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Optimal matching auctions

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Suppose a firm (or a government) wants to buy a given object from one of two possible suppliers. The firm's optimal solution, i.e., the one which minimizes expected payment, has been characterized by Myerson (1981). We present a modified 'matching' mechanism which implements the optimal solution. Supplier 1 first submits a bid, and Supplier 2 is then given the option of matching a function of that bid.

1. Introduction

In a seminal paper, Myerson (1981) characterized the optimal solution for auctioning an object which valuation is privately known by each of N bidders. Based on the revelation principle, one can restrict to direct revelation mechanisms, and by optimizing over this set the characterization of an optimal mechanism is obtained. While the problem solved by Myerson (1981) was that of selling an object, an analogous solution applies to the problem of buying an object which production cost is privately known by each of N potential sellers. (In most of this paper, we will consider the latter, which arises frequently in the context of government and industry procurement.)

Direct revelation mechanisms, the method considered by Myerson 9181), are seldom used in practice, if ever. On the other hand, while capable of implementing the optimal solution, they are not the *only* way of doing it. In fact, if bidders are symmetric, then both the English auction and the first-price sealed bid auction implement the optimal solution. Even if bidders are not symmetric, a modified sealed-bid auction-a mechanism frequently used both in government and in industrial procurement-implements the optimal solution. [1]

In this note, we consider the implementation of the buyer's optimal solution by means of a modified 'matching' mechanism similar to the current practice of various corporations. In a (simple) matching mechanism, one of the suppliers first submits a bid. The other supplier is then given the option of matching that bid, or else the first supplier is chosen. The modefied matching mechanism works in the same way except that the second supplier (typically the incumbent supplier) has to

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[1] See McAfee and McMillan (1985). In a simple sealed-bid auction, sellers submit bids simultaneously, and the lowest bid is selected by the buyer. In a modified sealed-bid auction, the buyer credibly commits to select the seller based on a function which compares the various bids, the identity function corresponding to the simple sealed-bid auction.

match a function of the first suppliers's bid, the identity function corresponding to the simple matching mechanism. We show that the optimal solution can be implemented by means of a modified matching mechanism and derive the optimal matching function.

2. Main result

Consider a buyer who wants to acquire a given object, and two possible suppliers, 1 and 2. Each supplier privately knows his own cost. It is common knowledge that costs are independently drawn from the distribution functions $F_i(c_i)$, both continuously differentiable and with a positive derivative $f_i(c_i)$ on $[\alpha_i, \beta_i]$, $i = 1, 2$. Define

$$J_i(c_i) = c_i + F_i(c_i)/f_i(c_i) \quad i = 1, 2. \tag{1}$$

Throughout the analysis, we will make the regularity assumption that $J_i(c_i)$ is a differentiable increasing function and that $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$, $J_i(\alpha) = J(\alpha)$ and $J_1(\beta) = J_2(\beta)$. [2] For simplicity, we will also assume that in-house production is not a viable alternative, i.e., the reservation price is infinity. [3]

The optimal solution from the buyer's point of view, i.e., that which minimizes expected price paid to suppliers, has been characterized in Myerson (1981). [4] It was shown that the optimal solution is fully characterized by the following conditions. First, the buyer ought to select the supplier with the lowest value of $J_i(c_i)$. Second, a supplier with the highest cost (β) should receive zero expected payoff.

In practice, there are a number of ways by which the optimal solution can be implemented. [5] Here we consider implementation by means of a modified 'matching' mechanism. One of the suppliers first submits a bid. The other supplier is then given the option of matching a function of the first supplier's bid. Our main result is that the optimal solution can be implemented by means of a matching mechanism.

Define

$$Z(x) \equiv J_2^{-1}(J_1(x)), \tag{2}$$

$$B(x) \equiv x + \frac{\int_x^\beta (1 - F_2(Z(y)))\,\mathrm{d}y}{1 - F_2(Z(x))}. \tag{3}$$

Notice that $B(x)$ is the equilibrium bid function in a symmetric sealed-bid auction with costs distributed according to $F_2(Z(x))$ [McAfee and McMillan (1987)]. In particular, one can check that $B(x)$ is an increasing function of x (if $J_i(x)$ is increasing).

[2] This greatly simplifies the results and we conjecture that the same results hold true in a more general setting. Notice that we still allow the the distribution functions $F_i(c_i)$ to differ.

[3] If the buyer can produce the good at cost c_B, then in the optimal mechanism the buyer has a reservation price strictly lower than c_B.

[4] This was done under the assumption that both the buyer and the suppliers are rational, risk-neutral players, and that all of the bargaining power is concentrated on the supplier, who is a Stackelberg leader in the sense that he gets to move first by announcing a selection mechanism.

[5] The well-known revenue equivalence theorem [Myerson (1981) and others] is just an instance of this.

Theorem. The optimal solution can be implemented by means of a matching auction. The first player submits a bid b and the second player is given the option of matching a function $Z(B^{-1}(b))$ of that bid.

Proof. Since the second bidder's strategy is to accept the offer if and only if it is greater than its cost, the first bidders's expected payoff is given by

$$\Pi_1(c_1) = \max_b \left[(b - c_1)\left(1 - F_2\left(Z\left(B^{-1}(b)\right)\right)\right) \right]. \tag{4}$$

Notice that this is the same first-order condition as in a symmetric sealed-bid auction with costs distributed according to $F_2(Z(x))$ and an equilibrium bid function given by $B(x)$. But $B(x)$ was defined precisely as the equilibrium bid function in such an auction. Therefore, we conclude that the first bidder's optimal bidding function coincides with $B(x)$.

Given this, the condition for the first bidder to win the auction is

$$Z\left(B^{-1}(B(c_1))\right) < c_2, \quad Z(c_1) < c_2, \quad J_1(c_1) < J_2(c_2), \tag{5}$$

which is Myerson's condition for an optimal auction. The proof is completed by checking that both suppliers get a zero expected payoff if $c_i = \beta$. □

As an example of the theorem, consider the simple case when $F_1(x) = F_2(x) = F(x)$. Then, $J_1(x) = J_2(x)$ and the optimal matching function is simply $B^{-1}(x)$, where $B(x)$ is the equilibrium bid function in a symmetric sealed-bid auction with costs distributed according to $F(x)$. That is, the buyer correctly anticipates that the first supplier will bid according to $B(x)$ and, based on the inverted bid function, is able to offer the second supplier an option which (in equilibrium) is accepted if and only if the second supplier's cost is lower than the first supplier's cost. This satisfies the first condition for an optimal auction. Clearly, if the first supplier's cost is equal to β, then his 'inverted' bid will always be matched; on the other hand, if the second supplier's cost is β he will never be willing to match the 'inverted' bid. The second condition is satisfied, and thus we have an optimal auction. Notice that for specific values of c_1 and c_2 the payment made by the winning bid is different from that in other auction mechanisms which are also optimal. The exception is that if the first bid is the winning bid, than the price paid is the same as in a sealed-bid auction. However, if costs are asymmetrically distributed, not even this necessarily holds.

3. Concluding remark

As the preceding example shows, even for the simple case of symmetric costs, the optimal matching auction may be as complicated as the equilibrium bid function in a sealed-bid auction. An interesting open question is whether there are simple matching functions (e.g., the winning bid minus some fixed percentage) which attain the optimal solution approximately enough.

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