry models.

Table 1
Model's notation

π_i	Firm profit when there are i active firms
μ_i	Gross surplus when there are i active firms
K	Entry cost
W	Discounted expected social welfare
δ	Discount factor
p	Entry strategy under grab-the-dollar entry
$F(t)$	Entry strategy under war-of-attrition entry
t	Time

3. Sequential entry

Consider first the sequential entry model: At stage i, $(i=1,2)$, Firm i decides whether or not to enter. The equilibrium is very simple. If $\pi_2 > K$, then two firms enter; if $\pi_2 < K$ then only one firm enters. Notice that $K < \pi_1$ guarantees that at least one firm enters. One firm implies social welfare $\mu_1 - K$, whereas a duopoly implies social welfare $\mu_2 - 2K$. In summary,

$$\hat{n} = \begin{cases} 1 & \text{if } \pi_2 < K, \\ 2 & \text{if } \pi_2 > K, \end{cases}$$

and

$$W = \begin{cases} \mu_1 - K & \text{if } \pi_2 < K, \\ \mu_2 - 2K & \text{if } \pi_2 > K, \end{cases}$$

where $\hat{n}$ is the equilibrium number of firms and W the level of social welfare. Comparing the equilibrium values to the social welfare function, we obtain the following result:

Proposition 1. *Under sequential entry*,

(a) *for any* $K \in (0, \frac{1}{2}\pi_1)$, *there exists a* $\underline{\pi}_2(K)$ *such that*, *if* $\pi_2 > \underline{\pi}_2(K)$ *then there is excess entry*;
(b) *for any* $K \in (0, K^*)$, *there exists a* $\bar{\pi}_2(K)$ *such that if* $\pi_2 < \bar{\pi}_2(K)$ *then there is insufficient entry.*

Proof. The optimal number of entrants is two if and only if $\mu_2 - 2K > \mu_1 - K$, or simply $K < \mu_2 - \mu_1$. If the inequality is reversed, then the optimal number of entrants is one. If $\pi_2 = \frac{1}{2}\pi_1$, then $\mu_2 - \mu_1 = 0$ and the optimal number of firms is one. If moreover $K < \frac{1}{2}\pi_1$, then the equilibrium number of entrants is two. Part (a) follows by continuity.

If $\pi_2 = 0$ and $K < K^*$, then the optimal number of firms is two. However, as $\pi_2 < K$, the equilibrium number of entrants is one. Part (b) follows by continuity. □

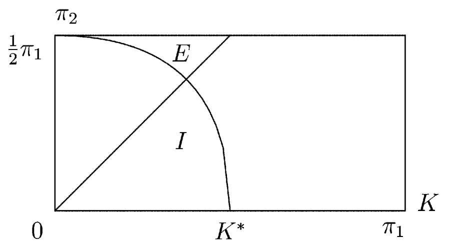

Fig. 1. Sequential entry with demand $Q=1-P$ and zero marginal cost. Excess entry in region E, insufficient entry in region I, second-best otherwise.

Fig. 1 divides the (K, π_2) space into different regions according to the relation between equilibrium entry and the social optimum, for the case when demand is linear and marginal cost is constant.[3] As suggested by Proposition 1(a), entry incentives are excessive when duopoly profits, π_2, are close to perfect collusion ($\pi_2 = \frac{1}{2}\pi_1$) and the entry cost, K, is below the critical level $K = \frac{1}{2}\pi_1$. Moreover, as suggested by Proposition 1(b), entry incentives are insufficient when duopoly profits are close to perfect competition ($\pi_2 = 0$) and the entry cost is below the critical level $\bar{K}$.[4]

4. Grab-the-dollar entry

Consider now the grab-the-dollar entry model.[5] In period t, potential entrant i ($i = 1, 2$) enters with probability p_{it}. I will consider the symmetric, stationary equilibrium where $p_{it} = p$. (Notice that, given symmetry, stationarity is a derived, not assumed, result.) If firms randomize between entering and not entering with an interior probability $p \in (0, 1)$, then it must be that the expected payoff from entry is equal to the entry cost:

$$p\pi_2 + (1-p)\pi_1 = K. \tag{1}$$

The left-hand side of (1) is the expected profit from entry: with probability p the rival enters and the firm earns profit π_2; with probability $1-p$ the rival does not enter and

[3] It can be shown that, when demand is $Q = 2 - P$ and marginal cost constant at zero,

$$\mu_1 = \tfrac{3}{2} \quad \text{and} \quad \mu_2 = 1 + \tfrac{1}{2}\pi_2 + \sqrt{1-\pi_2}.$$

These are the values used in the various figures in this paper.

[4] For the particular case of linear demand, constant marginal cost, $K^* = \frac{1}{2}\pi_1$.

[5] This model has been analyzed by a number of authors, including Gilbert and Stiglitz (1979), Dixit and Shapiro (1986), Cabral (1989) and Vettas (2000). However, none of these has considered the welfare calculations that I focus on in this paper.

the firm earns profits π_1. The right-hand side is the cost of entry. Since the firm earns zero by not entering, in order for a mixed-strategy equilibrium to exist the net profit from entering must be equal to zero, thus equality (1). Solving with respect to p we get $p = (\pi_1 - K)/(\pi_1 - \pi_2)$. If the resulting value of p is greater than 1, then firms strictly prefer to enter. It follows that, in equilibrium,

$$\hat{p} = \min\left(1, \frac{\pi_1 - K}{\pi_1 - \pi_2}\right).$$

This implies that $\hat{p} = 1$ if $K \leqslant \pi_2$ and $0 < \hat{p} < 1$ if $\pi_2 < K < \pi_1$.

Comparing sequential to grab-the-dollar entry, we notice that, if $K < \pi_2$, then both models predict a duopoly. If however $\pi_2 < K < \pi_1$, then sequential entry predicts monopoly, whereas grab-the-dollar entry predicts a probability distribution over time and over the possibilities of no entry, monopoly and duopoly.

Social welfare under grab-the-dollar entry is computed recursively:

$$W = p^2(\mu_2 - 2K) + 2p(1 - p)(\mu_1 - K) + (1 - p)^2\delta W,$$

where δ is the discount factor. Solving for W we get

$$W = \frac{p^2(\mu_2 - 2K) + 2p(1 - p)(\mu_1 - K)}{1 - (1 - p)^2\delta}.$$

The main result in this section is as follows.

Proposition 2. *Under grab-the-dollar entry,*

(a) *for any $K \in (0, \frac{1}{2}\pi_1)$, there exists a $\underline{\pi}_2(K)$ such that, if $\pi_2 > \underline{\pi}_2(K)$, then there is excess entry;*
(b) *there exists a $\bar{K}$ such that, for any $K \in (0, \bar{K})$, there exists an $\bar{\pi}_2(K)$ such that, if $\pi_2 < \bar{\pi}_2(K)$, then there is insufficient entry.*
(c) *there exists a $\underline{K}$ such that, if $K > \underline{K}$, then there is insufficient entry.*

Proof. From the analysis of (1), it is clear that there exists an entry tax such that the entry probability $\hat{p}$ is decreased. Likewise, if $0 < \hat{p} < 1$, then there exists an entry subsidy such that $\hat{p}$ is increased. Accordingly, I derive the conditions for excess and insufficient entry as a function of the derivative $\partial W/\partial p$.

From the analysis before, we know that $\pi_2 = \frac{1}{2}\pi_1, K < \frac{1}{2}\pi_1$ implies $p = 1$. Taking the derivative of W when $p = 1$, $\pi_2 = \frac{1}{2}\pi_1$ and thus $\mu_2 = \mu_1$ (which follows from $p = 1$, $\pi_2 = \frac{1}{2}\pi_1$), we get

$$\left.\frac{\partial W}{\partial p}\right|_{\substack{p=1\\ \pi_2=\pi_1/2\\ \mu_2=\mu_1}} = -2K < 0,$$

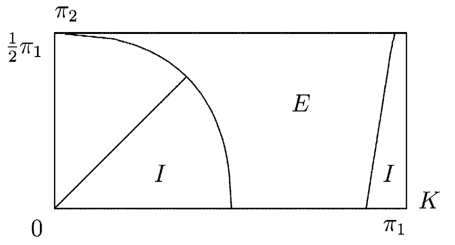

Fig. 2. Grab-the-dollar entry with demand $Q=1-P$, zero marginal cost and $\delta=0.99$. Excess entry in region E, insufficient entry in region I, second-best otherwise.

which proves part (a). Consider now the case when π_2 and K are close to zero. Straightforward derivation implies that

$$\left.\frac{\partial W}{\partial p}\right|_{\substack{p=\frac{\pi_1-K}{\pi_1-\pi_2}\\ \pi_2=0\\ K=0}} = 2(\mu_2-\mu_1) > 0.$$

This proves part (b). Finally,

$$\left.\frac{\partial W}{\partial p}\right|_{p=0} = 2\,\frac{\mu_1-\pi_1}{1-\delta} > 0, \tag{2}$$

which proves part (c). □

Fig. 2 divides the (K, π_2) space into different regions according to the relation between equilibrium entry and the social optimum, for the case when demand is linear, marginal cost is constant, and the discount factor is $\delta=0.99$.[6] As suggested by Proposition 2(a), entry incentives are excessive when duopoly profits, π_2, are close to perfect collusion ($\pi_2=\frac{1}{2}\pi_1$). Moreover, as suggested by Proposition 2(b), entry incentives are insufficient when duopoly profits are close to perfect competition ($\pi_2=0$) and the entry cost, K, is below the critical level K^*. Finally, as suggested by Proposition 2(c), entry incentives are excessive when entry costs are very high, $K \approx \pi_1$.

5. War-of-attrition entry

Consider now the case of war-of-attrition entry.[7] Suppose that two potential entrants start to invest at time zero at a rate of \$1 per period. Entry thus takes K periods of time to occur. Consider the symmetric equilibrium where firms start investing at time zero and each firm drops out of the entry race according to the cdf $F(t)$. So long as

[6] For lower values of δ, e.g., $\delta=0.9$, there is a single insufficient-entry region, and for any given value of $K \in (0, \pi_1)$, there is insufficient entry for a sufficiently low π_2.

[7] This model is similar to the models of war-of-attrition considered by Fudenberg and Tirole (1986), Bulow and Klemperer (1999) and others. My contribution here is to the welfare analysis of entry incentives.

$0 < F(t) < 1$, an equilibrium condition is that the firm be indifferent between giving up and continuing on. Since dropping out at time zero implies a payoff of zero, the expected payoff from waiting until time t must be zero:

$$F(t)(\pi_1 - K) + (1 - F(t))(-t) = 0.$$

The left-hand side is the expected payoff from waiting until time t and then dropping out of the entry race in case the rival has not dropped out by then. Specifically, if the rival is playing according to $F(t)$, then, with probability $F(t)$, the rival will have dropped out, in which case the firm gets net profit $\pi_1 - K$. With probability $1 - F(t)$, the rival will not have dropped out, in which case by dropping out the firm gets a negative payoff $-t$, the total investment up to time t.

If a firm invests until time $t^* = K - \pi_2$, then it is strictly better off by continuing to invest: the worst that can happen to it is to receive duopoly profits π_2, and non-sunk entry costs by time t^* are π_2. We thus have

$$F(t) = \begin{cases} \frac{t}{\pi_1 - K + t}, & t \leqslant K - \pi_2, \\ \frac{K - \pi_2}{\pi_1 - \pi_2}, & t > K - \pi_2. \end{cases}$$

In words, if the firm has not dropped out by time $t = K - \pi_2$ then it will not drop out after that time, so $F(t)$ is flat for $t > K - \pi_2$.

Welfare under war-of-attrition entry is a little more difficult to compute than in the previous cases. Let ψ be the probability of a duopoly and let ϕ be the expected value of entry costs that do not lead to entry (*wasted entry costs*). Then expected welfare is given by

$$W = \psi(\mu_2 - 2K) + (1 - \psi)(\mu_1 - K) - \phi.$$

As suggested by Definition 1, I consider the thought experiment whereby the government taxes or subsidizes entry. Specifically, I suppose that, in order to enter, each firm must invest at a rate τ for K periods of time, for a total cost of τK. According to Definition 1, if the derivative of W with respect to τ is positive then there is excess entry. If the derivative is negative, then there is insufficient entry.

Given the entry tax rate τ, the equilibrium indifference condition is now

$$F(t)(\pi_1 - \tau K) + (1 - F(t))(-\tau t) = 0.$$

Accordingly, we get the equilibrium strategy

$$F(t) = \begin{cases} \frac{\tau t}{\pi_1 - \tau K + \tau t}, & t \leqslant K - \pi_2/\tau, \\ \frac{\tau K - \pi_2}{\pi_1 - \pi_2}, & t > K - \pi_2/\tau. \end{cases} \tag{3}$$

The critical value where the expression for K changes is now given by $t^* = (1/\tau)(\tau K - \pi_2) = K - \pi_2/\tau$. A firm will decide to enter for sure when there is only π_2 left to invest from the total τK. Since the firm invests at a rate τ, this will take place after $(1/\tau)(\tau K - \pi_2)$ periods of investment.

The probability that a firm will invest all the way even if the rival does not drop out is given by $1 - F(t^*) = 1 - F(K - \pi_2/\tau)$. We therefore conclude that the probability of duopoly is given by

$$\psi = (1 - F(t^*))^2 = \left(\frac{\pi_1 - \tau K}{\pi_1 - \pi_2} \right)^2.$$

Using (3) and noting that $f(t) = \partial F(t)/\partial t = \tau(\pi_1 - \tau K)/(\pi_1 - \tau K + \tau t)^2$, the expected value of wasted entry costs is given by

$$\begin{aligned} \phi &= \int_0^{t^*} 2 f(t)(1 - F(t)) t \, \mathrm{d}t \\ &= \int_0^{K - \pi_2/\tau} 2 \frac{\tau(\pi_1 - \tau K)}{(\pi_1 - \tau K + \tau t)^2} \left(1 - \frac{\tau t}{\pi_1 - \tau K + \tau t} \right) t \, \mathrm{d}t \\ &= \frac{(\pi_1 - \tau K)(\tau K - \pi_2)^2}{(\pi_1 - \pi_2)^2 \tau}. \end{aligned}$$

In summary, we have

$$\begin{aligned} W = {} & \left(\frac{\pi_1 - \tau K}{\pi_1 - \pi_2} \right)^2 (\mu_2 - 2K) + \left(1 - \left(\frac{\pi_1 - \tau K}{\pi_1 - \pi_2} \right)^2 \right) (\mu_1 - K) \\ & - \frac{(\pi_1 - \tau K)(\tau K - \pi_2)^2}{(\pi_1 - \pi_2)^2 \tau}. \end{aligned} \tag{4}$$

The main result of this section is as follows.

Proposition 3. *Under war-of-attrition entry,*

(a) *for any $K \in (0, \pi_1)$, there exists a $\underline{\pi}_2(K)$ such that, if $\pi_2 > \underline{\pi}_2(K)$, then there is excess entry;*
(b) *if $0 < K < K^* < \frac{1}{2}\pi_1$, then there exists a $\bar{\pi}_2(K)$ such that, if $\pi_2 < \bar{\pi}_2(K)$, then there is insufficient entry.*
(c) *if $\pi_1 > K > K^* > \frac{1}{2}\pi_1$, then there exists a $\bar{\pi}_2(K)$ such that, if $\pi_2 < \bar{\pi}_2(K)$, then there is excess entry.*

Proof. Consider first the case when duopoly profits are high, specifically $\pi_2 = \frac{1}{2}\pi_1$. If $K < \frac{1}{2}\pi_1$, then we are at a corner solution. Changing entrants strategies implies at least $\tau = \pi_2/K$. Accordingly, we compute

$$\left. \frac{\partial W}{\partial \tau} \right|_{\substack{\tau = \pi_2/K \\ \pi_2 = \pi_1/2 \\ \mu_2 = \mu_1}} = 4 \frac{K^2}{\pi_1} > 0.$$

If $K > \frac{1}{2}\pi_1$, then we are at an interior solution. We then have

$$\left. \frac{\partial W}{\partial \tau} \right|_{\substack{\tau = 1 \\ \pi_2 = \pi_1/2 \\ \mu_2 = \mu_1}} = \pi_1 > 0.$$

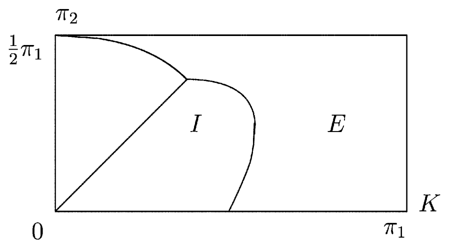

Fig. 3. War-of-attrition entry with demand $Q = 1 - P$ and zero marginal cost. Excess entry in region E, insufficient entry in region I, second-best otherwise.

Part (a) follows by continuity. Consider now the case when duopoly profits are low. It can be shown that

$$\left.\frac{\partial W}{\partial \tau}\right|_{\substack{\tau=1\\ \pi_2=0}} = \frac{K}{\pi_1^2}\,(2(\pi_1 - K)K^* - \pi_1 K).$$

Solving for K, we get

$$\left.\frac{\partial W}{\partial \tau}\right|_{\substack{\tau=1\\ \pi_2=0}} < 0 \quad \text{iff} \quad K < K' \equiv \frac{2\pi_1 K^*}{\pi_1 + 2K^*} = \frac{2K^*}{1 + 2(K^*/\pi_1)}.$$

Notice that $K^* < \frac{1}{2}\pi_1$ implies $K' > K^*$. Conversely, $K^* > \frac{1}{2}\pi_1$ implies $K' < K^*$. It follows that if $K^* < \frac{1}{2}\pi_1$ then $K \leqslant K^*$ is a sufficient condition for $\frac{\partial W}{\partial \tau}\big|_{\tau=1,\pi_2=0} < 0$, whereas if $K^* > \frac{1}{2}\pi_1$ then $K \geqslant K^*$ is a sufficient condition for $\frac{\partial W}{\partial \tau}\big|_{\tau=1,\pi_2=0} > 0$. This proves parts (b) and (c) by continuity. □

Fig. 3 divides the (K, π_2) space into different regions according to the relation between equilibrium entry and the social optimum, for the case when demand is linear and marginal cost is constant. As suggested by Proposition 3(a), entry incentives are excessive when duopoly profits are close to perfect collusion ($\pi_2 = \frac{1}{2}\pi_1$). Moreover, as suggested by Proposition 3(b), entry incentives are insufficient when duopoly profits are close to perfect competition ($\pi_2 = 0$) and the entry cost, K, is below some critical level. Finally, as suggested by Proposition 3(c), entry incentives are excessive when duopoly profits are close to zero and the entry cost, K, is above some critical level.

6. Discussion

Figs. 1–3 suggest that there are both similarities and differences in the welfare effects of free entry according to each of the models. In this section, I highlight the main similarities and differences.

6.1. Low entry costs: The business stealing effect and the consumer surplus externality

The first striking result of the comparison of Propositions 1–3 is that they all agree when sunk costs are low. Specifically, all models predict that entry incentives are excessive when duopoly profits are high and insufficient when duopoly profits are low. The excess entry result is akin to the results in Perry (1984), Mankiw and Whinston (1986) and Suzumura and Kiyone (1987). The intuition is that, if there is little duopoly competition, then an additional competitor only brings in extra entry costs, no extra surplus. All of the gain for the second entrant is a transfer of rents from the first entrant: the business stealing effect, in the words of Mankiw and Whinston (1986).

At the other extreme, when duopoly profits are close to zero, insufficient entry occurs; that is, for a given low value of K there is insufficient entry if π_2 is sufficiently low. Since duopoly profits are lower than monopoly profits, it must be that society would receive a greater gross surplus with a second entrant. Since entry costs are low, there would be a net increase in social welfare. However, since entry costs are still greater than duopoly profits, no second entry takes place. In this case, instead of the business-stealing externality, we have a surplus-creation externality: the second entrant does not take into account the increase in consumer surplus that its entry would imply.

6.2. High entry costs: Wasteful entry resources and the delay effect

The important difference between the various entry models occurs for high values of the entry cost, $K \approx \pi_1$. Under sequential entry, we obtain the second best: the second firm does not enter and society would not want it to enter. The same is not true for war-of-attrition entry or grab-the-dollar entry.

Under war-of-attrition entry we get excess entry. The reason is that, almost surely, only one firm will enter. However, both firms will invest a strictly positive amount with probability one, implying a waste of entry costs. The virtue of an entry tax is to reduce the degree of these wasteful entry resources. The question might then be asked, why do we not *always* get excess entry? There is a second effect to take into account. An entry subsidy implies that a firm that was going to give up at time $t^* = K - \pi_2$ is now going to invest all the way even if the rival does not drop out. In terms of gross social surplus, this implies an additional contribution of $\mu_2 - \mu_1$. However, in terms of *marginal* entry cost, society (and the firm) will only need to pay π_2. In other words, an entry subsidy has the virtue of reducing wasteful entry costs at the margin, that is, entry costs that do not translate into actual entry. However, if the entry cost is very high, $K \approx \pi_1$, then the probability that this happens is very small. Specifically, this situation only happens if the rival firm has not dropped out by time t^*, which happens with probability $1 - F(t^*)$; and if $K \approx \pi_1$ then $1 - F(t^*) \approx 0$.

Finally, and perhaps somewhat surprisingly, when entry costs are very high there is insufficient entry under grab-the-dollar entry. The intuition for this result is that, from society's viewpoint, the probability of duopoly, p^2, is very small: "entry mistakes" are a second-order effect. (A duopoly would be an "entry mistake" because neither the firm nor society would be able to recoup a second large entry cost.) In fact, since K

is close to π_1, p is close to zero. However, for a given firm, *conditional on entering,* the probability of an "entry mistake" is p, a first-order effect. For this reason, the risk of an "entry mistake" is disproportionately large from a private perspective, implying a delay effect, in the terminology of Bolton and Farrell (1988): firms are too cautious (too slow) in their entry decision, from a social welfare perspective.

7. The case of *n* potential entrants

So far, I have considered the case of two potential entrants. There are many situations when this is an appropriate assumption. The examples presented in Section 1 provide two such cases. When American Airlines and others indicated their need for a new wide-body aircraft, it was clear to all that Lockheed and McDonnell Douglas were the only potential competitors, with Boeing already committed to the development of the B-747 (see Newhouse, 1988). Likewise, when the race for the British Satellite industry got under way, it was also clear that BSB and Sky were the relevant potential competitors.

The above examples notwithstanding, there are situations when the number of potential entrants is unknown. In this case, robust policy predictions must be based on robust theoretical models. Accordingly, in this section, I consider the extension of grab-the-dollar and war-of-attrition to the n firm case. [8]

As discussed in Section 6, there are both similarities and differences between sequential entry and simultaneous entry. For low values of K, all three models have similar predictions. The differences appear when we consider the large K case: whereas the sequential entry equilibrium is second-best, grab-the-dollar entry implies insufficient entry and war-of-attrition entry implies excessive entry. Accordingly, I focus on the generalization of the results when the entry cost is large.

Let us first consider the case of grab-the-dollar entry. Notice that, whatever the shape of the profit function, π_n, if K is close to π_1 then it will be the case that the optimal response to previous entry by at least one rival is not to enter. This means that, for $K \approx \pi_1$, the welfare function is recursively defined by

$$W = (1-p)^n \delta W + \sum_{i=1}^{n} \binom{n}{i} p^i (1-p)^{n-i} (\mu_i - iK).$$

Solving for W, we get

$$W = \frac{\sum_{i=1}^{n} \binom{n}{i} p^i (1-p)^{n-i} (\mu_i - iK)}{1-(1-p)^n \delta}.$$

[8] As mentioned before, the welfare properties of sequential entry with n potential entrants have been analyzed extensively by Mankiw and Whinston (1986) and others; see Vives (1999) for a synthesis. Arvan (1988) and Cabral (1988) present results for a large number of potential entrants under simultaneous entry, but not welfare results.

Taking the derivative with respect to p and making $p=0$, $K=\pi_1$, we get

$$\left.\frac{\mathrm{d}W}{\mathrm{d}p}\right|_{\substack{p=0\\K=\pi_1}} = n\,\frac{\mu_1-K}{1-\delta}. \tag{5}$$

Notice that, as expected, (2) follows as a particular case of (5). Clearly, then, part (c) of Proposition 2 generalizes for any n: if sunk costs are sufficiently high, then there is insufficient entry under grab-the-dollar entry.

Let us now consider the case of war-of-attrition. The "surprising" part of Proposition 3 is that, if sunk costs are high, then entry is excessive even if duopoly profits are very low. I thus focus on the case of low oligopoly profits and high entry costs. Haigh and Cannings (1989) show that, in a war-of-attrition with one prize, all but two contestants drop out at time zero.[9] Since these $n-2$ "exiters" (or, rather, non-entrants) incur no entry costs, the welfare function, in equilibrium, is identical to the one considered in Section 5. We thus conclude that the results for small π_2 generalize for n potential entrants and $\pi_i=0$, $i>1$: (i) if K is sufficiently low, then there is insufficient entry; (ii) if K is sufficiently high, then there is excess entry.

In summary, the "interesting" results in the comparison of the three entry models are valid for a general number of firms: if entry costs are large, then grab-the-dollar entry implies insufficient entry even if product market competition is very soft; and war-of-attrition entry implies excess entry even if product market competition is very tough.

The complete generalization to the n case is quite difficult. In particular, in the grab-the-dollar case one is not even guaranteed that the value function from being in the market is monotonic in the number of active firms. Monotonicity was shown by Vettas (2000) for a very particular profit function π_i. Cabral (1993) considered the case of small discount factor, effectively avoiding the problem. In the case of war-of-attrition, the analysis also becomes quite complicated once we consider general π_i functions.

8. Conclusion

In this paper, I addressed the question of whether free entry leads to an optimal number of entrants. As Vives (1988) suggested, this question is not of purely academic interest. In many countries, central and local governments take measures that either foster entry or hinder entry into particular industries. My analysis suggests that, when entry costs are low, the relevant test is the intensity of product market competition: tough competition implies that entry incentives are insufficient, whereas soft competition implies excessive entry incentives. If entry costs are high, however, then the policy prescription varies considerably across entry models.

[9] Haigh and Cannings (1989), like this paper, consider a game of complete information. Bulow and Klemperer (1999) extend the analysis to the case of incomplete information. It should be noted that, strictly speaking, the game has no symmetric equilibrium. Haigh and Cannings (1989) solve this problem by imposing a series of instantaneous randomizations to eliminate all the extra entrants.

There is one caveat that should be taken into account: I considered the case of a homogeneous product industry. Early research suggested that product differentiation might work in the direction of insufficient entry. However, recent work by Anderson et al. (1995), in the context of sequential entry, suggests that this is not the case and that excess entry obtains within a fairly general setting. More work still needs to be done in the case of simultaneous entry.

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